Four-fermion production at the $\gamma\gamma$ collider

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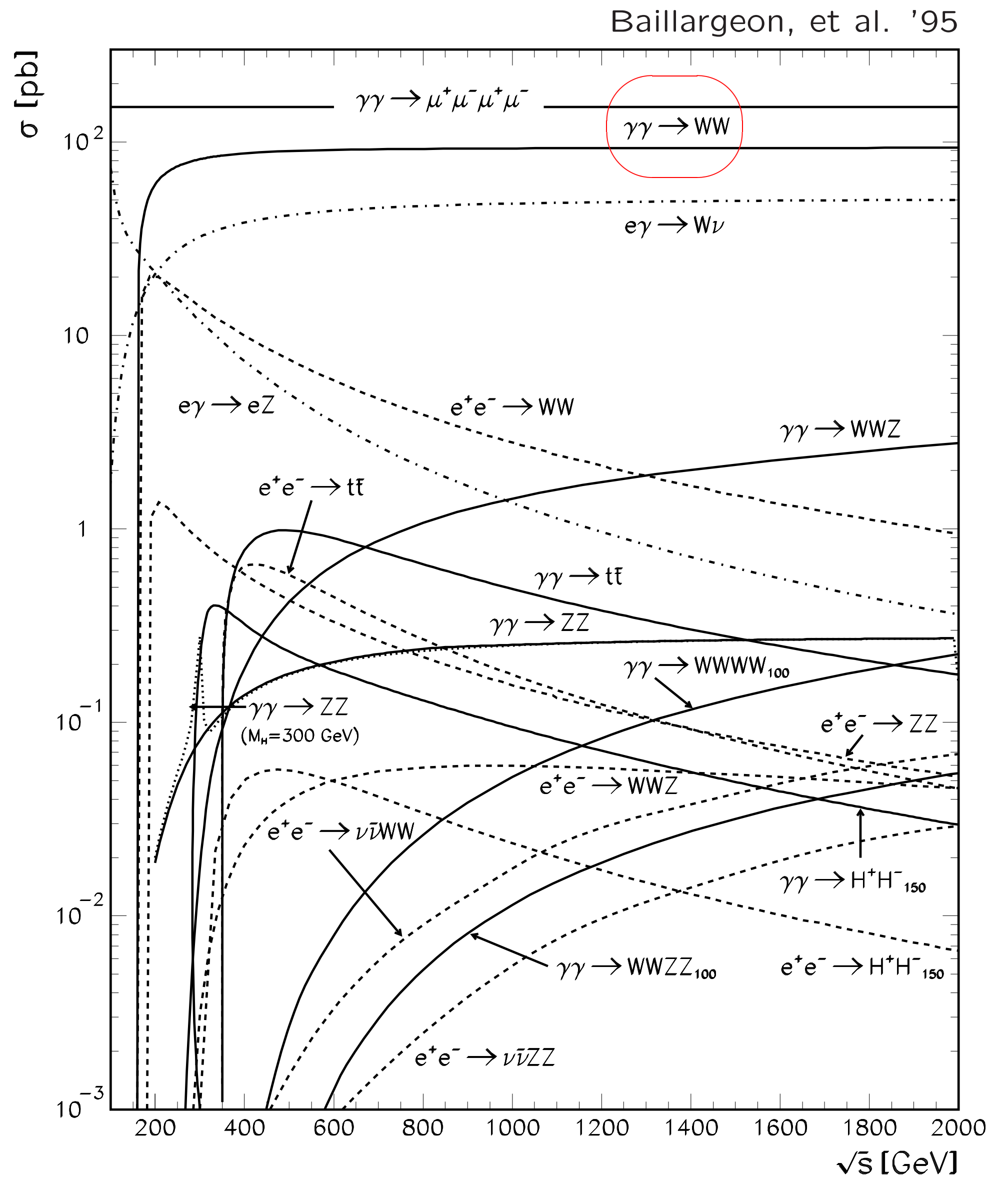
and Eur. Phys. J. C **44** (2005) 27 [arXiv:hep-ph/0506005]

Motivation

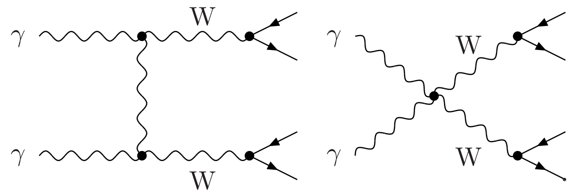
$\gamma\gamma \to \mathrm{WW}$

- one of the largest cross sections
- contains gauge-boson couplings $\gamma\mathrm{WW}$ and $\gamma\gamma\mathrm{WW}$, insight into EW sector of SM, limits on anomalous couplings
- s-channel Higgs resonance $\gamma\gamma \to \mathrm{H} \to \mathrm{WW}$ possible through loops of charged massive particles
- sensitive to extra dimensions



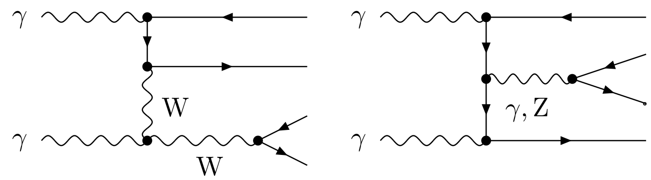
Motivation

W bosons are unstable $\Rightarrow$ $\gamma\gamma \to \mathrm{WW} \to 4f$ ("W-pair signal diagrams") $\mathcal{O}(1)$



experimental precision requires

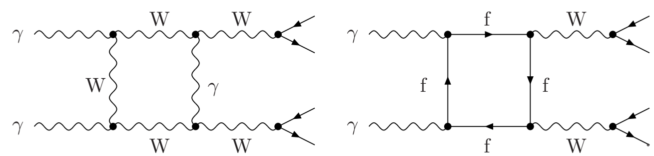
- inclusion of single and non-resonant diagrams ("background diagrams") in lowest order $\mathcal{O}(\Gamma_{\mathrm{W}}/M_{\mathrm{W}})$, $\mathcal{O}(\Gamma_{\mathrm{W}}/M_{\mathrm{W}})^2$



$\Rightarrow$ $\gamma\gamma \to 4f$ (Existing studies: Moretti '96; Baillargeon et al. '97; Boos, Ohl '97)

- inclusion of radiative corrections to signal diagrams

$\mathcal{O}(\alpha) \sim \mathcal{O}(\Gamma_{\mathrm{W}}/M_{\mathrm{W}})$

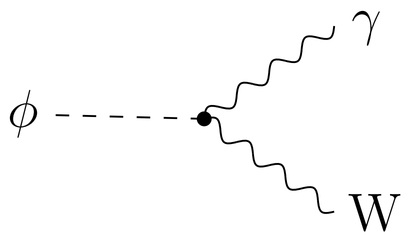


real corrections to $\gamma\gamma \to \mathrm{WW} \to 4f \Rightarrow \gamma\gamma \to 4f + \gamma$

first step: $\gamma\gamma \to 4f$ and $\gamma\gamma \to 4f + \gamma$ at tree level

second step: radiative corrections in double-pole approximation

Amplitudes for $\gamma\gamma \to 4f(\gamma)$

- Helicity amplitudes
- # diagrams: 6 ($\mathrm{e}^-\mathrm{e}^+\nu_\mu\bar{\nu}_\mu$) to 588 ($\mathrm{u}\bar{\mathrm{u}}\mathrm{d}\bar{\mathrm{d}} + \gamma$)
- Weyl–van-der-Waerden formalism
- Fermion masses neglected (mass effects restored in corrections)
- Non-linear gauge: 

vanishes
- Check against Madgraph (Stelzer, Long ’94)

Phase-space integration

Problem: rich peaking structure of integrand

RacoonWW
Denner,Dittmaier,Roth,Wackeroth '01

"importance sampling" : more points near peaks

$$\int \underbrace{dx}_{\downarrow} \, f(x) = \int dx\, g(x) \frac{f(x)}{g(x)} = \int dy\, \underbrace{\frac{f(x(y))}{g(x(y))}}$$

random numbers "weight" $\sim \mathrm{const}$

$\int_0^x d\bar{x}\, g(\bar{x}) = y(x)$: "mapping" $\rightarrow$ integrand flattened

many Feynman diagrams/propagators $\rightarrow$ "multi-channel" + adaptive optimization

Berends, Kleiss, Pittau '94 Kleiss, Pittau '94

one phase-space generator per diagram with appropriate "mapping"

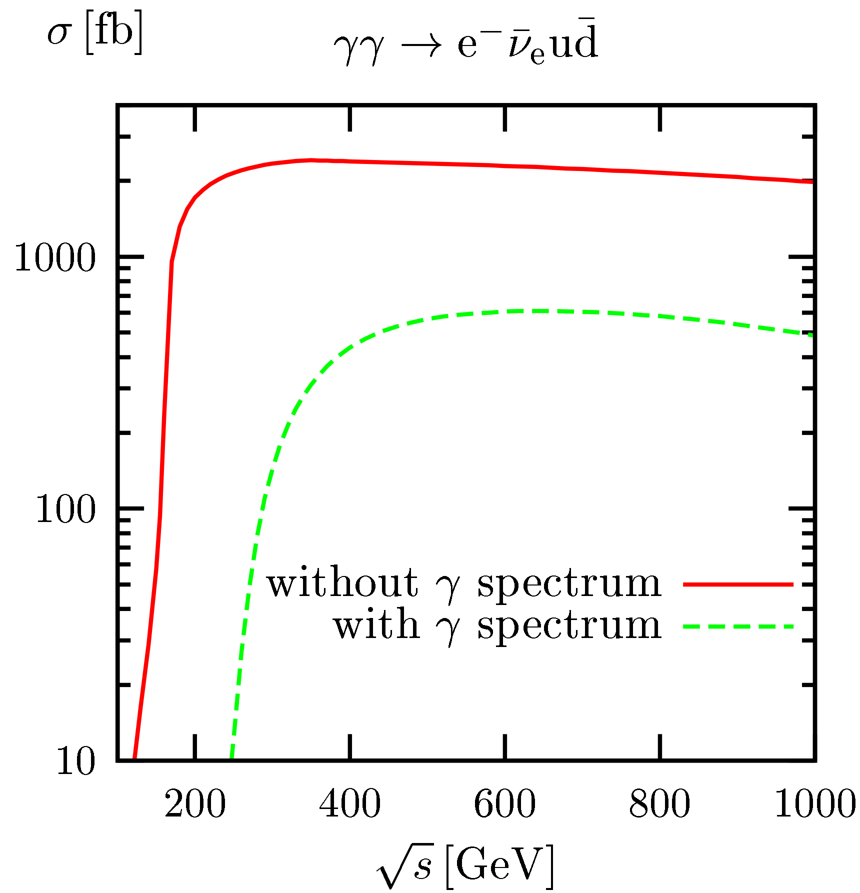
Photon spectrum (CompAZ Zarnecki '02; Telnov '95; Chen et al. '95):

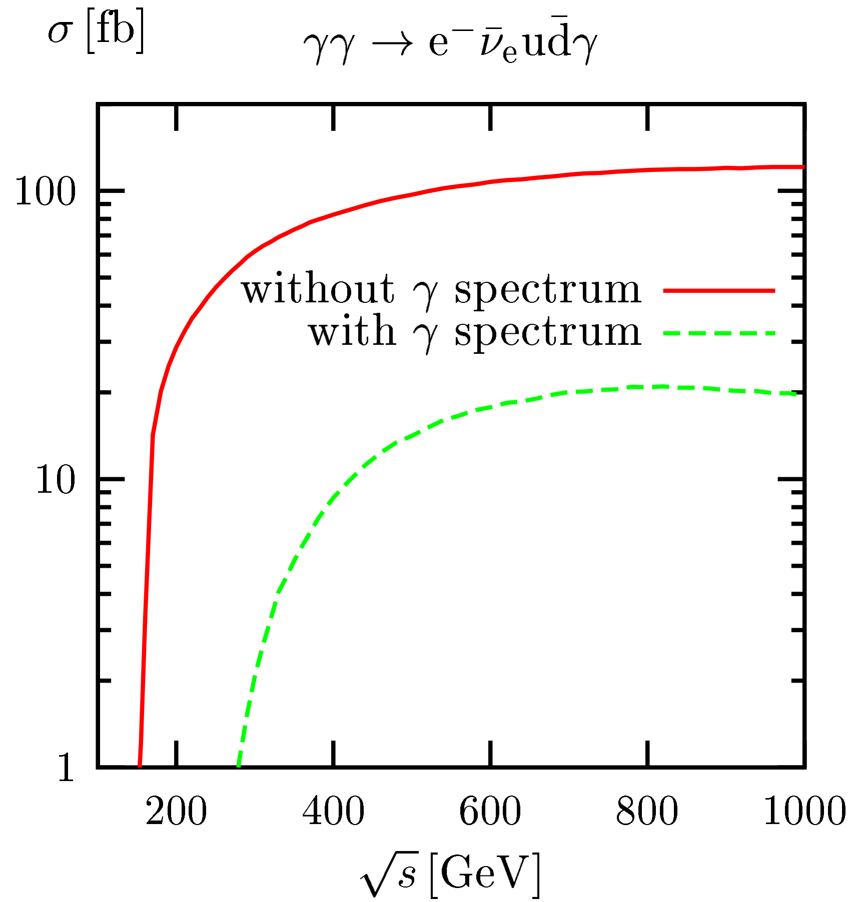
$$\mathrm{d}\sigma = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\, f(x_1) f(x_2) \mathrm{d}\sigma(x_1 P_1, x_2 P_2)$$

"stratified sampling" + adaptive optimization

Lowest-order results

Integrated cross section for $\gamma\gamma \to \mathrm{e}\bar{\nu}_\mathrm{e}\mathrm{u}\bar{\mathrm{d}}(\gamma)$





Comparison with Whizard&Madgraph Kilian '01; Stelzer, Long '94 $\longrightarrow$ good agreement

Anomalous triple couplings

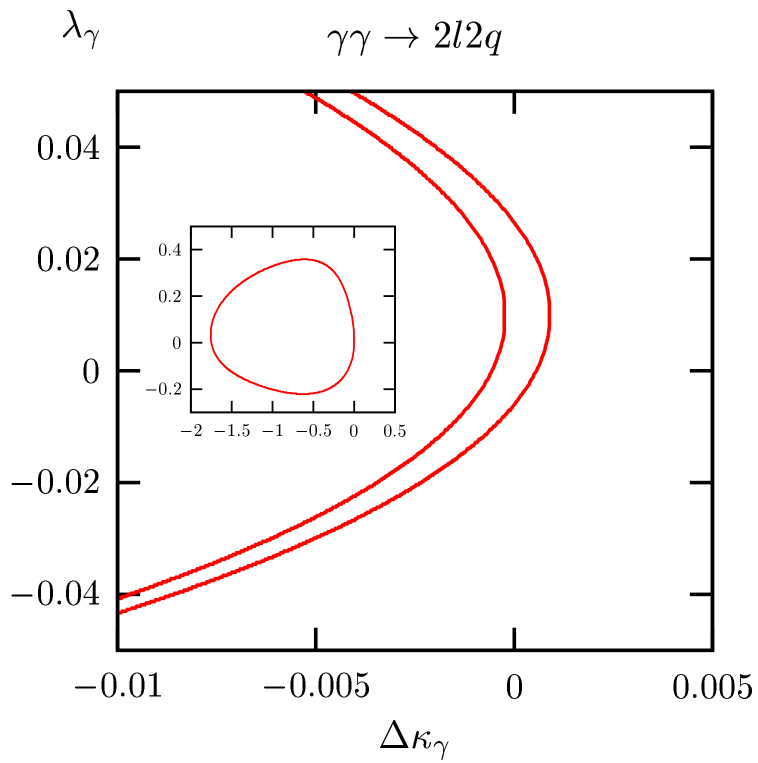
$\gamma\gamma \to 4f$ all semi-leptonic final states photon spectrum included

$\sqrt{s_{ee}} = 500\,\mathrm{GeV}$ $\int L dt = 100\,\mathrm{fb}^{-1}$ $\chi^2 = 1$ $\chi^2 \equiv \frac{(N(a_i) - N_{\mathrm{SM}})^2}{N_{\mathrm{SM}}}$

$$\begin{aligned}\mathcal{L} &= \mathrm{i}g_1 \frac{\alpha_{\mathrm{B}\phi}}{M_{\mathrm{W}}^2} (D_\mu\Phi)^\dagger B^{\mu\nu}(D_\nu\Phi) \\ &\quad -\mathrm{i}g_2 \frac{\alpha_{\mathrm{W}\phi}}{M_{\mathrm{W}}^2} (D_\mu\Phi)^\dagger \boldsymbol{\sigma}\cdot\mathbf{W}^{\mu\nu}(D_\nu\Phi) \\ &\quad -g_2 \frac{\alpha_{\mathrm{W}}}{6M_{\mathrm{W}}^2} \mathbf{W}^{\mu}{}_{\nu}\cdot(\mathbf{W}^{\nu}{}_{\rho}\times\mathbf{W}^{\rho}{}_{\mu}),\end{aligned}$$

$\to$ $\gamma\mathrm{WW}$ (and related $\gamma\gamma\mathrm{WW}$)

$\Delta\kappa_\gamma = \alpha_{\mathrm{W}\phi} + \alpha_{\mathrm{B}\phi}, \lambda_\gamma = \alpha_{\mathrm{W}}$ (LEP2)



$\to$ large interference with SM amplitude

expected limits comparable to $\mathrm{e}^+\mathrm{e}^-$-mode (see also Baillargeon et al. ’97; Bozovic-Jelisavcic et al. ’02)

full study requires consideration of distributions

Anomalous quartic couplings

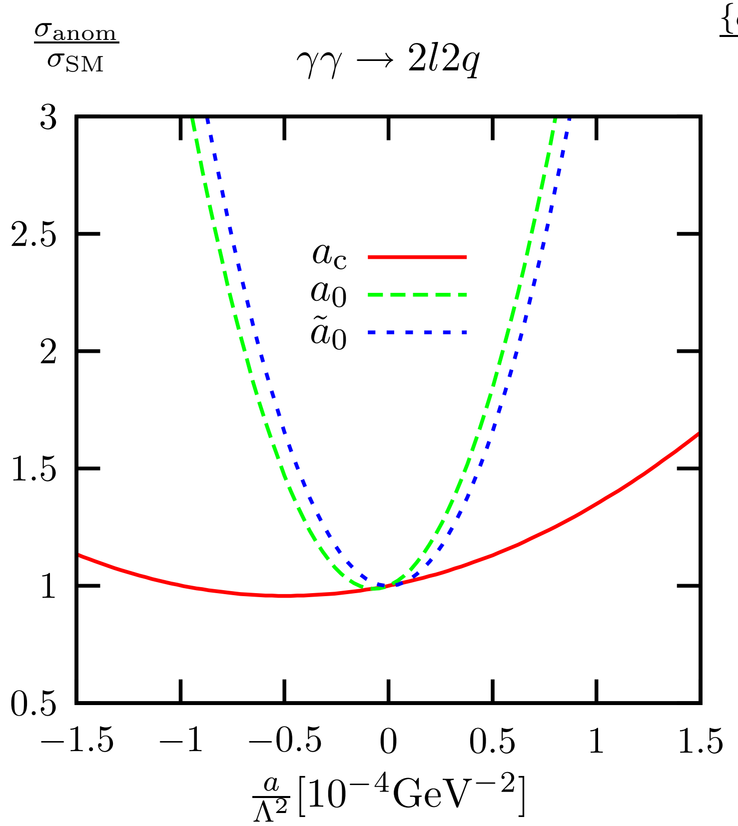
Assumption: CP, $\mathrm{U}(1)_{\mathrm{em}}$, $\mathrm{SU}(2)_{\mathrm{cust}}$

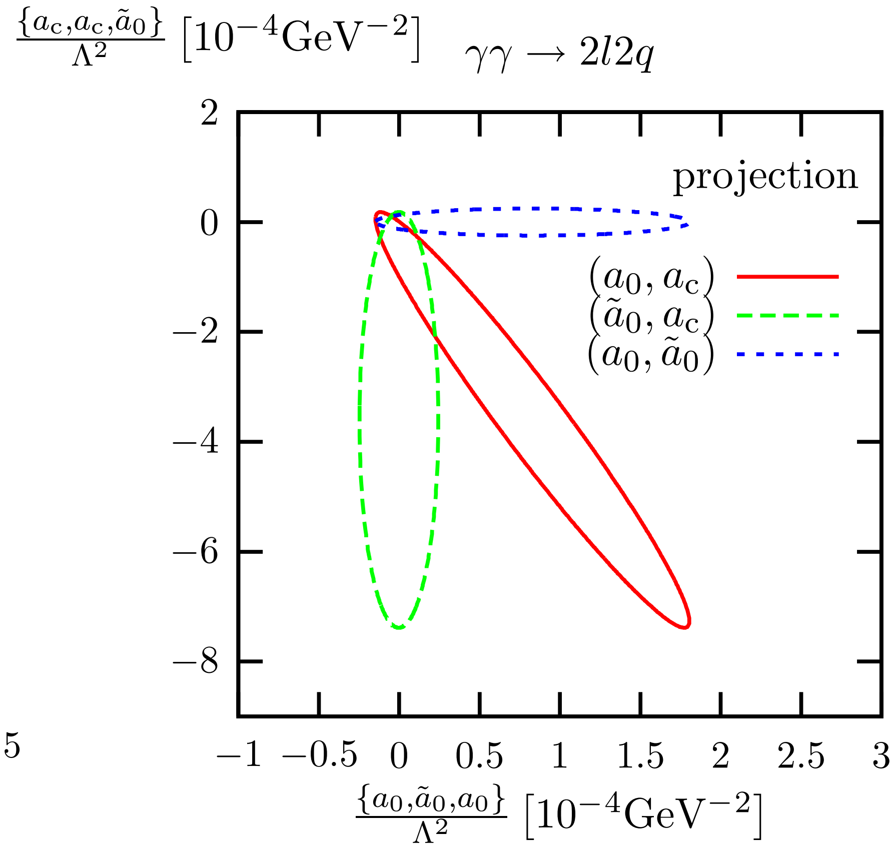
Bélanger,Boudjema '92; Abu Leil,Stirling '95;
Stirling,Werthenbach '00; Denner,Dittmaier,Roth,Wackeroth '01

$$\mathcal{L}_{\mathrm{anom}} = -\frac{e^2}{16\Lambda^2}\left(a_0\, F^{\mu\nu} F_{\mu\nu} \overline{\mathbf{W}}_\alpha \overline{\mathbf{W}}^\alpha + a_c\, F^{\mu\alpha} F_{\mu\beta} \overline{\mathbf{W}}^\beta \overline{\mathbf{W}}_\alpha + \tilde{a}_0\, F^{\mu\nu} \tilde{F}_{\mu\nu} \overline{\mathbf{W}}_\alpha \overline{\mathbf{W}}^\alpha \right)$$

$$\overline{\mathbf{W}}_\mu = (\overline{W}^1_\mu, \overline{W}^2_\mu, \overline{W}^3_\mu) = \left(\frac{1}{\sqrt{2}}(W^+ + W^-)_\mu, \frac{\mathrm{i}}{\sqrt{2}}(W^+ - W^-)_\mu, \frac{1}{c_{\mathrm{W}}} Z_\mu \right)$$

$\rightarrow \gamma\gamma\mathrm{WW}$ and $\gamma\gamma\mathrm{ZZ}$





better than in

$\mathrm{e}^+\mathrm{e}^- \to 4f\gamma$

Double-pole approximation at tree level

Naive W-pair signal:

only diagrams with two resonant

W propagators (not gauge invariant)

not sufficient

DPA = signal + “on-shell projection”

gauge invariant

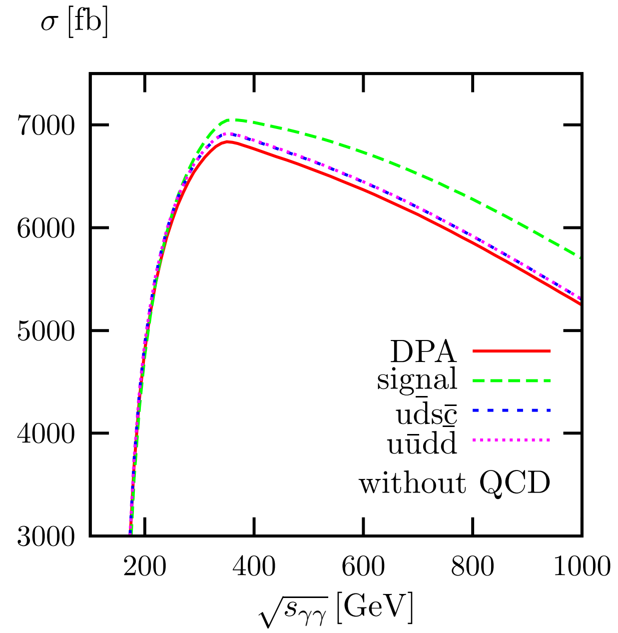
uncertainty of $\mathcal{O}(\Gamma_\mathrm{W}/M_\mathrm{W}) \sim 1-3\%$

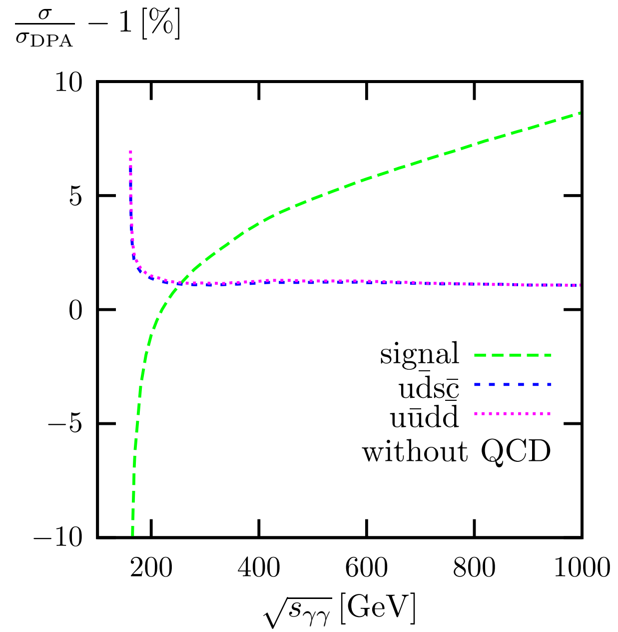
breakdown at WW threshold

$\rightarrow$ promising approach:

radiative corrections in DPA

uncertainty of $\mathcal{O}(\alpha/\pi \times \Gamma_\mathrm{W}/M_\mathrm{W}) \sim 0.5\%$





Virtual corrections in DPA

“Signal diagrams” are not gauge invariant

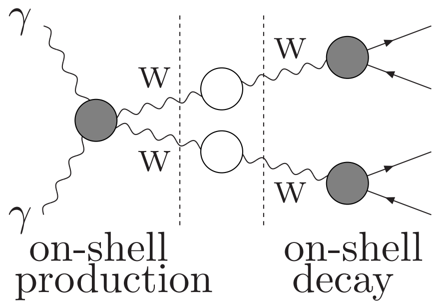
$\rightarrow$ expansion around resonant poles (expansion in $\alpha, p^2 - m^2$)

$$\mathcal{M} = \frac{R(p^2)}{p^2 - m^2} + N(p^2) \quad \rightarrow \quad \underbrace{\frac{R(m^2)}{p^2 - m^2 + \mathrm{i}m\Gamma}}_{\text{resonant}} + \underbrace{\frac{R(p^2) - R(m^2)}{p^2 - m^2} + N(p^2)}_{\text{non resonant}}$$

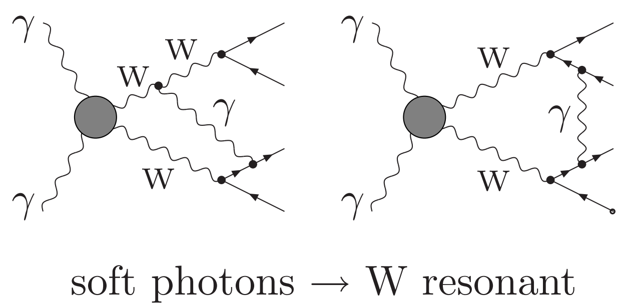
$R(p^2) \rightarrow R(m^2)$: on-shell projection

resonant radiative corrections:

factorizable



non factorizable



DPA applicable for $\sqrt{s} \gtrsim 170\,\mathrm{GeV} > 2M_{\mathrm{W}}$

(comparison with complete $\mathcal{O}(\alpha)$ calculation for $\mathrm{e^+e^-} \rightarrow 4f \rightarrow \mathcal{O}(0.5\%)$)

(Denner, Dittmaier, Roth, Wieders ’05)

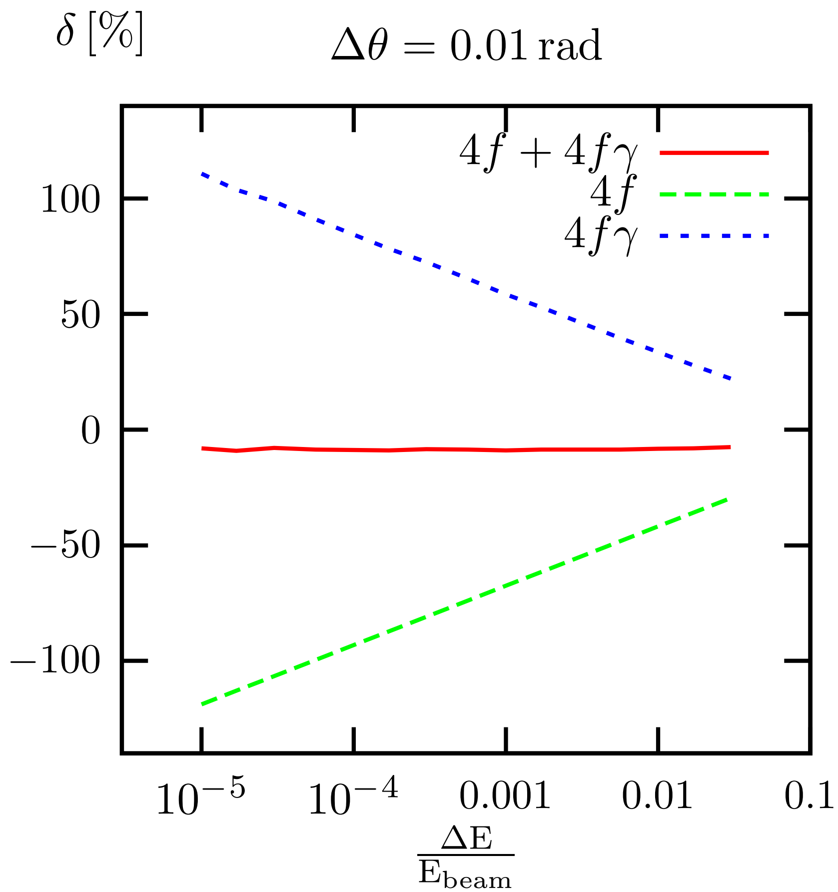
below $170\,\mathrm{GeV}$: IBA (Coulomb singularity, Higgs resonance, effective couplings)

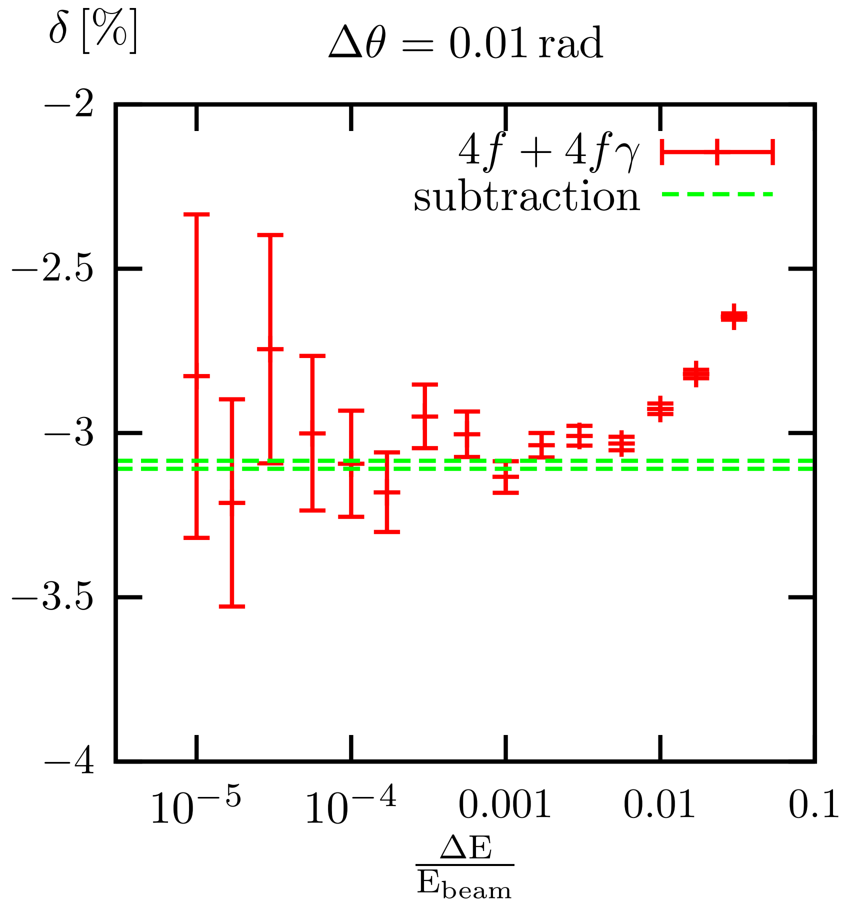
Phase-space slicing

$$\gamma\gamma \to \nu_e e^+ d\bar{u}, \quad E_{beam} = 250\,\mathrm{GeV}$$

$-ln\Delta E$
numerical

$+ln\Delta E$
analytical





good agreement between
slicing and subtraction method

Subtraction method

Basic idea: subtract and re-add the quantity $|M_{\rm sub}|^2$

$$\int {\rm d}\phi_{4f\gamma}\ \left(|\mathcal{M}_{\rm real}|^2 - |\mathcal{M}_{\rm sub}|^2\right)\ + \int {\rm d}\phi_{4f}|\mathcal{M}_{\rm virt}|^2 + \int {\rm d}\phi_{4f\gamma}|\mathcal{M}_{\rm sub}|^2$$

$|\mathcal{M}_{\rm sub}|^2 \sim |\mathcal{M}_{\rm real}|^2$ for $k \to 0$ or $p_i k \to 0$ $\quad k = \gamma$ momentum

$\Rightarrow \int {\rm d}\phi_{4f\gamma}\left(|\mathcal{M}_{\rm real}|^2 - |\mathcal{M}_{\rm sub}|^2\right)$ is finite (no regulators needed, $m_f = 0, m_\gamma = 0$)

define mapping $\phi_{4f\gamma} \to \tilde{\phi}_{4f}$ such that

$$p_i \underset{k\to 0}{\widetilde{\quad}} \tilde{p}_i, \qquad p_i + k \underset{kp_i\to 0}{\widetilde{\quad}} \tilde{p}_i, \qquad p_j \underset{kp_i\to 0}{\widetilde{\quad}} \tilde{p}_j \qquad\qquad \int {\rm d}\phi_{4f\gamma} = \int {\rm d}\tilde{\phi}_{4f} \otimes {\rm d}\phi_\gamma$$

$$\int {\rm d}\phi_{4f\gamma}|\mathcal{M}_{\rm sub}(\phi_{4f\gamma})|^2 = \int {\rm d}\tilde{\phi}_{4f} \otimes {\rm d}\phi_\gamma \underbrace{g(p_i, p_j, k)}_{\text{universal}} |\mathcal{M}_0(\tilde{\phi}_{4f})|^2$$

$$G = \int {\rm d}\phi_\gamma g(p_i, p_j, k)$$

$\Rightarrow \int {\rm d}\tilde{\phi}_{4f}\left(|\mathcal{M}_{\rm virt}|^2 + G|\mathcal{M}_0|^2\right)$ is finite due to KLN theorem

G contains singularities analytically

Explicit algorithm/method: dipole subtraction — Catani,Seymour ’96; Dittmaier ’99; Roth ’00

Subtraction method

Generalization to non-collinear safe observables

KLN theorem: no mass singularities for inclusive quantities

inclusive: fermion+photon = one quasi particle for $p_i k \to 0$

energy fraction $z_i = \frac{p_i^0}{p_i^0 + k^0}$ is fully integrated over

inclusiveness achieved e.g. by photon recombination ($p_i + k = \tilde{p}_i$ for $p_i k \to 0$)

cuts or histogram bins $\to$ integration over z_i is constrained,

mass singularities do not cancel between real and virtual corrections

$\to \alpha \log m_f$ terms

$\Rightarrow z_i$ cannot be integrated analytically,

has to be part of numerical phase-space integration

generalization straightforward for phase-space slicing,

but more involved for dipole subtraction

Subtraction method

Dipole subtraction has to be generalized (A. B., Dittmaier, Roth '05)

step function $\Theta(\phi)$ describes cuts or histogram bins:

$$\int \mathrm{d}\phi_{4f\gamma} \left(|\mathcal{M}_{\mathrm{real}}|^2 \Theta(\phi_{4f\gamma}) - |\mathcal{M}_{\mathrm{sub}}|^2 \Theta(\tilde{\phi}_{4f}) \right)$$

remember: subtraction function defined via mapping $\phi_{4f\gamma} \to \tilde{\phi}_{4f}$

photon recombination $\Rightarrow$ collinear-safe observable, $\Theta(\phi_{4f\gamma}) \xrightarrow[p_i k \to 0]{} \Theta(\tilde{\phi}_{4f})$

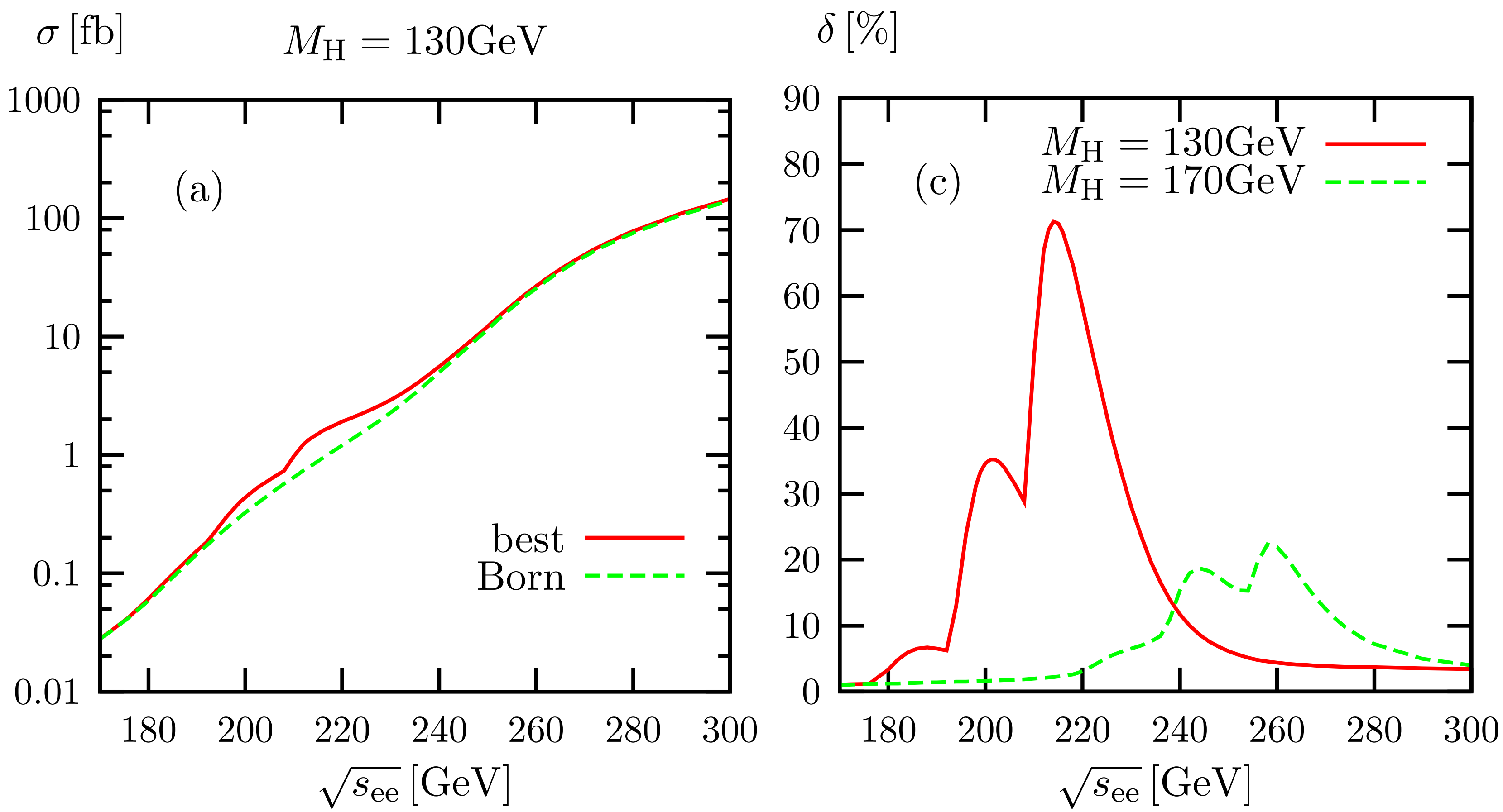
non-collinear safe observables:

keep information on energy fraction z_i in each part of the subtraction function:

- $\Theta(\tilde{\phi}_{4f}) \to \Theta\Big(p_i = z_{ij}\tilde{p}_i, k = (1 - z_{ij})\tilde{p}_i, \{\tilde{p}_{k\neq i}\}\Big)$ ($z_{ij} \xrightarrow[p_i k \to 0]{} z_i$)
- new subtraction functions $\int \mathrm{d}z_{ij} G(z_{ij}) = \int \mathrm{d}\phi_\gamma g(p_i, p_j, k)$
- numerical integration over z_{ij}

Impact of $\mathcal{O}(\alpha)$ corrections

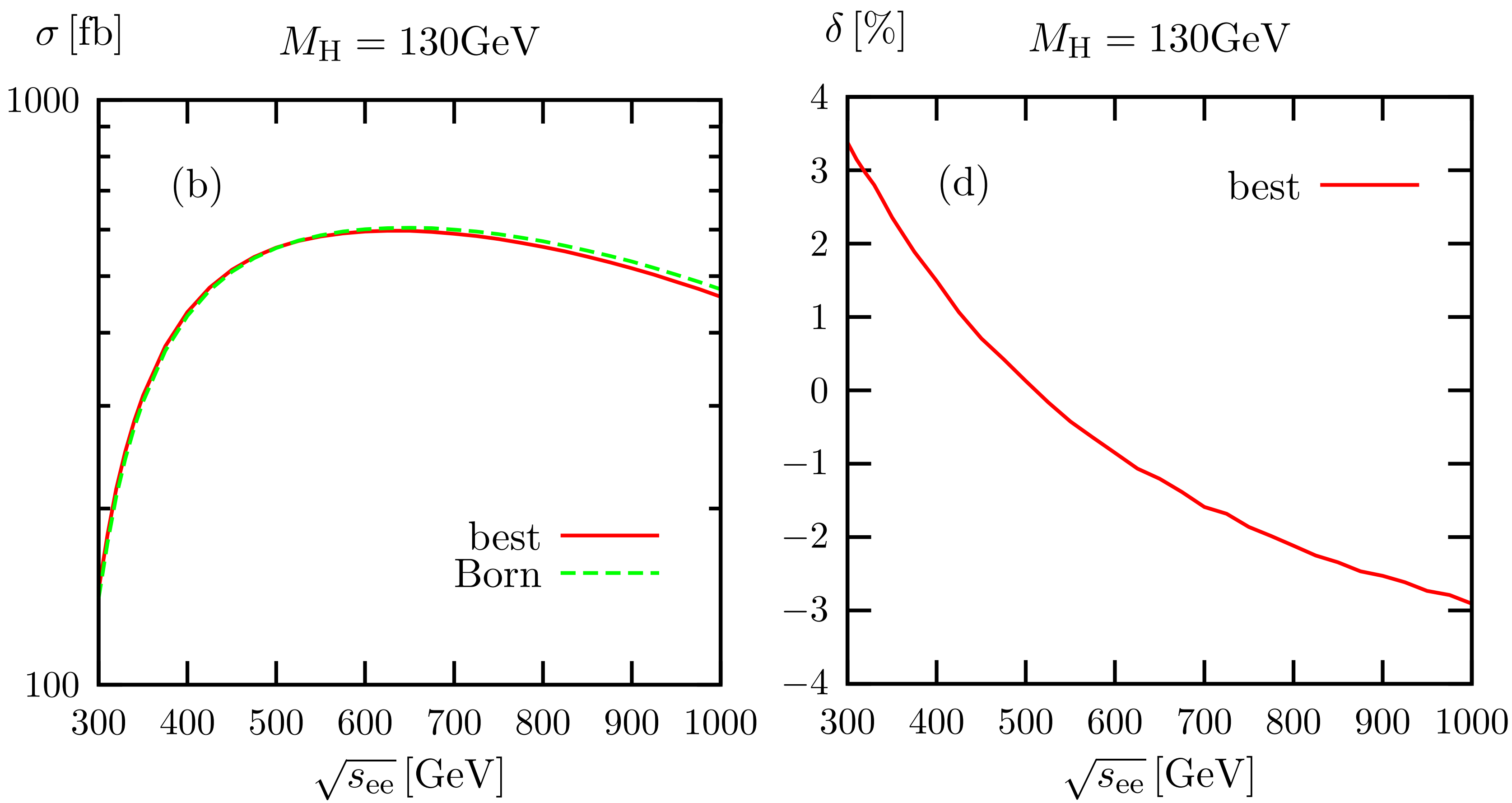
$\mathcal{O}(\alpha)$-corrected integrated cross section for $\gamma\gamma \to \nu_e e^+ d\bar{u}$ G_μ-scheme



photon spectrum determines shape of Higgs resonance
as function of e^-e^- CM energy $\sqrt{ee}$

Impact of $\mathcal{O}(\alpha)$ corrections

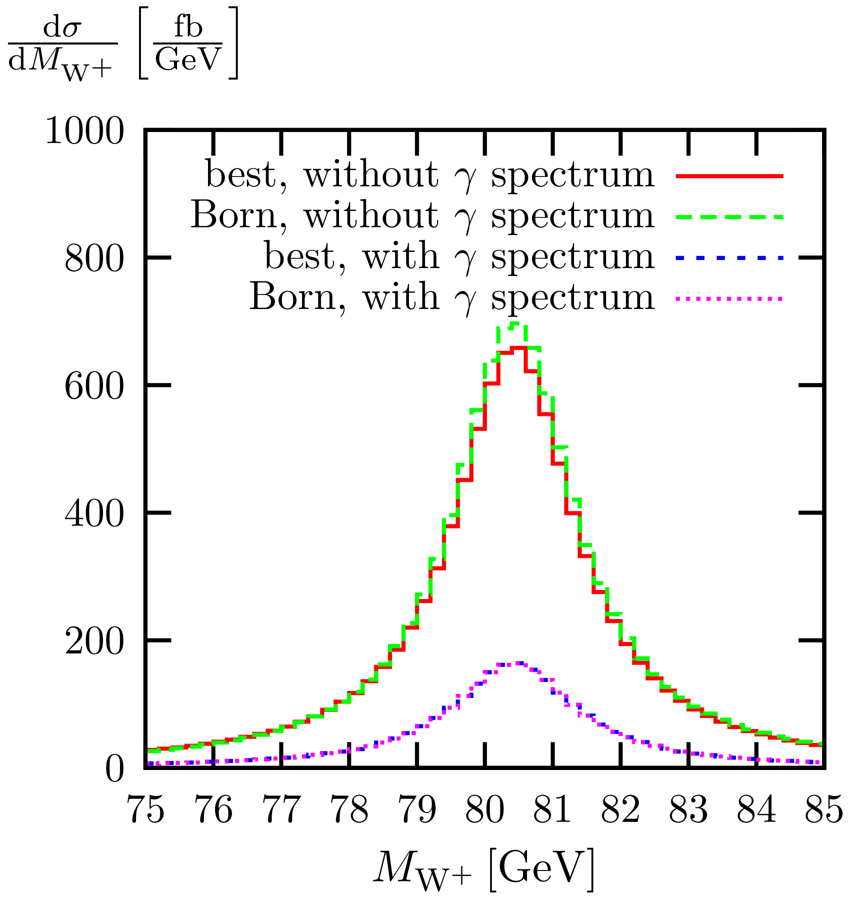
$\mathcal{O}(\alpha)$-corrected integrated cross section for $\gamma\gamma \to \nu_e e^+ d\bar{u}$ G_μ-scheme

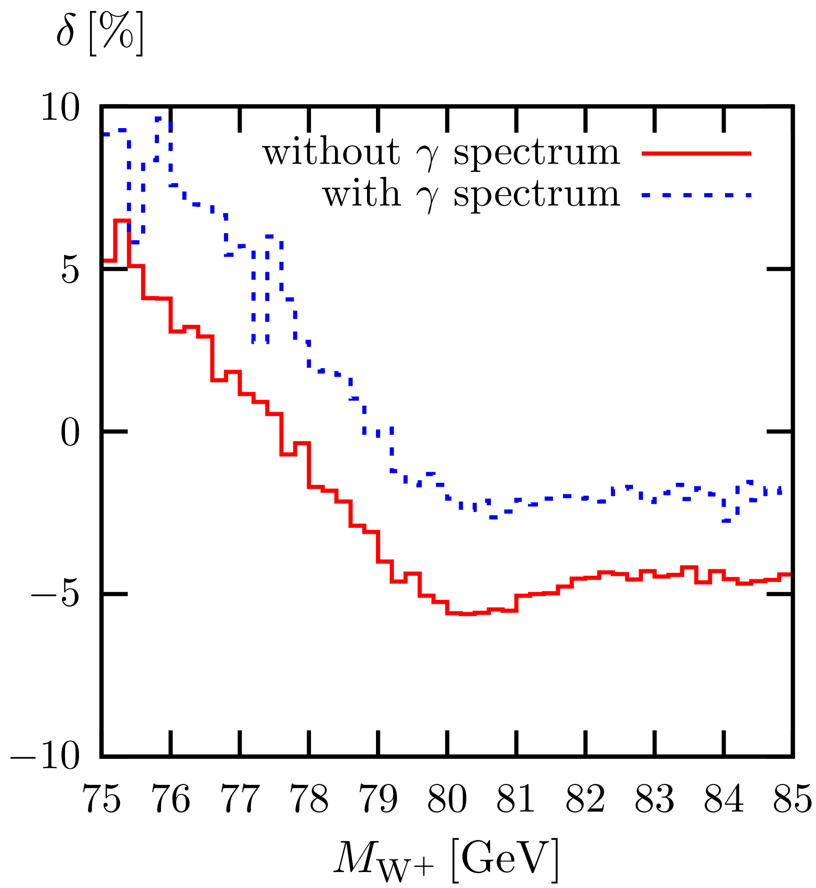


$\mathcal{O}(\alpha)$ corrections $\sim 3\%$ away from Higgs resonance

Impact of $\mathcal{O}(\alpha)$ corrections

$\mathcal{O}(\alpha)$-corrected W-invariant-mass distribution for $\gamma\gamma \to \nu_e e^+ d\bar{u}$ $\quad G_\mu$-scheme

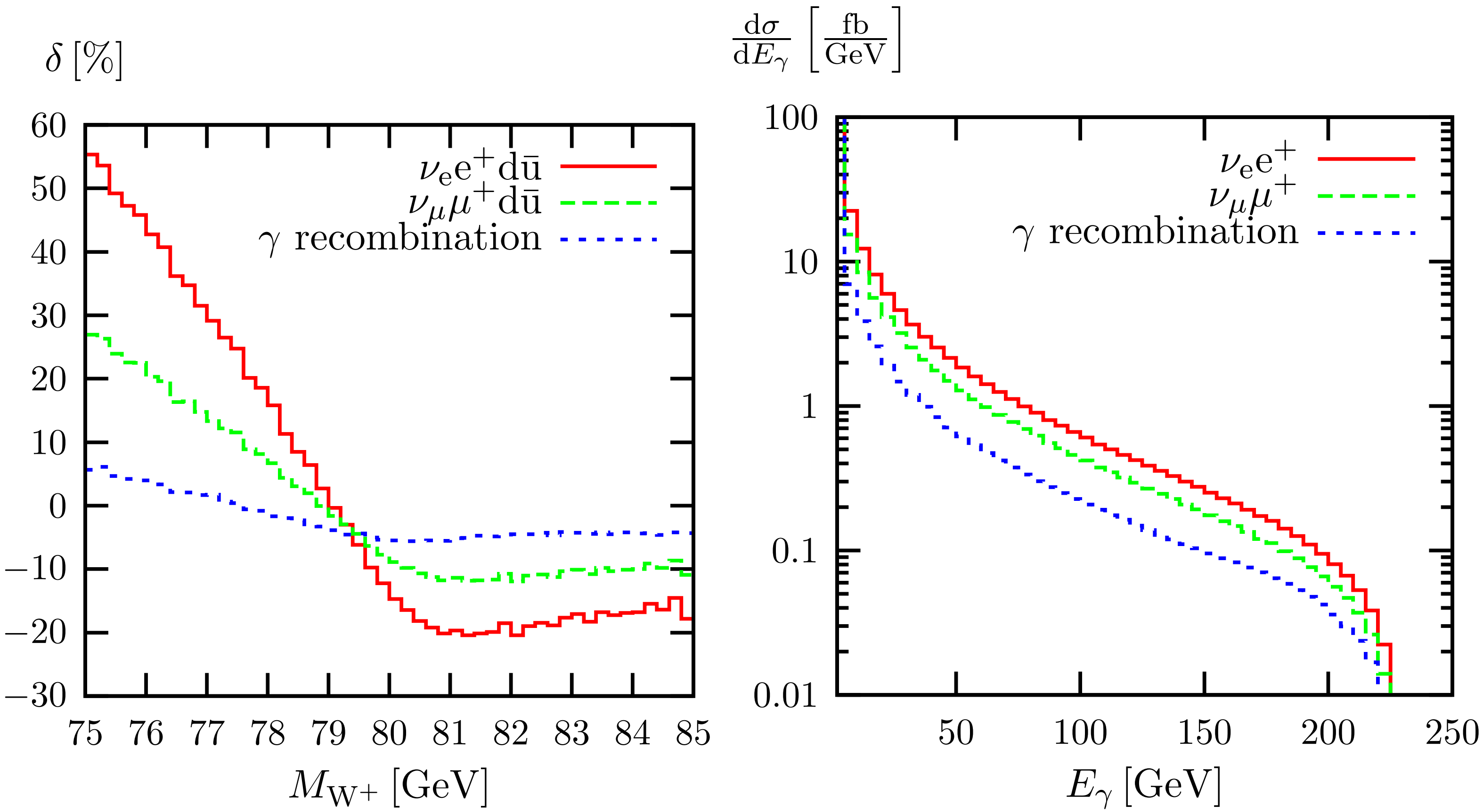




important corrections to W-line shape

Non-collinear safe observables

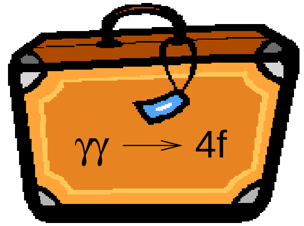
$\mathcal{O}(\alpha)$-corrected W-invariant-mass distribution and photon energy distribution for $\gamma\gamma \to \nu_e e^+ d\bar{u}$ ($\gamma\gamma \to \nu_\mu \mu^+ d\bar{u}$) (without convolution over photon spectrum)



$\alpha \log m_f$ terms visible without γ recombination,

γ recombination rearranges events

Monte Carlo generator



A.B.,Dittmaier,
Roth '04-'05

Coffer$\gamma\gamma$ (COrrections to
Four-FERmion production in $\gamma\gamma$ collisions)

- complete Born matrix elements
  - $\gamma\gamma \to 4f$ and $\gamma\gamma \to 4f + \gamma$ with massless fermions
  - anomalous $\gamma\mathrm{WW}$, $\gamma\gamma\mathrm{WW}$ and $\gamma\gamma\mathrm{ZZ}$ couplings
- radiative corrections to $\gamma\gamma \to \mathrm{WW} \to 4f$ in double-pole approximation

  similar to $\mathrm{e}^+\mathrm{e}^-$ case: Aeppli, v.Oldenborgh, Wyler '93; Beenakker, Berends, Chapovsky '98; Jadach et al. '99; Denner, Dittmaier, Roth, Wackeroth '99
- integration with multi-channel Monte Carlo with adaptive optimization
- treatment of real corrections with dipole subtraction or phase-space slicing (including generalization to non-collinear observables)
- realistic photon spectrum e.g. CompAZ, Zarnecki '02