PROPHECY4f:
a PROPer description
for the Higgs dECaY into 4 Fermions

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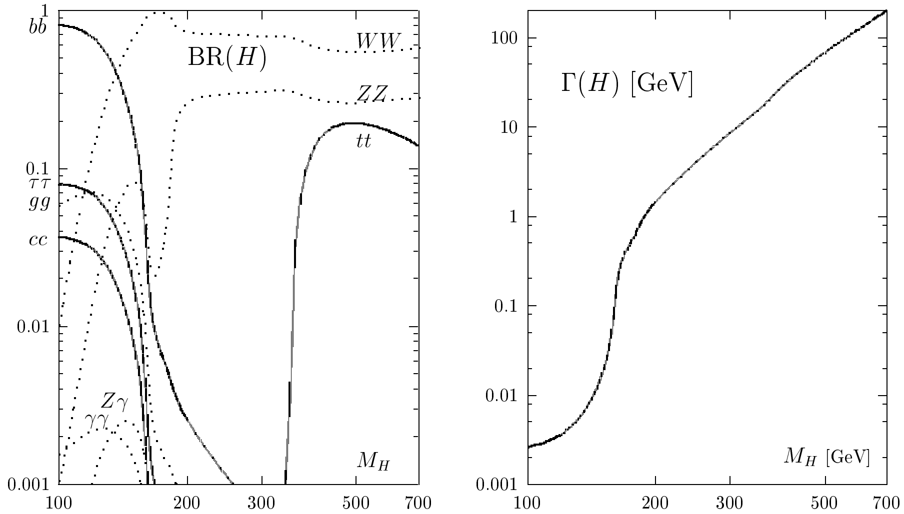
Contents

- Relevance of $\mathrm{H} \to \mathrm{WW}^{(*)}/\mathrm{ZZ}^{(*)}$
- Virtual corrections: tensor reduction, unstable particles
- Handling of soft and collinear divergences:
  dipole subtraction and phase-space slicing
- Numerical results for $\mathrm{H} \to \mathrm{WW}/\mathrm{ZZ} \to$ 4 leptons

$\to$ see also talk of M.M. Weber

$\mathbf{H \rightarrow WW^{(*)}/ZZ^{(*)}}$



Tesla TDR

LHC: $\mathrm{H} \rightarrow \mathrm{WW}^{(*)}/\mathrm{ZZ}^{(*)}$:

largest discovery potential

for $M_{\mathrm{H}} \gtrsim 125\,\mathrm{GeV}$

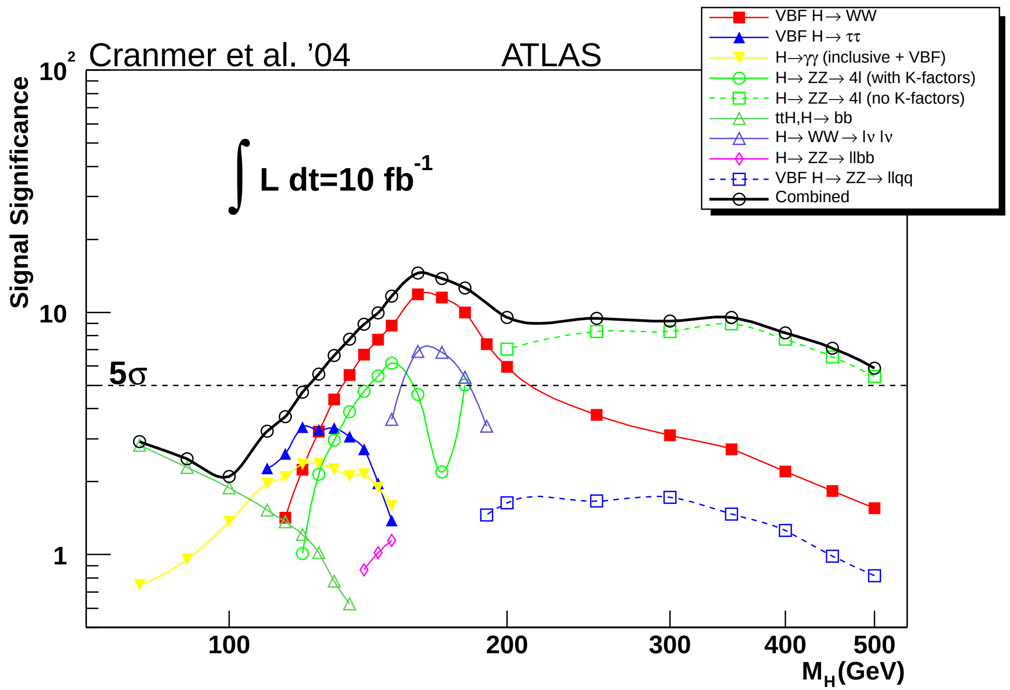
$\mathrm{H} \rightarrow \mathrm{ZZ} \rightarrow 4l$:

most accurate measurement

of Higgs mass for $M_{\mathrm{H}} \gtrsim 130\,\mathrm{GeV}$

ILC: measurement of branching ratios etc.

at percent level



$\mathbf{H \rightarrow WW^{(*)}/ZZ^{(*)}}$

Current theoretical predictions:

above $\mathrm{WW/ZZ}$ threshold:

$\mathcal{O}(\alpha)$ corrections known for stable $\mathrm{W/Z}$ (Kniehl '91; Bardin et al. '91)

below threshold (and in transition region): only tree-level predictions

(e.g. by HDecay: Djouadi, Kalinowski, Spira '97)

$\rightarrow$ corrections to $\mathrm{H} \rightarrow \mathrm{WW/ZZ} \rightarrow 4f$ necessary

Relevance of distributions:

- reconstruction (γ radiation)
- correlation of decay angles $\rightarrow$ verification of spin 0 (Choi et al. '02)

$\rightarrow$ Monte Carlo generator for $\mathrm{H} \rightarrow \mathrm{WW/ZZ} \rightarrow 4f$ required

This talk: Monte Carlo generator with $\mathcal{O}(\alpha)$ EW corrections

Virtual corrections: technical challenges

Reduction of tensor integrals:

appearance of small Gram determinants in denominator in standard Passarino-Veltman reduction

two different approaches to circumvent this problem $\rightarrow$ talk of S. Dittmaier

Same methods used as in $\mathrm{e}^+\mathrm{e}^- \rightarrow 4f$ Denner, Dittmaier, Roth, Wieders '05 :

(see Denner, Dittmaier '05)

- 5-point integrals are reduced to 4-point integrals without inverse Gram determinant
- 3-/4-point integrals:

  special treatment of phase-space points with small Gram determinant

  2 different methods:

  - semi-numerical method + analytical special cases

    $\rightarrow$ avoids inverse Gram determinant
  - expansion in small Gram and other kinematical determinants

Virtual corrections: conceptual issues

Unstable particles:

Dyson summation for description of resonances

$\rightarrow$ potential violation of gauge invariance

tree level: various schemes (e.g. naive fixed width, complex-mass scheme, fermion-loop scheme)

one loop: pole expansion (Aeppli et al. '93, '94; Stuart '91; Beenakker et al. '98; Denner et al. '00; Beneke et al. '04)

$M_{\mathrm{H}} \ll 2\mathrm{M}_{\mathrm{W/Z}}$: single-pole approximation

$M_{\mathrm{H}} \gg 2\mathrm{M}_{\mathrm{W/Z}}$: double-pole approximation

$M_{\mathrm{H}} \sim 2\mathrm{M}_{\mathrm{W/Z}}$: not reliable in threshold region

unified description: complex-mass scheme at one loop Denner, Dittmaier, Roth, Wieders '05

$M^2 \rightarrow \mu^2 = M^2 - \mathrm{i}M\Gamma$ everywhere in Feynman rules, also in loop integrals

renormalization conditions also modified:

e.g. $\hat{\Sigma}_T(\mu^2) = 0$ (on-shell scheme)

$\rightarrow$ talk of A. Denner

Phase-space integration

Multi-channel Monte Carlo integration with adaptive optimization

Berends, Kleiss, Pittau '94 Kleiss, Pittau '94

- mappings for propagators (e.g. resonances) $\rightarrow$ integrand is flattened
- "coherent" combination of different mappings (channels)
- adaptive optimization finds most important channels
- below threshold: additional mappings for non-resonant propagators

regularization of soft and collinear singularities: small mass parameters

matching of singularities between virtual and real corrections:

$\rightarrow$ dipole subtraction, phase-space slicing

Subtraction method

Basic idea: subtract and re-add the quantity $|M_{\mathrm{sub}}|^2$

$$\int \mathrm{d}\phi_{4f\gamma}\, \left(|\mathcal{M}_{\mathrm{real}}|^2 - |\mathcal{M}_{\mathrm{sub}}|^2\right) + \int \mathrm{d}\phi_{4f}|\mathcal{M}_{\mathrm{virt}}|^2 + \int \mathrm{d}\phi_{4f\gamma}|\mathcal{M}_{\mathrm{sub}}|^2$$

$|\mathcal{M}_{\mathrm{sub}}|^2 \sim |\mathcal{M}_{\mathrm{real}}|^2$ for $k \to 0$ or $p_i k \to 0$ $k = \gamma$ momentum

$\Rightarrow \int \mathrm{d}\phi_{4f\gamma}\left(|\mathcal{M}_{\mathrm{real}}|^2 - |\mathcal{M}_{\mathrm{sub}}|^2\right)$ is finite (no regulators needed, $m_f = 0, m_\gamma = 0$)

define mapping $\phi_{4f\gamma} \to \tilde{\phi}_{4f}$ such that

$$p_i \underset{k\to 0}{\widetilde{\quad}} \tilde{p}_i, \quad p_i + k \underset{kp_i\to 0}{\widetilde{\quad}} \tilde{p}_i, \quad p_j \underset{kp_i\to 0}{\widetilde{\quad}} \tilde{p}_j \qquad \int \mathrm{d}\phi_{4f\gamma} = \int \mathrm{d}\tilde{\phi}_{4f} \otimes \mathrm{d}\phi_\gamma$$

$$\int \mathrm{d}\phi_{4f\gamma}|\mathcal{M}_{\mathrm{sub}}(\phi_{4f\gamma})|^2 = \int \mathrm{d}\tilde{\phi}_{4f} \otimes \mathrm{d}\phi_\gamma \underbrace{g(p_i, p_j, k)}_{\text{universal}} |\mathcal{M}_0(\tilde{\phi}_{4f})|^2$$

$$G = \int \mathrm{d}\phi_\gamma g(p_i, p_j, k)$$

$\Rightarrow \int \mathrm{d}\tilde{\phi}_{4f}\left(|\mathcal{M}_{\mathrm{virt}}|^2 + G|\mathcal{M}_0|^2\right)$ is finite due to KLN theorem

G contains singularities analytically

Explicit algorithm/method: dipole subtraction Catani,Seymour '96; Dittmaier '99; Roth '00

Subtraction method

Generalization to non-collinear safe observables

KLN theorem: no mass singularities for inclusive quantities

inclusive: fermion+photon = one quasi particle for $p_i k \to 0$

energy fraction $z_i = \frac{p_i^0}{p_i^0 + k^0}$ is fully integrated over

inclusiveness achieved e.g. by photon recombination ($p_i + k = \tilde{p}_i$ for $p_i k \to 0$)

cuts or histogram bins $\to$ integration over z_i is constrained,

mass singularities do not cancel between real and virtual corrections

$\to \alpha \log m_f$ terms

$\Rightarrow z_i$ cannot be integrated analytically,

has to be part of numerical phase-space integration

generalization straightforward for phase-space slicing,

but more involved for dipole subtraction

Subtraction method

Dipole subtraction has to be generalized (A.B., Dittmaier, Roth ’05)

step function $\Theta(\phi)$ describes cuts or histogram bins:

$\int \mathrm{d}\phi_{4f\gamma} \left(|\mathcal{M}_{\mathrm{real}}|^2 \Theta(\phi_{4f\gamma}) - |\mathcal{M}_{\mathrm{sub}}|^2 \Theta(\tilde{\phi}_{4f}) \right)$

remember: subtraction function defined via mapping $\phi_{4f\gamma} \to \tilde{\phi}_{4f}$

photon recombination $\Rightarrow$ collinear-safe observable, $\Theta(\phi_{4f\gamma}) \xrightarrow[p_i k \to 0]{} \Theta(\tilde{\phi}_{4f})$

non-collinear safe observables:

keep information on energy fraction z_i in each part of the subtraction function:

- $\Theta(\tilde{\phi}_{4f}) \to \Theta\Big(p_i = z_{ij}\tilde{p}_i, k = (1 - z_{ij})\tilde{p}_i, \{\tilde{p}_{k \neq i}\} \Big)$ $\quad$ ($z_{ij} \xrightarrow[p_i k \to 0]{} z_i$)
- new subtraction functions $\int \mathrm{d}z_{ij} G(z_{ij}) = \int \mathrm{d}\phi_\gamma g(p_i, p_j, k)$
- numerical integration over z_{ij}

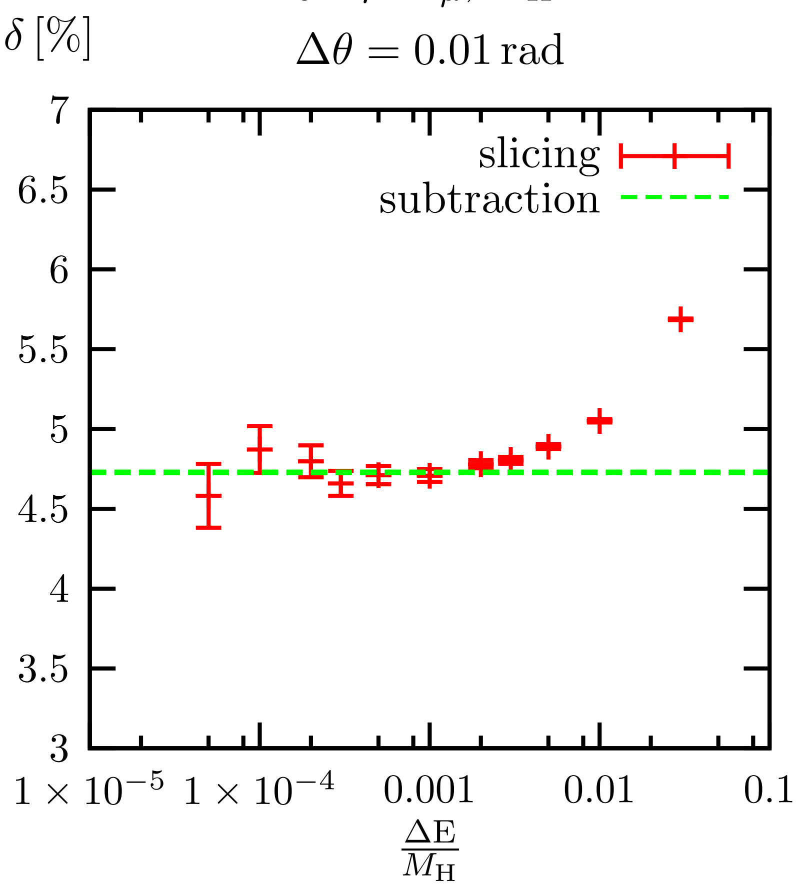
Phase-space slicing

$$\int \mathrm{d}\phi_{4f\gamma}\,|\mathcal{M}_{\mathrm{real}}|^2 \;=\; \int \mathrm{d}\phi^{\mathrm{finite}}_{4f\gamma}\,|\mathcal{M}_{\mathrm{real}}|^2 \;+\; \int_{\substack{E_\gamma<\Delta E \\ \text{or } \theta(\gamma,f_i)<\Delta\theta}} \mathrm{d}\phi^{\mathrm{sing}}_{4f\gamma}\,|\mathcal{M}_{\mathrm{real}}|^2$$

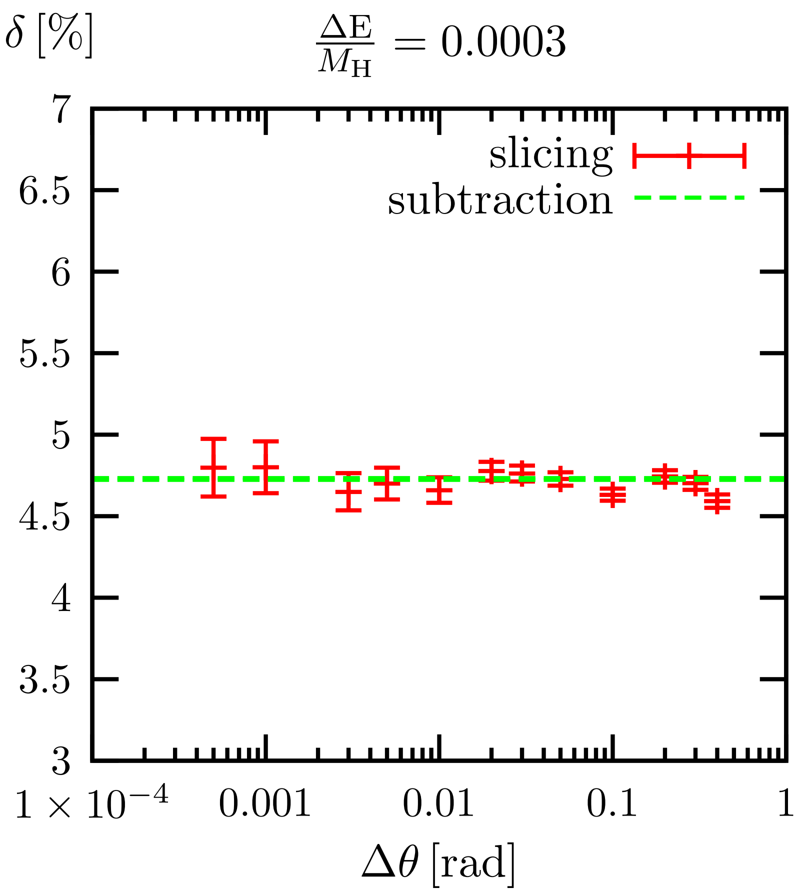
analytical integration over $\mathrm{d}\phi_\gamma$

$\mathrm{H} \to \nu_{\mathrm{e}}\mathrm{e}^+\mu^-\nu_\mu,\, M_{\mathrm{H}} = 200$

$\Delta\theta = 0.01\,\mathrm{rad}$



$\frac{\Delta \mathrm{E}}{M_{\mathrm{H}}} = 0.0003$



good agreement between subtraction and slicing method

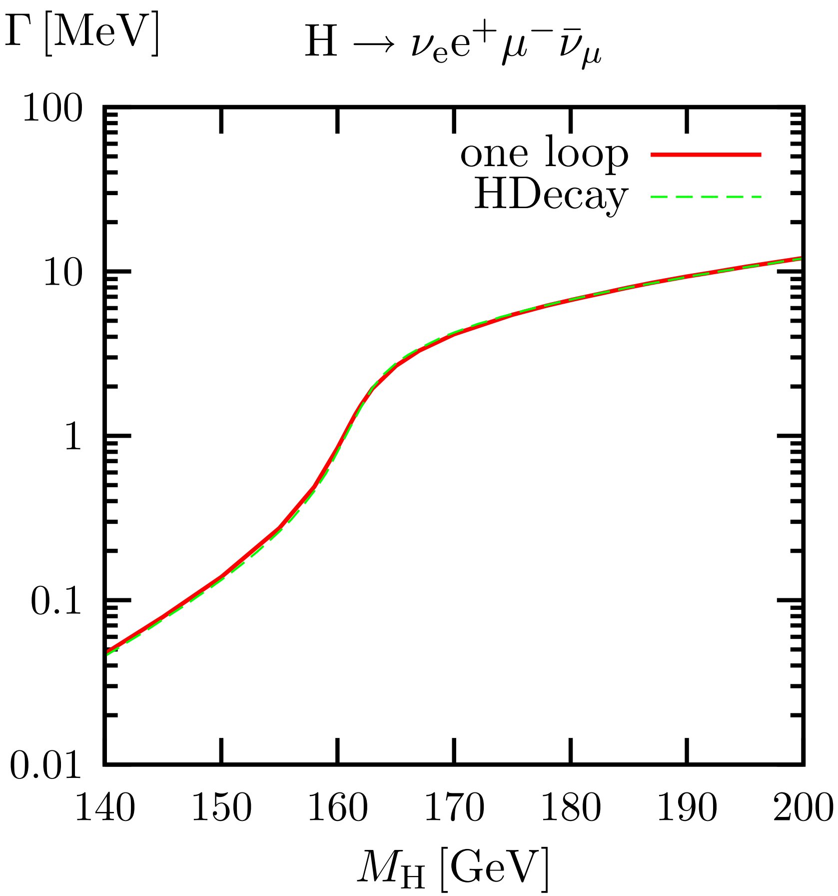
$5 \cdot 10^7$ events

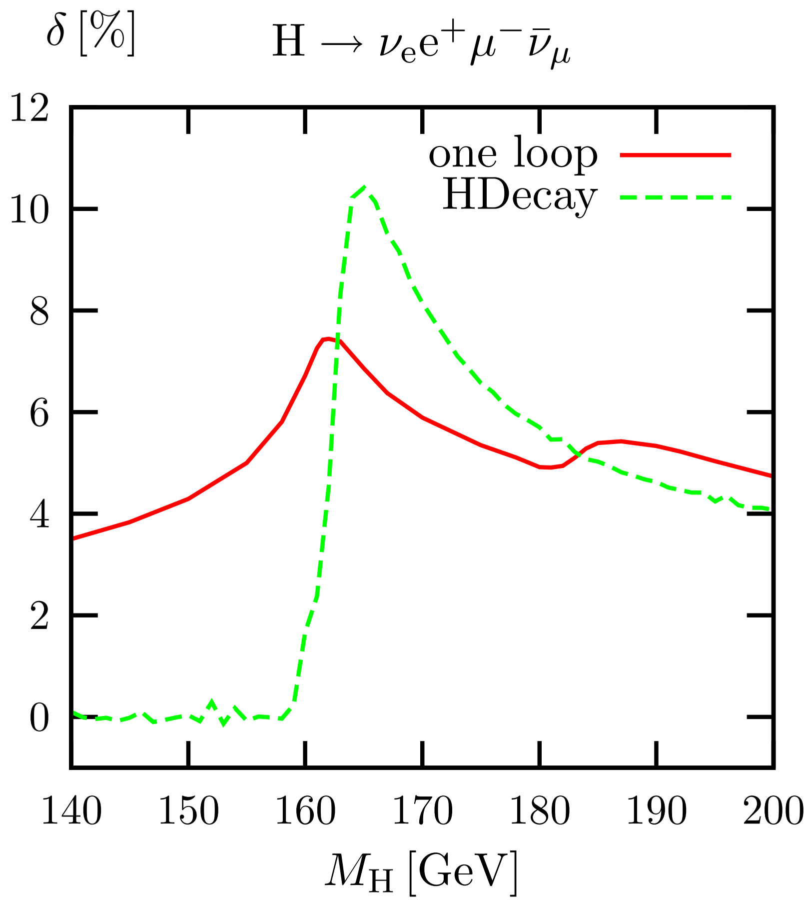
Consistency checks

- UV: independence of μ in dimensional regularization
- Soft IR: independence of photon mass (regulator for soft singularities)
- Collinear IR: independence of external fermion masses (needed to describe collinear singularities) in inclusive case
- Gauge independence: calculation in ’t Hooft–Feynman and background-field gauge (Denner, Dittmaier, Weiglein ’94)
- Different methods for combining soft and collinear singularities: dipole subtraction and phase-space slicing
- Last but not least: two completely independent calculations

Impact of $\mathcal{O}(\alpha)$ corrections

Partial decay width for $\mathrm{H} \to \mathrm{WW} \to \nu_\mathrm{e}\mathrm{e}^+\mu^-\bar{\nu}_\mu$ $\quad$ G_μ-scheme

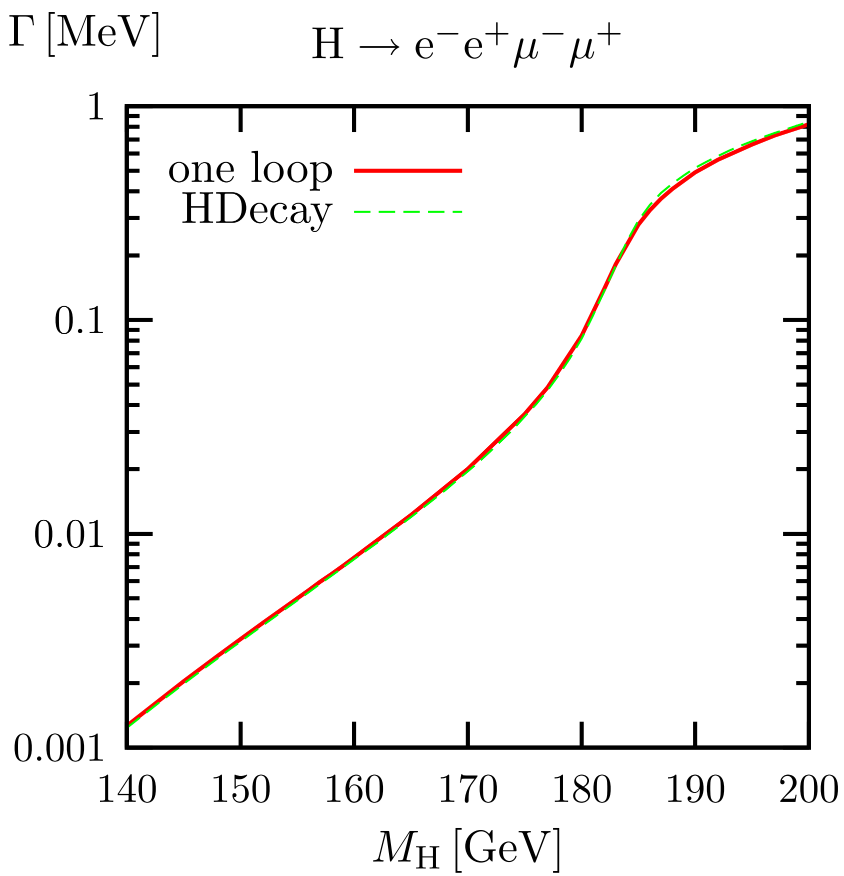


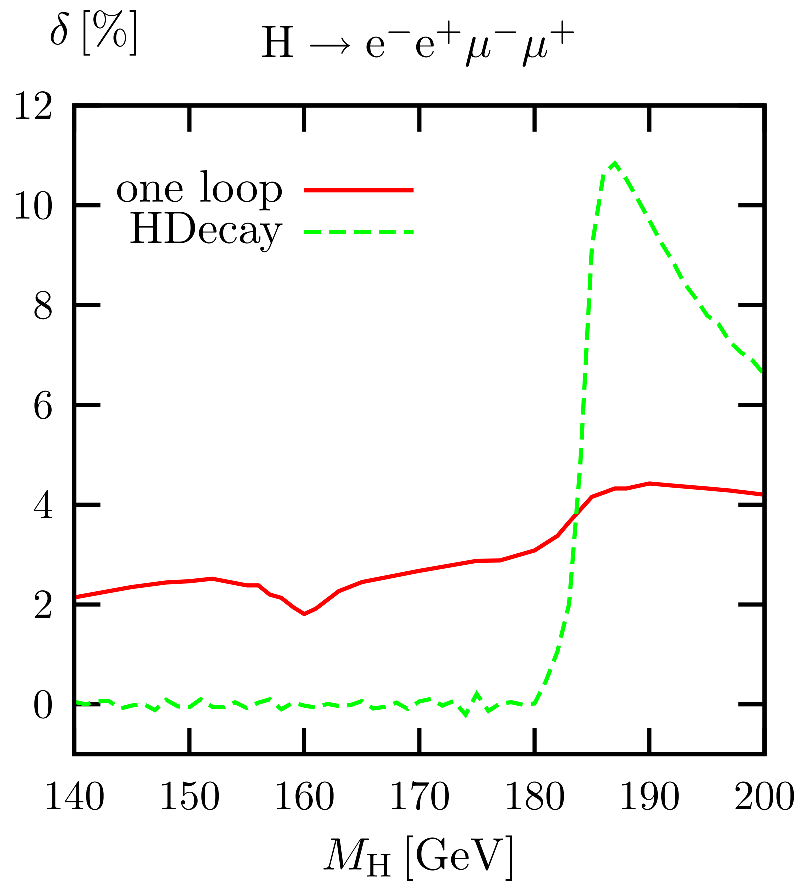


$$\delta = \frac{\Gamma}{\Gamma_{\mathrm{H}\to 4f,\mathrm{Born}}} - 1$$

Impact of $\mathcal{O}(\alpha)$-corrections

Partial decay width for $\mathrm{H} \to \mathrm{ZZ} \to \mathrm{e}^-\mathrm{e}^+\mu^-\mu^+$ $\qquad G_\mu$-scheme

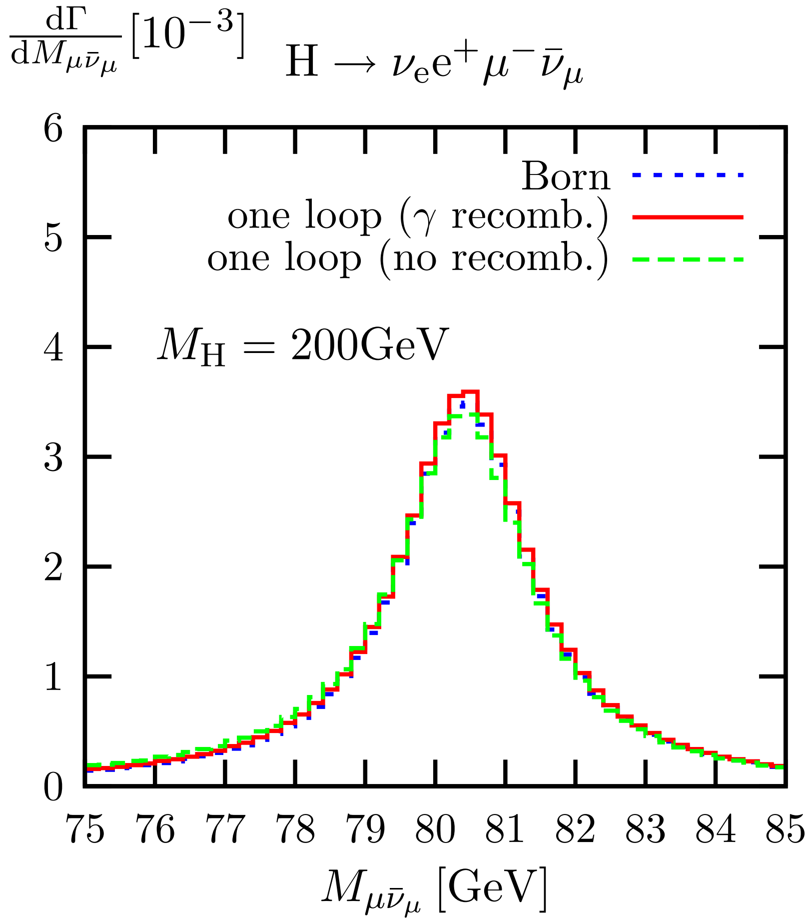


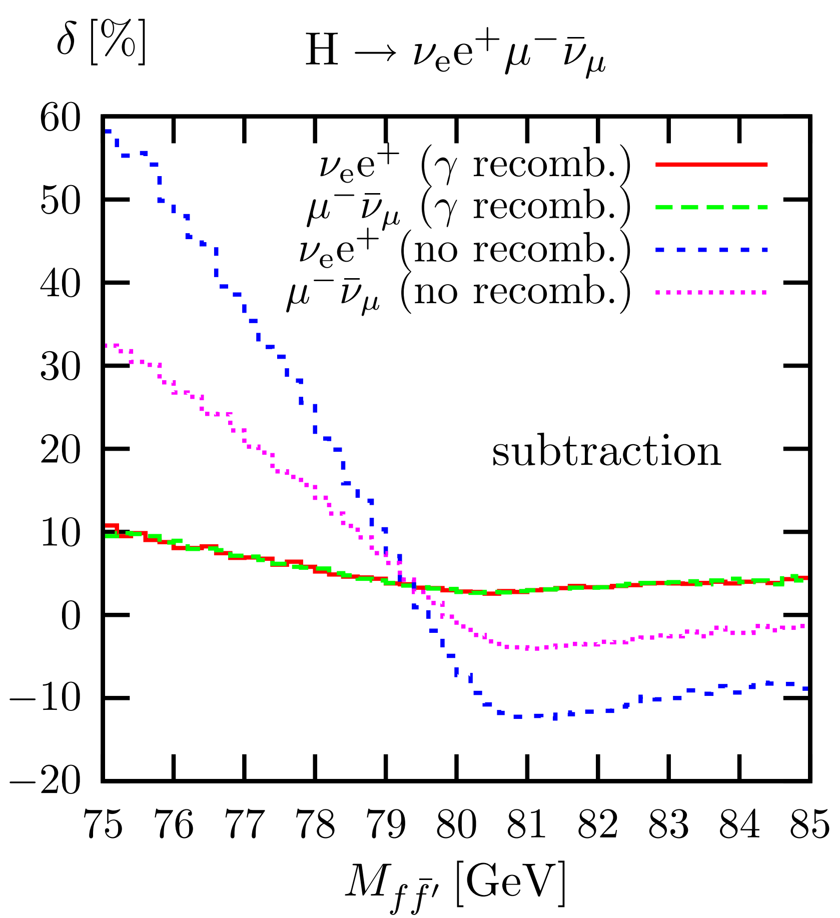


$$\delta = \frac{\Gamma}{\Gamma_{\mathrm{H}\to 4f,\mathrm{Born}}} - 1$$

Impact of $\mathcal{O}(\alpha)$-corrections

W-invariant-mass distribution for $\mathrm{H} \to \mathrm{WW} \to \nu_e e^+ \mu^- \bar{\nu}_\mu$ $\quad G_\mu$-scheme

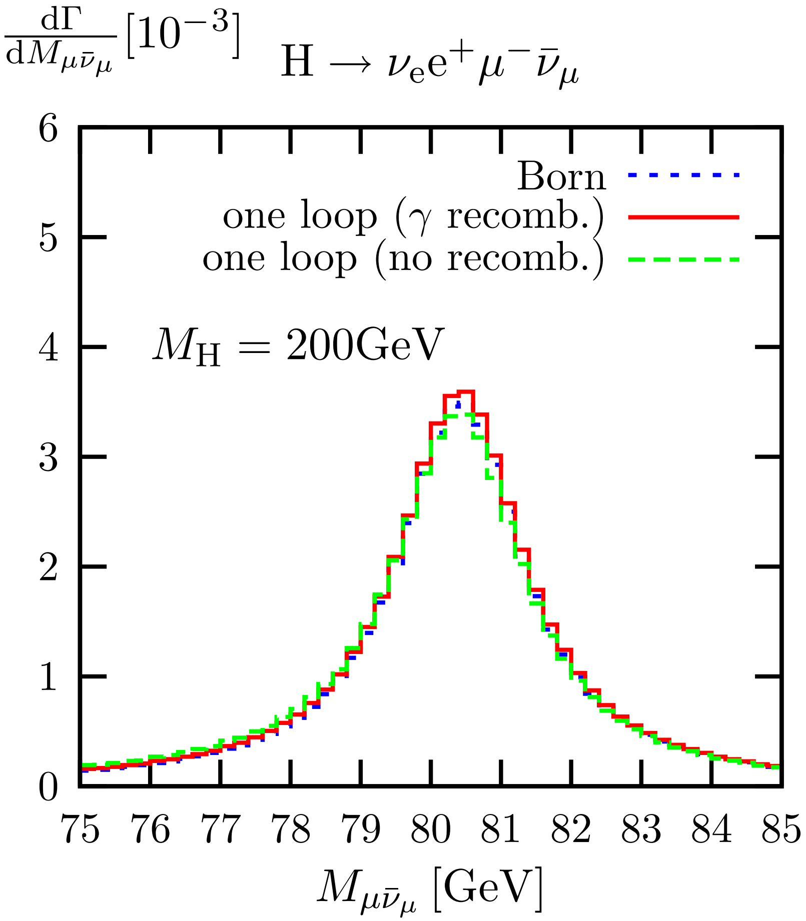


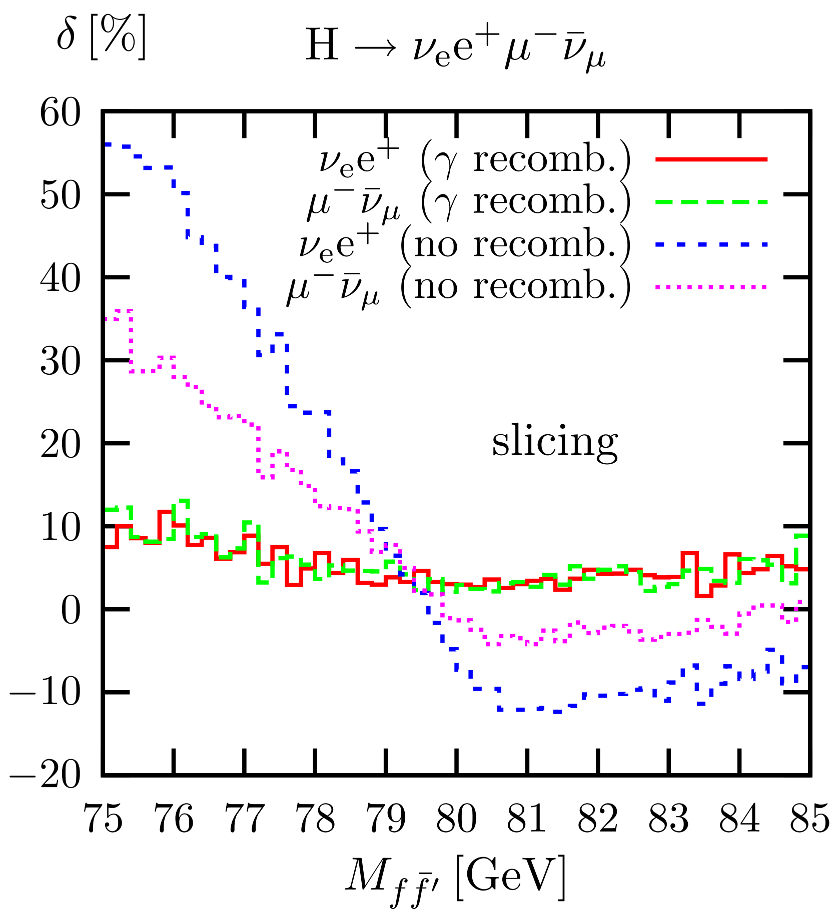


γ recombination if $M_{e^+\gamma/\mu^-\gamma} < 5\,\mathrm{GeV}$

Impact of $\mathcal{O}(\alpha)$-corrections

W-invariant-mass distribution for $\mathrm{H} \to \mathrm{WW} \to \nu_\mathrm{e}\mathrm{e}^+\mu^-\bar{\nu}_\mu$ $\qquad G_\mu$-scheme

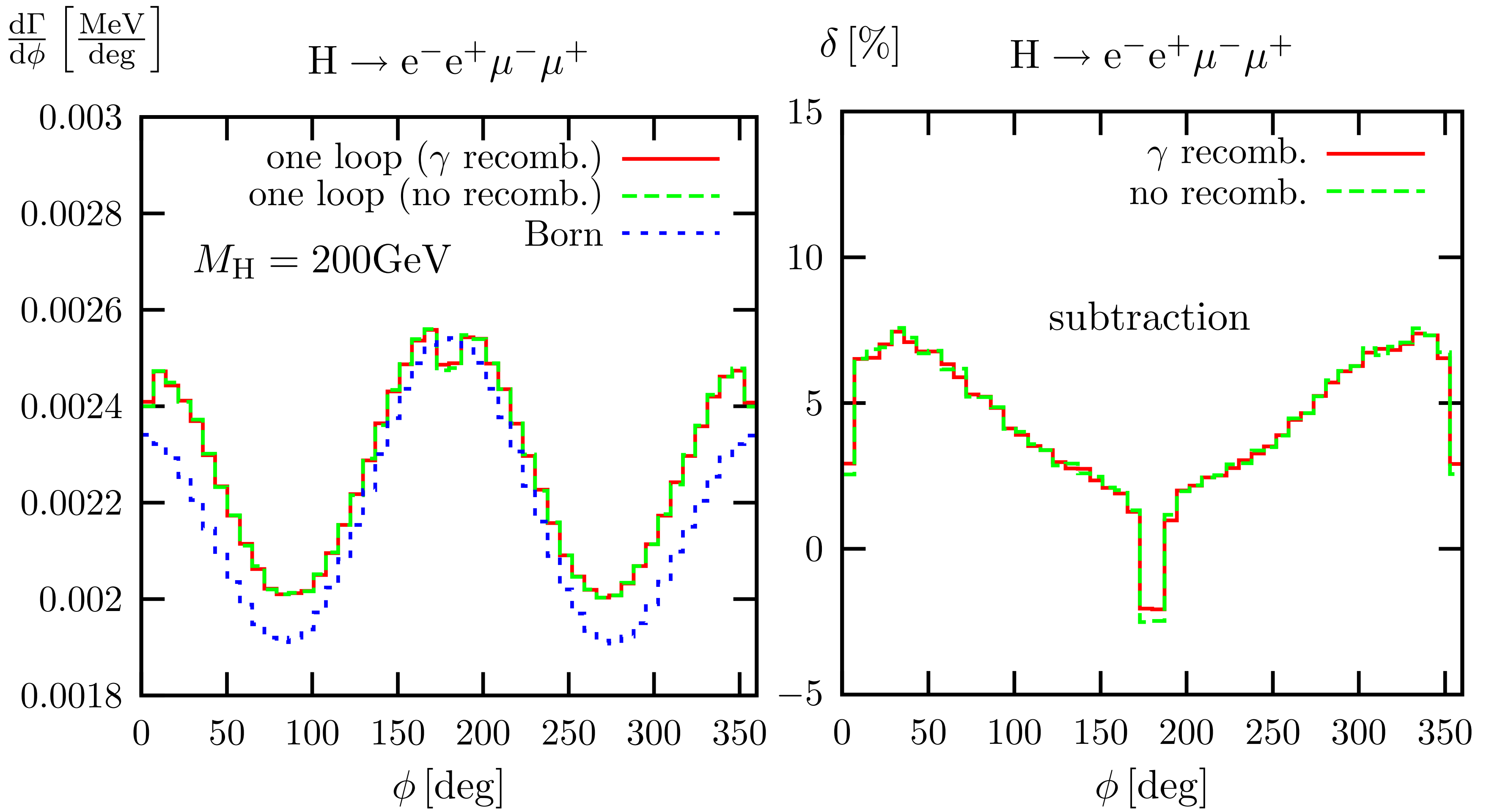




γ recombination if $M_{\mathrm{e}^+\gamma/\mu^-\gamma} < 5\,\mathrm{GeV}$

Impact of $\mathcal{O}(\alpha)$-corrections

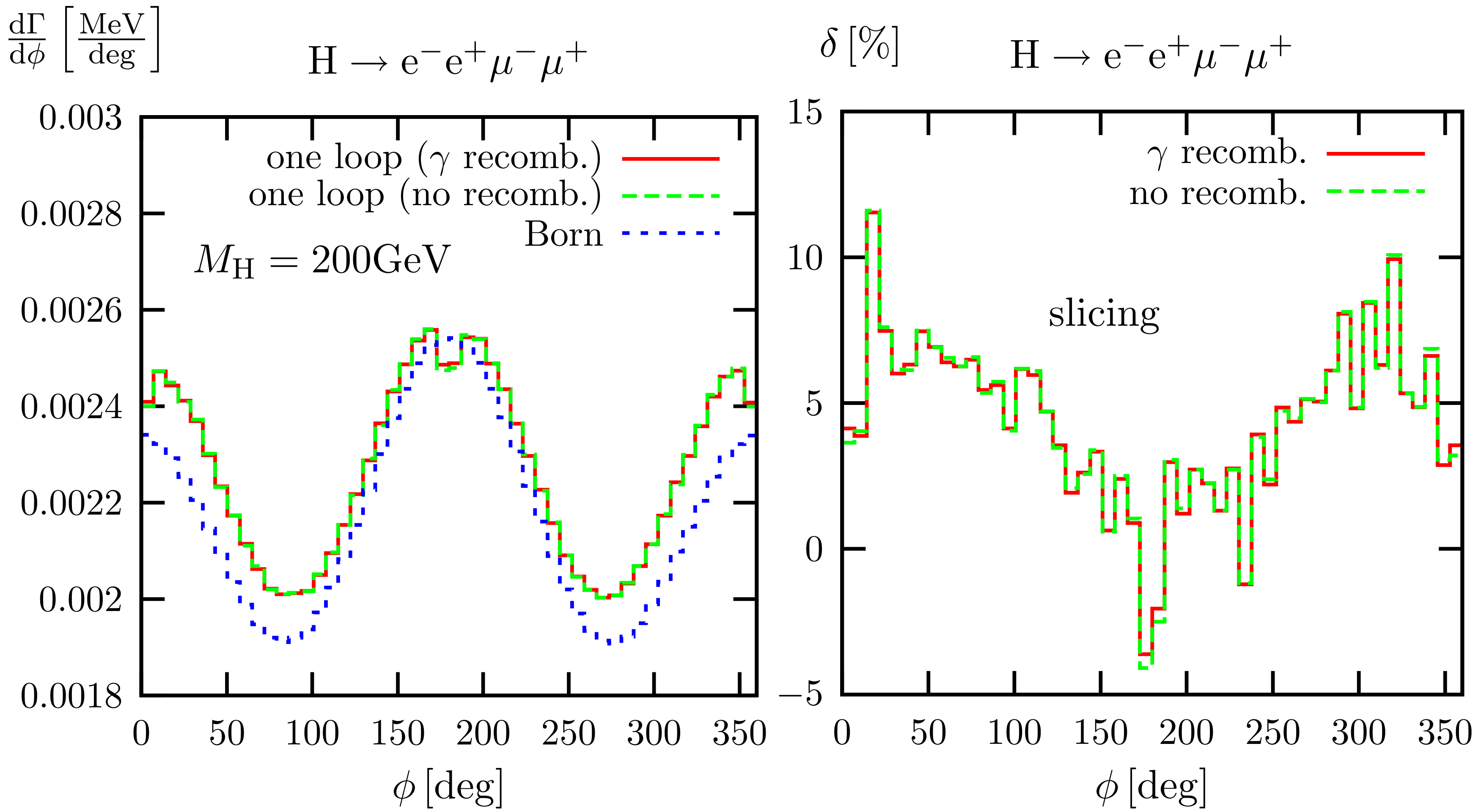
Angle between decay planes for $\mathrm{H} \to \mathrm{ZZ} \to \mathrm{e}^-\mathrm{e}^+\mu^-\mu^+$ $\qquad G_\mu$-scheme



$$\cos\phi = \frac{((p_1+p_2)\times p_1)(-(p_3+p_4)\times p_3)}{|(p_1+p_2)\times p_1||-(p_3+p_4)\times p_3|}, \quad p_\mathrm{H} = p_1+p_2+p_3+p_4(+p_\gamma)$$

Impact of $\mathcal{O}(\alpha)$-corrections

Angle between decay planes for $\mathrm{H} \to \mathrm{ZZ} \to \mathrm{e^-e^+}\mu^-\mu^+$ G_μ-scheme



$$\cos\phi = \frac{((p_1+p_2)\times p_1)(-(p_3+p_4)\times p_3)}{|(p_1+p_2)\times p_1||-(p_3+p_4)\times p_3|}, \quad p_\mathrm{H} = p_1+p_2+p_3+p_4(+p_\gamma)$$

Summary

- Precise description of $\mathrm{H} \to \mathrm{WW}^{(*)}/\mathrm{ZZ}^{(*)}$ requires calculation of radiative corrections to $\mathrm{H} \to \mathrm{WW}/\mathrm{ZZ} \to 4f$, in particular at and below threshold
- Implementation of new techniques for tensor reduction (Denner, Dittmaier ’05) and complex-mass scheme at one loop
- Monte Carlo generator with multi-channel integration with adaptive optim.
- Matching of soft and collinear singularities is done with the dipole subtraction or phase-space slicing method
- First results presented, RCs $\sim \mathcal{O}(2-8\%)$ in $\Gamma_{\mathrm{H}\to 4f}$, much larger in distributions (depending on photon treatment)
- Things to be done:
  - ◇ QCD corrections to $\mathrm{H} \to 2q2l, 4q$
  - ◇ final-state radiation beyond leading order
  - ◇ comparison with narrow-width approximation