Effective-Lagrangian approach to WW production

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Outline

Basic Idea: Usually, one introduces non-standard gauge-boson interactions by general form factors for every single vertex after EWSB.
Here, add terms satisfying certain symmetries to $\mathcal{L}_{\mathrm{SM}}$ before EWSB.
Advantage: less general but much less parameters and a consistent analysis taking constraints from $e^+e^- \to WW$, $\gamma\gamma \to WW$ and high-precision observ. into account

- 1. The effective Lagrangian and the anomalous couplings
  - Limit on the couplings from EW precision data
  - Relation to the "usual" anomalous couplings
- 2. The process $\gamma\gamma \to WW$
  - Anomalous contributions
  - Total cross sections
- 3. Reachable sensitivities at an ILC
  - Optimal observables
  - Sensitivity in the various modes and comparison to the current limits

1. The effective Lagrangian and the anomalous couplings

- New physics effects at scales $\Lambda \gg v$ parameterised by an effective Lagrangian
  - EW scale set by vacuum expectation value $v \approx 246$ GeV of the Higgs field
- Huge amount of possibility for such Lagrangian. Used restrictions:
  - Only SM gauge-boson and Higgs fields contribute
  - Terms should be invariant under the SM gauge group SU(3) × SU(2) × U(1)
  - Operators up to dimension-6
- ⇒ 10 non-SM terms [W. Buchmüller and D. Wyler, Nucl. Phys. B **268** (1986) 621.]:

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \mathcal{L}_2 \,, \qquad \widetilde{W}^i_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} W^{i\,\rho\sigma}$$

$$\mathcal{L}_2 = \Big(h_W O_W + h_{\tilde{W}} O_{\tilde{W}} + h_{\varphi W} O_{\varphi W} + h_{\varphi \tilde{W}} O_{\varphi \tilde{W}} + h_{\varphi B} O_{\varphi B} + h_{\varphi \tilde{B}} O_{\varphi \tilde{B}} + h_{WB} O_{WB} + h_{\tilde{W}B} O_{\tilde{W}B} + h^{(1)}_{\varphi} O^{(1)}_{\varphi} + h^{(3)}_{\varphi} O^{(3)}_{\varphi} \Big) / v^2$$

$$e.g.: \mathcal{O}_{\tilde{W}} = \epsilon^{ijk} \widetilde{W}^{i\,\nu}_{\mu} W^{j\,\lambda}_{\nu} W^{k\,\mu}_{\lambda} \,, \qquad O_{WB} = \left(\varphi^\dagger \tau^i \varphi\right) W^i_{\mu\nu} B^{\mu\nu} \,,$$

Diagonalisation and EW parameters

- Additional terms in the Lagrangian ⇒ (primed) electroweak SM-parameter change
  - Gauge-boson part of the Lagrangian (after EWSB)

$$\mathcal{L}_V^{(2)} = -\tfrac{1}{4}\begin{pmatrix} Z'_{\mu\nu} & A'_{\mu\nu} \end{pmatrix} T' \begin{pmatrix} Z'_{\mu\nu} \\ A'_{\mu\nu} \end{pmatrix} + \tfrac{1}{2}\, m'^{\,2}_Z \left(1 + \tfrac{1}{2}(h_\varphi^{(1)} + h_\varphi^{(3)})\right) Z'_\mu Z'^{\,\mu}$$

$$\mathcal{L}_Z^{(2)} = -\tfrac{1}{2}(1 - h_{\varphi W}) W'^{+}_{\mu\nu} W'^{+\,\mu\nu} + m'^{\,2}_W (1 + h_\varphi^{(1)}) W'^{+}_\mu W'^{-\,\mu}$$

  - **non-diagonal** kinetic matrix ⇒ transformation of the fields required

$$T' = \begin{pmatrix} a & b \\ b & d \end{pmatrix} \qquad a = 1 + \mathcal{O}(h)\,, \quad b = \mathcal{O}(h) \quad d = 1 + \mathcal{O}(h)\,.$$

  - Re-definition of fields and parameters ⇒ physical (unprimed) fields and param.

$$\mathcal{L}^{(2)} = -\tfrac{1}{4}(Z_{\mu\nu} Z^{\mu\nu} + A_{\mu\nu} A^{\mu\nu}) - \tfrac{1}{2} W^{+}_{\mu\nu} W^{-\,\mu\nu} + \tfrac{1}{2}\, m_Z^2 Z_\mu Z^\mu + m_W^2 W^{+}_\mu W^{-\,\mu}$$

$$m_W^2 = \frac{m'^{\,2}_W}{1 - h_{\varphi W}}\,, \qquad m_Z^2 = \frac{d}{ad - b^2} m'^{\,2}_Z$$

Limits on couplings from EW precision data

- Re-defined fields $A_\mu, Z_\mu, W_\mu^\pm$ will also enter the interaction SM Lagrangian
  $\Rightarrow$ anomalous contribution to $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}, \ldots$

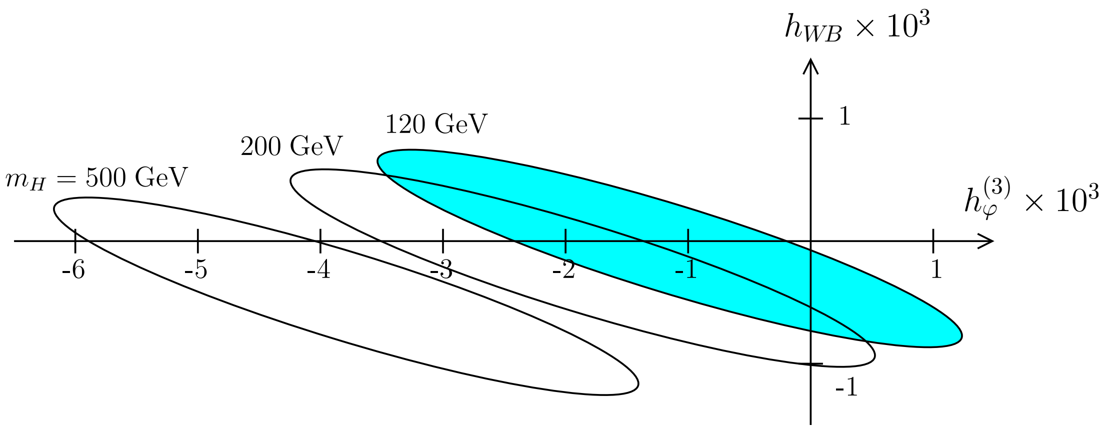
- In the end many EW observables will depend on h_{WB} and $h_\varphi^{(3)}$

$$m_W, \Gamma_W, \sigma^0_{\mathrm{had}}, \ldots$$

- Using **measured** and **SM** value (including higher corrections)

– $\Rightarrow$ expectation values for $m_H = 120$ GeV:

$$h_W = -6.2 \pm 3.6 \times 10^{-2}$$
$$h_{WB} = -0.6 \pm 7.9 \times 10^{-4}$$
$$h_\varphi^{(3)} = -1.2 \pm 2.4 \times 10^{-3}$$



Anam. gauge-boson couplings in $e^+e^- \to WW$ and $\gamma\gamma \to WW$

- Usually, introd. via form factor e.g. $(V = \gamma, Z g_{\gamma WW} = -e, g_{ZWW} = -e\cot\theta_W)$:
  [K. Hagiwara, R. D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. B **282** (1987) 253.]

$$\frac{\mathcal{L}_{VWW}}{i\, g_{VWW}} = g_1^V (W^+_{\mu\nu} W^{-\,\mu} - W^-_{\mu\nu} W^{+\,\mu}) V^\mu + \kappa_V W^+_\mu W^-_\nu V^{\mu\nu} + \dots ,$$

  - In SM: $g_1^V = \kappa_V = 1$, $\lambda_V = g_4^V = g_5^V = \widetilde{\kappa}_V = \widetilde{\lambda}_V = 0$
  - Advantage: very general (only Lorentz invariance and hermiticity required)
  - Disadvantage: many parameters 14 just for the TGCs,
    In comp.: 10param. in eff.-$\mathcal{L}$ approach to describe $e^+e^- \to WW, \gamma\gamma \to WW \dots$

- Identify in the effective $\mathcal{L}$ after EWSB the anomalous TGCs

$$\Delta g_1^\gamma = 0\,, \quad \Delta g_1^Z = \frac{s_0}{c_0(s_0^2 - c_0)} h_{WB} + \frac{h_\varphi^{(3)}}{4(s_0^2 - c_0^2)}\,, \quad s_0^2 = \frac{1}{2}\left(1 - \sqrt{1 - \frac{e^2}{\sqrt{2} G_F m_Z^2}}\right)\,,$$

$$\Delta\kappa_\gamma = \frac{c_0}{s_0} h_{WB}\,, \qquad \Delta\kappa_Z = \frac{2 s_0 c_0}{s_0^2 - c_0} h_{WB} + \frac{h_\varphi^{(3)}}{4(s_0^2 - c_0^2)}\,, \qquad c_0^2 = 1 - s_0^2\,,$$

$$\lambda_{\gamma,Z} \propto h_W\,, \qquad \widetilde{\kappa}_{\gamma,Z} \propto h_{\widetilde{W}B}\,, \quad \widetilde{\lambda}_{\gamma,Z} \propto h_{\widetilde{W}}\,, \qquad g_4^{\gamma,Z} = g_5^{\gamma,Z} = 0\,.$$

- 14 (complex) couplings described by 5 real parameters $\Rightarrow$
  constraints on h_i from LEP 2 measurement of $\Delta g_1^Z, \Delta\kappa_\gamma, \lambda_\gamma, \widetilde{\lambda}_Z$ and $\widetilde{\kappa}_Z$

2. The process $\gamma\gamma \to WW$

- Measurement of h_i using the **unpolarised** process

$$\gamma\gamma \to W^+W^- \to (f_1\bar{f}_2)(f_3\bar{f}_4)$$

  - Not completely realistic since one can produce polarised photons
  - Angular distrib. of f's essential for reconstruction of spin dependence of W's

- Unpolarised differential cross section

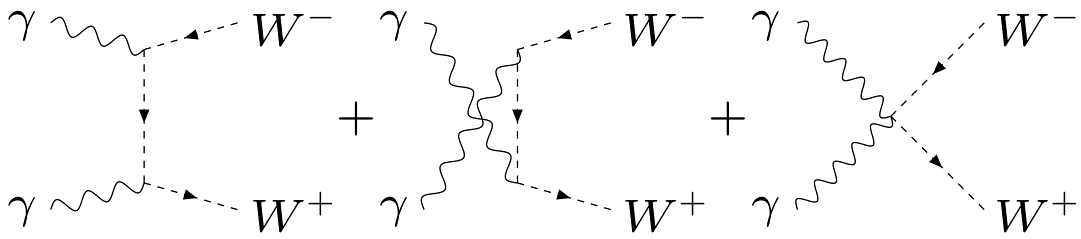
$$\frac{\mathrm{d}\sigma}{\mathrm{dcos}\,\Theta\;\mathrm{dcos}\,\vartheta\;\mathrm{d}\varphi\;\mathrm{dcos}\,\overline{\vartheta}\;\mathrm{d}\overline{\varphi}} = \frac{3\beta}{2^{13}\pi^3 s}\,B_{12}\,B_{34}\,\mathcal{P}^{\lambda_3\lambda_4}_{\lambda_3'\lambda_4'}\,\mathcal{D}^{\lambda_3}_{\lambda_3'}\,\overline{\mathcal{D}}^{\lambda_4}_{\lambda_4'}\,.$$

  - Θ: angle between W^- and beam pipe in $\gamma\gamma$ rest frame
  - $\varphi, \vartheta(\overline{\varphi}, \overline{\vartheta})$: angles describe direction of f_1 ($\bar{f}_4$) in W^- (W^+) rest frame
  - $\mathcal{P}^{\lambda_3\lambda_4}_{\lambda_3'\lambda_4'}(\Theta) = \sum_{\lambda_1,\lambda_2}\mathcal{M}(\lambda_1,\lambda_2;\lambda_3,\lambda_4)\mathcal{M}^*(\lambda_1,\lambda_2;\lambda_3',\lambda_4')$: W-prod. tensor
  - $\mathcal{M}(\lambda_1,\lambda_2;\lambda_3,\lambda_4)$: **Helicity amplitudes** for W-pair production
  - B_{ij} branching ratios for W decay

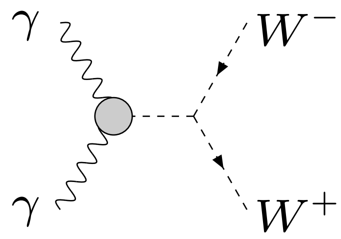
Anomalous contributions

- Anomalous contributions to the helicity amplitudes $\mathcal{M} = \mathcal{M}_{\mathsf{SM}} + \sum_i h_i \mathcal{M}_i$

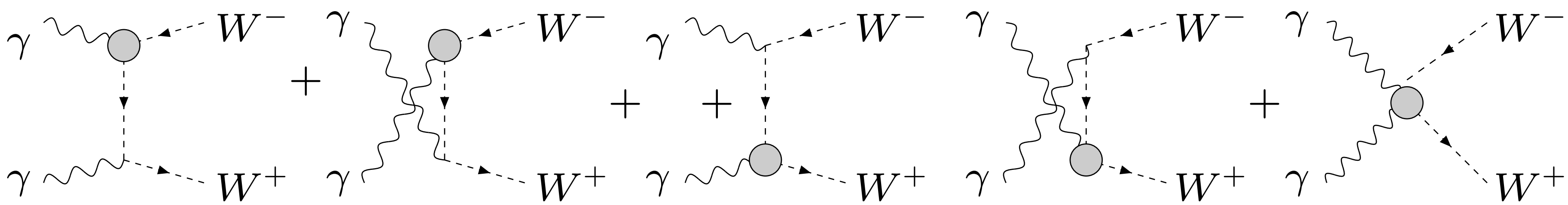
SM diagrams



Higgs exchange



anomalous triple and quartic gauge couplings



- Other vertices contribute on tree level as in $e^+e^- \to WW$
  - Gauge-boson couplings: γWW and $\gamma\gamma WW$ instead of γWW and ZWW
  - Only in $\gamma\gamma \to WW$ an anom. Higgs coupling $\gamma\gamma H$ contribute

Measurable couplings h_i in $\gamma\gamma \to WW$

	SM	h_W	$h_{\tilde{W}}$	$h_{\varphi W}$	$h_{\varphi \tilde{W}}$	$h_{\varphi B}$	$h_{\varphi \tilde{B}}$	h_{WB}	$h_{\tilde{W}B}$	$h_\varphi^{(1)}$	$h_\varphi^{(3)}$
γWW	√	√	√					√	√		
ZWW	√	√	√					√	√		P_Z
$\gamma\gamma WW$	√	√	√								
$\gamma\gamma H$				√	√	√	√	√	√		

- $\Rightarrow h_W$, $h_{\tilde{W}}$, h_{WB} and $h_{\tilde{W}B}$ measurable in $e^+e^- \to WW$ and $\gamma\gamma \to WW$
- Couplings $h_{\varphi W}$, $h_{\varphi B}$, $h_{\varphi \tilde{W}}$ and $h_{\varphi \tilde{B}}$ only measurable in $\gamma\gamma \to WW$
- But some anomalous contribution related in a trivial way

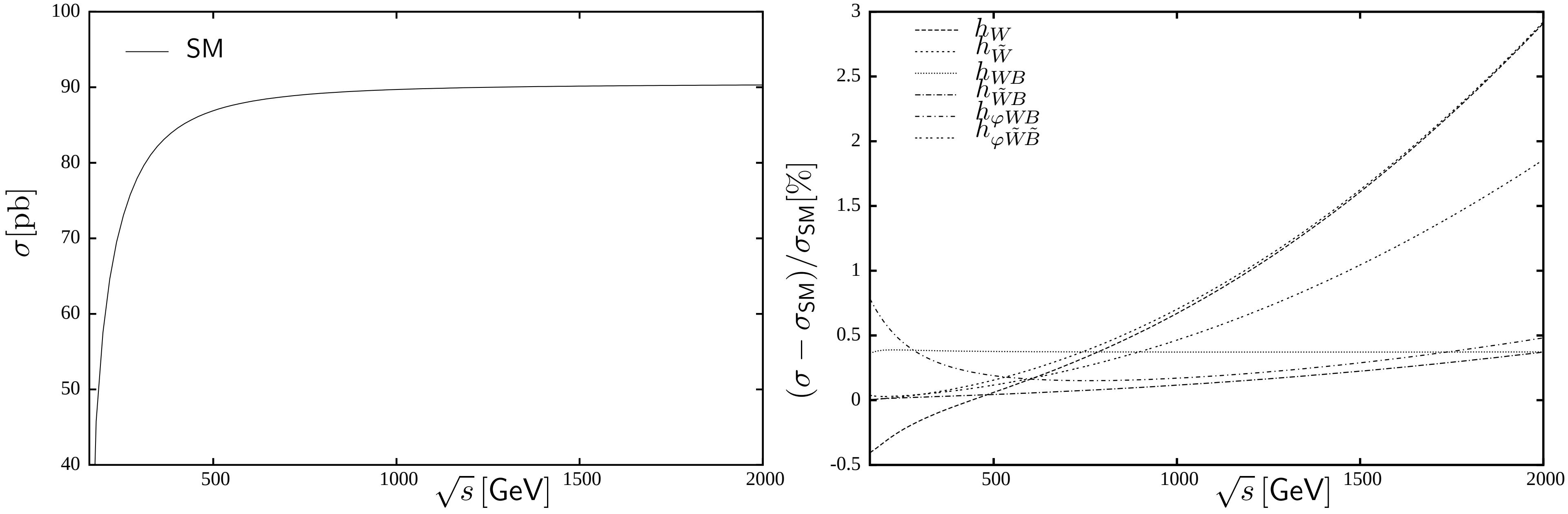
$$s_1^2 \mathcal{M}_{\varphi B} = c_1^2 \mathcal{M}_{\varphi W}\,, \quad s_1^2 \mathcal{M}_{\varphi \tilde{B}} = c_1^2 \mathcal{M}_{\varphi \tilde{W}}\,, \quad s_1^2 = \frac{e^2}{4\sqrt{2} G_F m_W^2}\,, \quad c_1^2 = 1 - s_1^2$$

$\Rightarrow$ only one CP-cons. and one CP-viol. combination is measurable

$$h_{\varphi WB} \equiv s_1^2 h_{\varphi W} + c_1^2 h_{\varphi B} \quad h_{\varphi \tilde{W}\tilde{B}} \equiv s_1^2 h_{\varphi \tilde{W}} + c_1^2 h_{\varphi \tilde{B}}$$

- In addition: $h_\varphi^{(3)}$ measurable through electroweak precision data

Total cross section for $\gamma\gamma \to WW$



- Left hand side: LO SM contribution to $\gamma\gamma \to WW$
- Right hand side: anom. corrections: $h_{WB} = 10^{-3}$ all other coupl. $h_i = 10^{-2}$
  - Anomalous couplings leads to a cross section growing with s
  - For realistic energies only relevant correction related to h_{WB}
    (Note, recent constraints from LEP of the order 10^{-2}
  - Hence measurement of total cross section only helpful for constraining h_{WB}

3. Reachable sensitivities at an ILC

- $\Rightarrow$ **Differential** information are needed; the **optimal observable** $\mathcal{O}_i$ is:

$$\mathcal{O}_i = \frac{S_i(\phi)}{S_0(\phi)}, \quad \text{where} \quad \frac{\mathrm{d}\sigma}{\mathrm{d}\phi} = S_0(\phi) + \sum_i h_i S_i(\phi)$$

  - "optimal" means: for $h_i \to 0$ errors on h_i are minimal using expectation values

$$\langle \mathcal{O}_i \rangle = \frac{1}{\sigma_0} \int \mathrm{d}\phi \, S_0 \mathcal{O}_i = \langle \mathcal{O}_i \rangle|_{h_j=0} + \sum_j c_{ij} h_j + \mathcal{O}(h^2)$$

- For this observ.: covariance matrix V, sensit. δh_i and correl. matrix W read:

$$V = \frac{1}{N} c^{-1} \quad \delta h_i = \sqrt{V_{ii}} \quad W_{ij} = \frac{V_{ij}}{\sqrt{V_{ii} V_{jj}}}$$

$$c_{ij} = \frac{1}{\sigma_0} \int \mathrm{d}\phi \, (S_i(\phi) \, S_j(\phi))/S_0(\phi) - \frac{1}{\sigma_0^2} \int \mathrm{d}\phi \, S_i(\phi) \int \mathrm{d}\phi \, S_j(\phi)$$

  - N number of events: $\delta h_i \propto 1/\sqrt{N}$
  - Normalisation of $\mathrm{d}\sigma/\mathrm{d}\phi$ does not enter

Details of the analysis

- Some issues we cannot discuss in detail
  - Photon energy is not fixed; we use a Compton spectrum
  - $\Rightarrow$ set of variables ϕ in $\mathrm{d}\sigma/\mathrm{d}\phi$ involves $\phi \equiv (\Theta, \varphi, \vartheta, \overline{\varphi}, \overline{\vartheta}, E_1, E_2)$
  - Only semi-leptonic decays of the W pairs in our analysis
  - In these events two types of two-fold ambiguities
    - Hadronic decay: charge of the jets cannot be identified
    - Leptonic decay: neutrino energy not measurable $\Rightarrow$ ambiguity in $\sqrt{s_{\gamma\gamma}}$
  - Ambiguities $\Rightarrow$ we have to introduce a new set of physical parameters ξ
    $\Rightarrow$ due to change of variables $\xi \leftrightarrow \phi$ integrals $\int \mathrm{d}\phi \, S(\phi) \ldots$ more involved
- **Results** $\Rightarrow$

Reachable sensitivities at a $\gamma\gamma$ collider for fixed $\sqrt{s_{\gamma\gamma}}$ and Compton spectrum

CP-conserving couplings					CP-violating couplings				
500 GeV									
h	δh $\times 10^3$	$W(h)$			h	δh $\times 10^3$	$W(h)$		
		h_W	h_{WB}	$h_{\varphi WB}$			$h_{\tilde{W}}$	$h_{\tilde{W}B}$	$h_{\varphi \tilde{W}\tilde{B}}$
h_W	0.36	1	0.519	-0.120	$h_{\tilde{W}}$	0.46	1	0.630	-0.238
h_{WB}	1.08	0.519	1	-0.299	$h_{\tilde{W}B}$	3.17	0.630	1	-0.550
$h_{\varphi WB}$	1.17	-0.120	-0.299	1	$h_{\varphi \tilde{W}\tilde{B}}$	1.01	-0.238	-0.550	1
800 GeV									
h	δh $\times 10^3$	$W(h)$			h	δh $\times 10^3$	$W(h)$		
		h_W	h_{WB}	$h_{\varphi WB}$			$h_{\tilde{W}}$	$h_{\tilde{W}B}$	$h_{\varphi \tilde{W}\tilde{B}}$
h_W	0.13	1	0.407	-0.256	$h_{\tilde{W}}$	0.17	1	0.553	-0.491
h_{WB}	0.60	0.407	1	-0.547	$h_{\tilde{W}B}$	2.64	0.553	1	-0.904
$h_{\varphi WB}$	0.74	-0.256	-0.547	1	$h_{\varphi \tilde{W}\tilde{B}}$	0.97	-0.491	-0.904	1
1500 GeV									
h	δh $\times 10^3$	$W(h)$			h	δh $\times 10^3$	$W(h)$		
		h_W	h_{WB}	$h_{\varphi WB}$			$h_{\tilde{W}}$	$h_{\tilde{W}B}$	$h_{\varphi \tilde{W}\tilde{B}}$
h_W	0.050	1	0.266	-0.232	$h_{\tilde{W}}$	0.073	1	0.519	-0.516
h_{WB}	0.40	0.266	1	-0.741	$h_{\tilde{W}B}$	3.38	0.519	1	-0.988
$h_{\varphi WB}$	0.44	-0.232	-0.741	1	$h_{\varphi \tilde{W}\tilde{B}}$	1.36	-0.516	-0.988	1
3000 GeV									
h	δh $\times 10^3$	$W(h)$			h	δh $\times 10^3$	$W(h)$		
		h_W	h_{WB}	$h_{\varphi WB}$			$h_{\tilde{W}}$	$h_{\tilde{W}B}$	$h_{\varphi \tilde{W}\tilde{B}}$
h_W	0.016	1	0.146	-0.146	$h_{\tilde{W}}$	0.030	1	0.679	-0.680
h_{WB}	0.23	0.146	1	-0.881	$h_{\tilde{W}B}$	6.81	0.679	1	-0.999
$h_{\varphi WB}$	0.22	-0.146	-0.881	1	$h_{\varphi \tilde{W}\tilde{B}}$	2.83	-0.680	-0.999	1

Comparison with $e^+e^- \to WW$ and recent bounds

	Constraints from LEP and SLD		Sensitivity at a LC					
			e^+e^- mode		$\gamma\gamma$ mode fixed $\sqrt{s_{\gamma\gamma}}$		$\gamma\gamma$ mode with CS	
	m_H [GeV]	h_i $\times 10^3$	$\sqrt{s_{ee}}$ [GeV]	δh_i $\times 10^3$	$\sqrt{s_{\gamma\gamma}}$ [GeV]	δh_i $\times 10^3$	$\sqrt{s_{ee}}$ [GeV]	δh_i $\times 10^3$
Measureable CP-conserving couplings								
h_W	-69 ± 39 Constraint from TGCs measurement at LEP 2		500 800 3000	0.28 0.12 0.018	400 640 1200 2400	0.23 0.083 0.033 0.011	500 800 1500 3000	0.36 0.13 0.050 0.016
h_{WB}	120 200 500	-0.06 ± 0.79 -0.22 ± 0.79 -0.45 ± 0.79	500 800 3000	0.32 0.16 0.015	400 640 1200 2400	0.89 0.50 0.32 0.18	500 800 1500 3000	1.08 0.60 0.40 0.23
$h_{\varphi WB}$	Does not contribute		Does not contribute		400 640 1200 2400	1.16 0.62 0.34 0.17	500 800 1500 3000	1.17 0.74 0.44 0.22
$h_{\varphi}^{(3)}$	120 200 500	-1.15 ± 2.39 -1.86 ± 2.39 -3.79 ± 2.39	500 800 3000	36.4 53.7 "∞"	Does not contribute			

- $h_{\varphi WB}$ only measurable in $\gamma\gamma \to WW$
- Precision rises with the energy
- h_{WB} already very well constrained $\Rightarrow$ best constraints from Giga-Z?

Comparison of the CP-violating couplings

	Constraints from LEP and SLD		Sensitivity at a LC: e^+e^- mode		Sensitivity at a LC: $\gamma\gamma$ mode fixed $\sqrt{s_{\gamma\gamma}}$		Sensitivity at a LC: $\gamma\gamma$ mode with CS	
	m_H [GeV]	h_i $\times 10^3$	$\sqrt{s_{ee}}$ [GeV]	δh_i $\times 10^3$	$\sqrt{s_{\gamma\gamma}}$ [GeV]	δh_i $\times 10^3$	$\sqrt{s_{ee}}$ [GeV]	δh_i $\times 10^3$
Measureable CP-violating couplings								
$h_{\tilde{W}}$		68 ± 81	500	0.28	400	0.31	500	0.46
		Constraint from	800	0.12	640	0.12	800	0.17
		TGCs measurement			1200	0.058	1500	0.073
		at LEP 2	3000	0.018	2400	0.015	3000	0.030
$h_{\tilde{W}B}$		33 ± 84	500	2.2	400	3.41	500	3.17
		Constraint from	800	1.4	640	3.29	800	2.64
		TGCs measurement			1200	6.58	1500	3.38
		at LEP 2	3000	0.77	2400	5.20	3000	6.81
$h_{\varphi\tilde{W}\tilde{B}}$		Does not contribute		Does not contribute	400	1.13	500	1.01
					640	1.26	800	0.97
					1200	2.69	1500	1.36
					2400	2.17	3000	2.83

- $h_{\varphi\tilde{W}\tilde{B}}$ only measurable in $\gamma\gamma \to WW$
- Precision comparable in $e^+e^- \to WW$ and $\gamma\gamma \to WW$
- Compton spectrum does not lead to a large reduction of sensitivity
- In comparison to LEP 2, sensitivity increase by $\mathcal{O}(100)$

Conclusion & Outlook

- The introduced $\mathcal{L}_{\text{eff}}$ describes the anomalous effects in $\gamma\gamma \to WW$, $e^+e^- \to WW$ and EW observables with few parameters (10) in a consistent way

- $h_{\varphi WB}$ and $h_{\varphi\tilde{W}\tilde{B}}$ are measurable only in the $\gamma\gamma$ mode

- $h_W, h_{WB}, h_{\tilde{W}}$ and $h_{\tilde{W}B}$ are measurable in both modes with a comparable precision

- EW high precision observables constrain h_{WB} quite well
  $\Rightarrow$ best constraints reachable in Giga-Z mode

- Calculation of the polarised case would be desirable