On the treatment of threshold effects in SUSY spectrum computations

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based on

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Overview

- Introduction, methods of computing on-shell sparticle masses from RG evolution
  - renormalization at a common scale
  - freeze-out at multiple scales
- Discussion of concrete implementations in
  - SPheno 2.2.3
  - Isajet 7.72

  for the examle of neutralino masses at SPS1a
- Improvements in Isajet 7.73
- Towards a consistent multiscale approach?
- Conclusions

Introduction

For computing the sparticle spectrum within a high-scale model of SUSY beaking, e.g. mSUGRA,

- boundary conditions for gauge and Yukawa couplings are applied at $Q_{\mathbf{EW}} = M_Z$, according to their experimental values;
- boundary conditions for the soft SUSY-breaking terms are applied at the high scale, e.g. at $Q_{\mathbf{high}} = M_{\mathbf{GUT}}$;
- the RGEs are run iteratively between $Q_{\mathbf{EW}}$ and $Q_{\mathbf{high}}$ until a convergent solution is found.

Such the spectrum in the $\overline{\mathbf{DR}}$ scheme is obtained. To get the *physical* masses, the shift to the on-shell scheme has to be added.

Two methods

There exist two principal methods to compute the pole masses:

- Common-scale approach (CSA)

  The $\overline{\mathbf{DR}}$ SUSY parameters are all extracted collectively at a scale $Q = M_{\mathbf{SUSY}}$, and the (logarithmic + finite) self-energy corrections are added at that scale.

  $\longrightarrow$ SoftSusy, SPheno and SuSpect

- Step-beta function approach (SFA)

  Each $\overline{\mathbf{DR}}$ SUSY parameter m_i is extracted at its own mass scale $m_i(m_i)$. For un-mixed sparticles, this corresponds to a leading-log approximtion of the pole mass. The finite corrections are added separatly.

  $\longrightarrow$ method followed in Isajet

Neutralino mass matrix

In the basis $\psi_j^0 = (-i\lambda', -i\lambda^3, \psi_{H_1}^0, \psi_{H_2}^0)$:

$$\mathcal{M}_N = \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix}$$

at lowest order.

$$N \mathcal{M}_N N^T = \mathrm{diag}(\epsilon_1 m_{\tilde{\chi}_1^0},\ \epsilon_2 m_{\tilde{\chi}_2^0},\ \epsilon_3 m_{\tilde{\chi}_3^0},\ \epsilon_4 m_{\tilde{\chi}_4^0})$$

gives the neutralino mass eigenstates $\tilde{\chi}_i^0 = N_{ij}\psi_j^0$.

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M_1, M_2, μ, etc. in $\mathcal{M}_N$ are $\overline{\mathrm{DR}}$ parameters from the RG evolution

$\rightarrow m_{\tilde{\chi}_i^0}$ are $\overline{\mathrm{DR}}$ masses

Shift to pole masses, CSA

In order to obtain the neutralino pole masses, one has to add self-energy corrections

$$\mathcal{M}_N^{\mathrm{onshell}} = \mathcal{M}_N(Q) + \Delta\mathcal{M}_N(Q)\,,$$

leading to corrections in the masses, $m_{\tilde{\chi}_i^0} \to m_{\tilde{\chi}_i^0} + \Delta m_{\tilde{\chi}_i^0}$, and in the mixing matrix $N \to N + \Delta N$.

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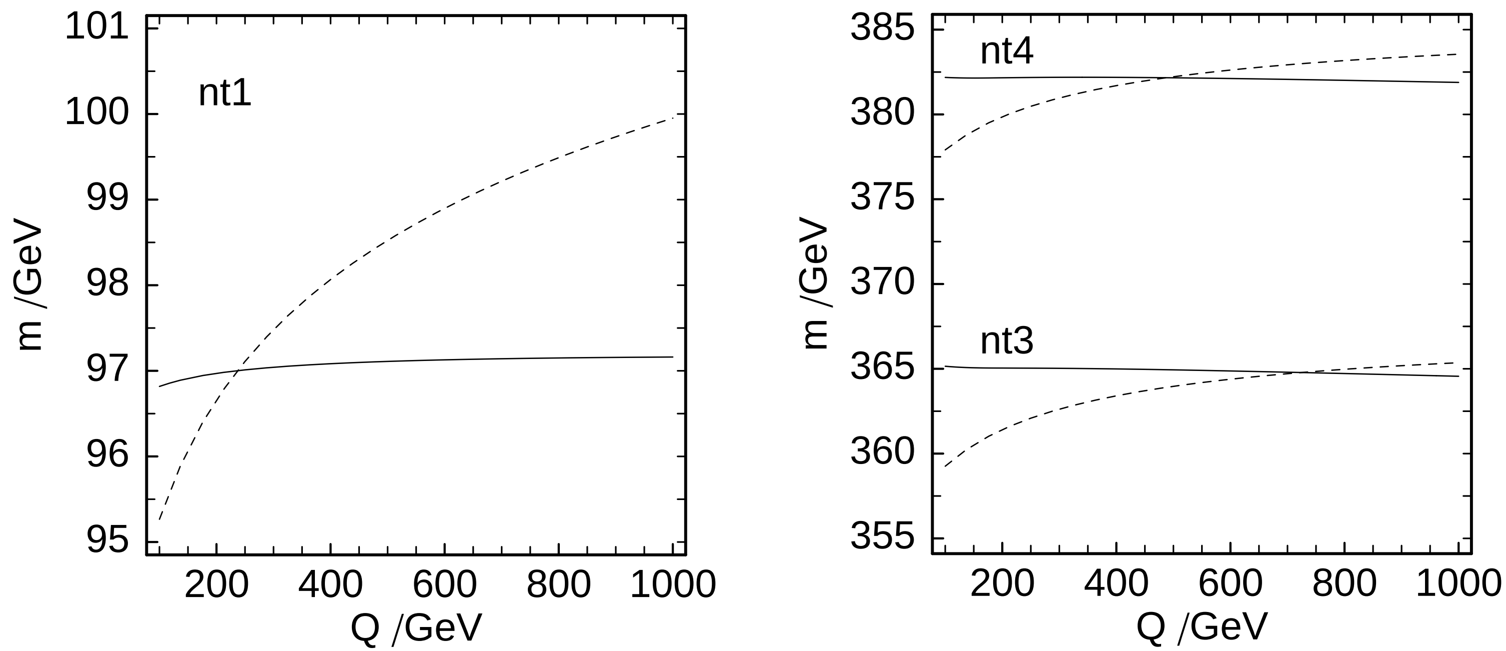
NB2: The on-shell condition $p^2 = m^2$ has to be fulfilled for each $m_{\tilde{\chi}_i^0}$ separately; so $\mathcal{M}_N^{\mathrm{onshell}}$ has to be computed 4 times ...

SPS1a, SPheno 2.2.3



dashed lines .... $\overline{\mathbf{DR}}$ masses

full lines .... pole masses @ 1 loop

Scale dependence at 1 loop is very small; e.g. 0.3% for $m_{\tilde{\chi}_1^0}$.

Caution:

This scale dependence is not the full theoretical uncertainty!

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For example:

- In SPheno 2.2.3, the SUSY-loop corrections to the gauge and Yukawa couplings are computed collectively at M_Z.
- They could also be computed at a different scale, e.g. at $m_{\mathbf{LSP}}$ or $M_{\mathbf{SUSY}}$.
- Since this results in a shift in the boundary conditions, it is not taken into account by the scale dependence.

The effect can be quite relevant and is under investigation.

Shift to pole masses, SFA

In the step-beta function approach, the parameters in $\mathcal{M}_N$ are

$M_1(M_1)$, $M_2(M_2)$, $\mu(M_{\rm SUSY})$.

According to [DEDES:1995SB] this should correspond to the on-shell mass matrix up to finite corrections, and hence

$$\mathcal{M}_N^{\rm onshell} = \mathcal{M}_N^{\rm log.corr} + \Delta\mathcal{M}_N^{\rm const}$$

Again, $p^2 = m^2_{\tilde{\chi}^0_i}\ \forall\ \tilde{\chi}^0_i$.

Not quite trivial to do this in a consistent way ....

SFA in Isajet 7.72

- The SUSY parameters are extracted from the RG running at their respective mass scales. They are, however, not formally integrated out.

  In contrast to the gauge and Yukawa couplings, where the beta functions change each time a threshold is passed, the soft-term RGEs remain those of the MSSM all the way from M_{GUT} to M_Z.

- For the finite shifts, the full expressions of [PIERCE:1996ZZ] for the 1-loop self-energies are used, with the renormalization scale for the A_0, B_0, B_1 functions set to $Q = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$.

  $\rightsquigarrow$ double counting of logs between M_{SUSY} and the actual mass scale of the sparticle.

  Important effect on LSP mass!

SPS1a, Isajet 7.72

Case	M_1	M_2	μ	$m^{\text{pole}}_{\tilde{\chi}^0_1}$	$m^{\text{pole}}_{\tilde{\chi}^0_2}$	$m^{\text{pole}}_{\tilde{\chi}^0_3}$	$m^{\text{pole}}_{\tilde{\chi}^0_4}$
A	99.5	192.4	351.2	95.2	180.5	357.0	377.5
B	99.5	192.4	351.2	97.9	182.0	357.6	377.9
C	103.2	192.9	345.1	101.5	181.7	351.6	372.6
D	101.7	192.1	350.9	97.3	180.2	356.7	377.2

A ... Isajet 7.72
B ... Isajet 7.72 with corrected renormalization scale
C ... same as B plus step-beta functions ∀ parameters
D ... all parameters frozen out at $Q = M_{\text{SUSY}}$, aequiv. CSA

Isajet 7.73

1. The $\overline{\mathbf{DR}}$ parameters of non-mixing sparticles, i.e. squarks & sleptons of the 1st+2nd generation, and the gluino, are extracted at their respective mass scales.
2. The $\overline{\mathbf{DR}}$ parameters of mixing sparticles i.e. neutralinos, charginos, stops, sbottoms, and staus, are all extracted at $Q = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$.
3. The renormalization scales for the 1-loop self-energy corrections are set to the respective freeze-out scales.
4. Gluino mass corrections depending on squark mixing have been added.
5. Variable beta functions for SUSY parameters are postponed until a consistent treatment of logarithmic and finite corrections for multiple scales is available.

Comparison for SPS1a

Mass	Isajet 7.72	Isajet 7.73	SPheno 2.2.3	δ^{scale}
$\tilde{\chi}_1^0$	95.19	97.39	97.11	0.3
$\tilde{\chi}_2^0$	180.5	180.4	180.7	1.1
$\tilde{\chi}_3^0$	356.7	358.7	364.9	0.6
$\tilde{\chi}_4^0$	377.2	379.0	382.2	0.3
$\tilde{\tau}_1$	134.8	134.6	134.4	0.6
$\tilde{\tau}_2$	206.7	205.9	206.4	0.3
$\vdots$				
$\tilde{u}_L$	559.5	564.9	565.1	9.8
$\tilde{u}_R$	544.0	548.6	547.8	8.9
$\tilde{t}_1$	401.8	395.2	400.6	5.0
$\tilde{t}_2$	583.5	584.4	586.0	9.0
$\tilde{g}$	611.4	605.9	604.3	1.3

Consistent multiscale treatment?

Technical difficulties:

- At each scale where a threshold is passed, the relevant field(s) have to be taken out of the RGEs and a new effective theory (EFT) has to be constructed.
- The shifts of the $\overline{\mathbf{DR}}$ to the pole-mass parameters of the sparticle that is integrated out involve contributions of all the particles that are degrees of freedom of the current EFT.
- In turn, the field(s) that are integrated out lead to finite shifts in the boundary conditions of the parameters which remain in the EFT $\longrightarrow$ non-trivial matching conditions.
- When the symmetry of the EFT is ‘smaller’ than the symmetry of the underlying theory, this leads to additional parameters, higher-dimensional operators ...

Well known from QCD, but much work needed for SUSY case

Conclusions

- Two principal methods of computing on-shell sparticle masses from RG evolution:
  - renormalization at a common scale
  - freeze-out at multiple scales
- Compared results of SPheno 2.2.3 and Isajet 7.72
  Examle: neutralino masses at SPS1a
- Concrete implementations, shortcomings, uncertainties, ways to improve...
- Isajet 7.73 incorporates several improvements
- Future challenge: consistent multiscale treatment