Computation of Top Quark Threshold Production

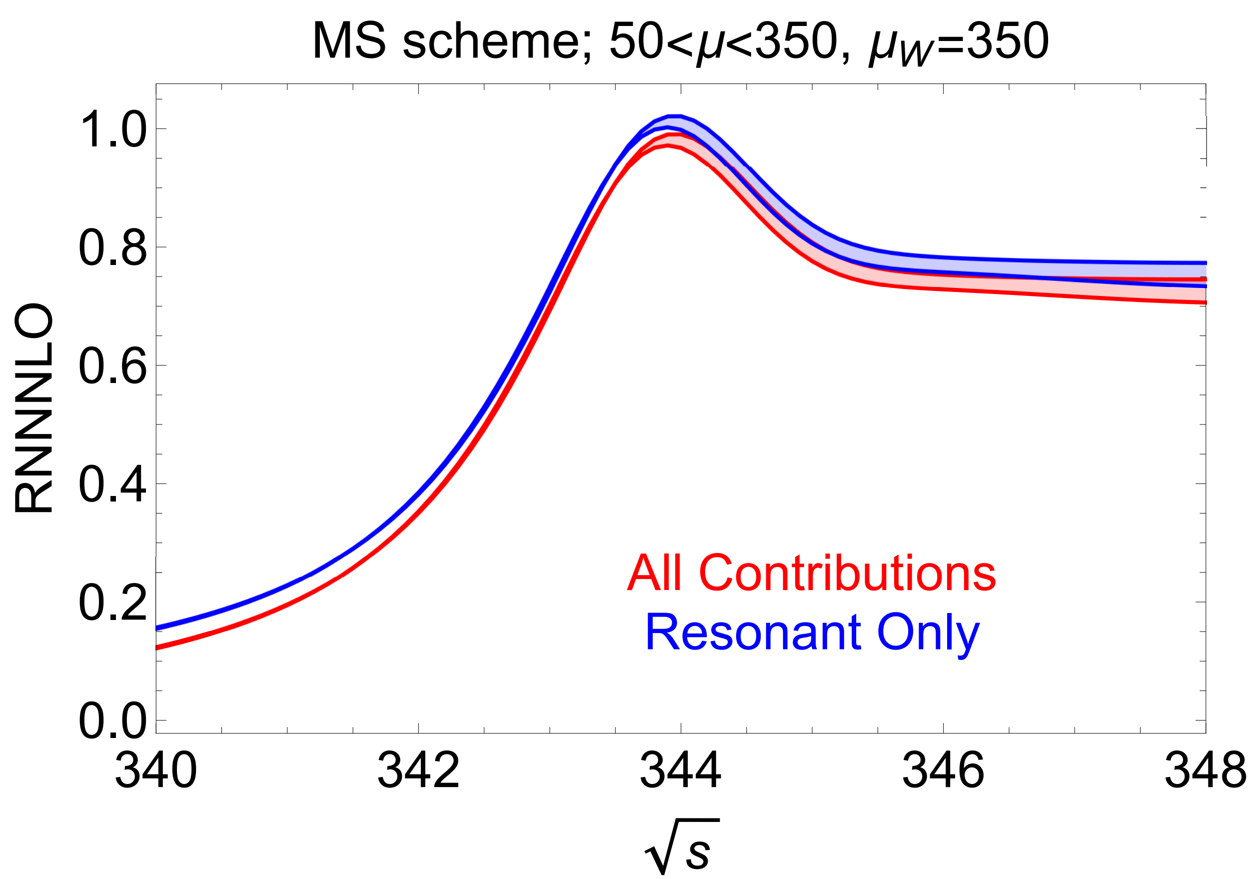
`QQbar_threshold`

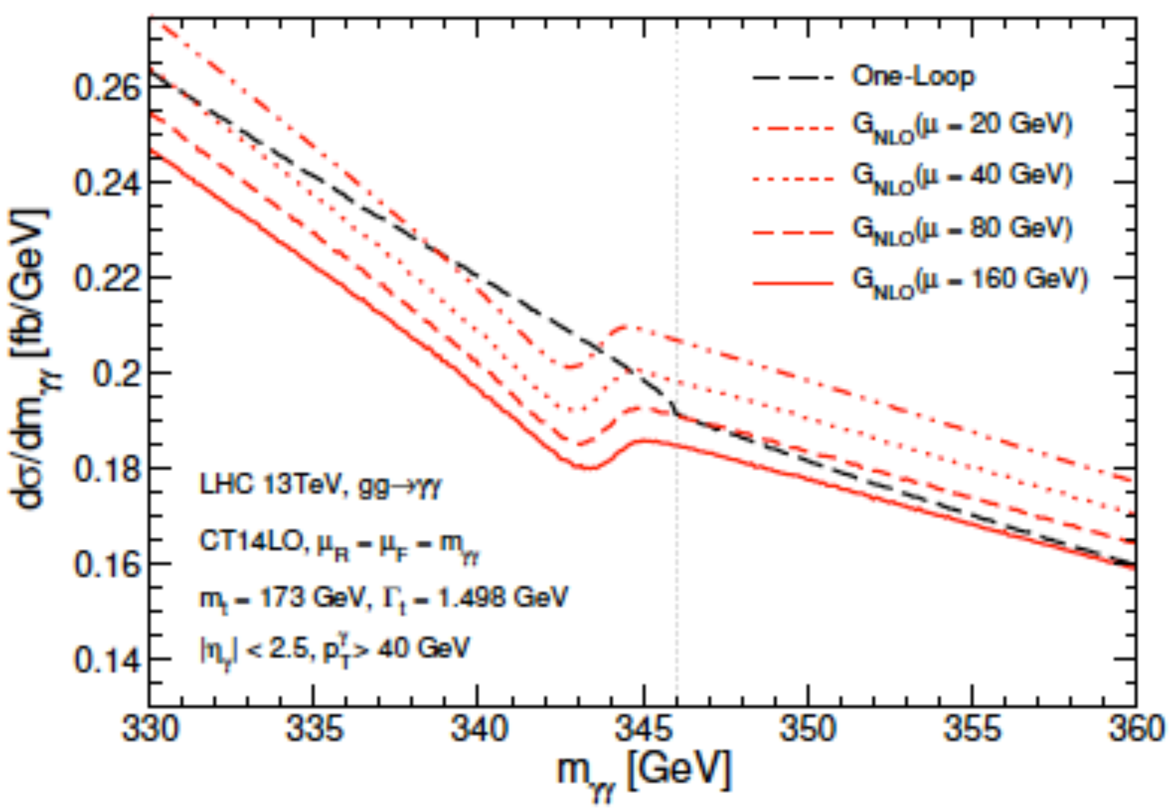
(C++ code for heavy quark threshold)

M.Beneke, Y.Kiyo, A.Maier, J.Piclum
arXiv:1605.03010v2[hep-ph]

Workshop on Top Physics at LC 2016,
KEK, July 6-8 2016

Top mass





Precision measurement

Threshold scan for top mass
at ILC in ee→tt

Looking at color singlet boundstate pole

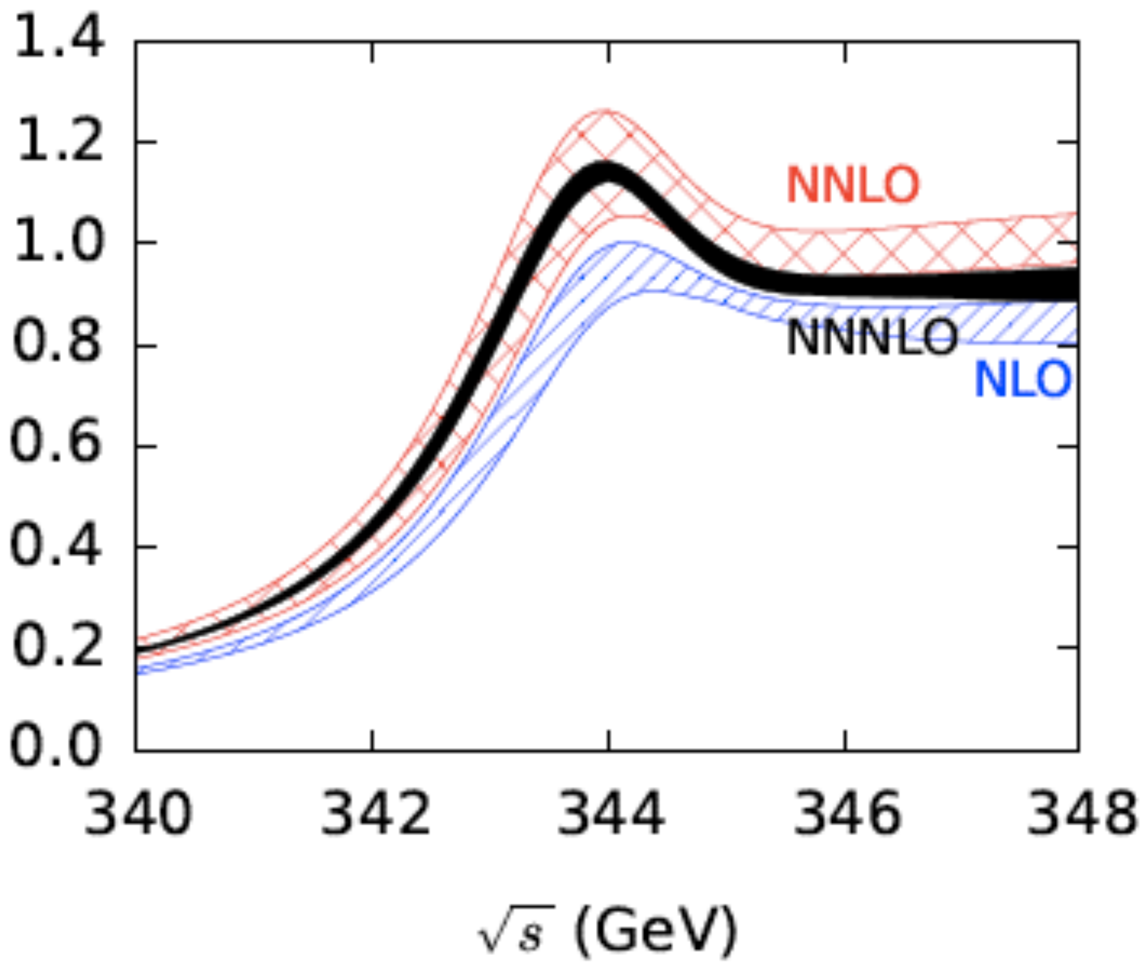
Top mass at LHC in gg→γγ
Kawabata-Yokoya, arXiv:1607.00990

Slide@Top WS 2015

top threshold

cross section near top threshold normalized to point particle one

$$R(\sqrt{s}) = \frac{\sigma(e^+e^- \to t\bar{t})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



Beneke-YK-Marquard-Penin-Piclum -Steinhauser:1506.06864[hep-ph]

- N^3LO threshold cross section computed

✓ three-loop matching coeff. C_v
Marquard-Piclum-Seidel-Steinhauser(14)

✓ three-loop QCD pot. V_{QCD}
Smirnov-Smirnov-Steinhauser(10); Anzai-YK-Sumino(10)

✓ two-loop 1/m pot. $V_{1/m}$
Kniehl-Penin-Smirnov-Steinhauser(02, 14);

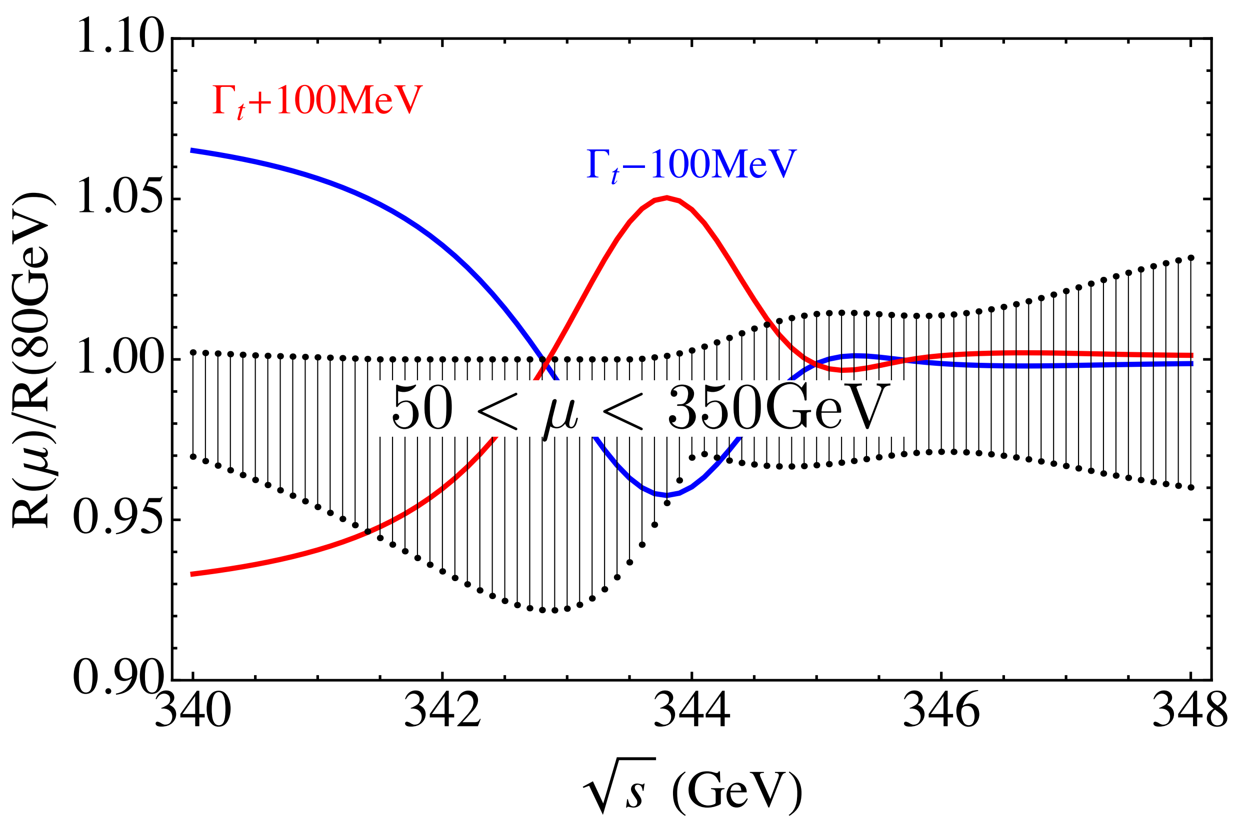
✓ N^3LO non-rela potential ins.
Beneke-YK-Schuller(05,14), Beneke-YK-Penin(07)

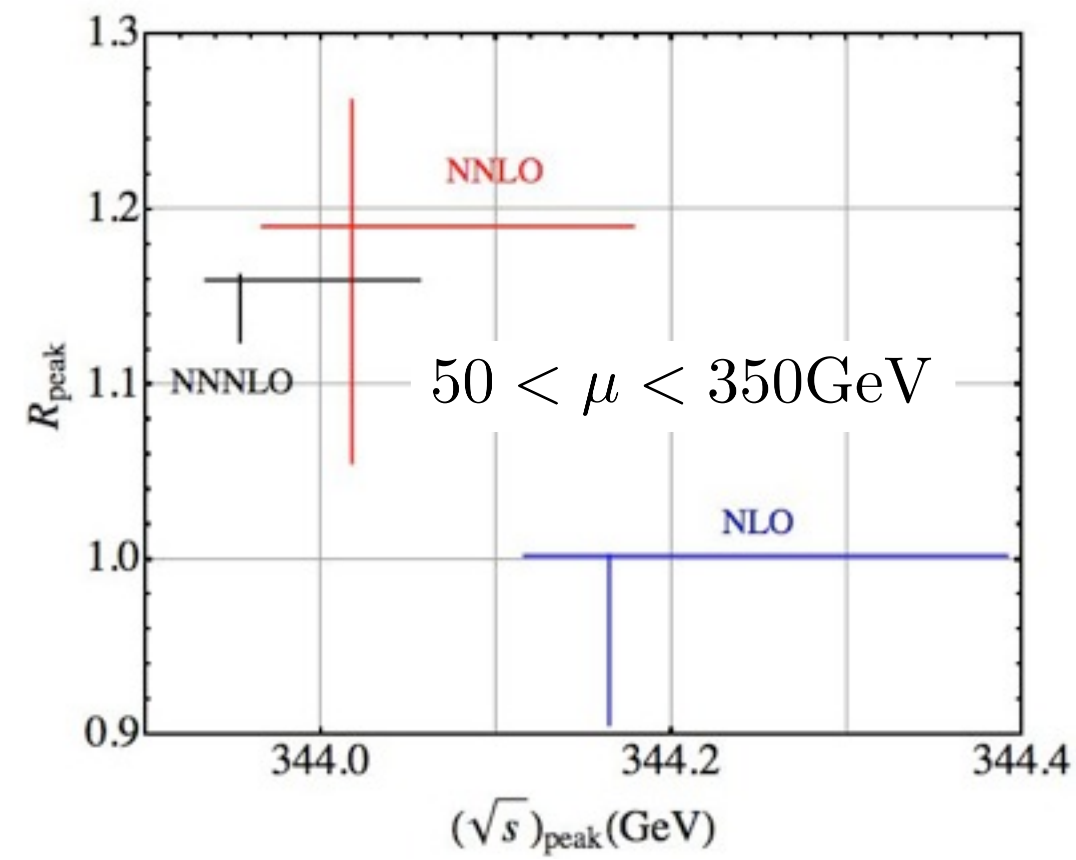
✤ Non-resonant, EW, Higgs......
Talk by Beneke (previous speaker)

Slide@Top WS 2015

NNNLO QCD result

Beneke-YK-Marquard-Penin-Piclum-Steinhauser(15)





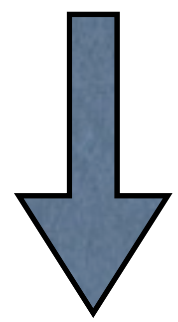
R(μ) normalized by R(80GeV)

- 3〜7 % μ-variation
- $\delta\Gamma$=±100MeV at μ=80GeV

$\sqrt{s}_{peak}$ and R_{peak} uncertainty

- N^1LO: $\delta E \sim 300\mathrm{MeV}, \delta R \sim 0.1$
- N^2LO: $\delta E \sim 200\mathrm{MeV}, \delta R \sim 0.2$
- N^3LO: $\delta E \sim 100\mathrm{MeV}, \delta R \sim 0.05$

- Our (private) NNNLO cross section code took few days to make a plots for threshold scan
- Mathematica package



- Fast C/C++ code
- Compatibility with Mathematica
- Public

QQbar_threshold

Beneke-Kiyo-Maier-Piclum(2016)

- c++ program
- Mathematica via MathLink
- top/bottom threshold cross section (5ms per parameter point)
- related quantities(energy, wavefunc. moments, top width) implemented

An example

examples/C++/xsection_1.cpp

```
#include "QQbar_threshold/load_grid.hpp"
#include "QQbar_threshold/xsection.hpp"
#include <iostream>

int main(){
  namespace QQt = QQbar_threshold;
  QQt::load_grid(QQt::grid_directory() + "ttbar_grid.tsv");
  const double mu = 50.;
  const double mu_width = 350.;
  const double mt_PS = 168.;
  const double width = 1.4;
  for (double sqrt_s = 330.; sqrt_s < 345.; sqrt_s += 1.0) {
    std::cout << sqrt_s << '\t'
      << QQt::ttbar_xsection(
          sqrt_s ,{mu, mu_width} ,{mt_PS , width}, QQt::N3LO
      )
      << '\n';
  }
}
```

An example

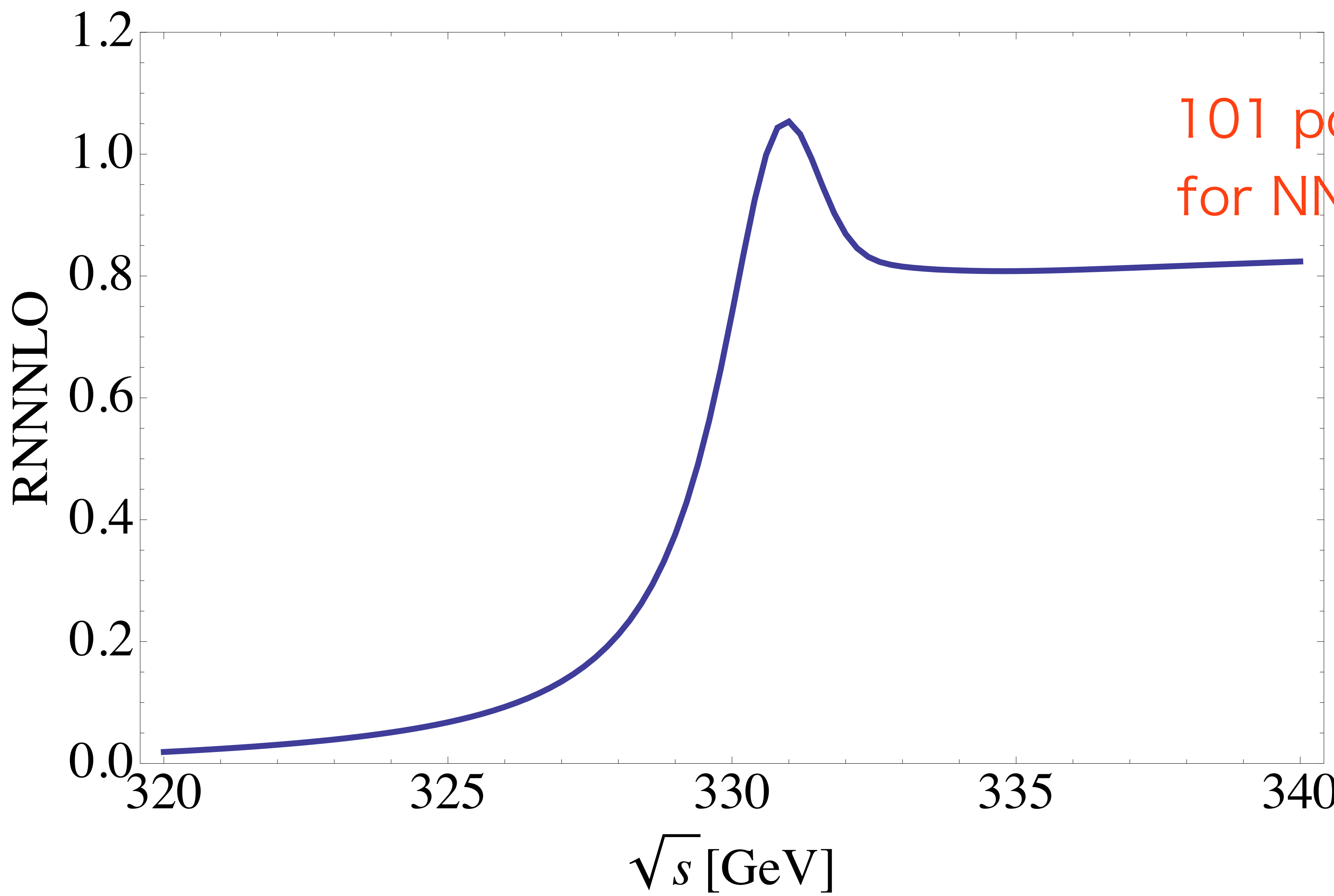
examples/Mathematica/xsection_1.m

```
Needs["QQbarThreshold`"];

LoadGrid[GridDirectory <> "ttbar_grid.tsv"];
With[
   {
     mu = 50.,
     mtPS = 168.,
     width = 1.4,
     muWidth = 350.,
     order = "N3LO"
   },
   Do[
      Print[
         sqrts, "\t",
         TTbarXSection[sqrts, {mu, muWidth}, {mtPS, width}, order]
      ],
      {sqrts, 330., 345., 1.}
   ]
];
```

An example

```
Table[{ rs,
TTbarXSection[rs, {100, 200}, {165, 1.33}, "N3LO" ]},
{rs, 320, 340, 0.2}]]
```



101 points takes 0.3 sec
for NNNLO cross section

X section formula

Because top quark is unstable, top and (bW) can not be distinguished. Therefore non-resonant (bW) production need to be taken into account in the definition of top quark production. Hoang-Reisser(04)

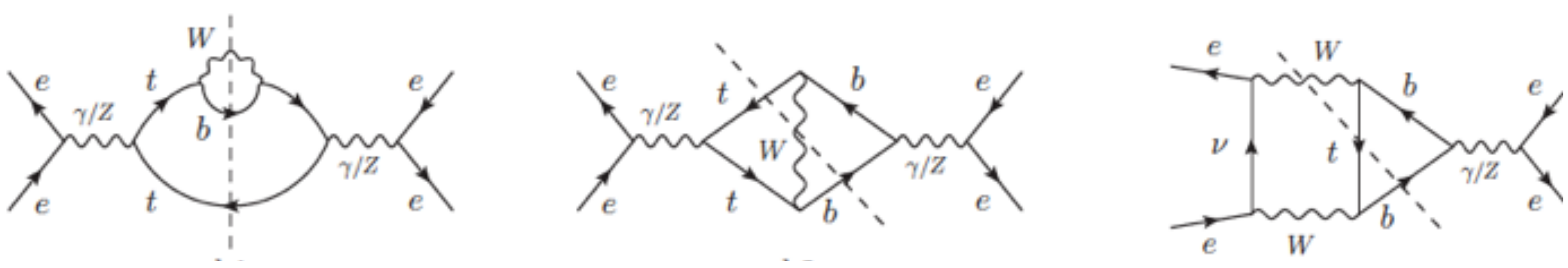
$$R(s) = R_{\rm res}(s) + R_{\rm non-res}(s)$$

$m_t, \Gamma_t, \alpha_s, \mu, \mu_W, \cdots$

scale dep due to unstable top

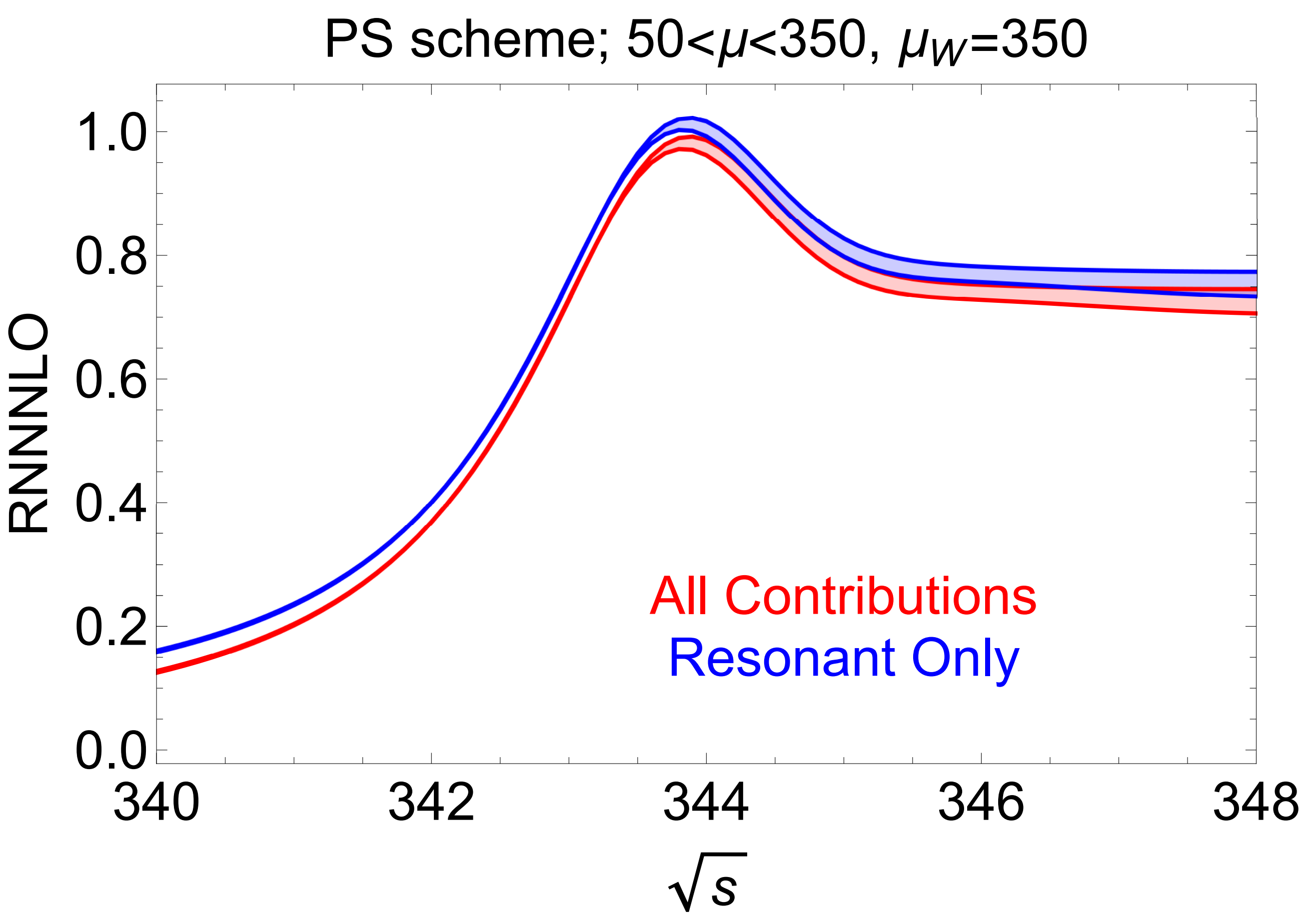
$$R_{\rm res}(s) = R_{\rm S-wave} + R_{\rm P-wave}$$

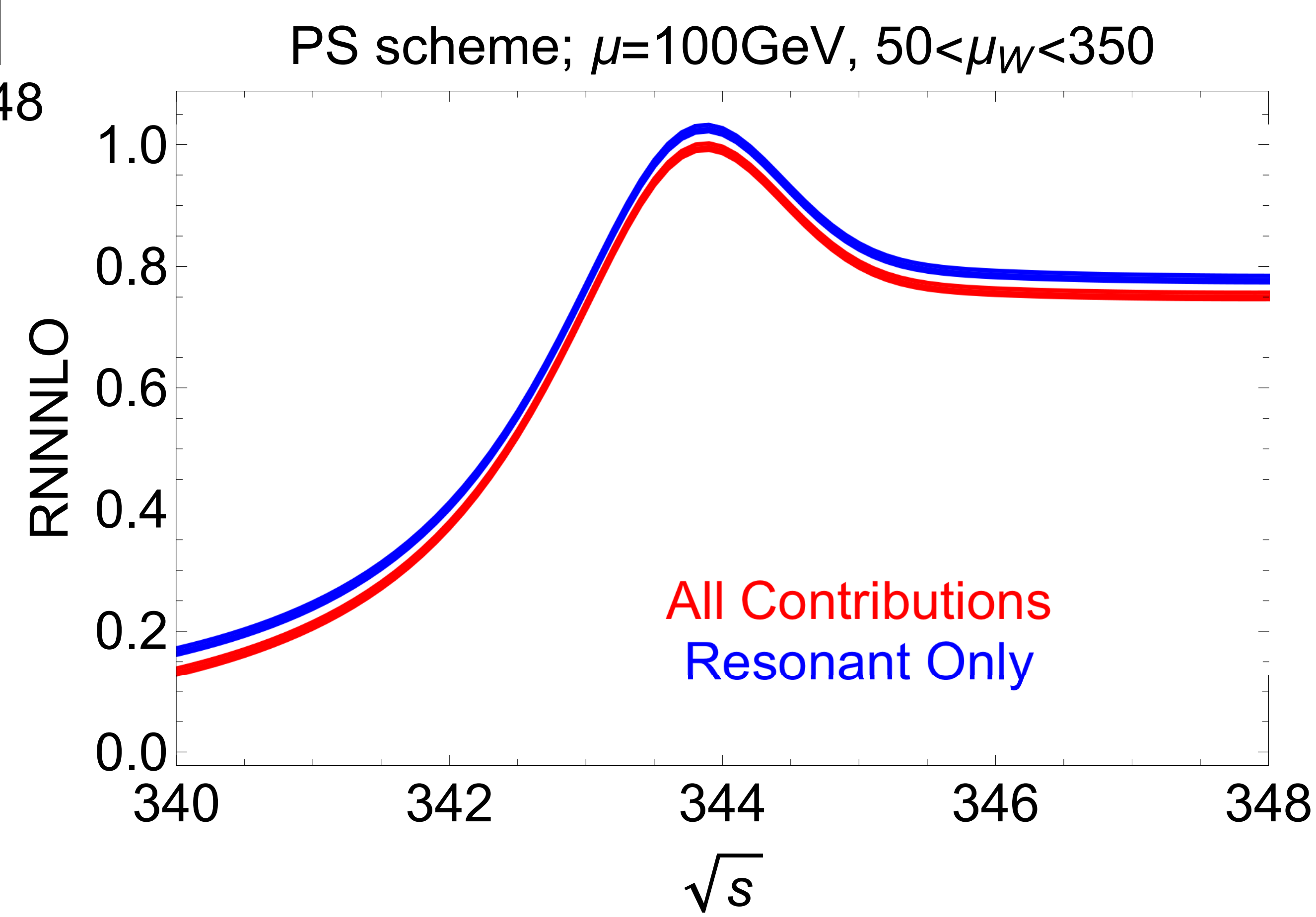
$$R_{\rm non-res}(s) = R(e^+e \to (bW^+)\bar{t}) + R(e^+e \to t(\bar{b}W^-)) + \cdots$$



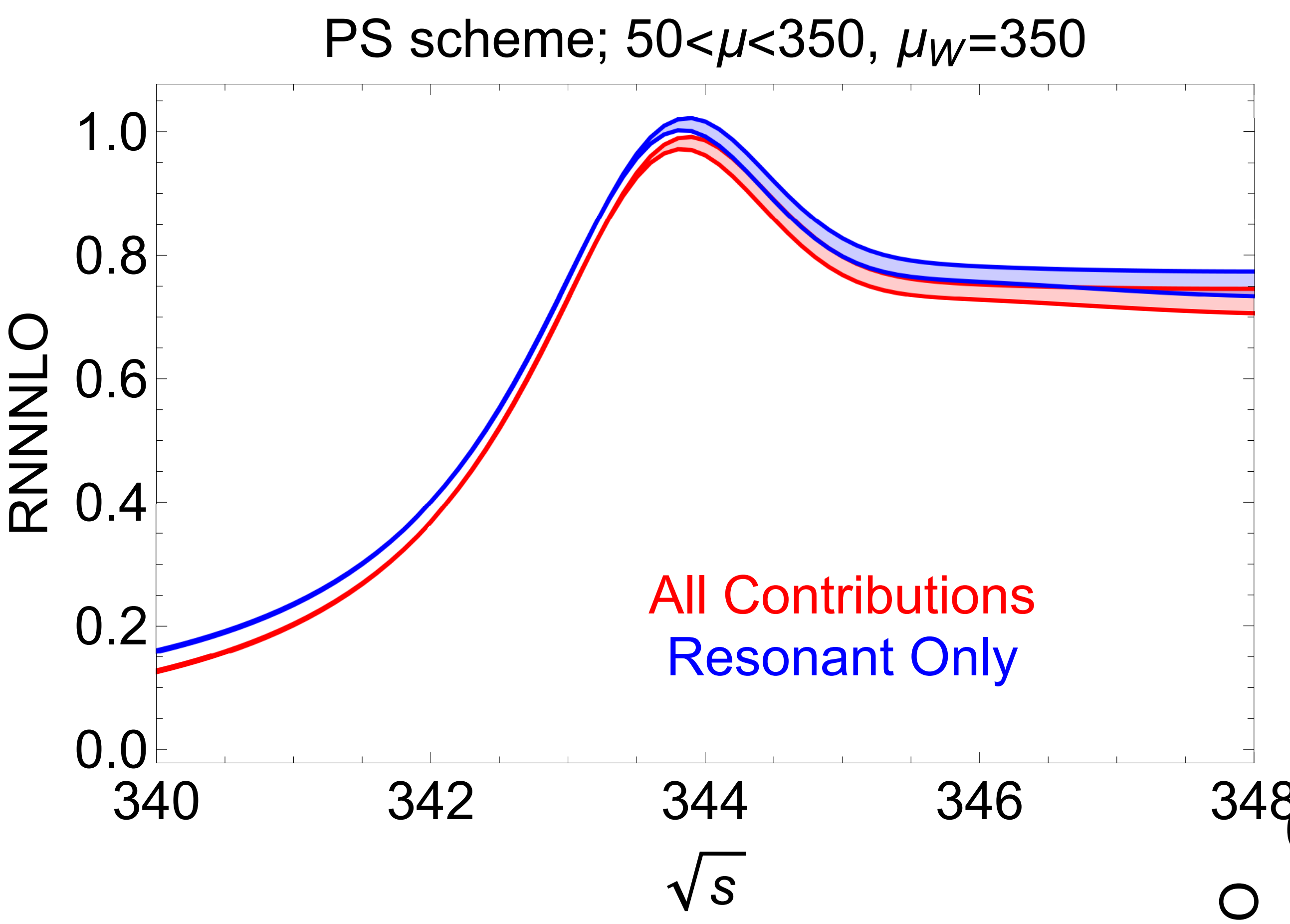
non-resonant diagrams(example) Beneke-Jantzen-Femenia(2010)

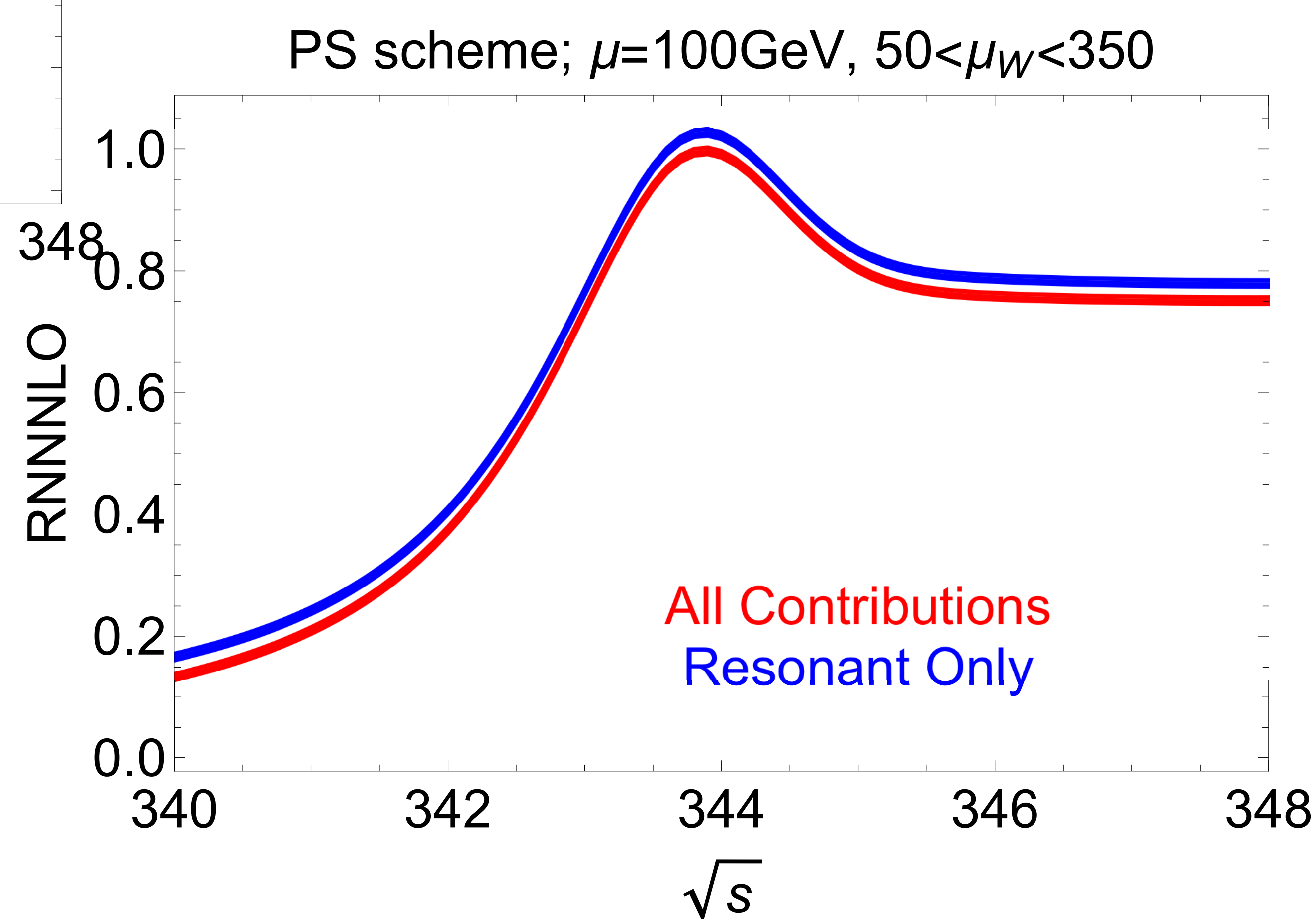
Scale dependence





PS


PS scheme; 50<μ<350, μW=350
1.0
0.8
0.6
0.4
0.2
0.0
RNNNLO
340
342
344
346
348
√s
All Contributions
Resonant Only


PS scheme; μ=100GeV, 50<μW<350
1.0
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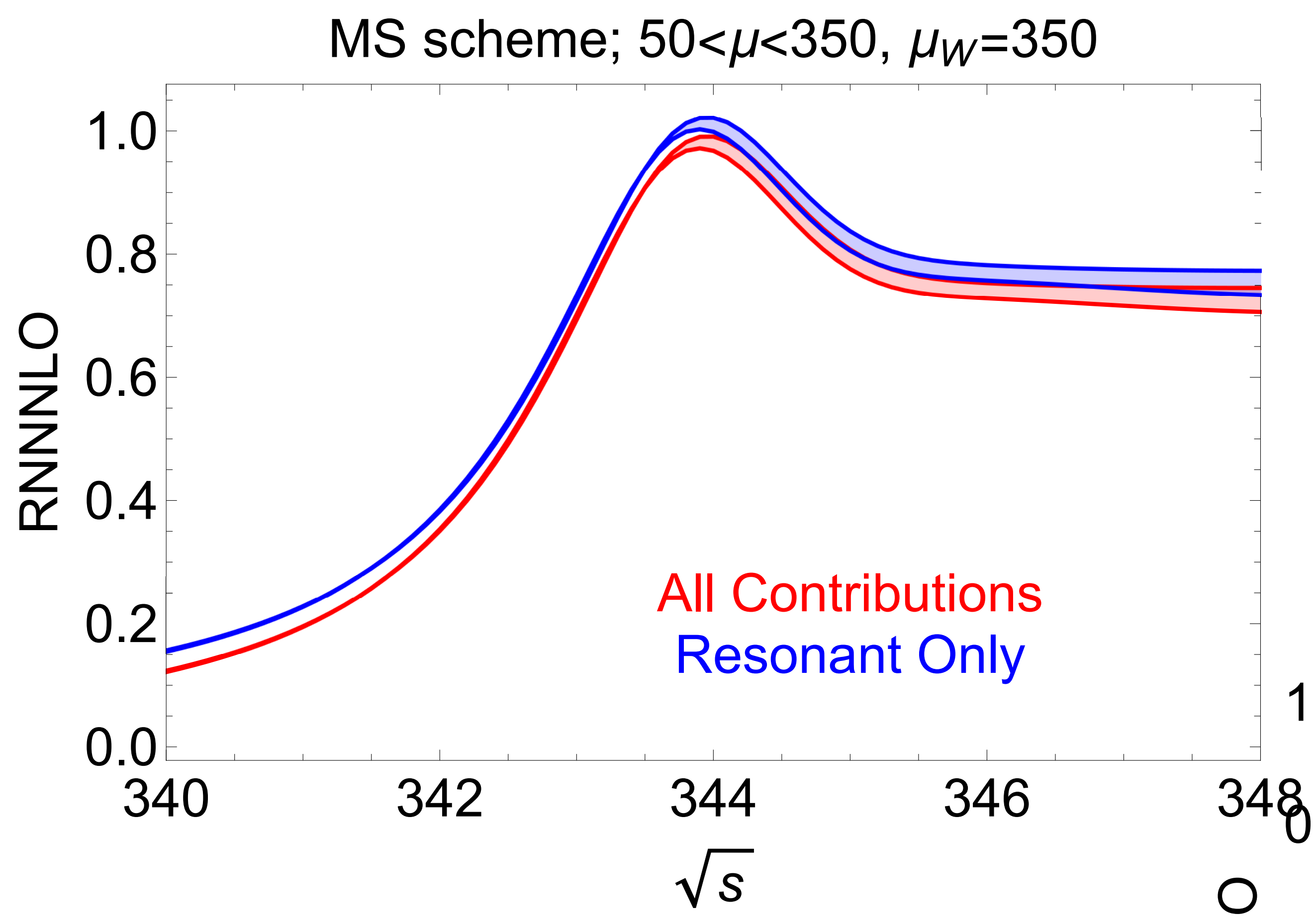

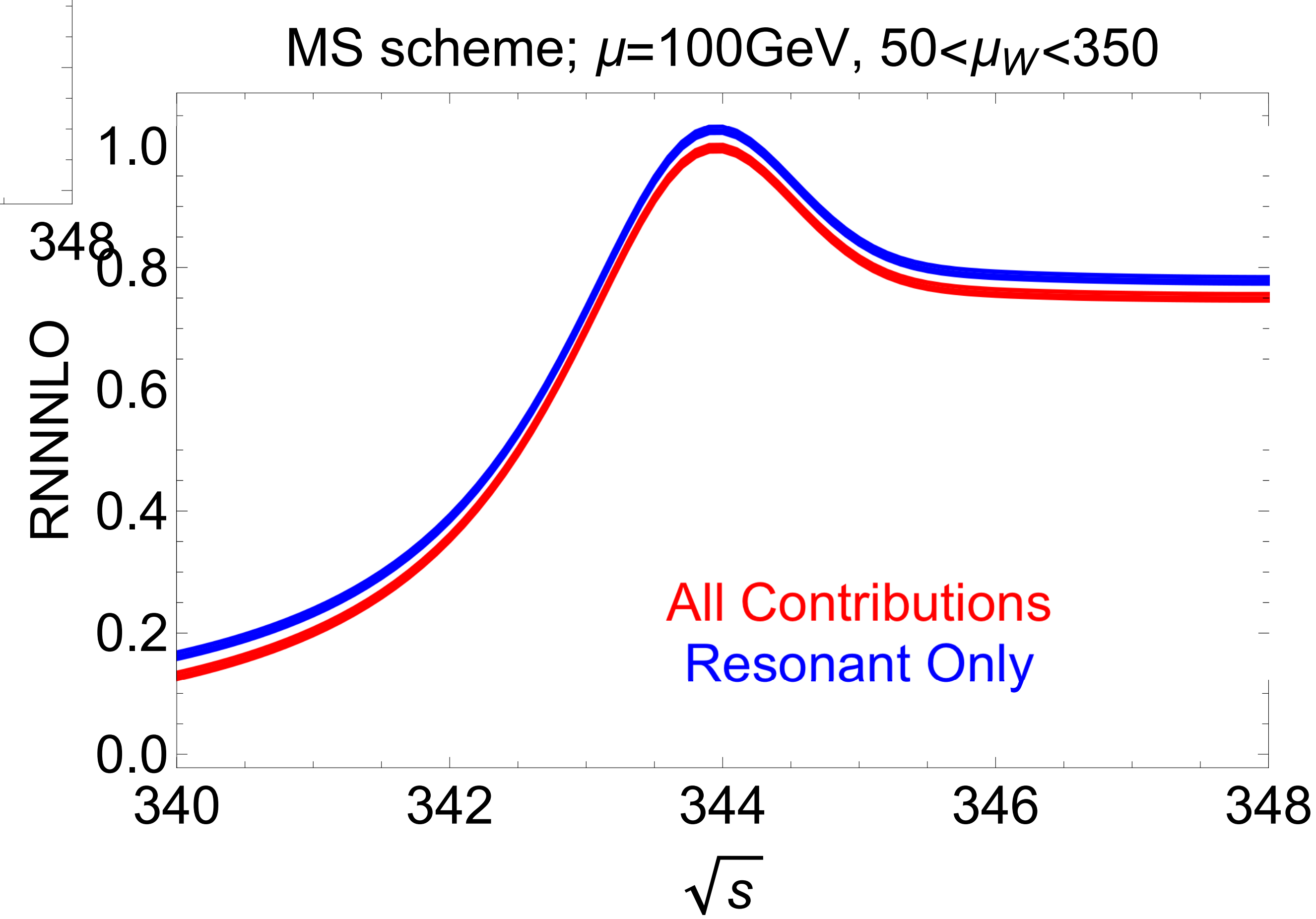

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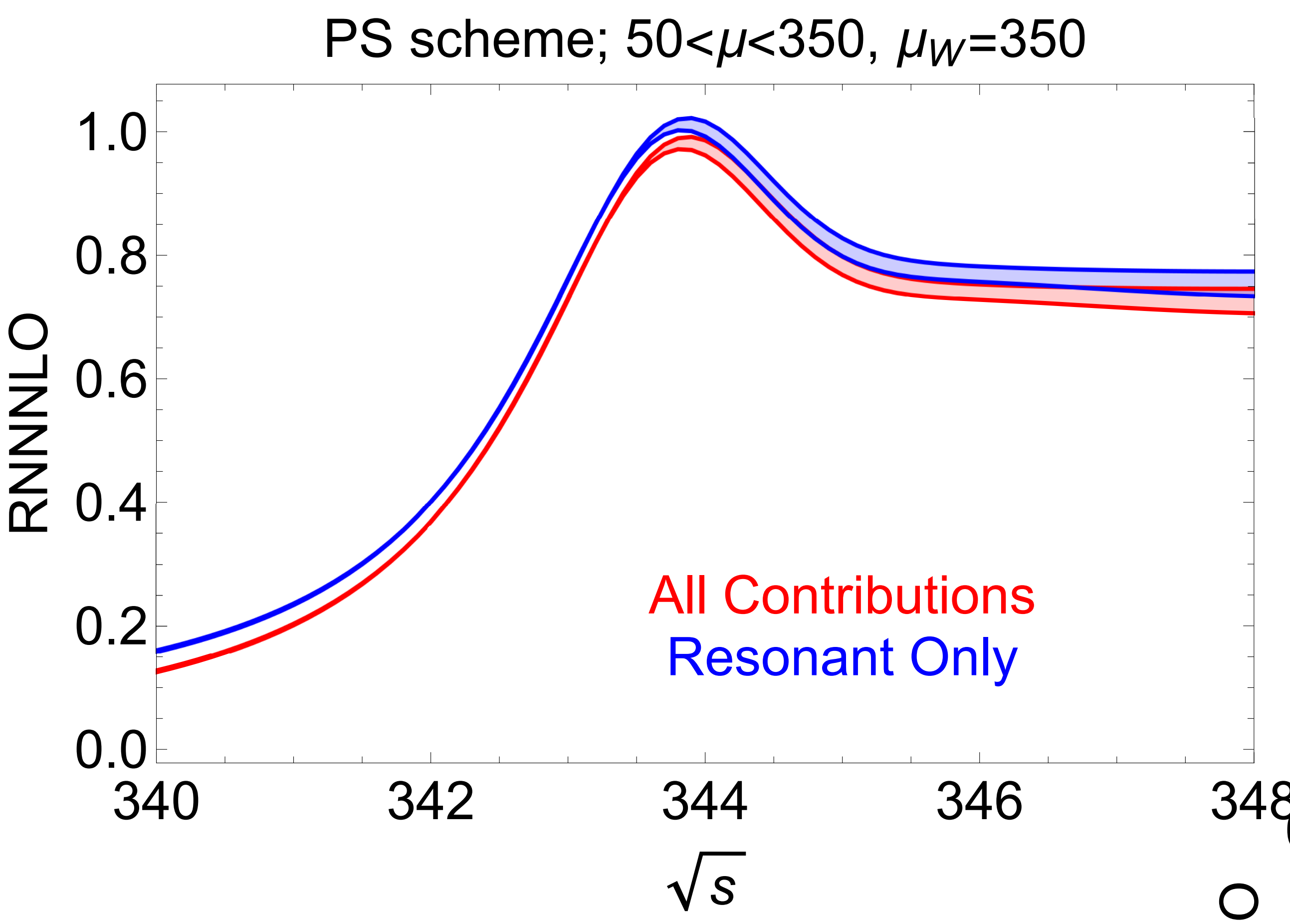
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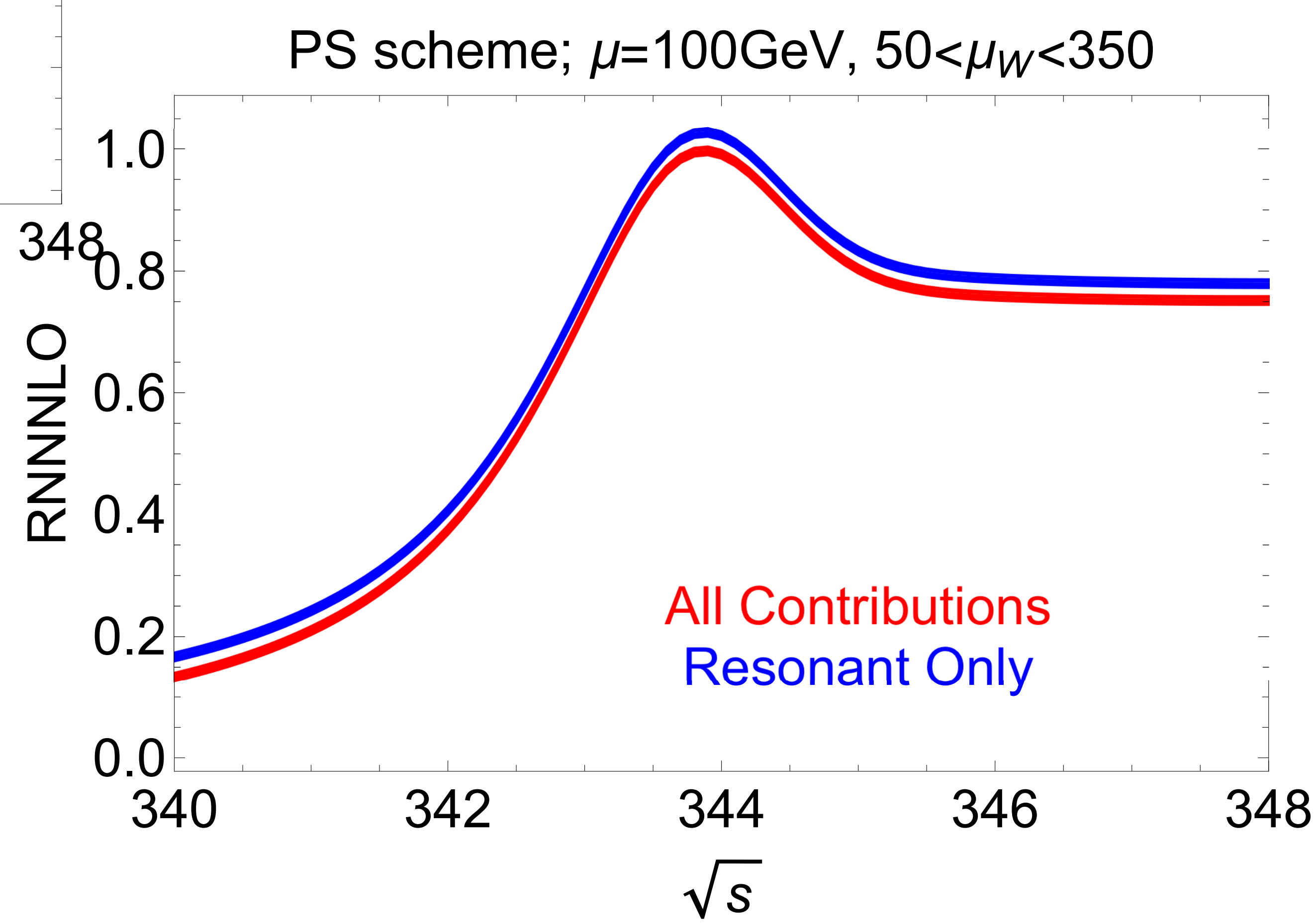

PS scheme; μ=100GeV, 50<μW<350
RNNNLO
√s
All Contributions
Resonant Only

Options

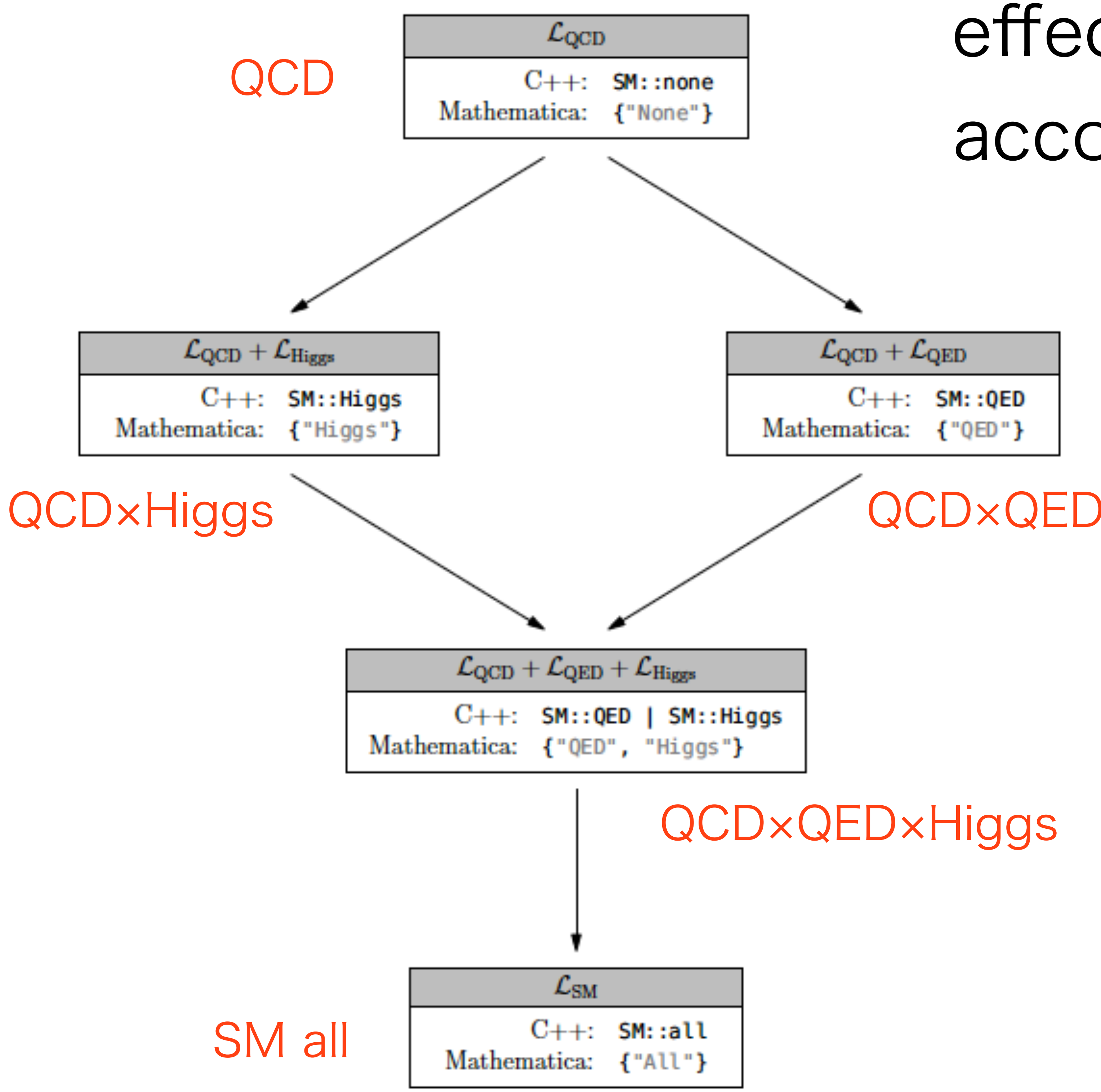
C++ name	Mathematica name	Description
contributions	Contributions	Fine-grained control over higher-order corrections.
alpha_s_mZ	alphaSmZ	Value of $\alpha_s(m_Z)$.
alpha_s_mu0	alphaSmu0	Values of μ_0 and $\alpha_s(\mu_0)$.
m_Higgs	mHiggs	Value of m_H.
Yukawa_factor	YukawaFactor	Multiplier for heavy-quark Yukawa coupling.
resonant_only	ResonantOnly	Toggle for non-resonant contribution.
invariant_mass_cut	InvariantMassCut	Cut on $W\,b$ invariant mass.
ml	ml	Value of light-quark mass.
r4	r4	Value of parameter in $\mathrm{N^3LO}$ $\overline{\mathrm{MS}}$ to pole scheme conversion.
alpha	alpha	Value of $\alpha(\mu_\alpha)$.
mu_alpha	muAlpha	Value of scale μ_α for QED coupling.
resum_poles	ResumPoles	Number of resummed poles.
beyond_QCD	BeyondQCD	Toggle for higher-order corrections beyond QCD.
mass_scheme	MassScheme	Mass renormalisation scheme.
production	Production	Toggle for production channels.
expand_s	ExpandEnergyFactor	Toggle for expansion of $1/s$ prefactors.
double_light_insertion	DoubleLightInsertion	Toggle for double insertions of light-quark potential.

Contributions

C++ name	Mathematica name	Corrections	Defined eq.
v_Coulomb	vCoulomb	$\{\delta_C V^{(1)\dagger}, \delta_C V^{(2)}, \delta_C V^{(3)}\}$	(24)
v_delta	vdelta	$\{\delta_\delta V^{(0)}, \delta_\delta V^{(1)}\}$	(24)
v_r2inv	vr2inv	$\{\delta_{1/r^2} V^{(1)}, \delta_{1/r^2} V^{(2)}\}$	(24)
v_p2	vp2	$\{\delta_p V^{(0)}, \delta_p V^{(1)}\}$	(24)
v_kinetic	vkinetic	$\{\delta \mathrm{kin}\}$	(24)
ultrasoft	ultrasoft	$\{\delta^{us} G(E)\}$	(23)
v_Higgs	vHiggs	$\{\delta_H V^{(0)}\}$	(24)
v_QED_Coulomb	vQEDCoulomb	$\{\delta_{\mathrm{QED}} V^{(0)}\}$	(24)
cv	cv	$\{c_v^{(1)}, c_v^{(2)}, c_v^{(3)}\}$	(17)
cv_Higgs	cvHiggs	$\{c_{vH}^{(2)}, c_{vH}^{(3)}\}$	(17)
Cv_QED	CvQED	$\{C_{\mathrm{QED}}^{(v)}\}$	(21)
Ca_QED	CaQED	$\{C_{\mathrm{QED}}^{(a)}\}$	(21)
Cv_WZ	CvWZ	$\{C_{\mathrm{WZ}}^{(v)}\}$	(21)
Ca_WZ	CaWZ	$\{C_{\mathrm{WZ}}^{(a)}\}$	(21)
dv	dv	$\{d_v^{(0)}, d_v^{(1)}\}$	(18)
ca	ca	$\{c_a^{(1)}\}$	(19)

Table 2: List of potential and matching coefficient corrections that can be modified with the `contributions` option. In general superscripts refer to the number of loops associated with a correction. For c_{vH} we instead follow the notation of [19], where the superscript indicates the PNRQCD order. † $\delta_C V^{(1)}$ multiplies both the contributions from the NLO colour Coulomb potential and the QED Coulomb potential.

Beyond QCD: Models

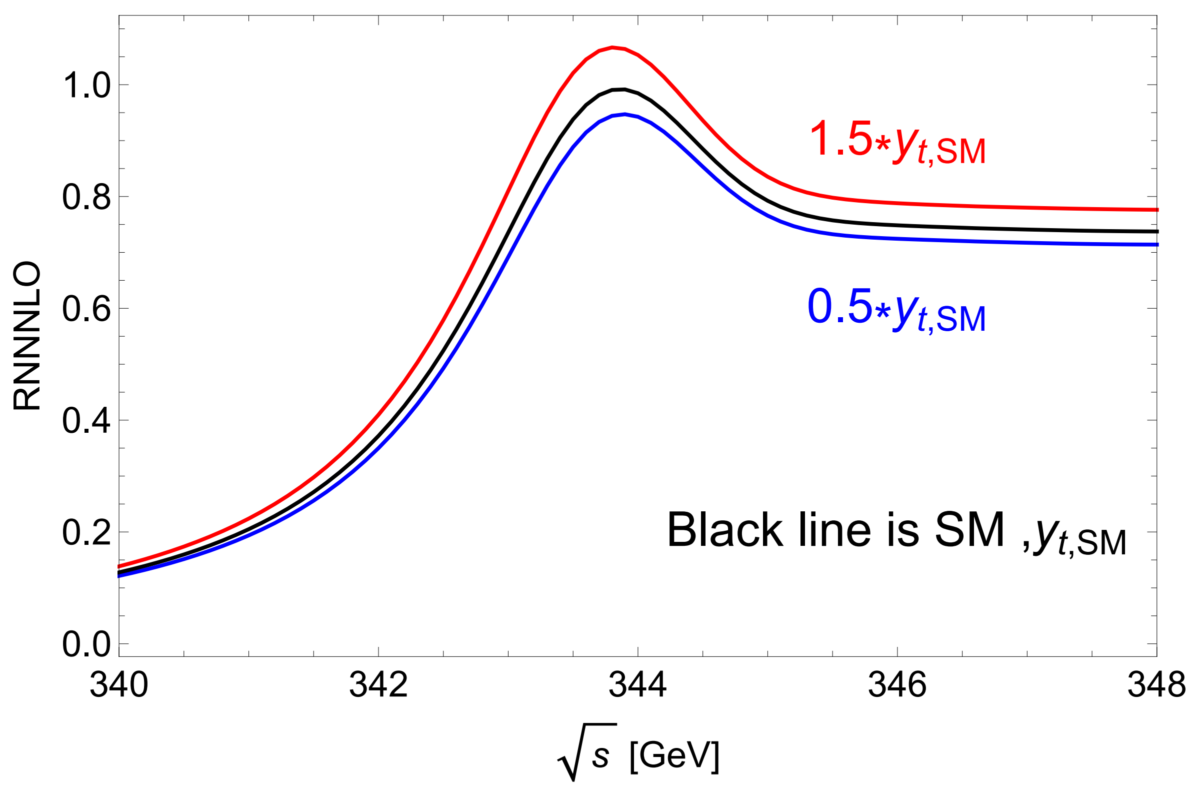


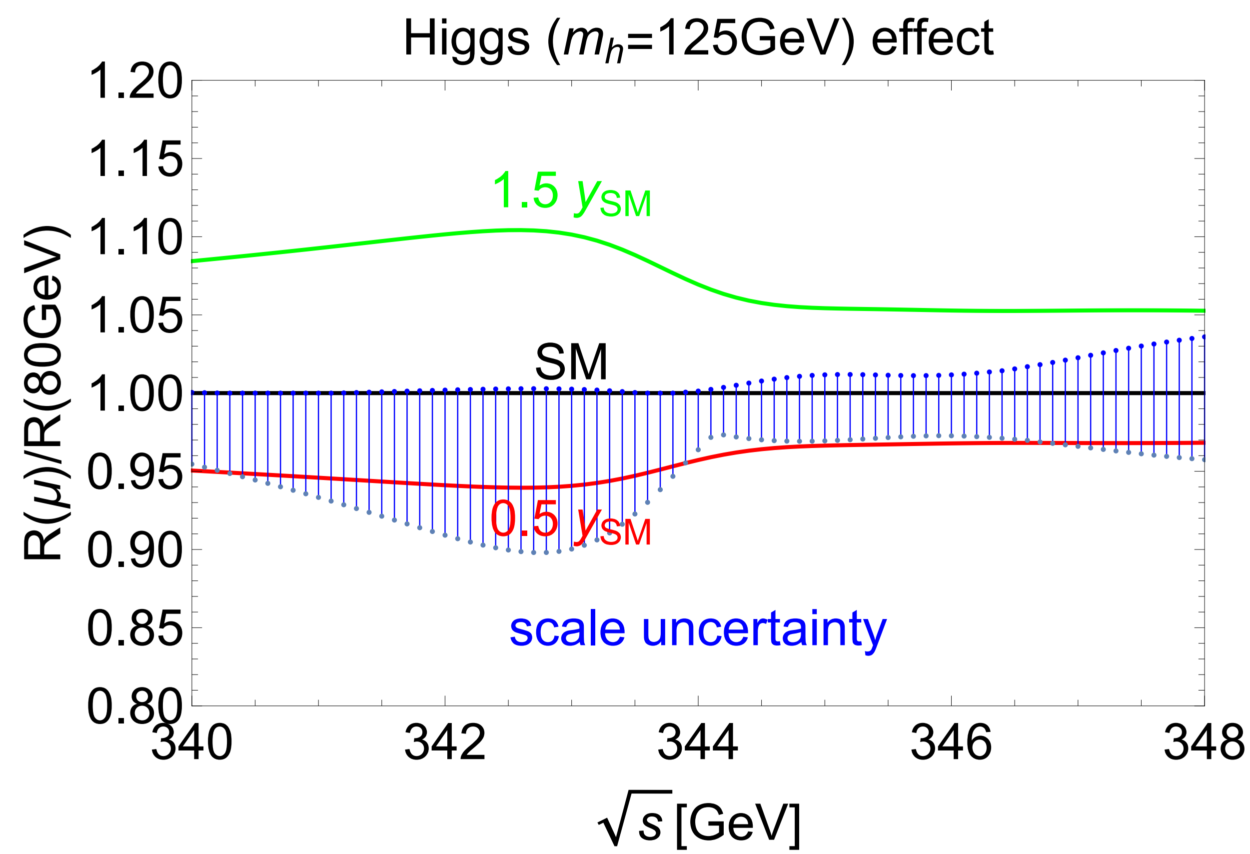
effects can be taken into account assuming model Lagrangian

dim-6 op

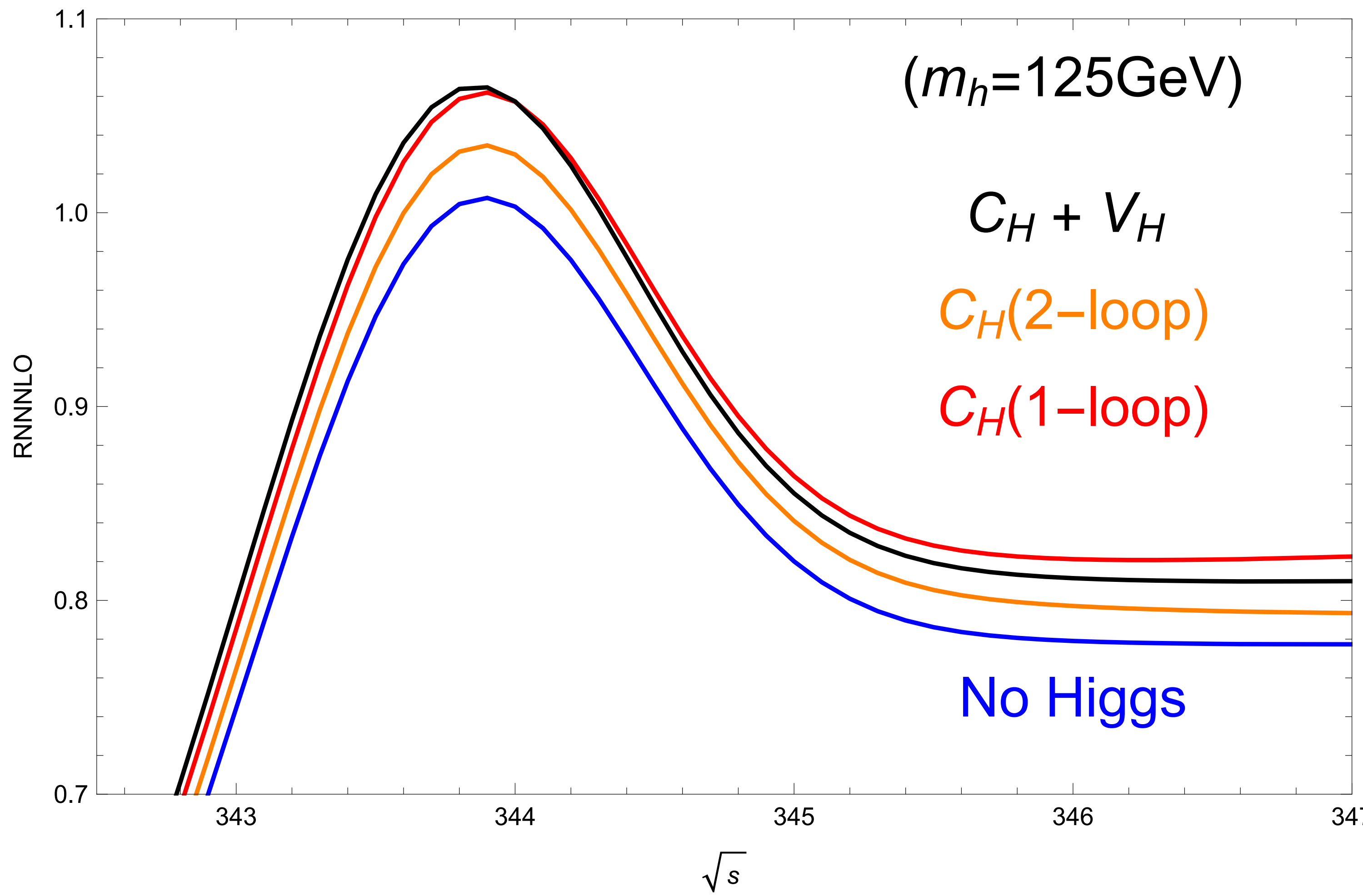
$$\mathcal{L}_{\rm Higgs} = -\frac{y_t}{\sqrt{2}}\bar{t}tH$$

Higgs effect: YukawaFactor

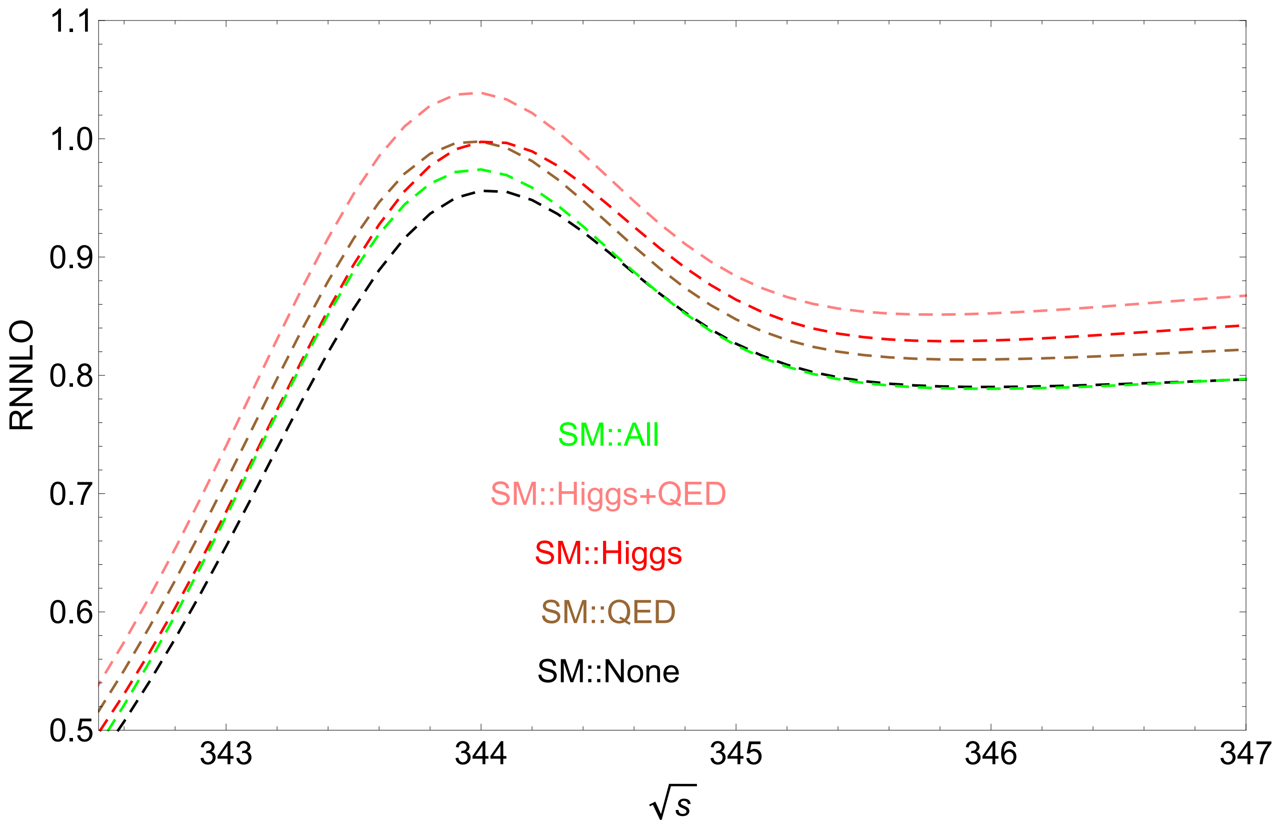




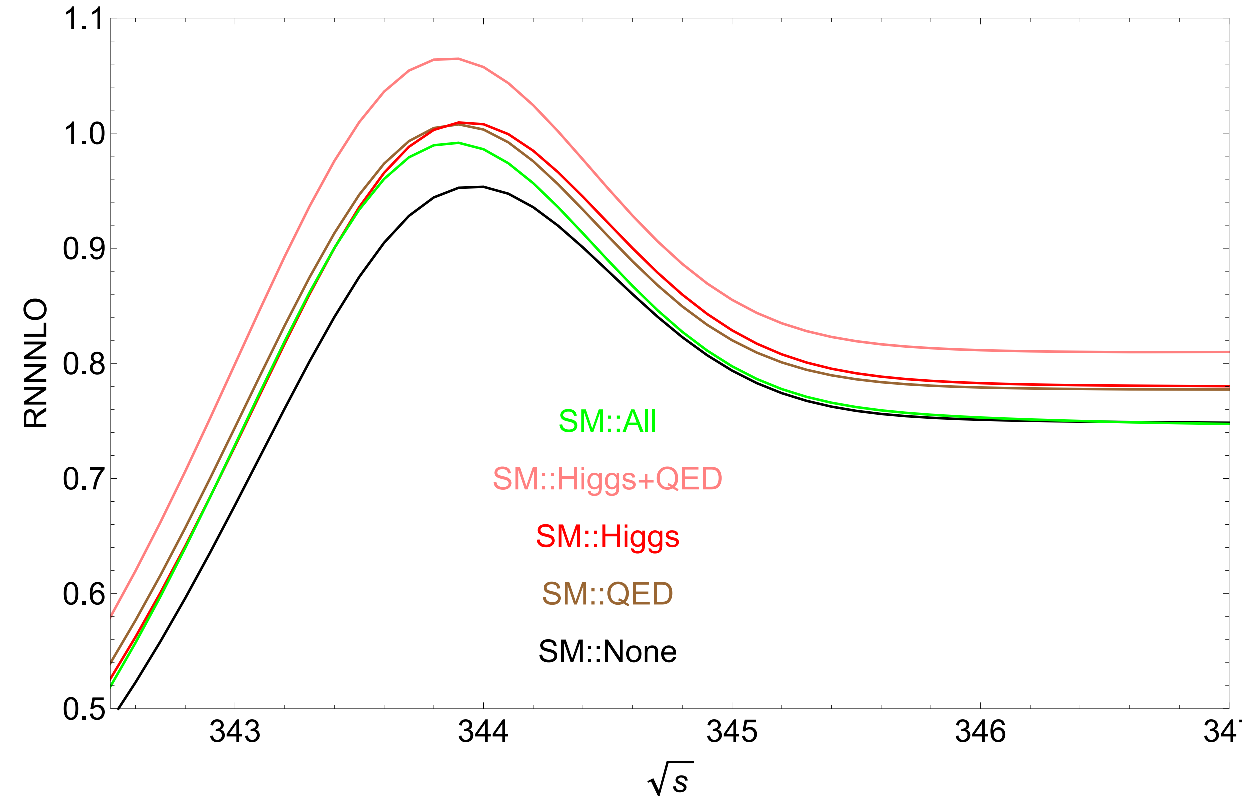
Higgs effect in SM



Models



1.1
1.0
0.9
0.8
0.7
0.6
0.5
RNNLO
343
344
345
346
347
$\sqrt{s}$
SM::All
SM::Higgs+QED
SM::Higgs
SM::QED
SM::None



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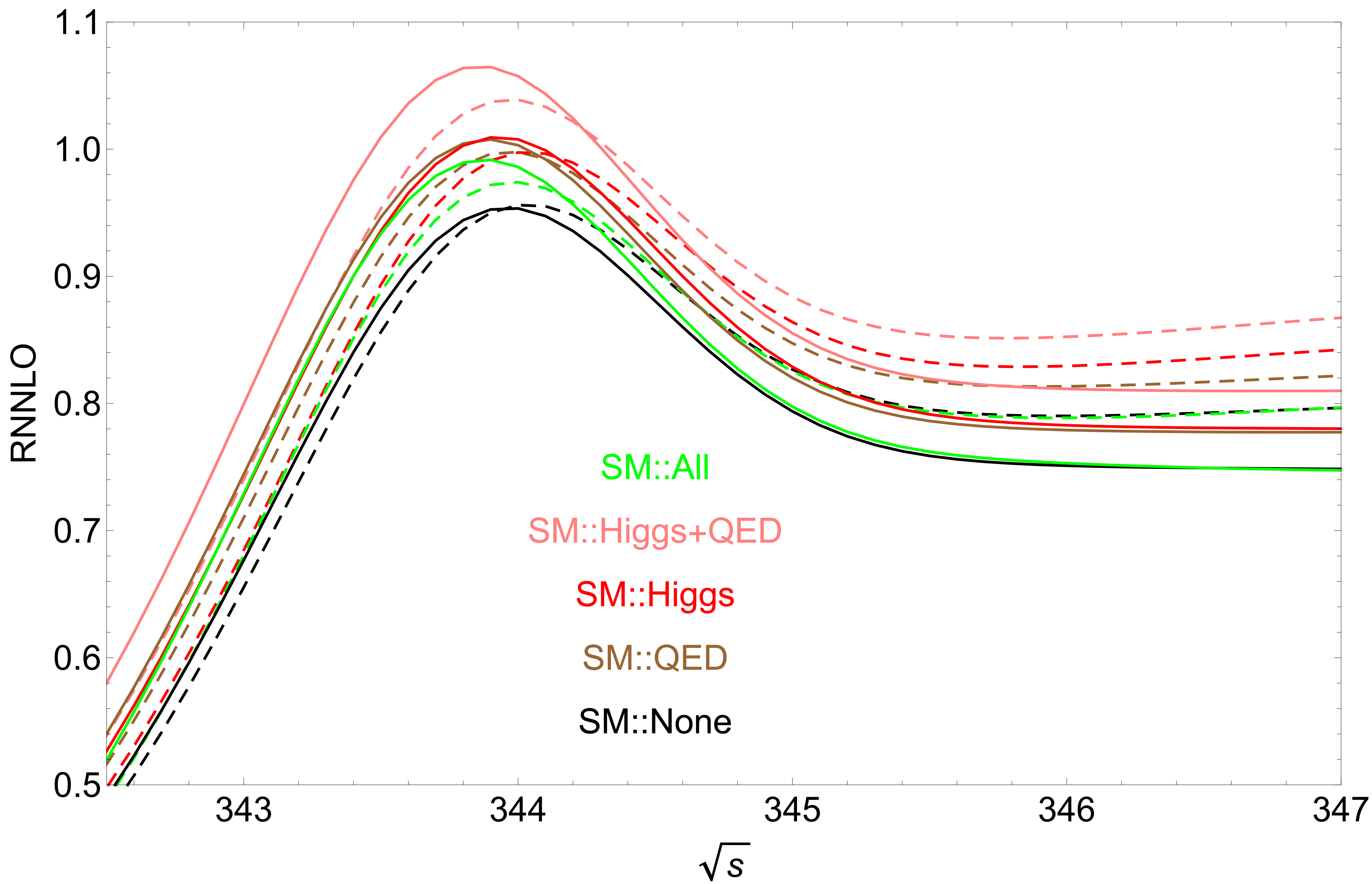
Model



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Summary

- Public C++ code: QQbar_threshold Published
- MathLink allows mathematica usage
- Assemble many (Nealy complete) Loop corrections
- Many options for model studies

Future development

- Complete NNNLO corrections
- NNLO and higher Non-resonant contributions
- e+e- ISR for simulation study
- Anything more?

Top WS@Valencia 2015

top threshold

cross section near top threshold normalized to point particle one

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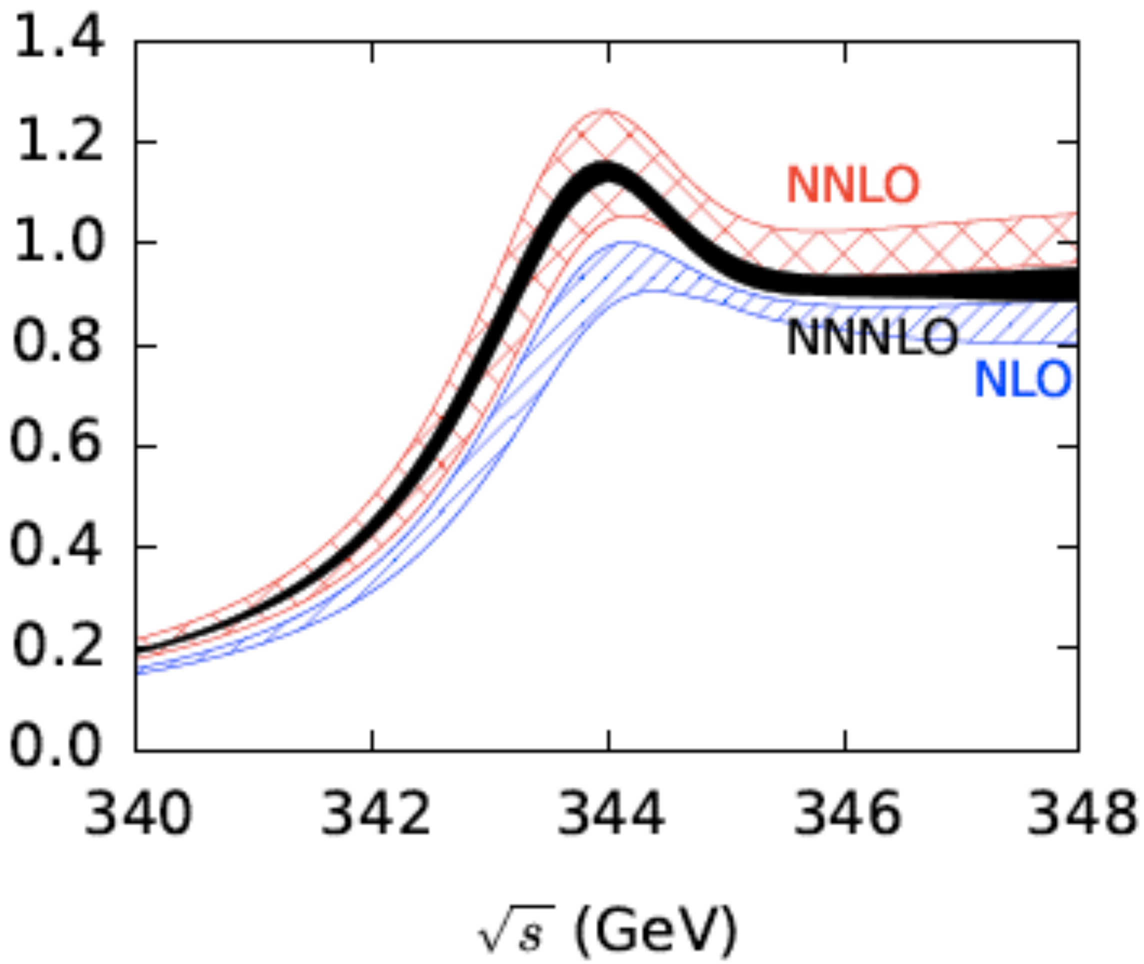

Beneke-YK-Marquard-Penin-Piclum -Steinhauser:1506.06864[hep-ph]

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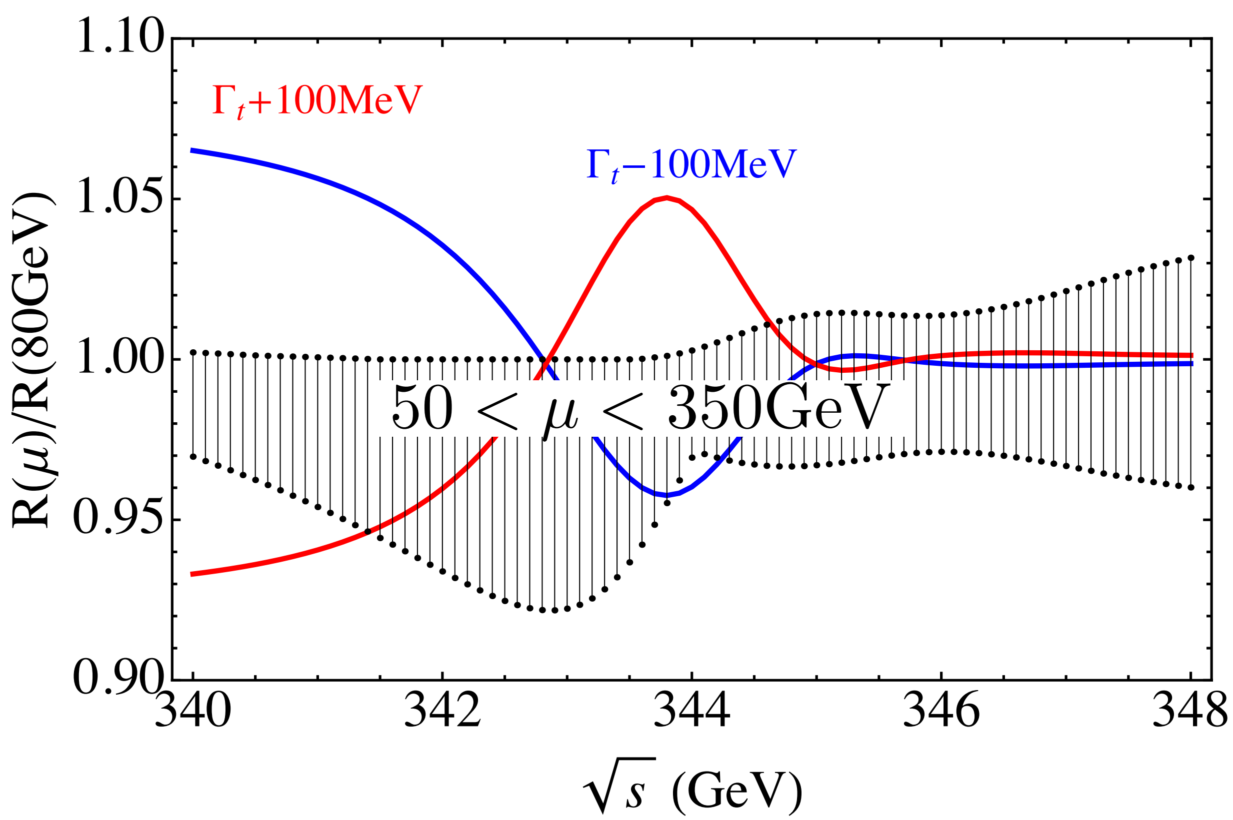
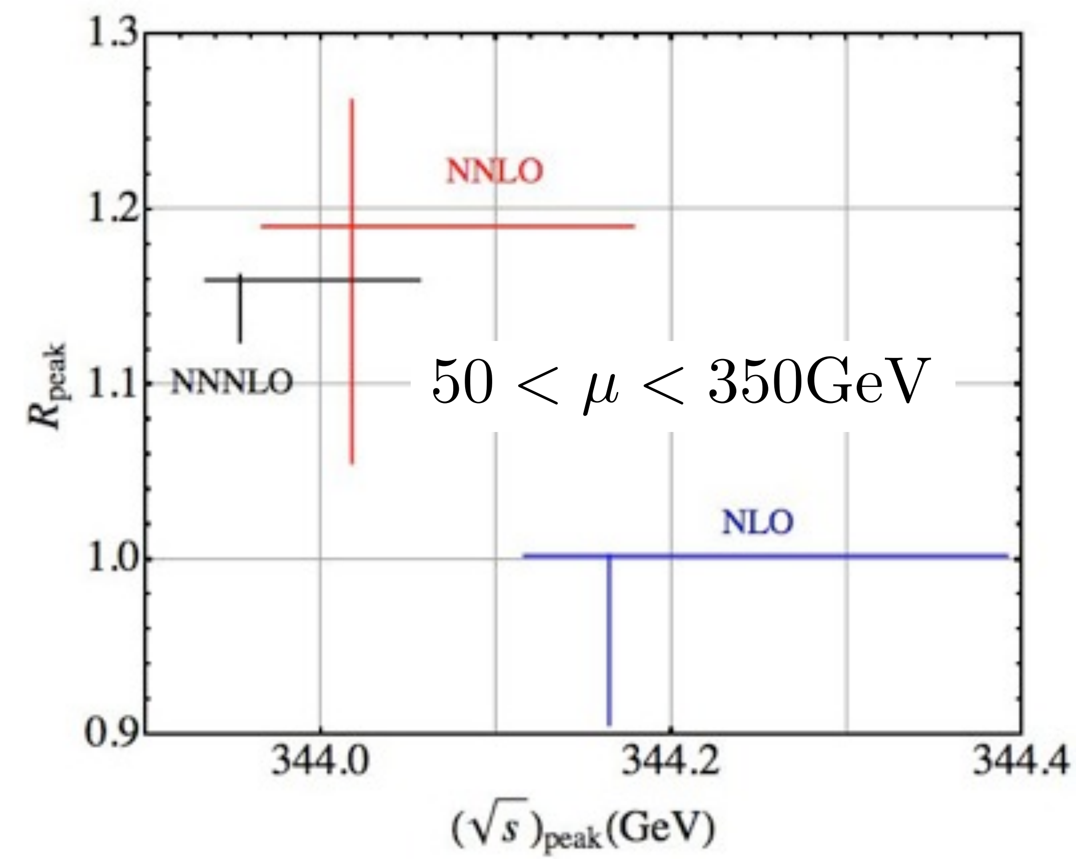
✤ Non-resonant, EW, Higgs......
Talk by Beneke (previous speaker)

NNNLO result

Beneke-YK-Marquard-Penin-Piclum-Steinhauser(15)





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- N^3LO: $\delta E \sim 100\mathrm{MeV}, \delta R \sim 0.05$

Part I

Recently the full $\mathcal{O}(\alpha_S^5 m, \alpha_S^5 m \log \alpha_S)$ correction to the heavy quarkonium $1S$ energy level has been computed (except the a_3-term in the QCD potential). We point out that the full correction (including the $\log \alpha_S$-term) is approximated well by the large-β_0 approximation. Based on the assumption that this feature holds up to higher orders, we discuss why the top quark pole mass cannot be determined to better than $\mathcal{O}(\Lambda_{\rm QCD})$ accuracy at a future e^+e^- collider, while the $\overline{\rm MS}$ mass can be determined to about 40 MeV accuracy (provided the 4-loop $\overline{\rm MS}$-pole mass relation will be computed in due time).

YK-Sumino(2002)

Combining recent perturbative analyses on the static QCD potential and the quark pole mass, we find that, for the heavy quarkonium states $c\bar{c}$, $b\bar{b}$ and $t\bar{t}$, (1) ultrasoft (US) corrections in the binding energies are small, and (2) there is a stronger cancellation of IR contributions than what has been predicted by renormalon dominance hypothesis. By contrast, for a hypothetical heavy quarkonium system with a small number of active quark flavors ($n_l \approx 0$), we observe evidence that renormalon dominance holds accurately and that non-negligible contributions from US corrections exist. As an important consequence, we improve on a previous prediction for possible achievable accuracy of top quark $\overline{\rm MS}$–mass measurement at a future linear collider and estimate that in principle about 20 MeV accuracy is reachable.

YK-Mishima-Sumino: 1506.06542[hep-ph]

Mass extraction@ILC

(Simplified Strategy diagram)

$M(1S)=\mathrm{Func.}(\alpha_s, m_X)$

Threshold Scan

$m_X=m_{PS}$ Beneke et al.

Mass concersion to other scheme

$m_{MS}=\mathrm{Func.}(\alpha_s, m_{PS})$

We(YK-Mishima-Sumino) suggest to work in MS from begining to get better precision

Pole-$\overline{\text{MS}}$ mass relation

$$m_{\text{pole}} = \overline{m}\left[1 + d_0 \frac{\alpha_s(\overline{m})}{\pi} + d_1 \left(\frac{\alpha_s(\overline{m})}{\pi}\right)^2 + d_2 \left(\frac{\alpha_s(\overline{m})}{\pi}\right)^3 + d_3 \left(\frac{\alpha_s(\overline{m})}{\pi}\right)^4\right]$$

$$= \overline{m}\left[1 + 0.4244\alpha_s + 0.8345\alpha_s^2 + 2.368\alpha_s^3 + 8.461\alpha_s^4\right]$$

→Talk by Marquard

d_3 in full QCD: Marquard-Smirnov-Smirnov-Steinhauser arXiv:1502.01030[hep-ph]

*numbers are slightly differernt form the one of QCD, because of decoupling

- In this talk, I use $\overline{m} = m_{\overline{\text{MS}}}(\overline{m})$
- We use effective field theory, in which the heavy quark decoupled,

i.e. n_l=5 for "topoinum" → renormalon cancellation

$$d_3 \equiv d_3^{(n_l=5)} = -0.67814 n_l^3 + 43.396 n_l^2 - 745.42 n_l + 3551.1$$

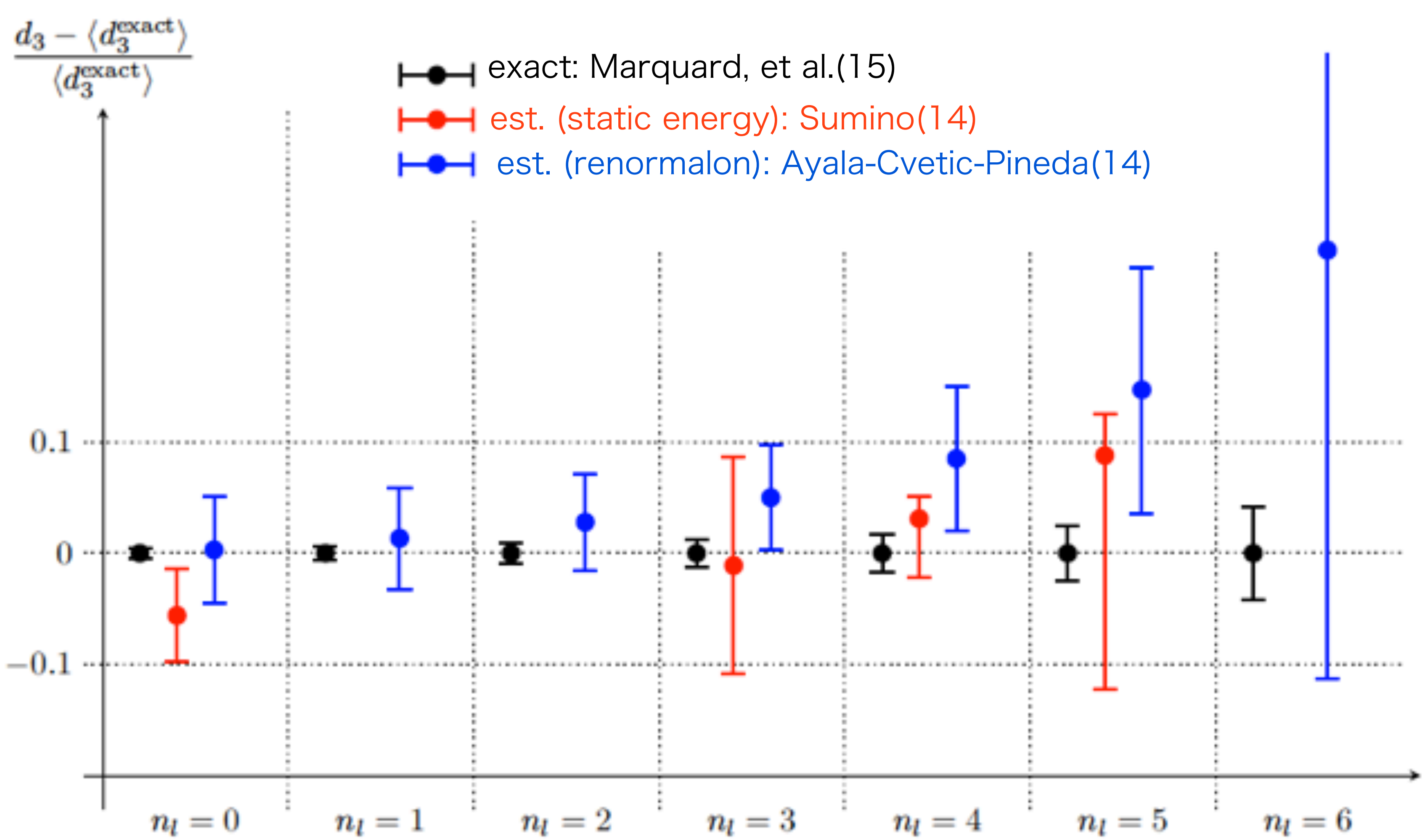
±21.5 (Marquard, et al.)

$$\alpha_s \equiv \alpha_s^{(n_l=5)} = 0.108855 \ (\mu = \overline{m})$$

d3 comparison

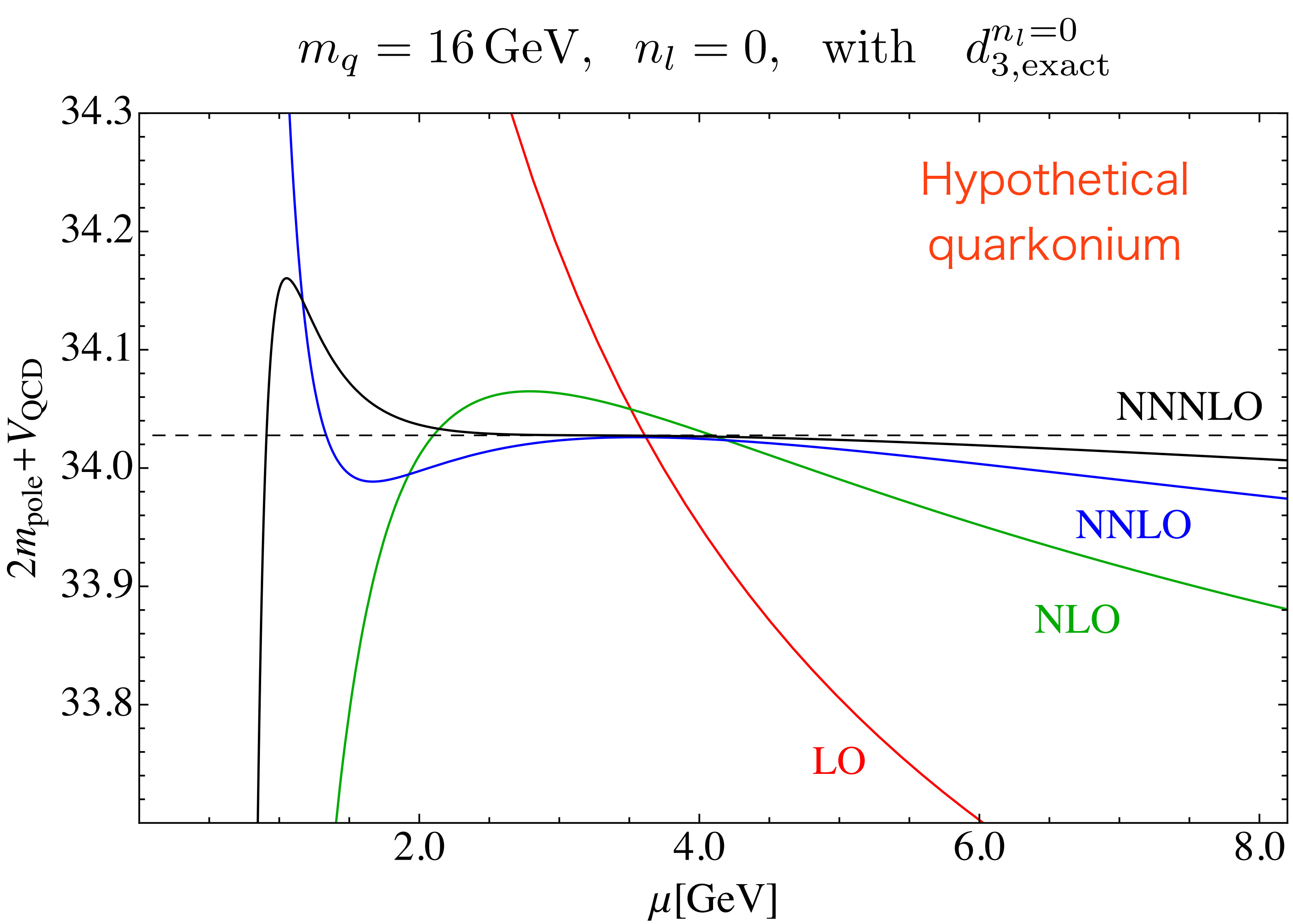
n_l	0	1	2	3	4	5	6
$d_3^{\rm est}[3]$	3351(152)	—	—	1668(167)	1258^{+26}_{-66}	897^{+31}_{-175}	—
$d_3^{\rm est}[4]$	3562(173)	2887(133)	2291(98)	1772(82)	1324(81)	945(92)	629(191)
$d_3^{\rm exact}[1]$	3551.1(21.5)	2848.4(21.5)	2228.4(21.5)	1687.1(21.5)	1220.3(21.5)	824.1(21.5)	494.3(21.5)

top quark



QCD Static Energy

Stability of the static energy can be seen/investigated for arbitrary n_f and m_q if a_i, d_i are known to required order.



Stability of E_{QCD}

→ est. for d_3 Sumino(14)

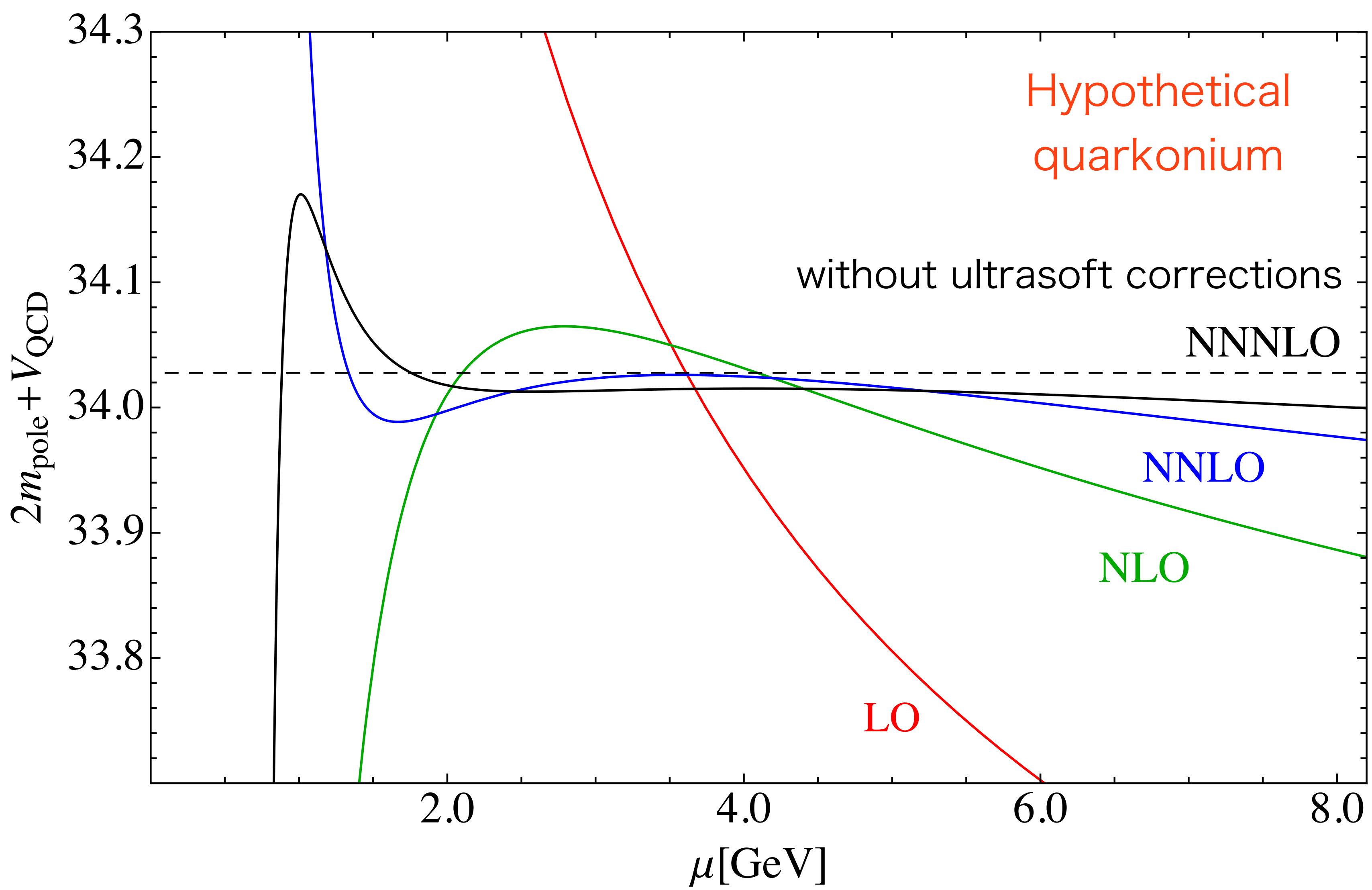
a3: Smirnov-Smirnov-Steinhauser,
Anzai-YK-Sumino

Vus: Anzai-YK-Sumino

d3: Marquard-Smirnov-Smirnov-Steinhauser

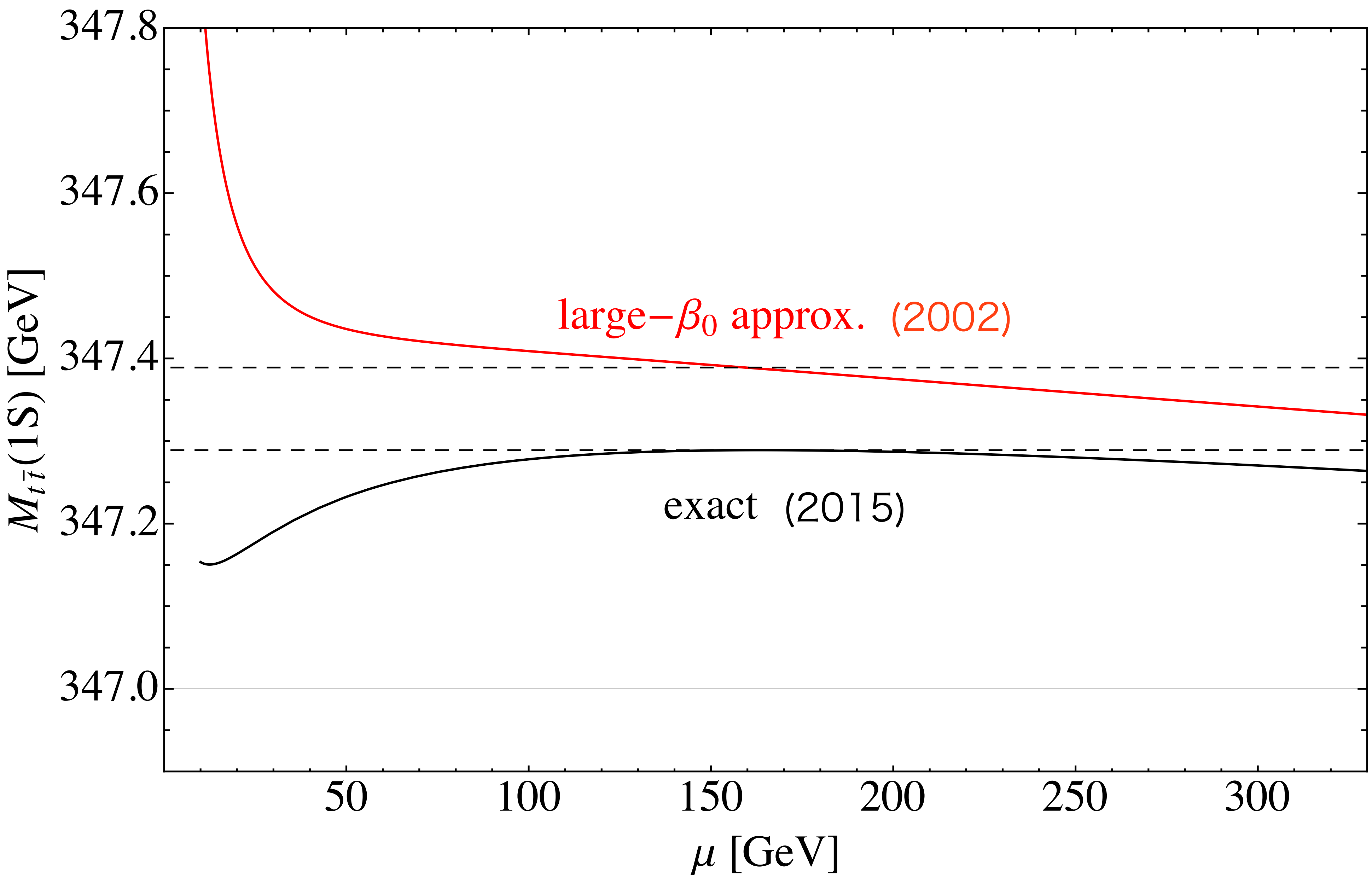
QCD Static Energy

(ultrasoft correction excluded)



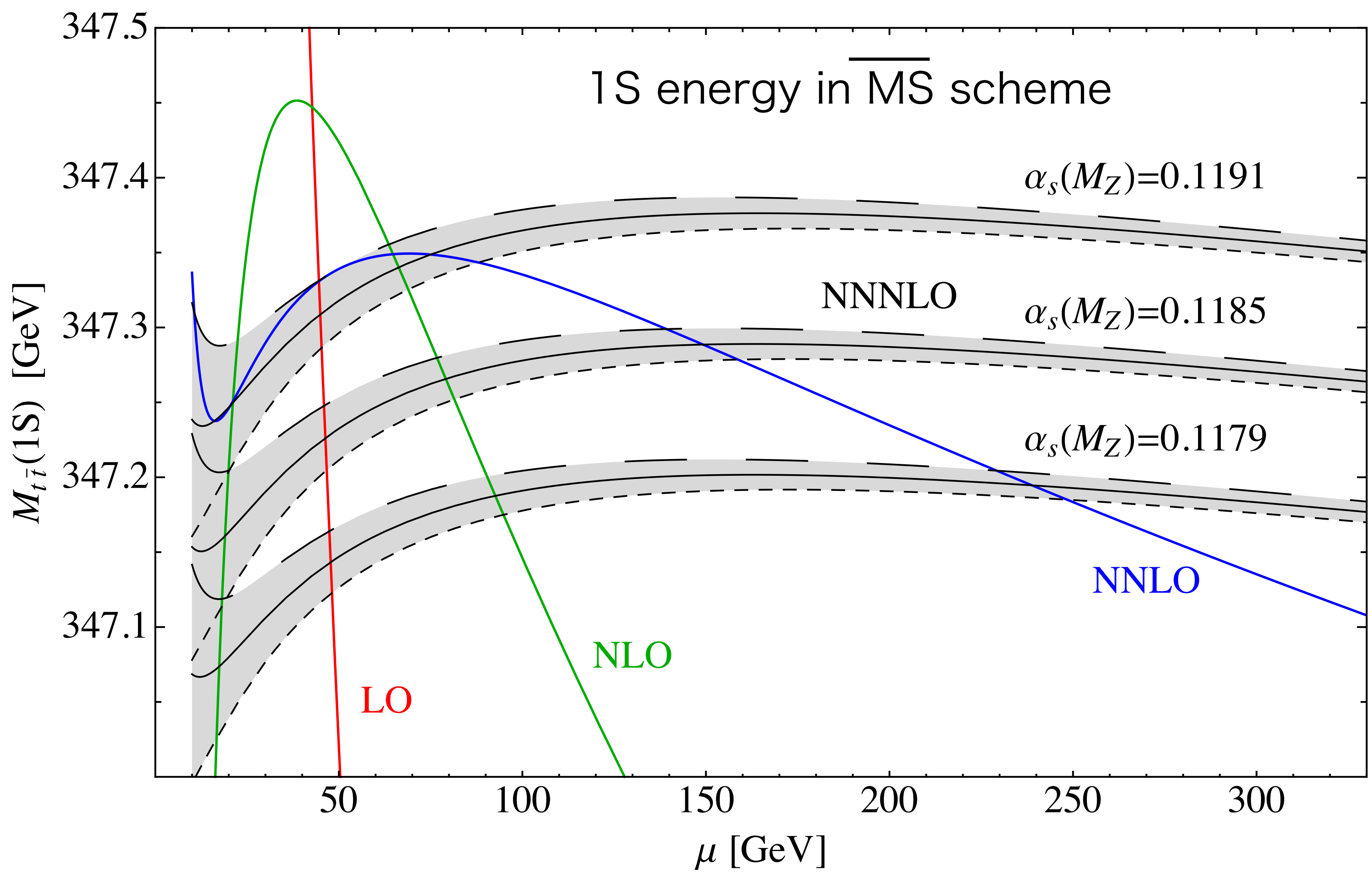
Stability of E_{QCD} holds without ultrasoft effect, but visible constant shift

Toponium energy



Existence of a minimum sensitivity point against scale variation with exact d3, which was not the case in large-beta0 approximation in 2002

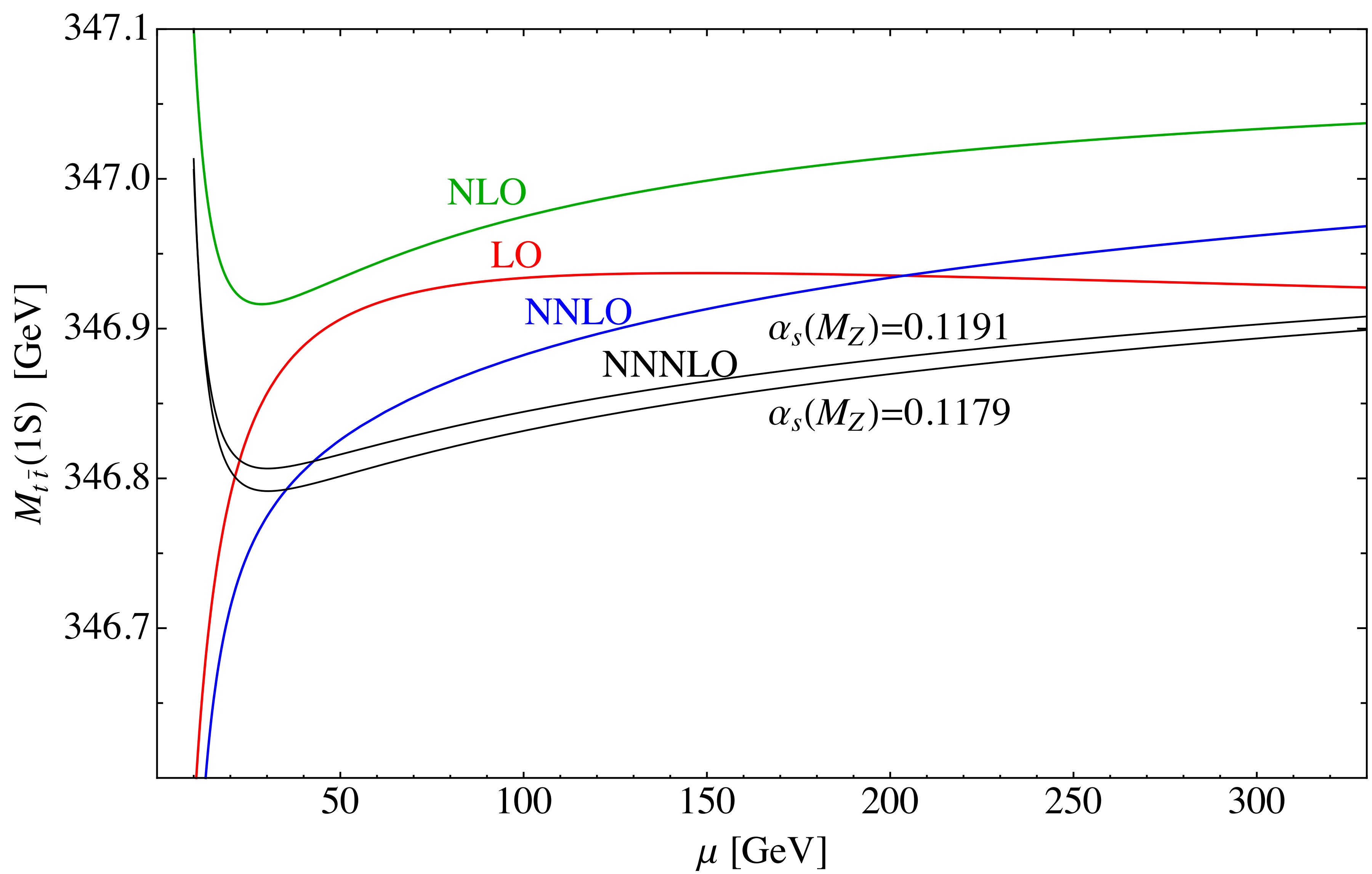
Toponium energy



- balck bands due to numerical error of d3(exact)
- three lines for NNNLO for $\alpha_s(M_z) = 0.1185 \pm 0.0006$

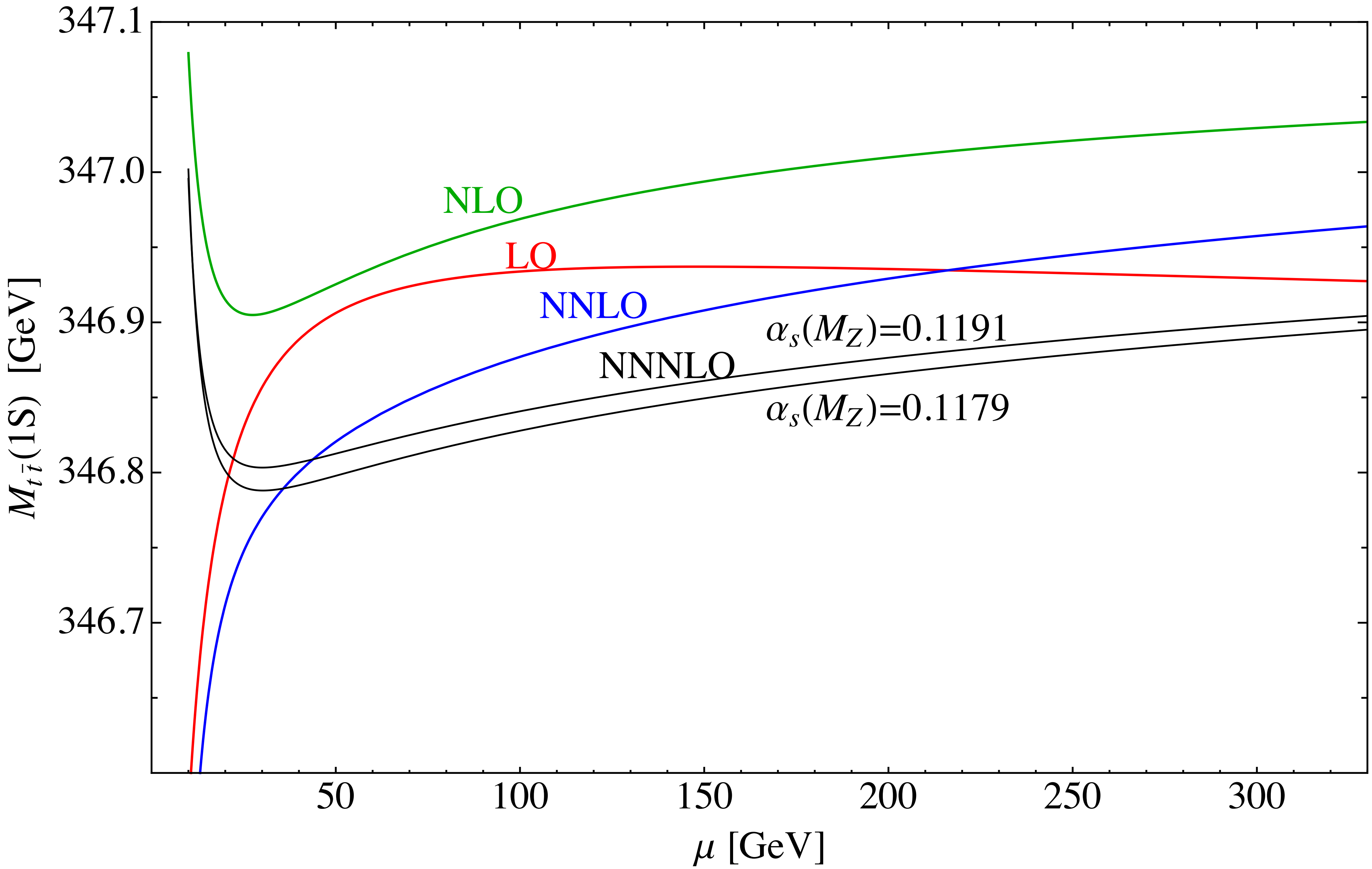
➡ $$\delta M_{1S} = 2\delta m_{\overline{\rm MS}} = (40_\mu + 10_{d_3} + 90_{\alpha_s})\ \ {\rm MeV}$$

E1S in PS scheme

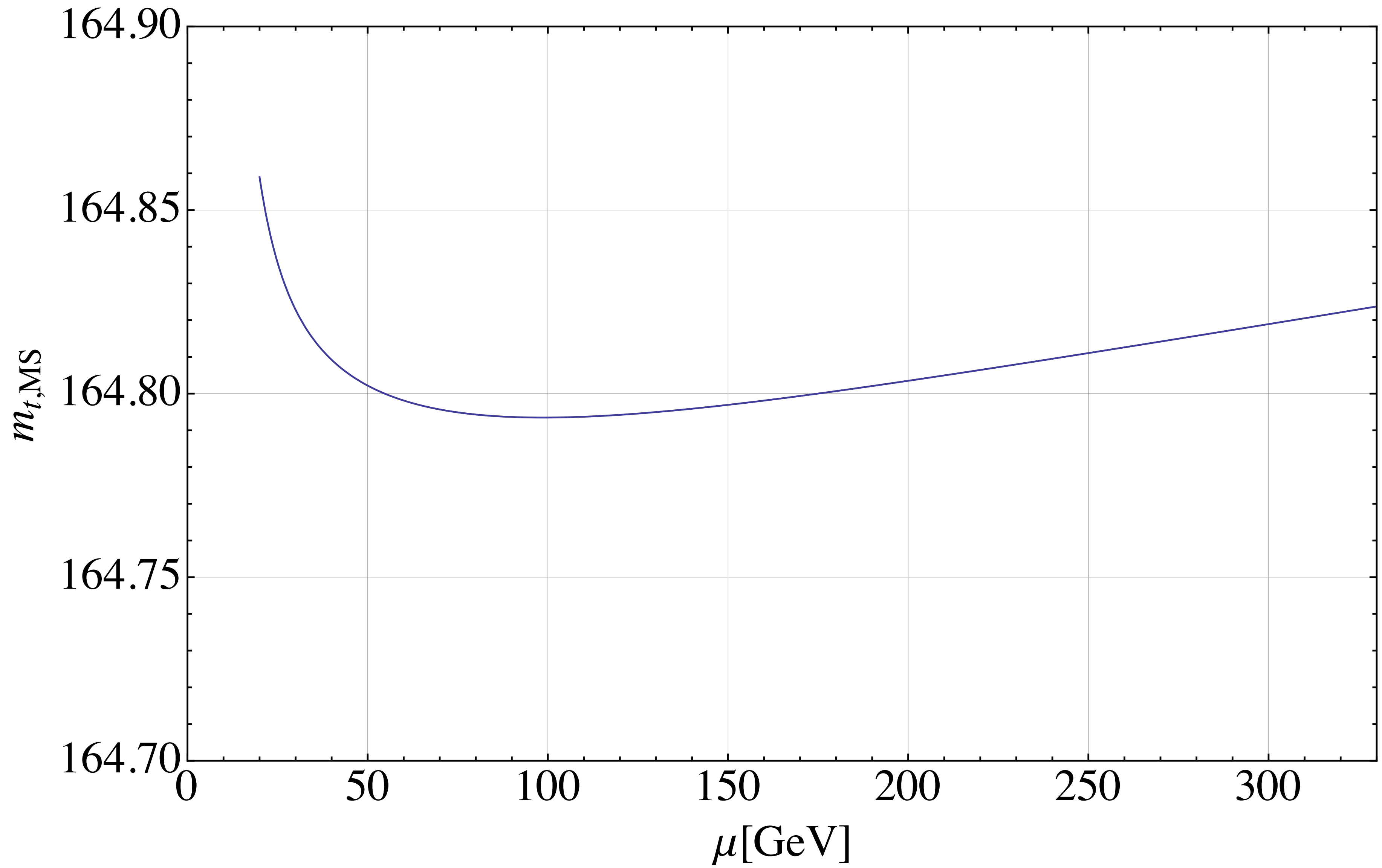


$$\delta M_{1S} = 2\delta m_{\rm PS} = (75_{\mu} + 16_{\alpha_s})\ \ {\rm MeV}\quad (80 < \mu < 320\ {\rm GeV})$$

E1S in PS' scheme



PS > MS



$$\delta m_{\mathrm{MS}} = 30_{\mu}\ \mathrm{GeV} + \cdots \quad (80 < \mu < 320\mathrm{GeV})$$

provided that m_{PS}, α_s has no significant error

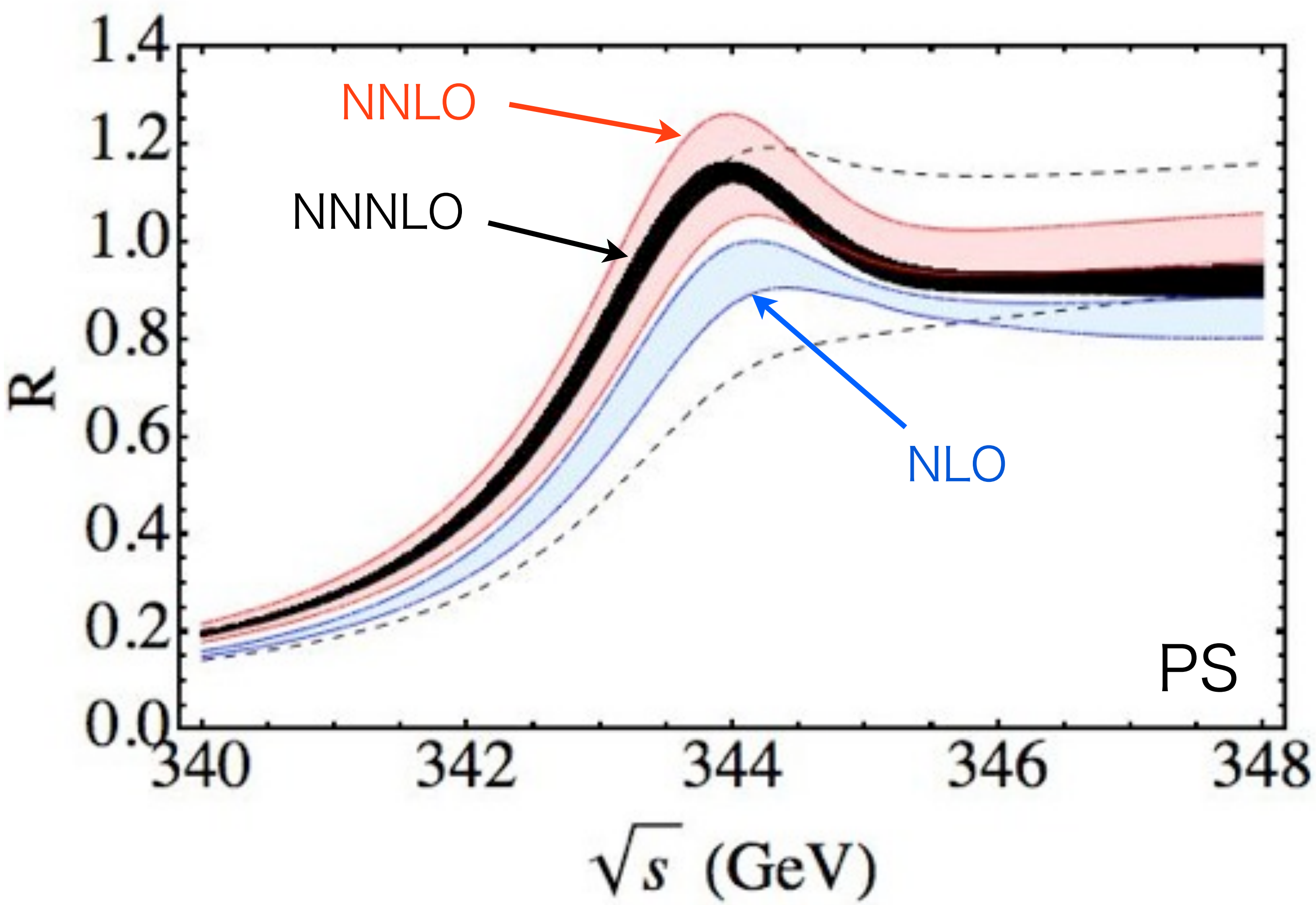
Part II

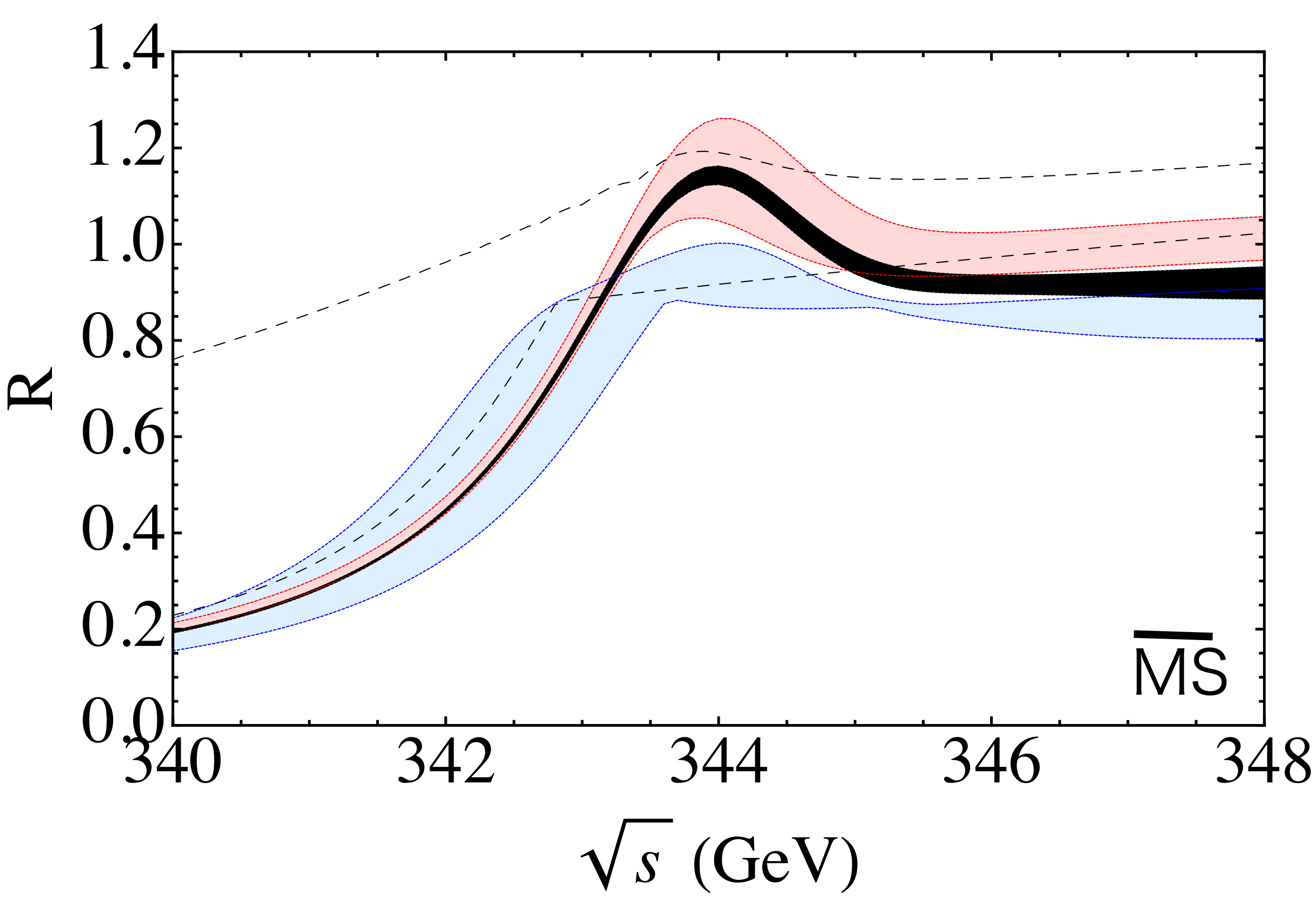
Threshold cross section in MS scheme using our code: TTbarXSection

Beneke-YK-Schuller (2008〜)

inputs:

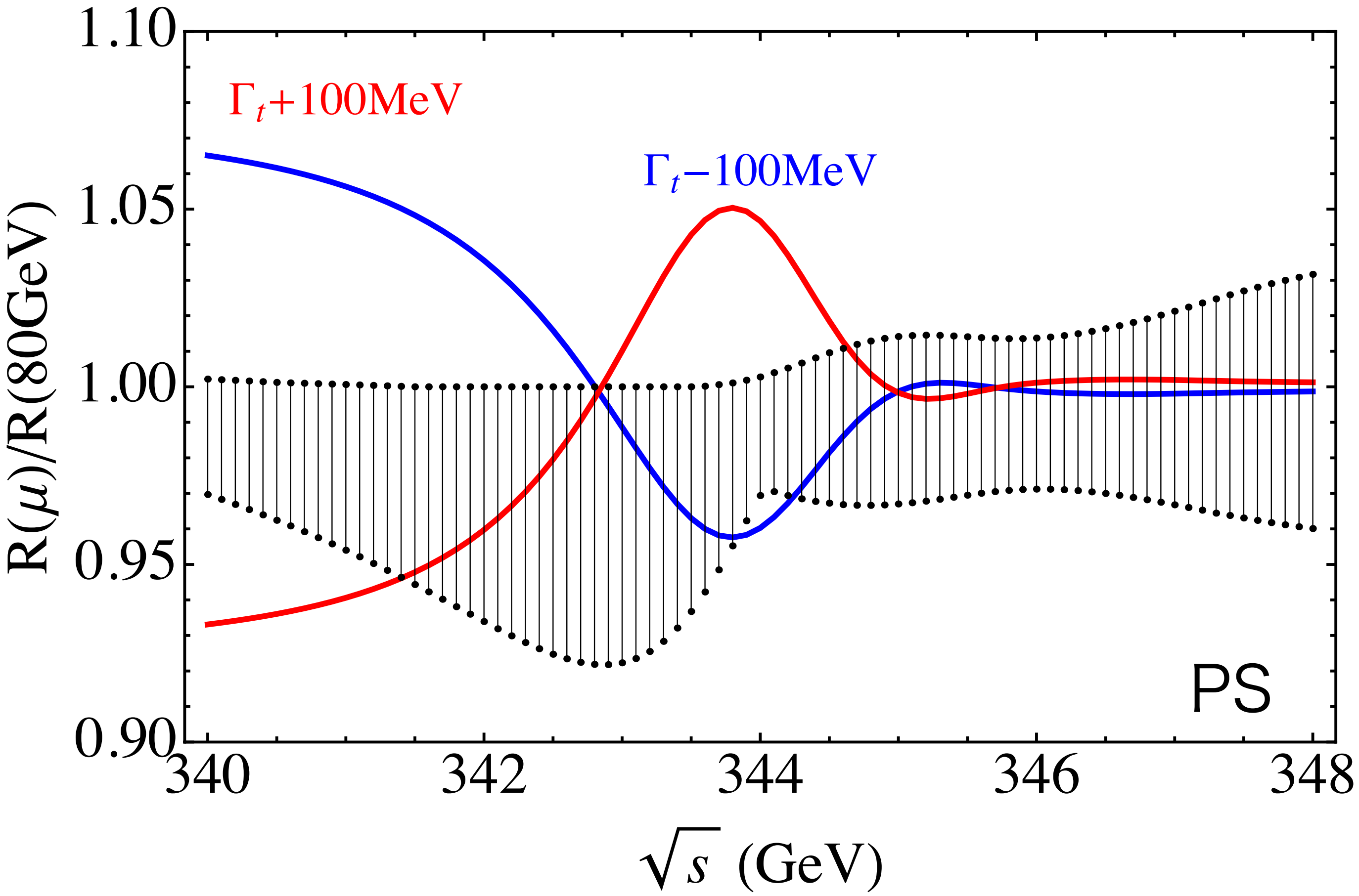
$$m_{\rm PS} = 173\,{\rm GeV}$$
$$m_{\overline{\rm MS}} = 163.3\,{\rm GeV}$$
$$\Gamma_t = 1.33\,{\rm GeV}$$
$$\alpha_s(M_z) = 0.1185$$

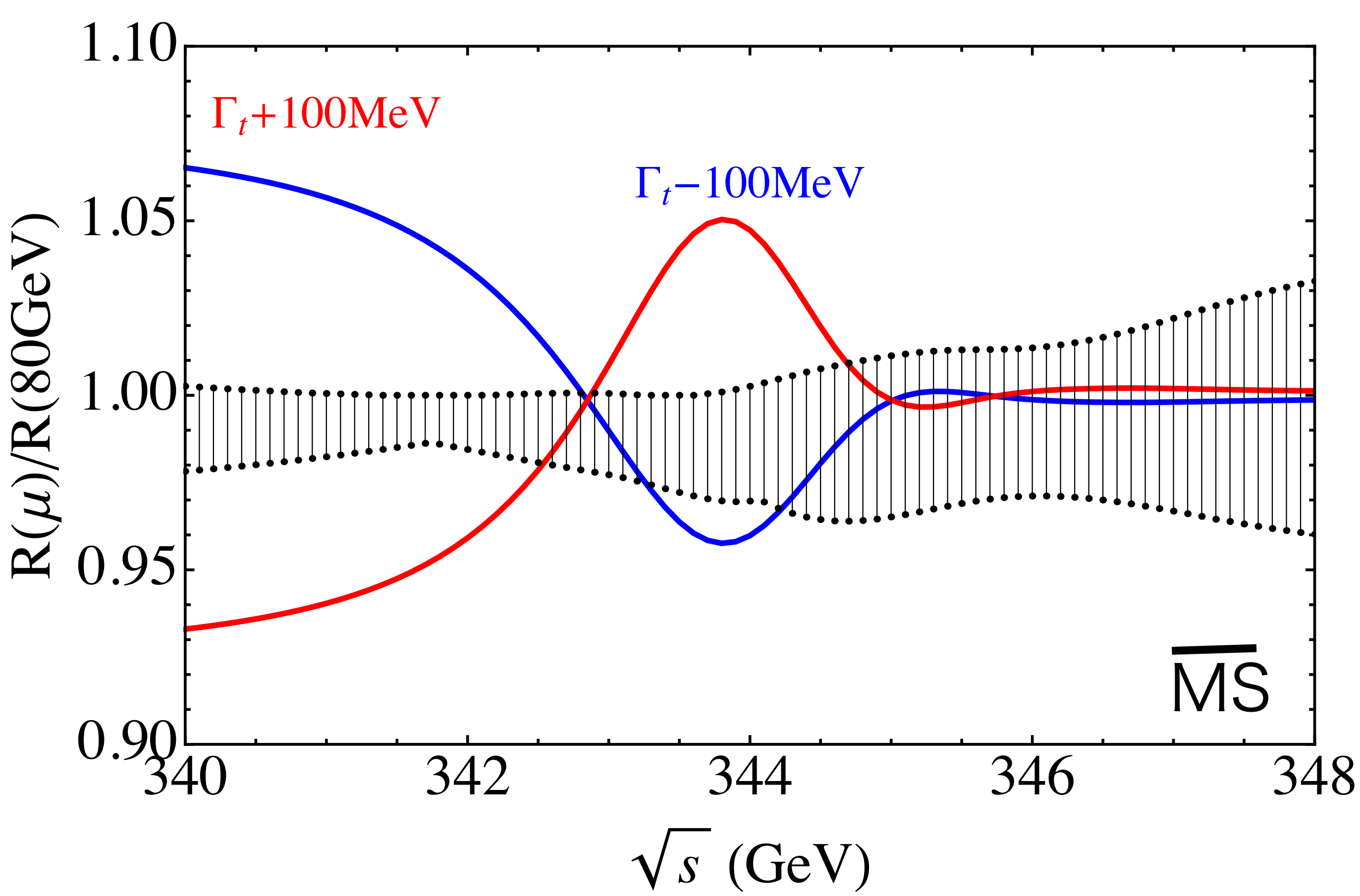




MSbar scheme

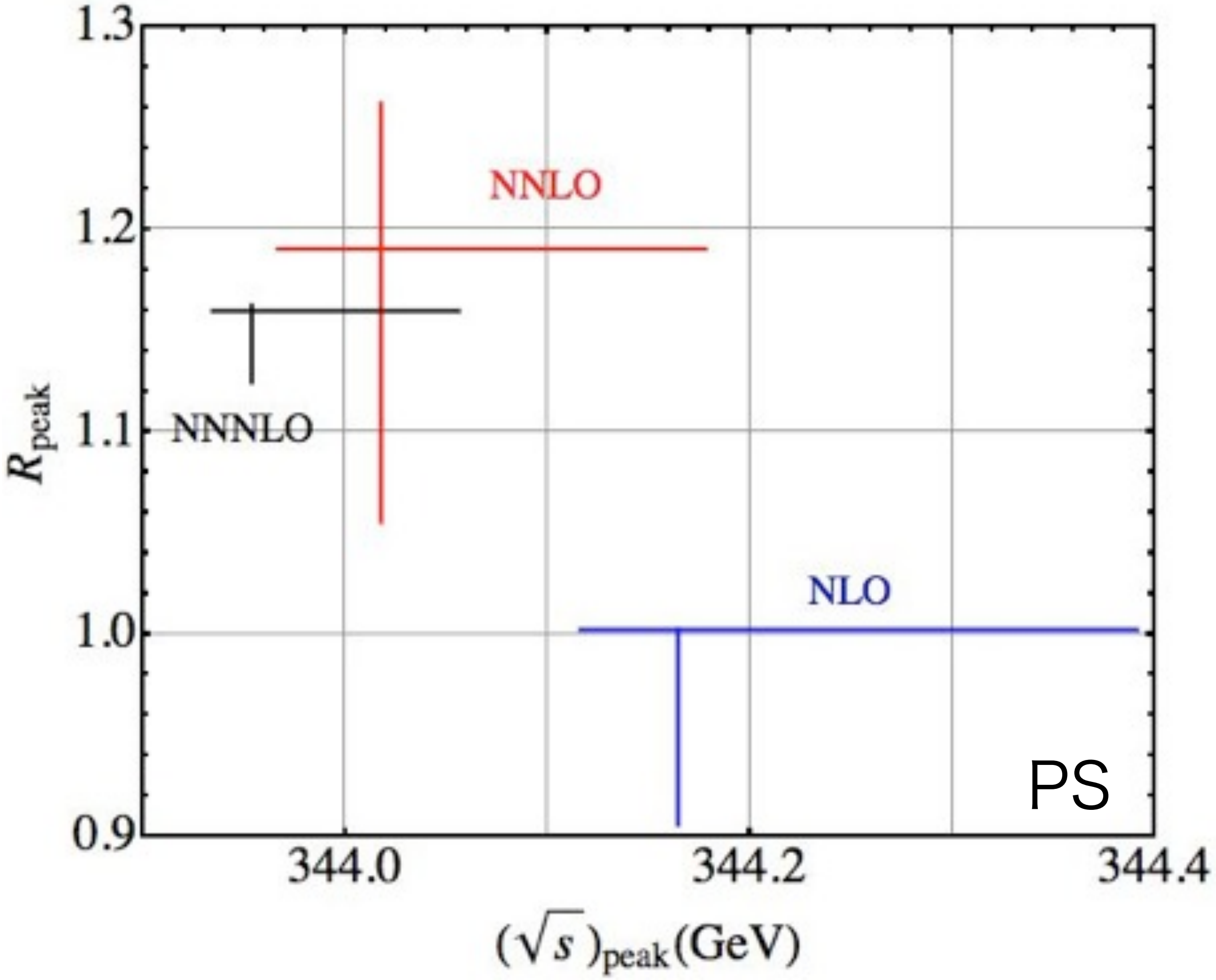
- large corrections at lower order
- but converge quickly
- scale dependence improved at NNNLO

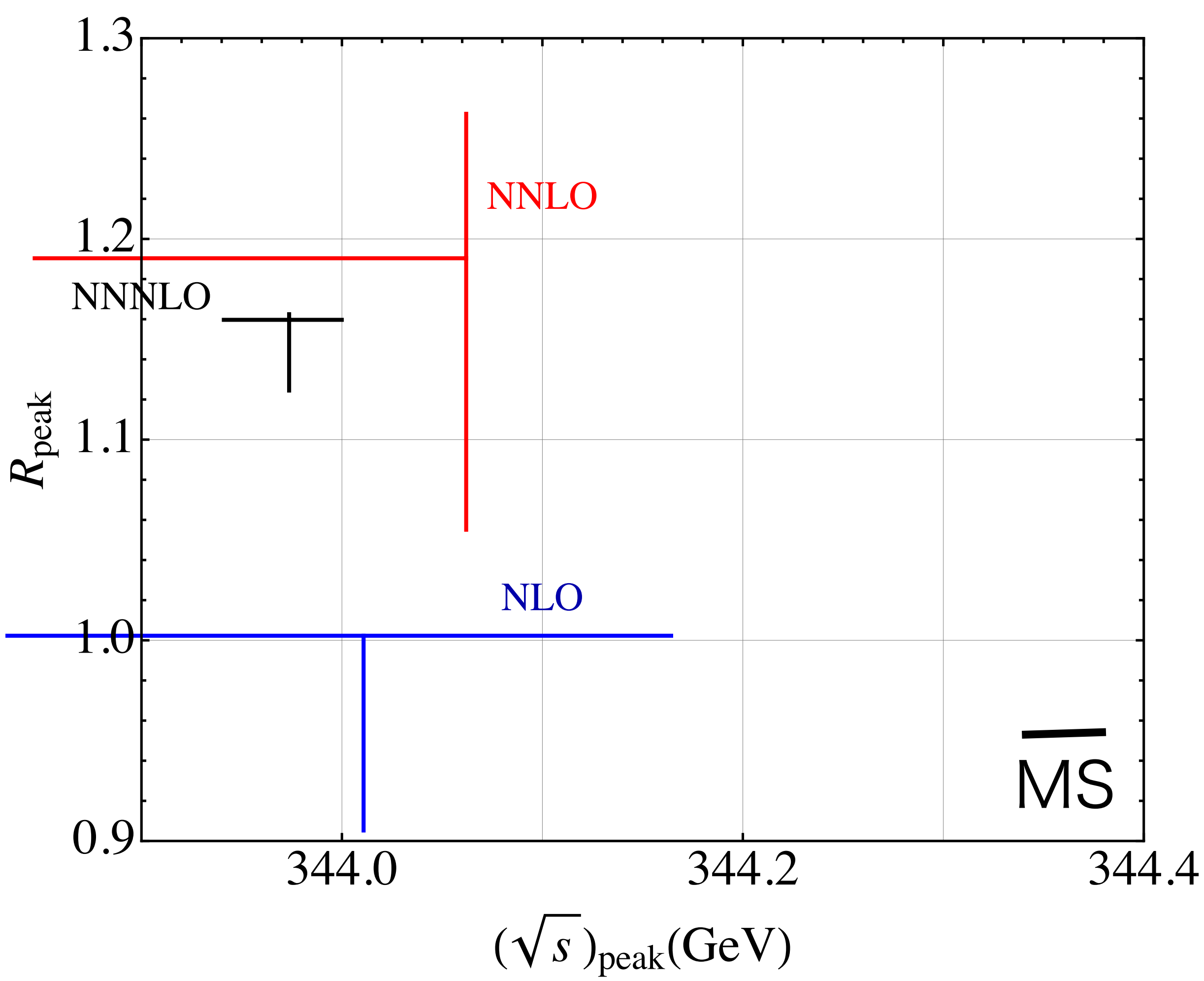




MSbar scheme NNNLO

- uncertainty band due to μ variation is about half or smaller at the peak and below peak position.
- no improvement above peak





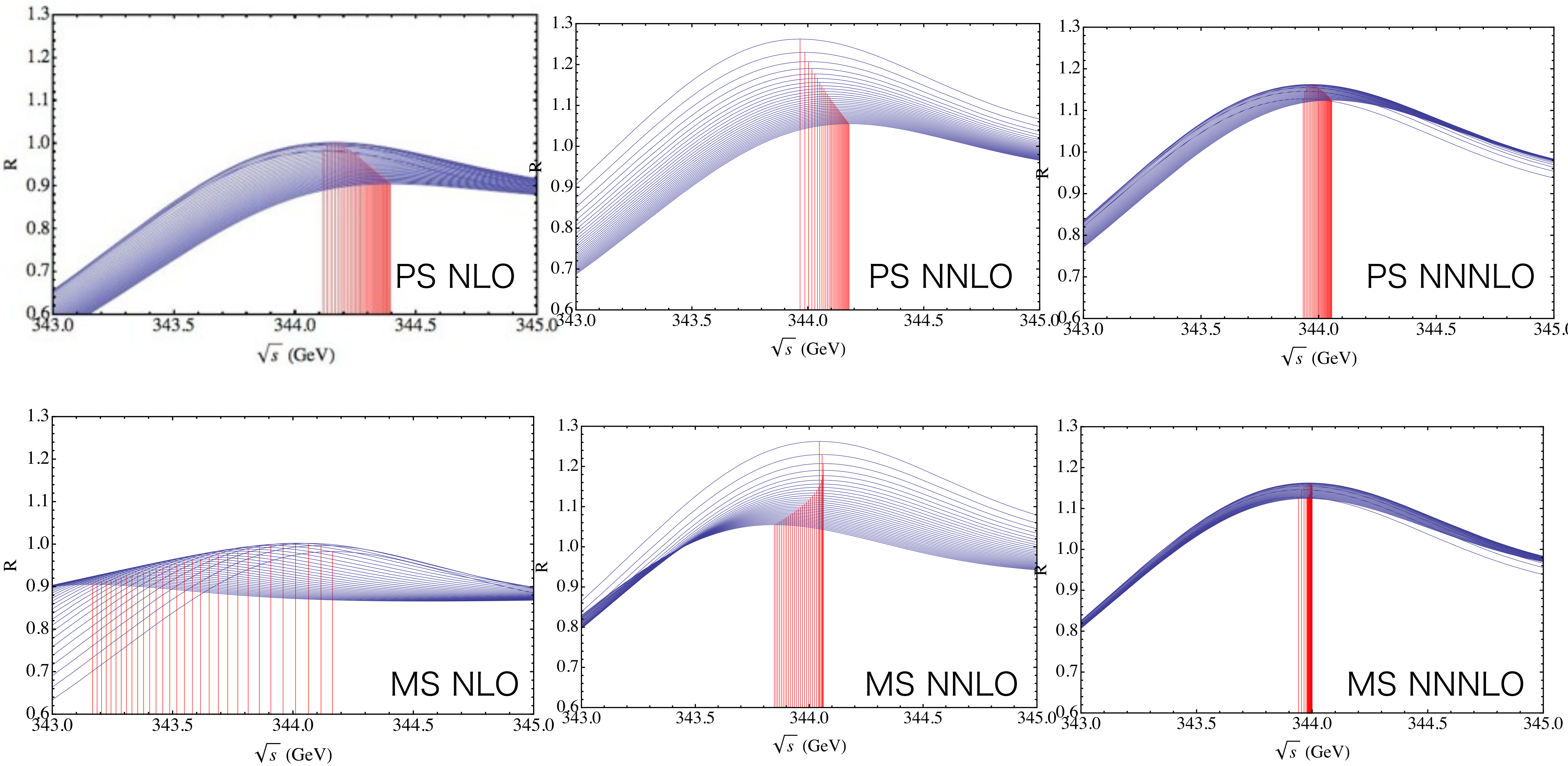
Peak (MSbar scheme) at NNNLO

- uncertainty due to μ for height of R is same with PS, but peak position is stable and uncertainty band get reduced by about a factor 2.

XS near peak

$$m_{\mathrm{PS}} = 173\mathrm{GeV}$$

$$m_{\overline{\mathrm{MS}}} = 163.3\mathrm{GeV}$$



Conclusion

- precision top mass measurement investigated based on NNNLO threshold cross section

$$\delta\sqrt{s}_{peak} \sim \pm 50\mathrm{MeV}$$
$$\delta R_{peak} \sim \pm 3\%$$

in PS scheme

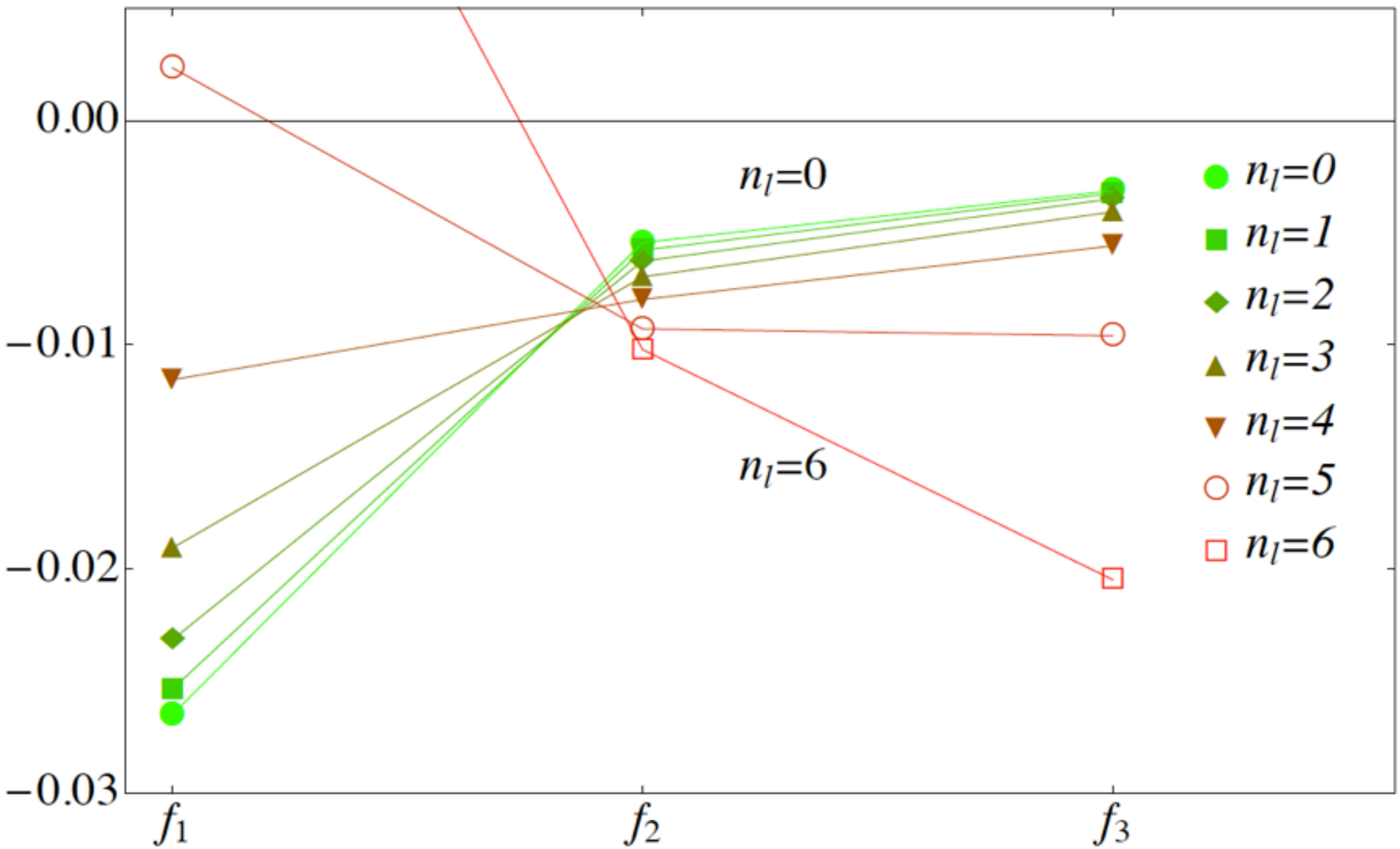
- Direct extraction of $\overline{\mathrm{MS}}$ mass suggested

$$\delta\sqrt{s}_{peak} \sim \pm 30\mathrm{MeV}$$

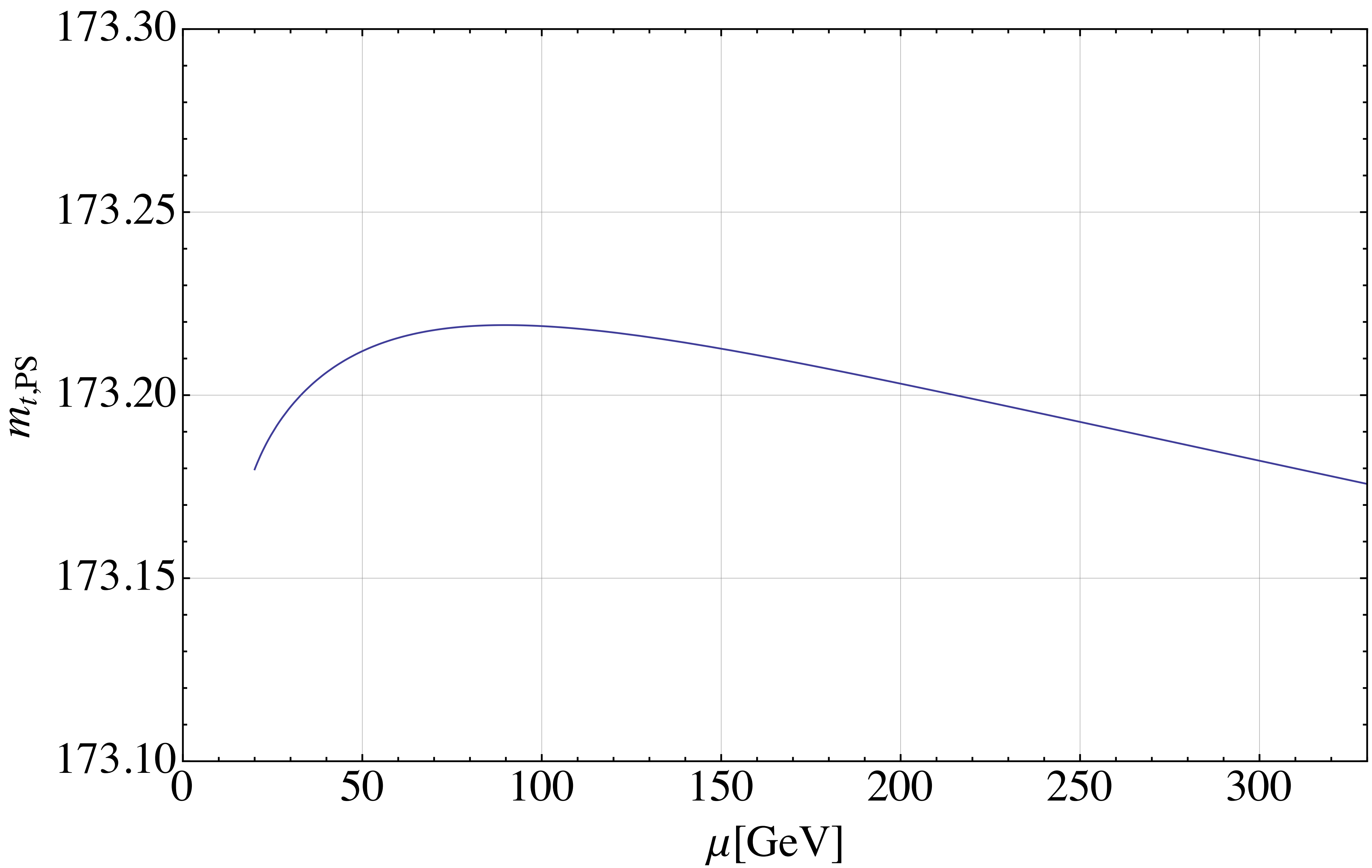
in $\overline{\mathrm{MS}}$ scheme

- QCD coupling should be known better than $\delta\alpha_s = \pm 0.0006$ for direct MSbar determination

Backup


0.00
−0.01
−0.02
−0.03
f_1
f_2
f_3
n_l=0
n_l=6
n_l=0
n_l=1
n_l=2
n_l=3
n_l=4
n_l=5
n_l=6

MS > PS


173.30
173.25
173.20
173.15
173.10
$m_{t,\mathrm{PS}}$
0
50
100
150
200
250
300
μ[GeV]