Two-loop electroweak corrections to the effective weak mixing angle

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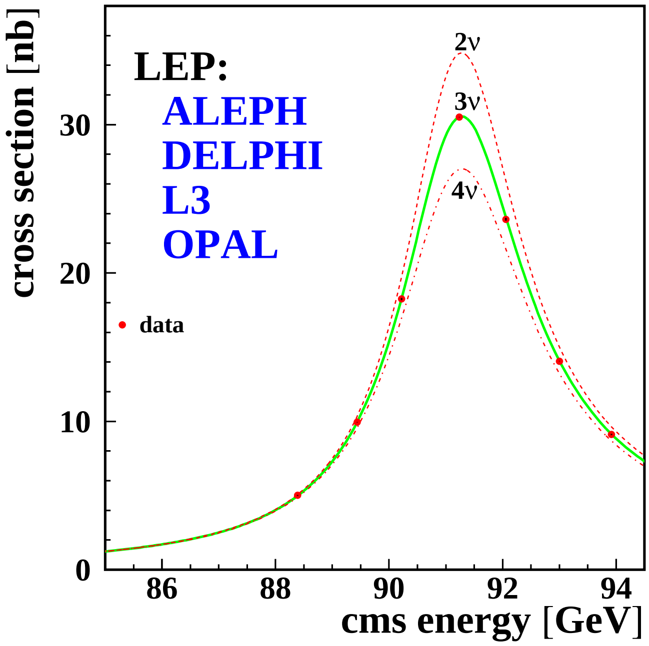
- **The effective mixing angle**
- **Fermionic 2-loop contributions**
- **Higgs-dependent bosonic 2-loop contributions**

Z resonance



Z-boson observables can be expressed in terms of

- effective Z boson couplings:

$$g_V^f \to g_V^f + \Delta g_V^f, \qquad g_A^f \to g_A^f + \Delta g_A^f$$

with higher order contributions in $\Delta g_{V,A}^f$

- effective ew mixing angle (for $f = e$):

$$\sin^2\theta_{\text{eff}} = \frac{1}{4}\left(1 - \text{Re}\,\frac{g_V^e}{g_A^e}\right)$$

complete at 1-loop order, 2-loop fermionic contributions

effective leptonic mixing angle

- very sensitive to M_H

- exp.: asymmetries of the Z resonance

  $\sin^2\theta_{\text{eff}} = 0.23153 \pm 0.00016$

- linear collider: expected precision 1.3×10^{-5}

$\Rightarrow$ precise theoretical prediction needed

$$\sin^2\theta_{\text{eff}} = \kappa \left(1 - \frac{M_W^2}{M_Z^2}\right) \equiv \kappa\, s_W^2$$

$$\kappa = 1 + \Delta\kappa$$

existing calculations

- One loop corrections $\mathcal{O}(\alpha)$
  Marciano, Sirlin '80

- QCD corrections $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}\left(\alpha\alpha_s^2\right)$
  Djouadi '88
  Chetyrkin, Kühn, Steinhauser '95

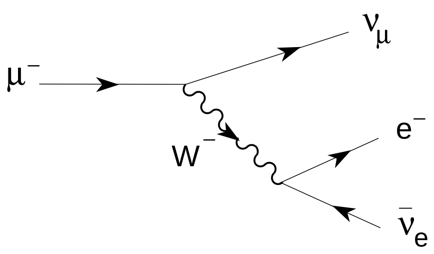
- electroweak higher-order corrections $\mathcal{O}\left(\alpha^2\right)$
  - two gauge-invariant subsets:
    fermionic/bosonic corrections
  - leading terms in top-mass expansion
    $\mathcal{O}\left(\alpha^2 m_t^4\right)$ and $\mathcal{O}\left(\alpha^2 m_t^2\right)$
    Degrassi, Gambino, Sirlin '97
  - 3-loop leading universal terms through
    ρ parameter
    Faisst, Kühn, Seidensticker, Veretin '03
    Boughezal, Tausk, van der Bij '04, '05
  - complete 2-loop fermionic contributions
    Awramik, Czakon, Freitas, Weiglein '04
    WH, Meier, Uccirati '05
  - 2-loop bosonic contributions,
    Higgs-dependent subset
    WH, Meier, Uccirati '05

$M_W - M_Z$ correlation

Definition of Fermi constant G_F via muon lifetime:

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)(1 + \Delta q)$$

Δq: QED corrections in Fermi Model,
included in definition



SM prediction:

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2\left(1 - M_W^2/M_Z^2\right)}(1 + \Delta r)$$

Δr: quantum correction, $\Delta r = \Delta r(m_t, M_H, \dots)$

complete at 1-loop and 2-loop order

$$\rightarrow \quad M_W = M_W(\alpha, G_F, M_Z, m_t, M_H)$$

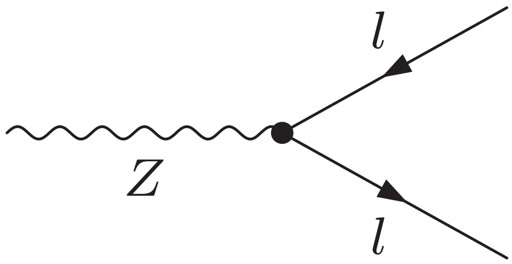
Δr complete at the electroweak 2-loop level

- fermionic 2-loop contributions:
  Freitas, WH, Walter, Weiglein '00, '02;
  Awramik, Czakon '03

- bosonic 2-loop contributions:
  Awramik, Czakon '02
  Onishchenko, Veretin '02
  Awramik, Czakon, Onishchenko, Veretin '02

$\Delta\kappa$ **from on-shell** Zll**-vertex**

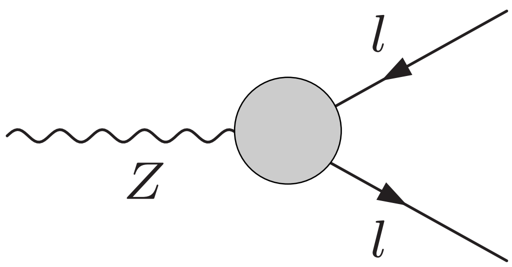
tree-level:



$$\Gamma_\mu^{(0)} \; = \; \gamma_\mu \, [g_V - g_A \gamma_5]$$

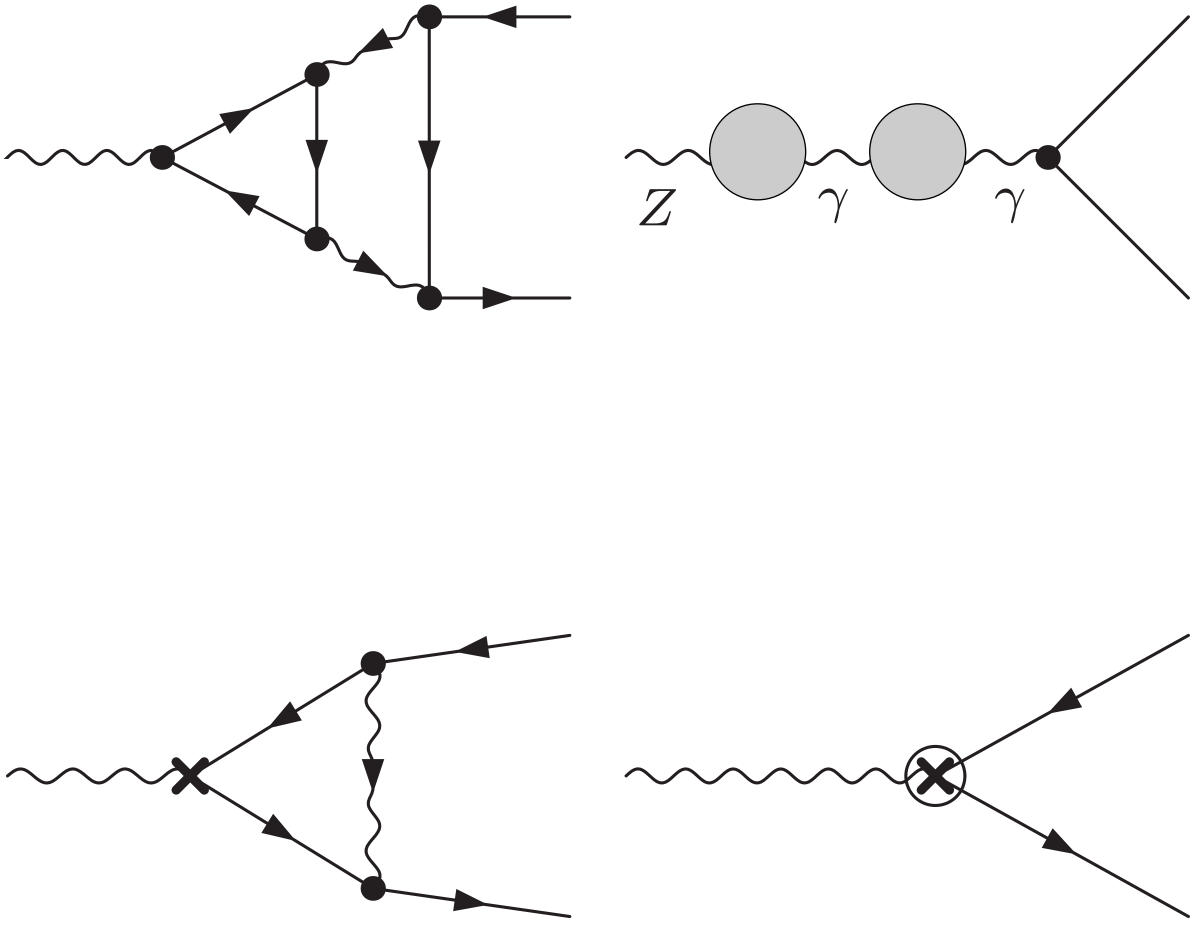
$g_{V/A}$: vector/axial born coupling

loop contributions:



$$\Delta\Gamma_\mu \; = \; \gamma_\mu \, [\Delta g_V - \Delta g_A \gamma_5]$$

classes of diagrams at 2-loop order



Calculation of two-loop corrections

- problems:
  - 2-loop renormalization
  - 2-loop vertices for $q^2 \neq 0$

- strategy: split into two UV-finite parts

$$\hat{\Gamma}^{Z\bar{l}l}_{(2)}\left(M_Z^2\right) = \Gamma^{Z\bar{l}l}_{(2)}\left(M_Z^2\right) + \delta Z^{Z\bar{l}l}_{(2)} =$$

$$\underbrace{\left(\Gamma^{Z\bar{l}l}_{(2)}(0) + \delta Z^{Z\bar{l}l}_{(2)}\right)}_{\text{finite}} + \underbrace{\left(\Gamma^{Z\bar{l}l}_{(2)}\left(M_Z^2\right) - \Gamma^{Z\bar{l}l}_{(2)}(0)\right)}_{\text{finite}}$$

  - first term : complete 2-loop renormalization but no genuine 2-loop vertices
  - second term : all 2-loop vertices but simple divergence-structure

Two-loop counterterm for $\sin^2\theta_{\text{eff}}$

$$
\begin{aligned}
\delta\sin^2\theta_{\text{eff}}^{(2)} &= \delta s_W^2 + \frac{1}{2 s_W c_W}\delta Z_{(2)}^{\gamma Z} \\
&\quad + s_W^2\left(s_W^2 - c_W^2\right)\delta Z_{(2)}^{\ell,L} \\
&\quad - s_W^2\left(s_W^2 - c_W^2\right)\delta Z_{(2)}^{\ell,R} \\
&\quad + \text{products of one loop counterterms}
\end{aligned}
$$

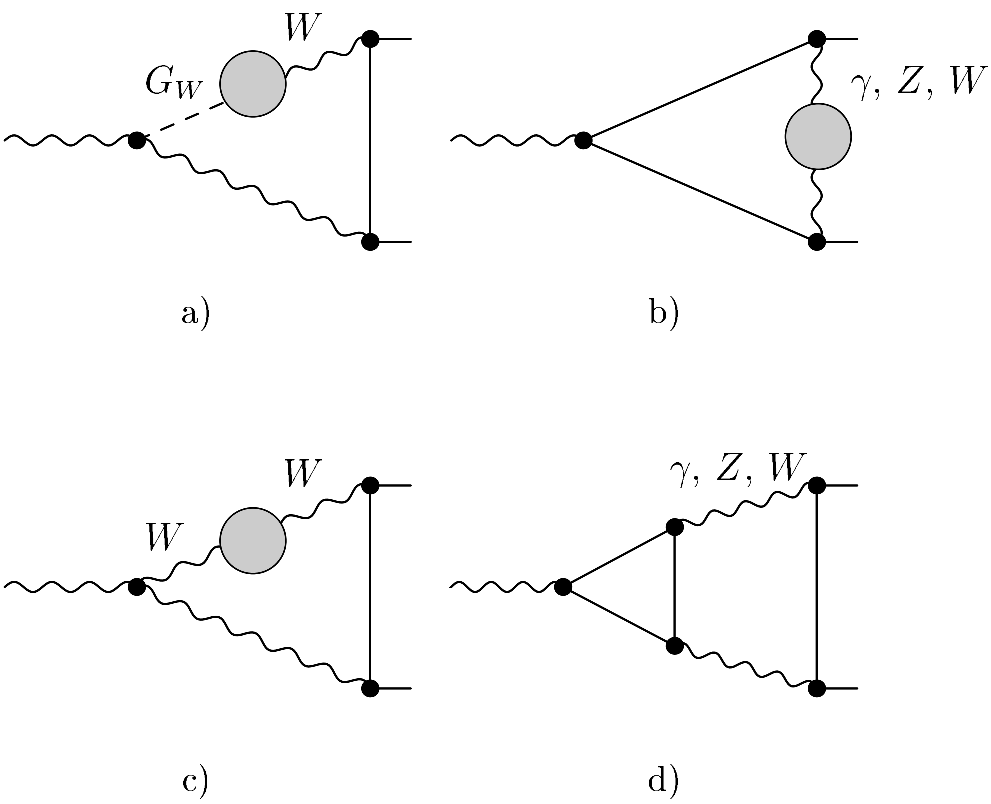
$$
\begin{aligned}
\delta s_W^2 &= \frac{M_W^2}{M_Z^2}\left(\frac{\delta M_{Z(2)}^2}{M_Z^2} - \frac{\delta M_{W(2)}^2}{M_W^2}\right) \\
&\quad - \frac{M_W^2}{M_Z^2}\frac{\delta M_{Z(1)}^2}{M_Z^2}\left(\frac{\delta M_{Z(1)}^2}{M_Z^2} - \frac{\delta M_{W(1)}^2}{M_W^2}\right)
\end{aligned}
$$

$\delta M_{W,Z}$: W, Z mass counterms in the on-shell scheme

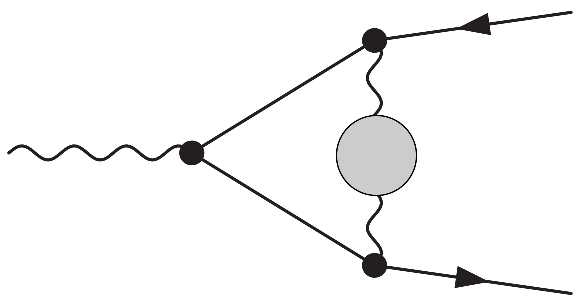
$\delta Z^{\ell,L}$, $\delta Z^{\ell,R}$: lepton field renormalization constants

$\delta Z^{\gamma Z}$: field renormalization for γZ 2-point function

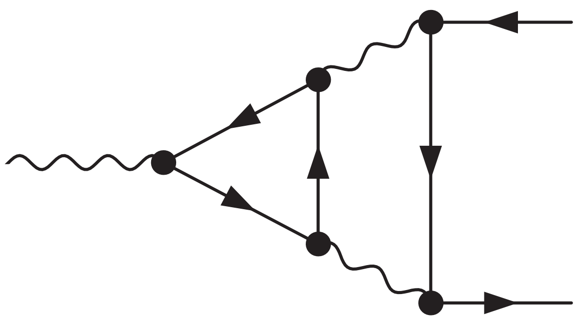
Irreducible 2-loop vertex diagrams



two types of vertex diagrams



dispersion relation
$\longrightarrow$ one-dimensional integral representation



Feynman-parameters, analytical manipulations
Ferroglia, Passera, Passarino, Uccirati
$\longrightarrow$ up to 4-dimensional integration

results for $\Delta\kappa$ $\left([\Delta\kappa] = 10^{-4}\right)$

$\sin^2\theta_{\text{eff}} =: (1 - M_W^2/M_Z^2)\,(1 + \Delta\kappa)$

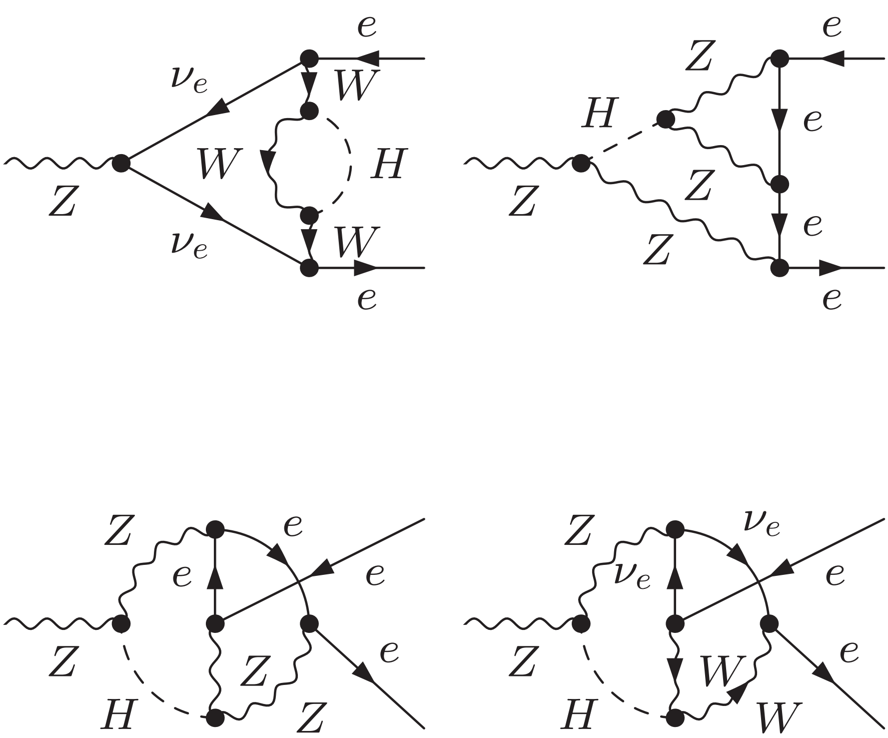
M_H [GeV]	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha^2)$	prev. calc.
100	438.937	-0.633(1)	-0.63
200	419.599	-2.161(1)	-2.16
600	379.560	-5.008(1)	-5.01
1000	358.619	-4.733(1)	-4.73

M_H [GeV]	2 ferm. loops	1 ferm. loop
100	13.758	-14.391(1)
200	13.758	-15.919(1)
600	13.758	-18.766(1)
1000	13.758	-18.491(1)

$M_W = 80.426$ GeV, $M_Z = 91.1876$ GeV,
$\Gamma_Z = 2.4952$ GeV, $m_t = 178.0$ GeV,
$\Delta\alpha\left(M_Z^2\right) = 0.05907$, $\alpha_s\left(M_Z^2\right) = 0.117$,
$G_F = 1.16637 \times 10^{-5}$.

Bosonic 2-loop contributions, Higgs-dependent part

examples for vertices:



difference $\Delta\kappa(M_H) - \Delta\kappa(M_H^0) = \Delta\kappa_{sub}$

UV finite

Numerical results ($M_H^0 = 100$ GeV)

$M_H\ [GeV]$	$\Delta\kappa^{(\alpha^2)}_{ferm,sub} \times 10^{-4}$	$\Delta\kappa^{(\alpha^2)}_{bos,sub} \times 10^{-4}$
100	0	0
200	-1.528	0.265
600	-4.375	0.914
1000	-4.100	1.849

$$\sin^2\theta_{\text{eff}} = (1 + \Delta\kappa)\left(1 - \frac{M_W^2}{M_Z^2}\right)$$

$\sin^2\theta_{\text{eff}}$ varies by $4 \cdot 10^{-5}$
when M_W is kept fixed ($\leftarrow$ exp)

G_F, $\Delta r \rightarrow M_W$:
strong compensation of M_H-dependence
between $\Delta\kappa(M_H)$ and $M_W(M_H)$
in the bosonic 2-loop terms

Conclusions

- effective leptonic weak mixing angle
  is an important precision observable

- electroweak 2-loop corrections
  are needed

- we performed an independent calculation of the fermionic corrections in agreement with Awramik, Czakon, Freitas, Weiglein

- bosonic corrections are approached with the same methods

  Higgs-dependent part completed

  remaining constant part at work
  preliminary result [Czakon, RADCOR 2005]