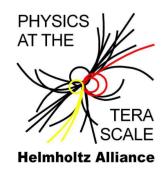


Track reconstruction for InGrid

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Outline:

- Goals
- RANSAC
- Super Pixel
- Linear regression
- Hough Transform
- Conclusion and outlook

Goals:

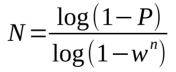
- Try to find an algorithm for track finding for pixel TPC
- Use the unit of the chip as a segment of the track calling **Tracklet**
- Find the tracklets and then using Kalman filter or General Broken Lines for connecting candidate tracklets



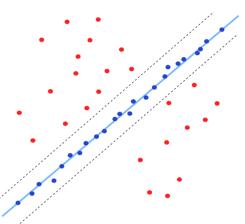
RANSAC: Introduction

Random Sample Consensus (RANSAC)

- Iterative method for estimating parameters of a model from a set of data containing outliers e.g. noise
- Model given by physics case e.g. straight line
- RANSAC tests this model by randomly choosing points from data set
- Number n of randomly chosen points depends on the model, n = 2 for straight line
- Repeat procedure N times and keep all solutions fulfilling threshold
- Inliers: all points with distance to line fit smaller than tolerance
- Tolerance: width of the distribution of the hits from the model
- Threshold: fraction of inliers greater than given value W
- W: number of inliers / number of points in data set
- Consensus: the set of inliers for the fitting model
- Finding the best fit based on the χ^2



P: probability of finding solution with N iterations (99%)



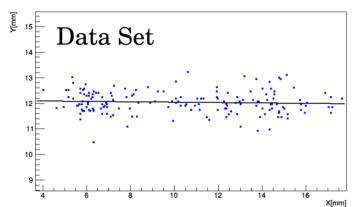
By Msm - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=2071406

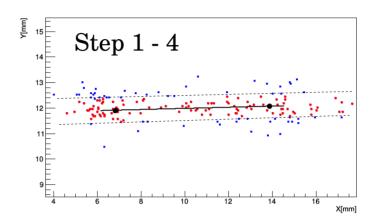
W	N			
0.1	458			
0.2	112			
0.3	48			
0.4	26			
0.5	16			
0.6	10			
0.7	6			
0.8	4			
0.9	2			
1	0			



RANSAC: MarlinTPC

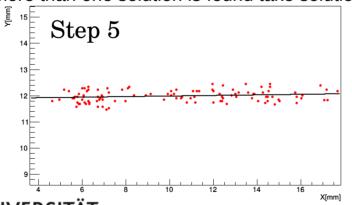
- 1) Choose two random hits
- 2) Define straight line based on these two hits
- 3) Calculate the distance of all other hits to the line

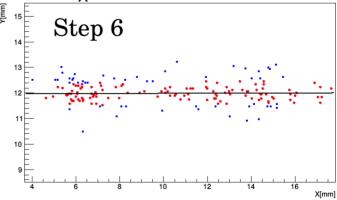




- 4) Collect those points with the distance less than the tolerance
- 5) If there are enough points in collection then a solution is found. Fit the line using all points in collection.
- 6) Recalculate inliers and outliers points based on this fit and then refit again based on new inlier points.
- 7) Repeat step 1 to 6 N times.

8) If more than one solution is found take solution with smallest χ^2 .







RANSAC: MarlinTPC

Optimization:



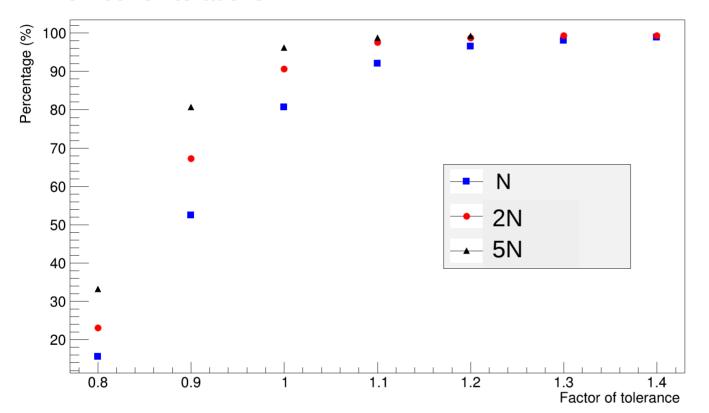
- Simulation of 1000 Events:
 - One InGrid Chip
 - 20 mm distance of the track from the readout plane
 - Parallel to the x axis from Y = 12 mm
 - The straight line model $Y = P \cdot 0 + P \cdot 1 \cdot X$
 - W= 0.6 => N=10
- Use width of the track as the tolerance (In this case ~0.44 mm)
- Width of the track is calculated based on the distance of the track from the readout plane and diffusion coefficient.
- Analysis done for different scaling factors of width of the track (0.8,0.9,...,1.4)
- Results: Distribution of P0 and P1 for all 1000 events
- For few events solution is found using scaling factor of 0.8 and for around 98% of events solution is found for scaling factor 1.4 with standard N iteration.
- In order to find the optimal tolerance, the previous step has been repeated for two larger number of Iterations.



RANSAC: Number of Iteration



Percentage of events with at least one solution for different number of iterations



$$N = \frac{\log(1-P)}{\log(1-w^n)}$$

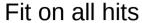
Expected percentage is 99%

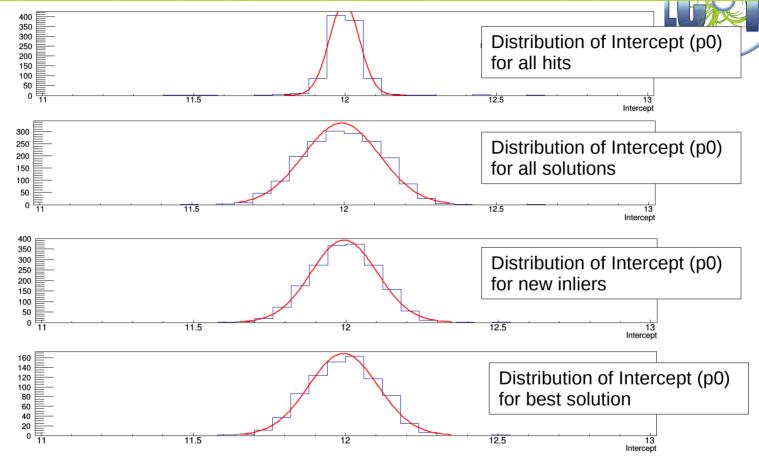
1.0	# candidate (%)		
N = 10	80.7		
2N	90.6		
5N	96.2		

- Keep the tolerance small in order to have better two track resolution
- In our case N iterations are not sufficient to reach optimal percentage.



RANSAC: Result for scaling factor of 1.0





	Nominal value [mm]	Mean value all solutions [mm]	Sigma all solutions [mm]	Number of solutions	Mean value best solution [mm]	Sigma best solution [mm]	Number of Best solution
р0	12.0	11.99 +/- 0.00	0.125 +/- 0.002	1781	11.99 +/- 0.00	0.114 +/- 0.003	807
p1	0.0	0.000054 +/- 0.007	0.027 +/- 0.0005	1781	0.00033 +/- 0.0010	0.028 +/- 0.0008	807

Super Pixel: Introduction



SuperPixel:

- Offered by Claus Kleinwort
- Each Super pixel includes several pixels.
 - a=2ⁿ Number of pixels in each row and column
 - w=256/a w² Number of super pixel for each chip
- Collect all hits inside of the each super pixels → calculate center of gravity → SuperHit
- CellID0 → SuperPixel number
- CellID1 → Chip number
- Covarinace Matrix → Calculated based on the super pixel
- Edep → number of hits in each SuperHit

Tracklet finding:

• Linear regression:
$$Y = X \beta + \epsilon \longrightarrow \beta = (X^T D^{-1} X)^{-1} X^T D^{-1} Y$$

$$\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} \longrightarrow \text{Intersect}$$
Slope

• Hough Transform:
$$d_0 = y \cos(\varphi) - x \sin(\varphi)$$



Linear regression: Covariance matrix



Covariance Matrix for a Super pixel:

Covariance matrix for all hits in each super pixel for x,y,z

$$V_{ij} \equiv cov(X_i, X_j) \equiv \langle (X_i - \bar{X}_i)(X_j - \bar{X}_j) \rangle$$

Spacial Transverse resolution for super hits

$$\sigma_D^2 = \left(a \cdot \frac{LPix}{\sqrt{12}}\right)^2 + \frac{ZD_T^2}{N}$$

Lpix: length of a pixel (0.055 mm)

Spacial Longitudinal resolution for super hits

$$\sigma_L^2 = 16 + \frac{(T_c v_d)^2}{12} + \frac{ZD_L^2}{N}$$

a: number of pixel in each row of a super pixel

N: number of hits

$$T_c = 1/f_c$$
, $f_c = 40 \text{ MHz}$

Covariance matrix for super hit

$$G = (V_{ij} + \sigma_D^2 + \sigma_L^2)$$

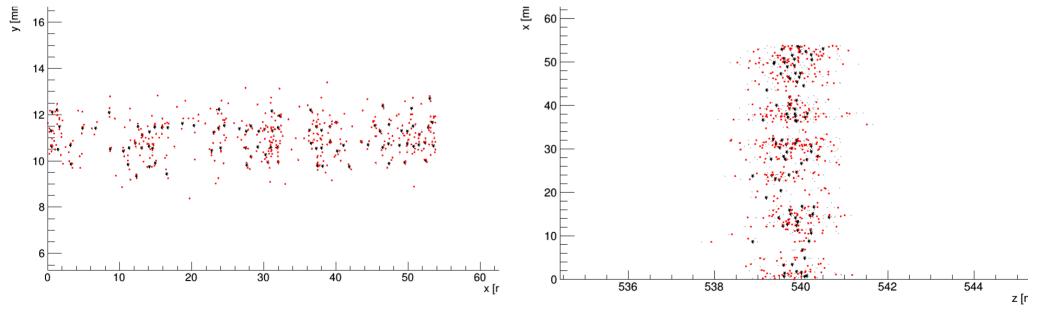
G: 3x3 matrix



Super Pixel: Simulation

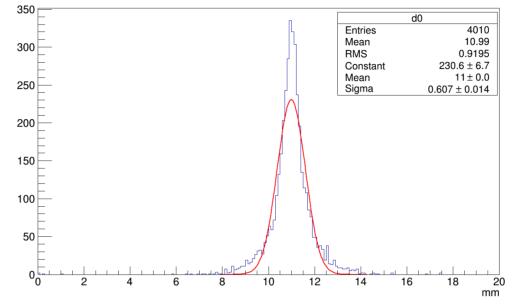
Simulation 1000 Events:

- Y0 = 12 mm, parallel to X-axis
- Z = 60 mm from the readout plane
- Weighted Least Square (XY- Plane): $G = V_{22} + \sigma_D^2$
- Weighted Least Square (ZX- Plane): $G = V_{33} + \sigma_L^2$
- 16 x 16 pixels in each super pixel

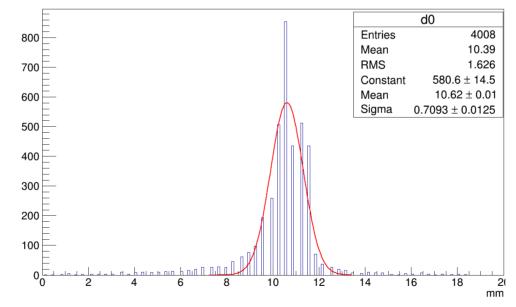




d0 from linear regression



d0 from Hough Transform

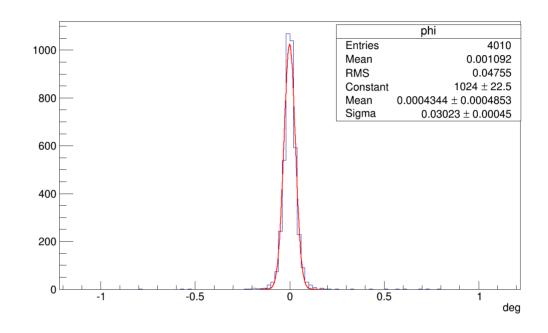




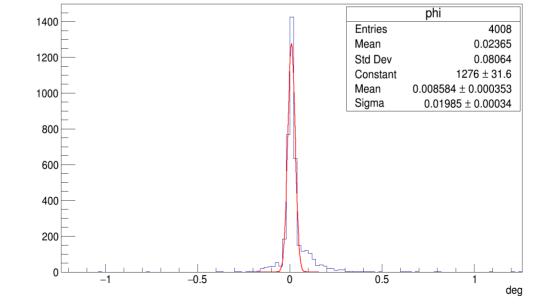
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φ from linear regression

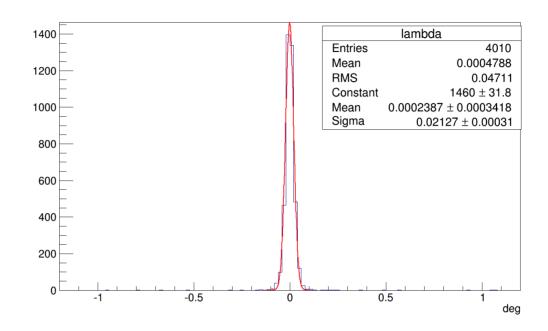


φ from Hough Transform

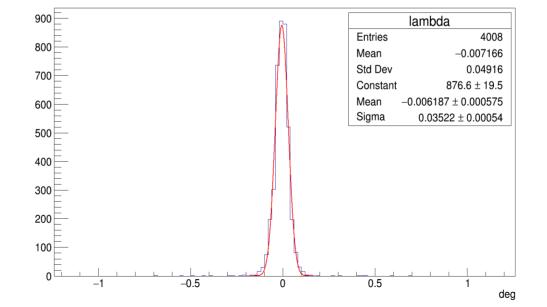




λ from linear regression

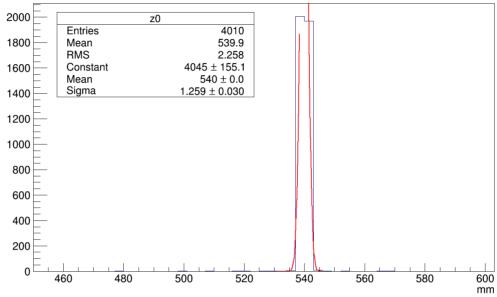


λ from Hough Transform

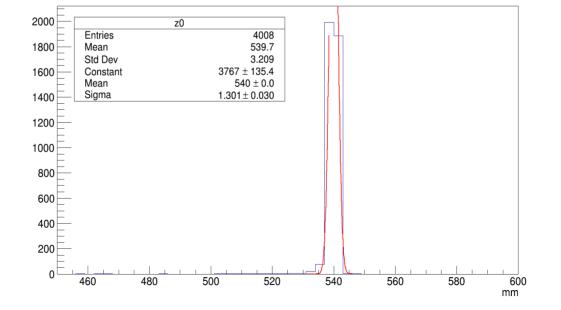




z0 from linear regression

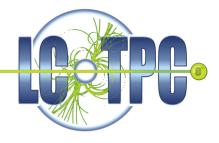


z0 from Hough Transform





Summary & Outlook



Conclusion:

- RANSAC for many chips is slow.
- Linear regression needs to know direction of the track. For ϕ > 45° should be interchanged.

Outlook:

- Find the direction of a track with Hough Transform and then using linear regression
- Use RANSAC for super pixel → Less number of iteration
- Find inliers and outliers to remove noise
- Simulation of 2 tracks with various distances from each other
- Use real data from test beam using up to 160 chips

