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Order $\alpha\alpha_s$ corrections to the quark propagator in the Standard Model

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Perspectives at ILC

A threshold scan of the top-antitop electromagnetic production at ILC will enable the extraction of m_{top} with < 100 MeV. (exp) accuracy.

For theoreticians to match this precision it is fundamental to fix the $\mathcal{O}(\alpha_s^3)$ corrections the NREFT matching coefficient for the vector current production:

$$\mathcal{L}_{\text{bilinear}}(x) = \sum_{\mathbf{p}} \psi_{\mathbf{p}}^\dagger(x)\left\{ iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m_t} + \frac{\mathbf{p}^4}{8m_t^3} + \frac{i}{2}\Gamma_t\left(1-\frac{\mathbf{p}^2}{2m_t^2}\right) - \delta m_t \right\}\psi_{\mathbf{p}}(x) + (\psi_{\mathbf{p}} \to \chi_{\mathbf{p}}),$$

$$\Delta\mathcal{L} = \sum_{\mathbf{p}} (C_V \mathcal{O}_{V,\mathbf{p}} + C_A \mathcal{O}_{A,\mathbf{p}}) + \text{h.c.}$$

$$\mathcal{O}_{V,\mathbf{p}} = [\bar{e}\,\gamma_j\, e]\,\mathcal{O}^j_{\mathbf{p},1}, \qquad \mathcal{O}_{A,\mathbf{p}} = [\bar{e}\,\gamma_j\gamma_5\, e]\,\mathcal{O}^j_{\mathbf{p},1}, \qquad \mathcal{O}^j_{\mathbf{p},1} = \left[\psi_{\mathbf{p}}^\dagger\,\sigma_j(i\sigma_2)\chi^*_{-\mathbf{p}}\right]$$

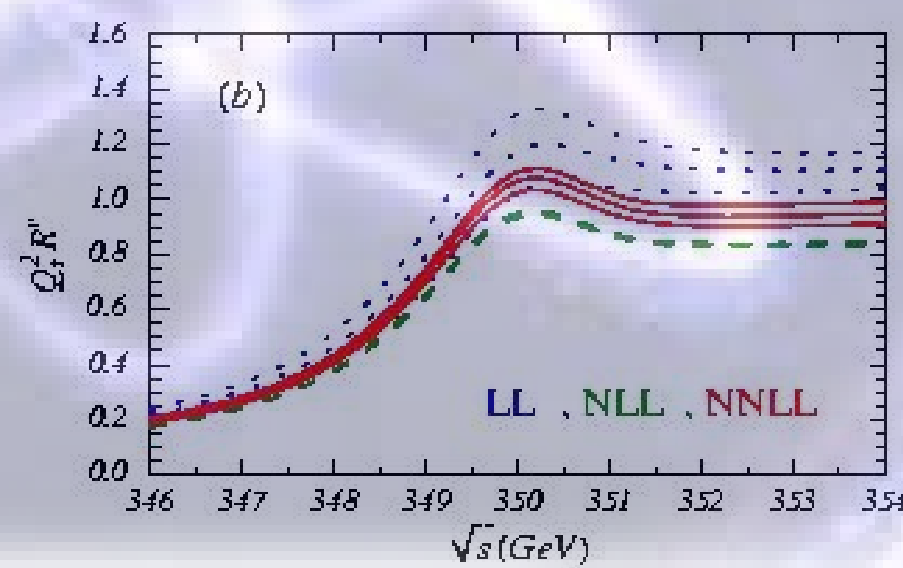
A.Hoang et al.

Besides:

- The pure strong, three-loop correction,

is needed also (parametrically of the same size):

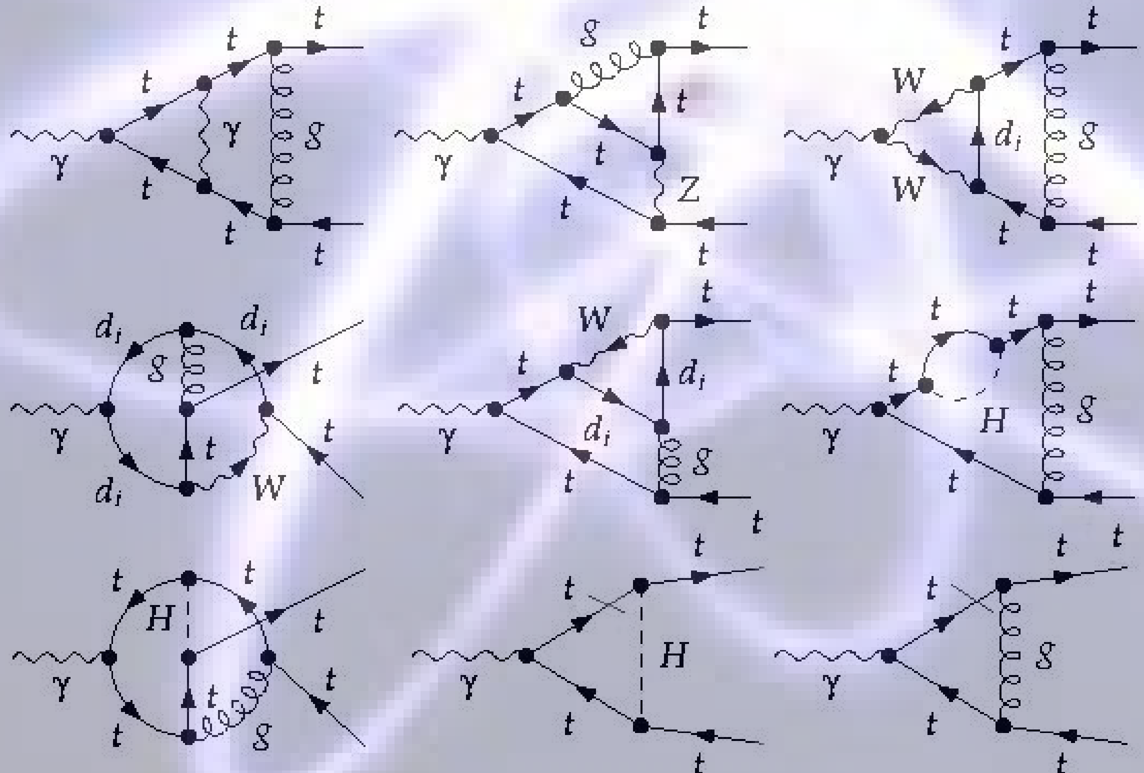
- The mixed strong-electroweak, two-loop correction.



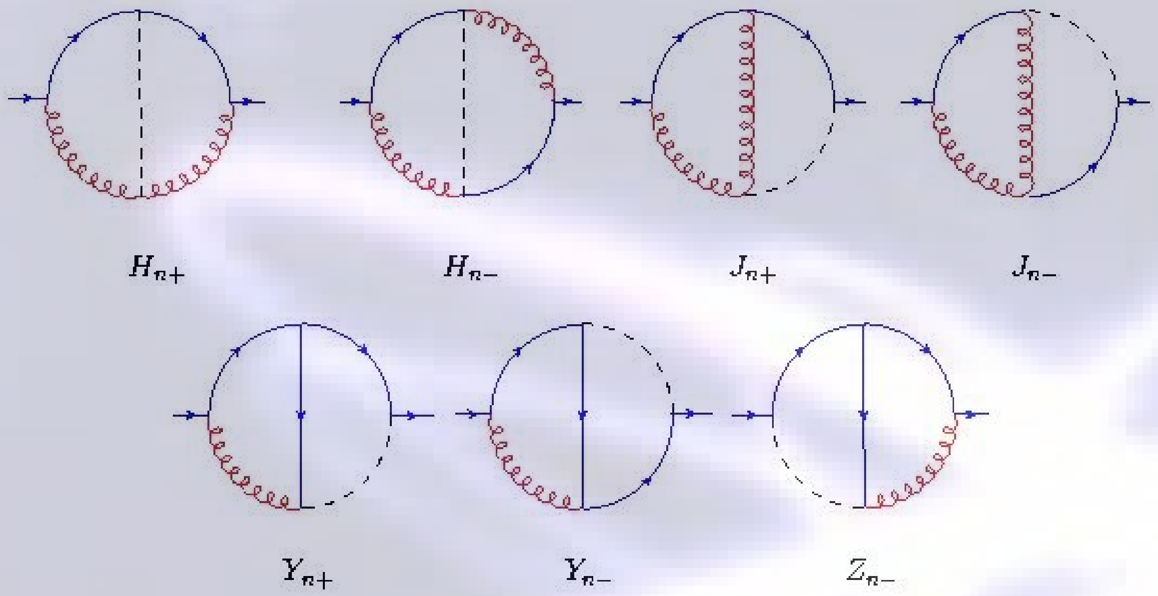
Vertex graphs

Ingredients of the $\mathcal{O}(\alpha\alpha_s)$ vertex correction (calculated at threshold):

- The pure two-loop diagrams (sample shown) with a gluon and a eweak boson: H, χ, Z (neutral) and ρ, W (charged).
- Mass counterterm in the one-loop vertex (last two graphs).
- Two-loop correction ($\alpha\alpha_s$) to the wave function renormalization (Z_2).
- Coupling renormalization (also implies one-loop diagrams).



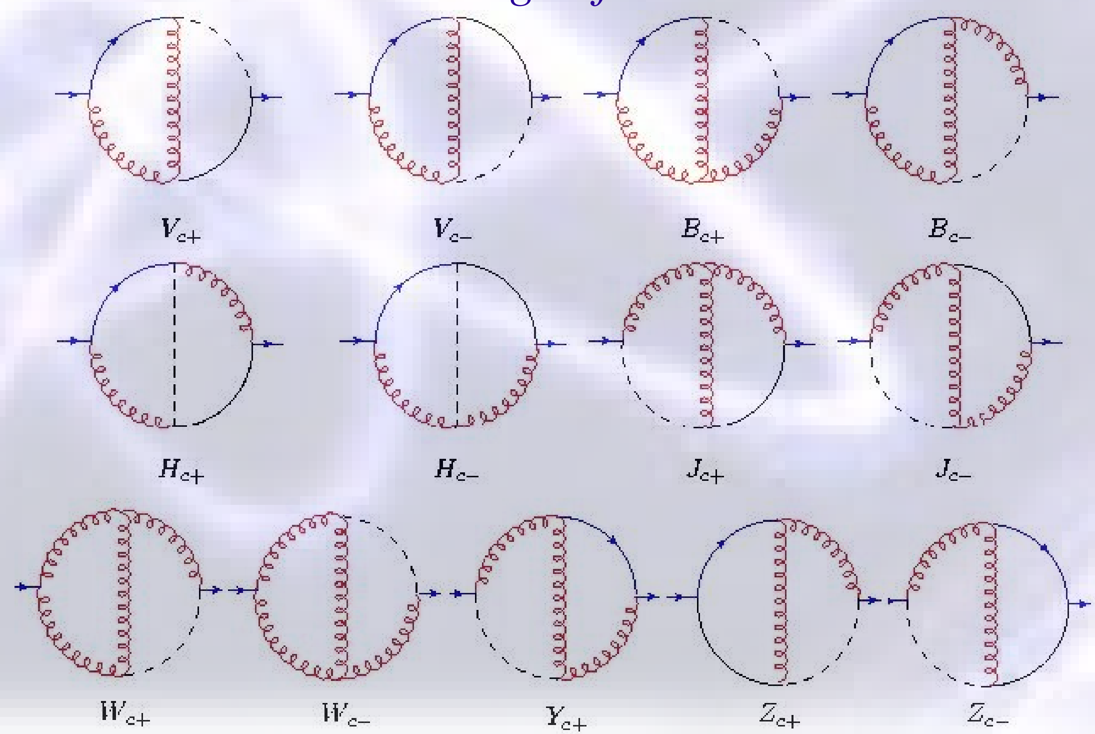
Integral types



Neutral integrals

All these types arise in the pure 2-loop vertex diagrams (after partial fractioning) and only some in the Z2 correction. The vertex ones show non-standard signs for the momentum.

Charged integrals



Example: one two-loop vertex graph

Diagrams are constructed using:

- Form: indexes of the integrals arbitrary high/low.
- AIR: performs integration by parts. Indexes become -1,0,1. Anastasiou, Lazopoulos.
- Identification work: minimal set of Master Integrals.

```
[H-](1,1,0,1,0)*M1^-2*M2^8-1088/81/(2*M1^2*M2^6-16*M1^4*M2^4+32*M1^6*M2^2)*[H-](1,1,0,1,0)*M1^2*
M2^4+1280/81/(2*M1^2*M2^6-16*M1^4*M2^4+32*M1^6*M2^2)*[H-](1,1,0,1,0)*M1^4*M2^2-1024/81/(2*M1^2*
M2^6-16*M1^4*M2^4+32*M1^6*M2^2)*[H-](1,1,0,1,0)*M1^6+496/81/(2*M1^2*M2^6-16*M1^4*M2^4+32*M1^6*
M2^2)*[H-](1,1,0,1,0)*M2^6-8/27/(2*M1^2*M2^6-16*M1^4*M2^4+32*M1^6*M2^2)*[Z+](1,-1,0,1,1)*M1^-2*
M2^6+32/27/(2*M1^2*M2^6-16*M1^4*M2^4+32*M1^6*M2^2)*[Z+](1,-1,0,1,1)*M1^2*M2^2-128/27/(2*M1^2*
M2^6-16*M1^4*M2^4+32*M1^6*M2^2)*[Z+](1,-1,0,1,1)*M1^4+32/27/(2*M1^2*M2^6-16*M1^4*M2^4+32*M1^6*
M2^2)*[Z+](1,-1,0,1,1)*M2^4+8/27/(2*M1^2*M2^6-16*M1^4*M2^4+32*M1^6*M2^2)*[Y+](0,-1,1,1,1)*M1^-2*
M2^6+32/27/(2*M1^2*M2^6-16*M1^4*M2^4+32*M1^6*M2^2)*[Y+](0,-1,1,1,1)*M1^2*M2^2+128/9/(2*M1^2*
M2^6-16*M1^4*M2^4+32*M1^6*M2^2)*[Y+](0,-1,1,1,1)*M1^4-64/27/(2*M1^2*M2^6-16*M1^4*M2^4+32*M1^6*
M2^2)*[Y+](0,-1,1,1,1)*M2^4-56/81/(2*M1^2*M2^6-16*M1^4*M2^4+32*M1^6*M2^2)*[Y-](1,0,0,0,1)*M1^-2*
M2^6+256/27/(2*M1^2*M2^6-16*M1^4*M2^4+32*M1^6*M2^2)*[Y-](1,0,0,0,1)*M1^2*M2^2-256/9/(2*M1^2*M2^6-
16*M1^4*M2^4+32*M1^6*M2^2)*[Y-](1,0,0,0,1)*M1^4+176/81/(2*M1^2*M2^6-16*M1^4*M2^4+32*M1^6*M2^2)*
[Y-](1,0,0,0,1)*M2^4+188/(-48*M1^2*M2^8+288*M1^4*M2^6-768*M1^6*M2^4+768*M1^8*M2^2+3*M2^10)*
[H+](0,0,1,1,1)*M1^2*M2^6*ri-336/(-48*M1^2*M2^8+288*M1^4*M2^6-768*M1^6*M2^4+768*M1^8*M2^2+3*M2^10)*
[H+](0,0,1,1,1)*M1^4*M2^4*ri+620/(-48*M1^2*M2^8+288*M1^4*M2^6-768*M1^6*M2^4+768*M1^8*M2^2+3*M2^10)*
[H+](0,0,1,1,1)*M1^6*M2^2*ri-488/(-48*M1^2*M2^8+288*M1^4*M2^6-768*M1^6*M2^4+768*M1^8*M2^2+3*M2^10)*
[H+](0,0,1,1,1)*M1^8*ri+16/(-48*M1^2*M2^8+288*M1^4*M2^6-768*M1^6*M2^4+768*M1^8*M2^2+3*M2^10)*
[H+](0,0,1,1,1)*M2^8*ri-648/(-20*M1^2*M2^8+160*M1^4*M2^6-640*M1^6*M2^4+1280*M1^8*M2^2-1024*M1^10+
M2^10)*[H+](0,0,1,1,1)*M1^2*M2^6*ri+1104/(-20*M1^2*M2^8+160*M1^4*M2^6-640*M1^6*M2^4+1280*M1^8*
M2^2-1024*M1^10+M2^10)*[H+](0,0,1,1,1)*M1^4*M2^4*ri-1434/(-20*M1^2*M2^8+160*M1^4*M2^6-640*M1^6*
M2^4+1280*M1^8*M2^2-1024*M1^10+M2^10)*[H+](0,0,1,1,1)*M1^6*M2^2*ri+780/(-20*M1^2*M2^8+160*M1^4*
M2^6-640*M1^6*M2^4+1280*M1^8*M2^2-1024*M1^10+M2^10)*[H+](0,0,1,1,1)*M1^8*ri+117/(-20*M1^2*M2^8+
160*M1^4*M2^6-640*M1^6*M2^4+1280*M1^8*M2^2-1024*M1^10+M2^10)*[H+](0,0,1,1,1)*M2^8*ri-4/81/(-16*
M1^2*M2^8+32*M1^4*M2^6+2*M2^10)*[J-](1,-1,0,1,1)*M1^-2*M2^8*ri+32/81/(-16*M1^2*M2^8+32*M1^4*M2^6+
2*M2^10)*[J-](1,-1,0,1,1)*M1^2*M2^4*ri+8/81/(-16*M1^2*M2^8+32*M1^4*M2^6+2*M2^10)*[J-](1,-1,0,1,1)*
M2^6*ri+180/(-16*M1^2*M2^8+96*M1^4*M2^6-256*M1^6*M2^4+256*M1^8*M2^2+M2^10)*[H+](0,0,1,1,1)*M1^2*
M2^6*ri-330/(-16*M1^2*M2^8+96*M1^4*M2^6-256*M1^6*M2^4+256*M1^8*M2^2+M2^10)*[H+](0,0,1,1,1)*M1^4*
M2^4*ri+420/(-16*M1^2*M2^8+96*M1^4*M2^6-256*M1^6*M2^4+256*M1^8*M2^2+M2^10)*[H+](0,0,1,1,1)*M1^6*
M2^2*ri-240/(-16*M1^2*M2^8+96*M1^4*M2^6-256*M1^6*M2^4+256*M1^8*M2^2+M2^10)*[H+](0,0,1,1,1)*M1^8*
ri-30/(-16*M1^2*M2^8+96*M1^4*M2^6-256*M1^6*M2^4+256*M1^8*M2^2+M2^10)*[H+](0,0,1,1,1)*M2^8*ri-
212/(-16*M1^2*M2^8+96*M1^4*M2^6-256*M1^6*M2^4+256*M1^8*M2^2+M2^10)*[H+](1,0,0,1,1)*M1^2*M2^6*ri
```

Changing strategy: resorting to asymptotic expansion

- The difficulty of this expressions does not come from its length.
- Also not from the large number of M.I. appearing.
- The problem is the intrinsic difficulty of two-scale M.I. with 4/5 propagators $\Rightarrow$ not calculated up to the date (non-standard signs).

Change of strategy: for the physical problem it would be enough having accurate, controlled expansions of the whole set of M.I. in the following limits:

- $y_h := m_t/m_h \sim 0, 1, \propto$,
- $y_Z := m_t/m_Z \sim \propto$, (accuracy below the % level)
- $y_W := m_t/m_W \sim \propto$ (accuracy below the % level).

Testing ground: Z_m and Z_2 corrections

The method can be proved for:

- Z_m (exact expression already known in the literature) and
- Z_2 (that we need anyway for renormalizing the two-loop vertex diagrams).

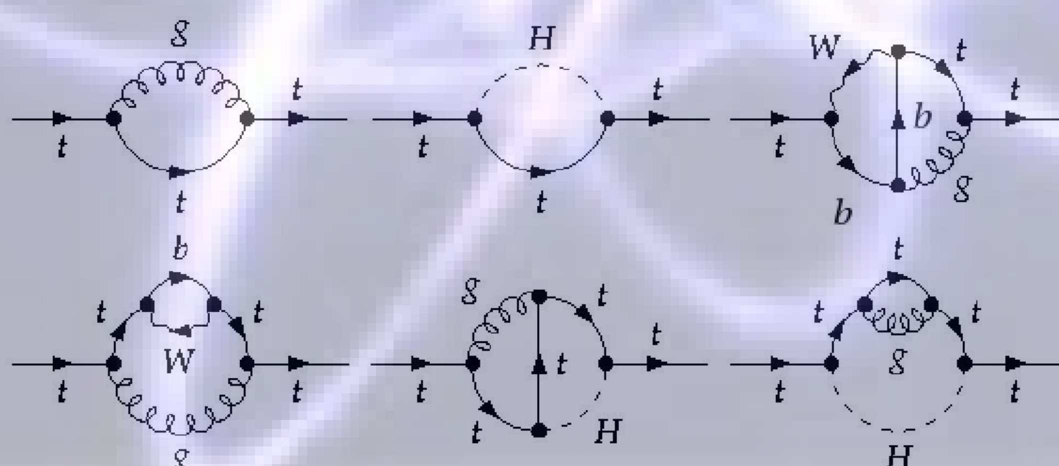
The number of M.I. in these cases is much more restricted:

Neutral

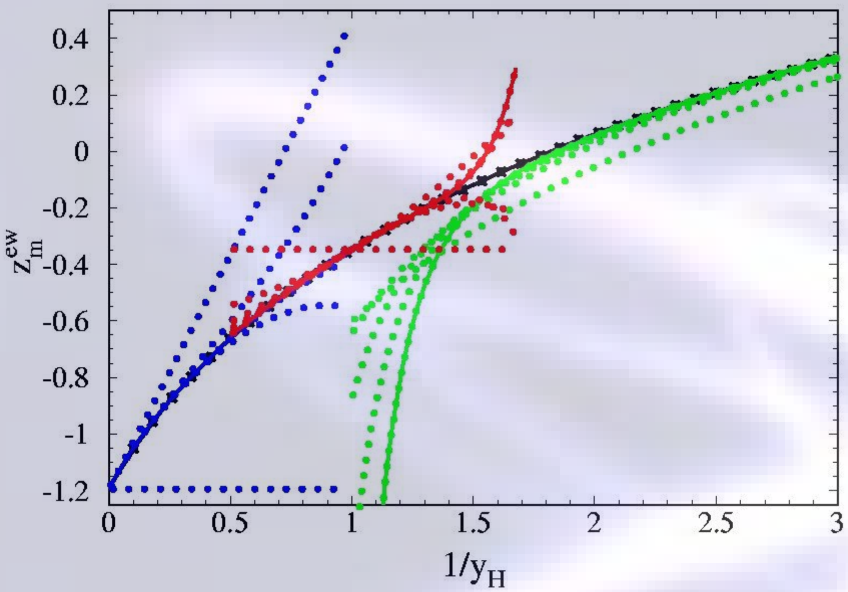
$$H_1 = H_N^+(1,1,0,0,0)\,,\quad H_2 = H_N^+(0,0,1,1,1)\,,\quad H_3 = H_N^+(1,1,0,1,0)\,,$$
$$Y_1 = Y_N^-(1,1,1,0,0)\,,\quad Y_2 = Y_N^-(1,0,0,0,1)\,,\quad Y_3 = Y_N^-(1,1,0,0,1)\,.$$
$$H_4 = H_N^+(1,0,0,1,1)\,,\quad H_5 = H_N^+(2,0,0,1,1)\,,\quad Y_4 = Y_N^-(1,1,1,0,1)\,.$$

Charged

$$H_6 = H_C^+(0,1,1,1,0)\,,\quad H_7 = H_C^+(0,1,1,1,-1)\,,\quad H_8 = H_C^+(1,1,0,1,0)$$
$$W_3 = W_C^-(0,-1,1,1,1)\,,\quad W_4 = W_C^-(1,1,1,0,1)\,.$$
$$W_1 = W_C^-(1,1,1,0,0)\,,\quad W_2 = W_C^-(1,1,0,0,1)$$



Accuracy in the reproduction of Zm^{OS}/Zm^{MS}

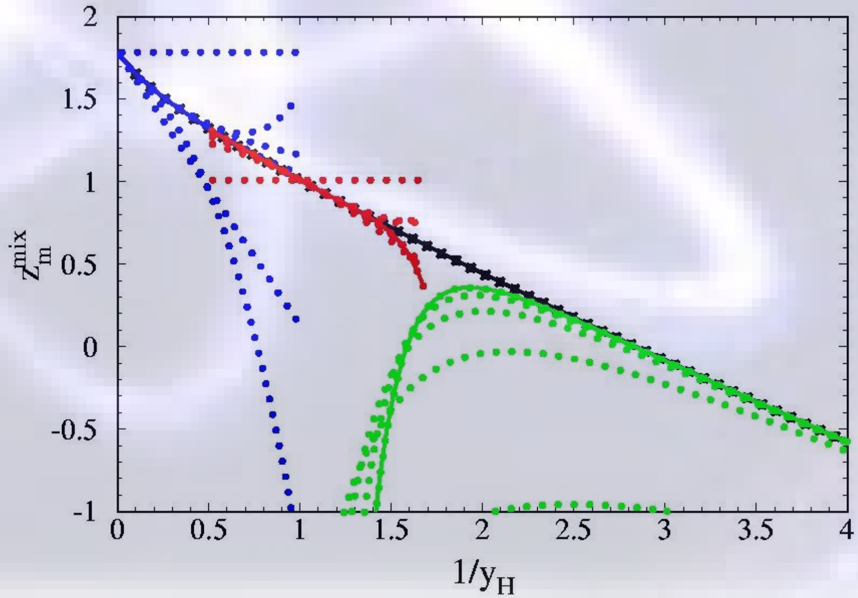


One-loop α expansion of the quotient

Higgs dependence: 1/yH = mh/mt

$$Z_m^{OS} = 1 + \left(\Sigma_S(m^2) + \frac{1}{2}\left(\Sigma_L(m^2) + \Sigma_R(m^2)\right)\right)$$

Two-loop $\alpha\alpha$s expansion of the quotient



Plots in agreement with the results of:
Jegerlehner and Kalmykov;
Faisst, Kuhn, Veretin;

Exact versus expanded

$$+\frac{1}{8}\ln\left(1-\frac{1}{\omega_t}\right)(1-\omega_t)\frac{(18\omega_t^2+21\omega_t+17)}{\omega_t}+\ln\omega_t\left[\frac{22\omega_t+17}{8\omega_t}\right]$$
$$+(1-\omega_t)\frac{(1+2\omega_t)(2+\omega_t)}{2\omega_t}\ln\omega_t\ln\left(1-\frac{1}{\omega_t}\right)+\frac{1+\omega_t-\omega_t^2}{2\omega_t}\ln^2\omega_t$$
$$-(1-\omega_t)^2\frac{4\omega_t+5}{8\omega_t}\ln^2\left(1-\frac{1}{\omega_t}\right)+\frac{(1+\omega_t)}{4\omega_t}\left(4\omega_t^2+7\omega_t-9\right)\mathrm{Li}_2\left(\frac{1}{\omega_t}\right)$$
$$-\frac{1}{\omega_t}(1-\omega_t)^2(1+2\omega_t)\left\{\frac{3}{2}\mathrm{S}_{1,2}\left(\frac{1}{\omega_t}\right)-\frac{3}{2}\mathrm{Li}_3\left(\frac{1}{\omega_t}\right)-\ln\omega_t\mathrm{Li}_2\left(\frac{1}{\omega_t}\right)\right.$$
$$\left.+\frac{1}{2}\ln\left(1-\frac{1}{\omega_t}\right)\mathrm{Li}_2\left(\frac{1}{\omega_t}\right)+\frac{1}{4}\ln\left(1-\frac{1}{\omega_t}\right)\left[\ln^2\omega_t+6\zeta_2\right]\right\}$$

$$+\frac{(1+y_Z)^2(1+y_Z^2)(17+41y_Z+17y_Z^2)}{18\omega_t y_Z^3}\left\{3\left(2\mathrm{Li}_3(y_Z)+\mathrm{Li}_3(-y_Z)\right)-3\zeta_2\ln(1+y_Z)\right.$$
$$\left.-2\ln y_Z\left(2\mathrm{Li}_2(y_Z)+\mathrm{Li}_2(-y_Z)\right)-\ln^2 y_Z\left(\ln(1-y_Z)+\frac{1}{2}\ln(1+y_Z)\right)\right\}$$
$$+\frac{(1-y_Z)(1+y_Z)^3(17+41y_Z+17y_Z^2)}{18\omega_t y_Z^3}\times$$
$$\left\{2\ln y_Z\left(\ln(1-y_Z)+\frac{1}{2}\ln(1+y_Z)\right)+2\mathrm{Li}_2(y_Z)+\mathrm{Li}_2(-y_Z)\right\}$$

$$-\frac{4}{9}\frac{(1+y_Z)}{y_Z^2}\left(5-4\frac{\omega_t y_Z}{(1+y_Z)^2}\right)\times$$
$$\left\{\frac{(1+y_Z^2)(1+4y_Z+y_Z^2)}{(1+y_Z)}\left[3\left(2\mathrm{Li}_3(y_Z)+\mathrm{Li}_3(-y_Z)\right)-3\zeta_2\ln(1+y_Z)\right.\right.$$
$$\left.-2\ln y_Z\left(2\mathrm{Li}_2(y_Z)+\mathrm{Li}_2(-y_Z)\right)-\ln^2 y_Z\left(\ln(1-y_Z)+\frac{1}{2}\ln(1+y_Z)\right)\right]$$
$$+(1-y_Z)(1+4y_Z+y_Z^2)\left[2\ln y_Z\left(\ln(1-y_Z)+\frac{1}{2}\ln(1+y_Z)\right)+2\mathrm{Li}_2(y_Z)\right]$$
$$+\frac{9}{4}(1-y_Z)^2(1+y_Z)\ln(1+y_Z)+\frac{(1+2y_Z-24y_Z^2+2y_Z^3+y_Z^4)}{(1-y_Z)}\mathrm{Li}_2(-y_Z)$$
$$\left.+\frac{3}{4}\frac{y_Z(1+3y_Z)(4-y_Z+y_Z^2)}{(1-y_Z)}\ln y_Z-\frac{1}{4}\frac{y_Z(2+9y_Z+3y_Z^2+16y_Z^3+6y_Z^4)}{(1+y_Z)(1-y_Z)}\ln^2 y_Z\right\}$$

$$-\frac{(1+y_Z)^3(9+32y_Z+9y_Z^2)}{4\omega_t(1-y_Z)y_Z^2}\mathrm{Li}_2(-y_Z)+\frac{447}{16}+\frac{125}{9}\frac{(1+y_Z^2)}{y_Z}$$
$$+\frac{32}{3}\left[1-\omega_t\frac{y_Z}{(1+y_Z)^2}\right]\left\{\zeta_3-4\zeta_2\ln 2\right\}+\frac{(1+y_Z)^4(17y_Z^2-19y_Z+17)}{8\omega_t y_Z^3}\ln(1+y_Z)$$

$$-\frac{1}{\omega_t}\ln^2 y_Z\left\{-\frac{685}{36}+\frac{17}{36y_Z^2}+\frac{67}{24y_Z}-\frac{335}{24}y_Z-\frac{497}{72}y_Z^2-\frac{17}{12}y_Z^3+\frac{25}{1-y_Z}\right\}$$
$$+\frac{(1+y_Z)}{24\omega_t y_Z^2(1-y_Z)}\left[51y_Z^5+113y_Z^4+134y_Z^3+237y_Z^2+197y_Z+68\right]\ln y_Z$$
$$-\frac{1}{3}\zeta_2\left\{\frac{20-39y_Z-y_Z^2-80y_Z^3-20y_Z^4}{y_Z(1-y_Z)}-\omega_t\frac{7-49y_Z+17y_Z^2-55y_Z^3-16y_Z^4}{(1-y_Z)(1+y_Z)^2}\right\}$$
$$-\frac{1}{\omega_t}\left\{\frac{4157}{72}+\frac{425(1+y_Z^4)}{72y_Z^2}+\frac{2561(1+y_Z^2)}{96y_Z}\right\}$$
$$+\frac{1}{\omega_t}\zeta_2\left\{\frac{187}{3}+\frac{17}{6y_Z^2}+\frac{133}{12y_Z}+\frac{535}{12}y_Z+\frac{211}{12}y_Z^2+\frac{17}{6}y_Z^3-\frac{50}{1-y_Z}\right\}$$

$$-3\omega_t\ln\omega_t\frac{(1+y_H+y_H^2)}{(1+y_H)^2}+\frac{3}{2\omega_t}\frac{y_H}{(1+y_H)^2}\frac{(1+y_Z)^4}{y_Z^2}\ln\frac{(1+y_Z)^2}{y_Z}$$
$$-\frac{1}{\omega_t}\frac{y_H}{(1+y_H)^2}\left\{\frac{11(1+y_H^2)(1+y_H)^2}{8y_H^2}+8N_c+\frac{1}{2}\frac{(1+y_Z)^4}{y_Z^2}\right\}$$
$$+\frac{1}{\omega_t}\zeta_2\left\{\frac{3}{2y_H}+\frac{9}{2}y_H+\frac{3}{4}y_H^2\right\}-\frac{\omega_t}{36}\frac{(625+1286y_H+625y_H^2)}{(1+y_H)^2}$$
$$+\frac{1}{\omega_t}\frac{(1-y_H)^2}{y_H^2}\ln y_H\left[\ln(1-y_H)+\frac{1}{2}\ln(1+y_H)\right]\left[(1-y_H^2)-\frac{1}{2}(1+y_H^2)\ln y_H\right]$$
$$-\frac{1}{8}\frac{1}{\omega_t}\frac{2+8y_H-10y_H^2-3y_H^3}{y_H}\ln^2 y_H+\frac{1}{8}\frac{1}{\omega_t}\frac{(1+y_H)(6-63y_H+5y_H^2)}{y_H}\ln y_H$$
$$-\frac{1}{8}\frac{1}{\omega_t}\frac{(1+y_H)^2(5-62y_H+5y_H^2)}{y_H^2}\ln(1+y_H)-\frac{3}{2}\frac{1}{\omega_t}\zeta_2\ln(1+y_H)\frac{(1-y_H)^2(1+y_H^2)}{y_H^2}$$
$$+\frac{1}{\omega_t}\frac{(1-y_H)(1+y_H)}{y_H^2}\left\{\frac{(5-28y_H+5y_H^2)}{4}\mathrm{Li}_2(-y_H)+(1-y_H)^2\mathrm{Li}_2(y_H)\right\}$$
$$+\frac{1}{\omega_t}\frac{(1-y_H)^2(1+y_H^2)}{y_H^2}\left\{\frac{3}{2}\left[2\mathrm{Li}_3(y_H)+\mathrm{Li}_3(-y_H)\right]-\ln y_H\left[2\mathrm{Li}_2(y_H)+\mathrm{Li}_2(-y_H)\right]\right\}\Bigg),$$

$$\omega_t = \frac{m_W^2}{m_t^2},$$
$$y_A = \frac{1-\sqrt{1-\frac{4m_t^2}{m_A^2}}}{1+\sqrt{1-\frac{4m_t^2}{m_A^2}}},\ A=H,Z\ .$$

(as calculated by Jegerlehner and Kalmykov)

$$z_{m,0}^{H,mix} = y_t^2\left[-\frac{9}{32}+\frac{33L_H}{128}-\frac{9L_H^2}{128}-\frac{3\pi^2}{128}+\left(-\frac{49}{1152}+\frac{165L_H}{288}+\frac{13L_H^2}{192}-\frac{\pi^2}{192}\right)y_H^2+\left(-\frac{21097}{19432}+\frac{665L_H}{768}+\frac{45L_H^2}{256}+\frac{15\pi^2}{256}\right)y_H^4\right],$$

$$\begin{aligned}
z_{m,1}^{H,mix} = y_t^2\Bigg[&-\frac{133}{256}+\frac{Y'}{32}+\left(\frac{\ln 3}{48}-\frac{35}{768}\right)\pi^2+\left(\frac{9\ln 3}{32}+\frac{531}{256}\right)S_2-\frac{\sqrt{3}}{16}\mathrm{Ls}_3\left(\frac{2\pi}{3}\right)-\frac{7\sqrt{3}}{1152}\pi^3+\left(-\frac{\ln 3}{24}-\frac{\ln^2 3}{192}+\frac{3S_2}{32}+\frac{53}{384}\right)\pi\sqrt{3}-\frac{\zeta(3)}{6}\\
&+\left(\frac{41}{96}+\frac{Y'}{24}+\left(\frac{\ln 3}{96}+\frac{1}{144}\right)\pi^2+\left(\frac{3\ln 3}{8}-\frac{93}{64}\right)S_2-\frac{\sqrt{3}}{12}\mathrm{Ls}_3\left(\frac{2\pi}{3}\right)-\frac{7\sqrt{3}}{864}\pi^3+\left(+\frac{\ln 3}{144}-\frac{\ln^2 3}{144}+\frac{S_2}{8}-\frac{13}{288}\right)\pi\sqrt{3}-\frac{3\zeta(3)}{9}\right)y_{H,1}\\
&+\left(\frac{1}{12}+\frac{Y'}{96}+\left(\frac{\ln 3}{144}+\frac{7}{216}\right)\pi^2+\left(\frac{3\ln 3}{32}-\frac{123}{128}\right)S_2-\frac{\sqrt{3}}{48}\mathrm{Ls}_3\left(\frac{2\pi}{3}\right)-\frac{7\sqrt{3}}{3456}\pi^3+\left(-\frac{\ln 3}{288}-\frac{\ln^2 3}{576}+\frac{S_2}{32}-\frac{1}{36}\right)\pi\sqrt{3}-\frac{\zeta(3)}{18}\right)y_{H,1}^2\\
&+\left(-\frac{1}{288}+\frac{143\pi^2}{7776}-\frac{19S_2}{64}-\left(\frac{\ln 3}{216}+\frac{59}{5184}\right)\pi\sqrt{3}\right)y_{H,1}^3\\
&+\left(\frac{55}{6912}+\frac{4145\pi^2}{373248}-\frac{257S_2}{1536}-\left(\frac{5\ln 3}{5184}+\frac{695}{62208}\right)\pi\sqrt{3}\right)y_{H,1}^4\\
&+\left(\frac{163}{34560}+\frac{3923\pi^2}{466560}-\frac{71S_2}{640}-\left(\frac{\ln 3}{2160}+\frac{269}{31104}\right)\pi\sqrt{3}\right)y_{H,1}^5\Bigg],
\end{aligned}$$

$$\begin{aligned}
z_{m,\infty}^{H,mix} = y_t^2\Bigg[&-\frac{187}{256}+\left(\frac{\ln 2}{4}-\frac{1}{64}\right)\pi^2-\frac{3\zeta(3)}{8}-\frac{\pi}{8}\frac{1}{y_H}+\left(\frac{39}{64}-\frac{15L_H}{64}-\frac{3\ln 2}{16}\pi^2+\frac{9\zeta(3)}{32}\right)\frac{1}{y_H^2}\\
&+\left(\frac{173}{576}-\frac{L_H}{24}-\frac{\ln 2}{6}\right)\pi\frac{1}{y_H^3}+\left(-\frac{281}{1536}+\frac{17L_H}{256}+\left(\frac{\ln 2}{32}+\frac{3}{256}\right)\pi^2-\frac{3\zeta(3)}{64}\right)\frac{1}{y_H^4}\\
&+\left(-\frac{4861}{76800}+\frac{3L_H}{320}+\frac{3\ln 2}{80}\right)\pi\frac{1}{y_H^5}+\left(\frac{85}{13824}-\frac{5\pi^2}{3072}\right)\frac{1}{y_H^6}\Bigg],
\end{aligned}$$

$$z_{m,\infty}^{t,mix} = y_t^2\left[-\frac{65}{256}+\frac{5\pi^2}{128}-\frac{3\zeta(3)}{32}-\left(\frac{19}{256}+\frac{L_t}{32}\right)\frac{1}{y_t^2}+\left(-\frac{15}{32}-\frac{15L_t}{64}+\frac{\pi^2}{64}+\frac{9\zeta(3)}{32}\right)\frac{1}{y_t^4}+\left(-\frac{71}{384}+\frac{15L_t}{64}-\frac{5\pi^2}{288}-\frac{3\zeta(3)}{16}\right)\frac{1}{y_t^6}\right],$$

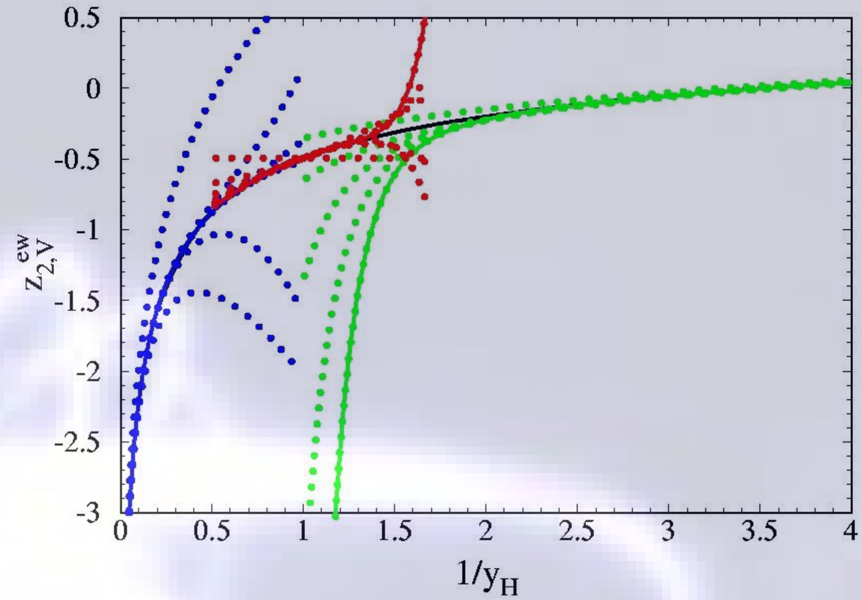
$$\begin{aligned}
z_{m,\infty}^{Z,mix} = y_t^2\Bigg[&\frac{37}{256}-\frac{\pi^2}{64}+\left(-\frac{1}{64}-\frac{L_Z}{64}+\left(-\frac{\ln 2}{16}+\frac{1}{32}\right)\pi^2+\frac{3\zeta(3)}{32}\right)\frac{1}{y_Z^2}+\frac{\pi}{32}\frac{1}{y_Z^3}\\
&+\left(-\frac{55}{512}+\frac{13L_Z}{256}+\frac{\ln 2}{32}\pi^2-\frac{3\zeta(3)}{64}\right)\frac{1}{y_Z^4}+\left(-\frac{667}{11520}+\frac{L_Z}{96}+\frac{\ln 2}{24}\right)\pi\frac{1}{y_Z^5}+\left(\frac{11}{1536}-\frac{3\pi^2}{1024}\right)\frac{1}{y_Z^6}\Bigg]\\
&+a_t^2 s_t^2\Bigg[\frac{57}{256}+\left(\frac{3\ln 2}{4}-\frac{11}{32}\right)\pi^2-\frac{9\zeta(3)}{8}-\frac{\pi}{8}\frac{1}{y_Z}+\left(\frac{7}{8}-\frac{13L_Z}{16}+\left(-\frac{\ln 2}{2}+\frac{3}{32}\right)\pi^2+\frac{3\zeta(3)}{4}\right)\frac{1}{y_Z^2}\\
&+\left(\frac{167}{192}-\frac{L_Z}{8}-\frac{\ln 2}{2}\right)\pi\frac{1}{y_Z^3}+\left(-\frac{71}{256}+\frac{15L_Z}{128}+\left(\frac{\ln 2}{16}+\frac{5}{256}\right)\pi^2-\frac{3\zeta(3)}{32}\right)\frac{1}{y_Z^4}\Bigg]\\
&+v_t^2 s_t^2\Bigg[-\frac{71}{256}+\left(-\frac{\ln 2}{4}+\frac{5}{32}\right)\pi^2+\frac{3\zeta(3)}{8}+\left(-\frac{3}{4}+\frac{3\pi^2}{32}\right)\frac{1}{y_Z^2}+\left(-\frac{19}{288}+\frac{L_Z}{24}+\frac{\ln 2}{6}\right)\pi\frac{1}{y_Z^3}\\
&+\left(-\frac{39}{256}+\frac{15L_Z}{128}+\left(\frac{\ln 2}{16}-\frac{7}{256}\right)\pi^2-\frac{3\zeta(3)}{32}\right)\frac{1}{y_Z^4}\Bigg],
\end{aligned}$$

$$z_m^{A,mix} = -\frac{71}{144}+\left(-\frac{4\ln 2}{9}+\frac{5}{18}\right)\pi^2+\frac{2\zeta(3)}{3},$$

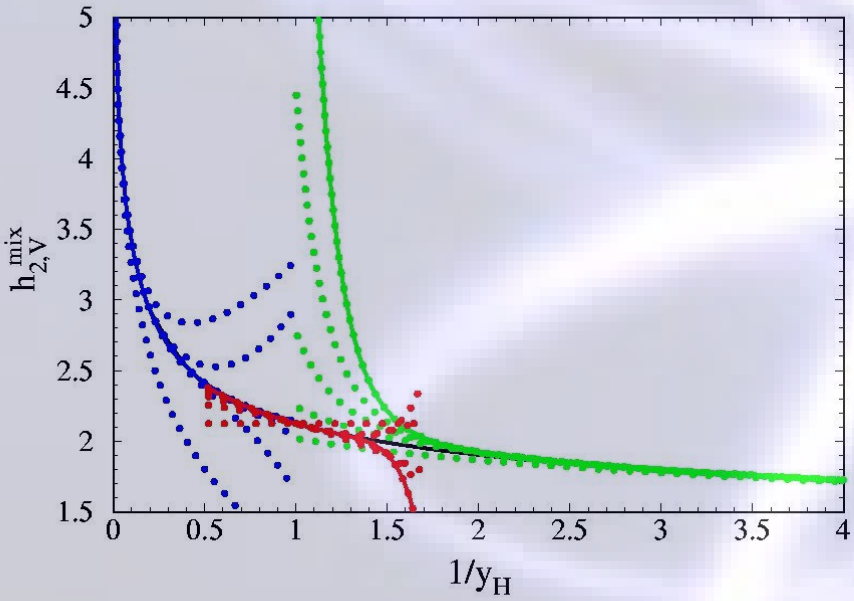
$$z_m^{tad,mix} = y_t^2\left[\left(-\frac{3}{64}+\frac{3L_Z}{64}\right)\frac{1}{y_H^2}+\left(-\frac{1}{16}+\frac{3}{16}+L_t\right)\frac{y_H^2}{y_t^4}+\left(-\frac{1}{32}+\frac{1}{32}L_t\right)\frac{1}{y_t^2}\right]+a_t^2 s_t^2\left(\left(-\frac{1}{8}+\frac{3L_Z}{8}\right)\frac{y_H^2}{y_Z^2}+\left(-\frac{1}{16}+\frac{L_Z}{16}\right)\right),$$

Z2 plots

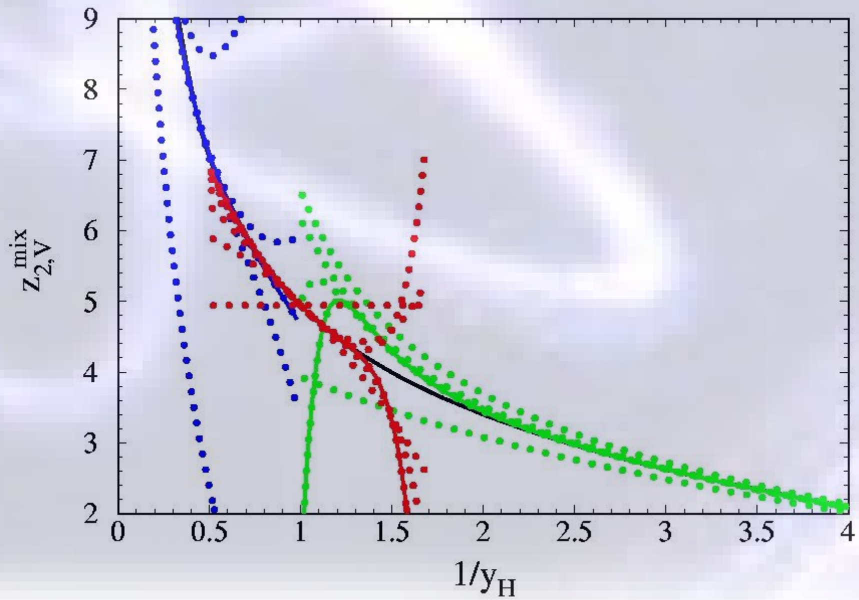
Higgs dependence: 1/yH = mh/mt



Z2 one-loop $\mathcal{O}(\alpha)$ correction



Z2 two-loop $\mathcal{O}(\alpha\alpha_s)$ corr - $1/\epsilon$ pole



Z2 two-loop $\mathcal{O}(\alpha\alpha_s)$ correction - constant term

Z2 exact and expanded

$$
\begin{aligned}
A_2^{H,\mathrm{mix}} = \; & \frac{y_W^2}{64}\Bigg[-\frac{(9-c+12c^2+26c^3)-(30+28c+22c^2+100c^3)y_H^2+(56c+16c^2-72c^3)y_H^4}{(4y_H^2-1)c}\frac{H_1}{m_t^4} \\
& +\frac{(-6+5c-13c^2+2c^3)+(14-22c+96c^2-160c^3)y_H^2}{2cy_H^2}\frac{H_2}{m_t^2} \\
& +\frac{(-6+5c-13c^2+2c^3)+(30+58c^2-16c^3)y_H^2+(-32-88c-24c^2+112c^3)y_H^4+(64c-320c^3)y_H^6}{2(4y_H^2-1)cy_H^2}\frac{H_3}{m_t^2} \\
& -\frac{-2+20c^2-54c^3}{c}\frac{H_4}{m_t^2}+\frac{-(3+6c^2+7c^3)+(14-12c+72c^2-72c^3)y_H^2}{cy_H^2}\frac{H_5}{m_t^2} \\
& +\frac{(6-16c+14c^2-4c^3)+(-20+58c-56c^2+16c^3)y_H^2}{(4y_H^2-1)c}\frac{Y_1}{m_t^2} \\
& -\frac{(-12-5c-13c^2-46c^3)+(40+88c-68c^2+380c^3)y_H^2+(-200c+352c^2-744c^3)y_H^4}{2(4y_H^2-1)c}\frac{Y_2}{m_t^4} \\
& +\frac{-(12+3c+11c^2+54c^3)+(60+54c-6c^2+348c^3)y_H^2+(-64-128c+136c^2-472c^3)y_H^4+(48c-80c^2+32c^3)y_H^6}{2(4y_H^2-1)cy_H^2}\frac{Y_3}{m_t^2} \\
& -\frac{(-6+16c-14c^2+4c^3)+(30-96c+88c^2-24c^3)y_H^2+(-32+144c-128c^2+32c^3)y_H^4}{(4y_H^2-1)cy_H^2}\frac{Y_4}{m_t^2}\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
B_2^{H,\mathrm{mix}} = \; & \frac{y_W^2}{4}\Bigg\{\Bigg[-\frac{(3-2c)+(-36+12c)y_H^2+(24+8c)y_H^4}{16(4y_H^2-1)}L_{\mu t}+\frac{3c(1-12y_H^2+8y_H^4)}{32(4y_H^2-1)}L_{\mu t}^2 \\
& +\frac{(3-2c+6c^2)+(-36+12c-52c^2)y_H^2+(24+8c+32c^2)y_H^4}{16(4y_H^2-1)c}+\frac{(1-12y_H^2+8y_H^4)c\pi^2}{64(4y_H^2-1)}\Bigg]\frac{B_p[0,1]}{m_t^2} \\
& +\Bigg[\frac{(3-2c)+(-36+12c)y_H^2+(60-4c)y_H^4}{16(4y_H^2-1)}L_{\mu t}-\frac{3c\,(1-12y_H^2+20y_H^4)}{32(4y_H^2-1)}L_{\mu t}^2 \\
& -\frac{(3-2c+6c^2)+(-36+12c-52c^2)y_H^2+(60-4c+72c^2)y_H^4}{16c(4y_H^2-1)}-\frac{(1-12y_H^2+20y_H^4)c\pi^2}{64(4y_H^2-1)}\Bigg]\frac{B_p[1,0]}{m_t^2} \\
& +\Bigg[\frac{3c\,(1-16y_H^2+44y_H^4-8y_H^6)}{32(4y_H^2-1)y_H^2}L_{\mu t}^2+\frac{(-3+2c)+(48-20c)y_H^2+(-132+28c)y_H^4+(24-16c)y_H^6}{16(4y_H^2-1)y_H^2}L_{\mu t} \\
& +\frac{(3-2c+6c^2)+(-48+20c-76c^2)y_H^2+(132-28c+200c^2)y_H^4+(-24+16c)y_H^6}{16(4y_H^2-1)cy_H^2} \\
& +\frac{(1-16y_H^2+44y_H^4-16y_H^6)c\pi^2}{64(4y_H^2-1)y_H^2}\Bigg]B_p[1,1]\Bigg\}.
\end{aligned}
$$

$$z_{2,1',0}^{H,\mathrm{mix}} = y_W^2\left[\frac{53}{512}-\frac{15L_H}{256}+\frac{15L_H^2}{256}+\frac{\pi^2}{64}+\left(\frac{571}{2304}-\frac{27L_H}{192}+\frac{3L_H^2}{128}-\frac{13\pi^2}{384}\right)y_H^2\right.$$
$$\left.+\left(\frac{3443}{9216}-\frac{527L_H}{512}+\frac{49L_H^2}{128}+\frac{71\pi^2}{768}\right)y_H^4+\left(\frac{12909}{6400}-\frac{621L_H}{320}+\frac{9L_H^2}{32}\right)y_H^6\right],$$

$$z_{2,1',1}^{H,\mathrm{mix}} = y_W^2\left[-\frac{689}{768}+\frac{V^\epsilon}{24}+\left(\frac{\ln 3}{36}-\frac{209}{2304}\right)\pi^2+\left(\frac{3\ln 3}{8}+\frac{1167}{256}\right)S_2-\frac{\sqrt{3}}{12}\mathrm{Ls}_3\left(\frac{2\pi}{3}\right)-\frac{7\sqrt{3}}{864}\pi^3+\left(-\frac{181\ln 3}{1152}-\frac{\ln^2 3}{144}+\frac{S_2}{8}+\frac{329}{1152}\right)\pi\sqrt{3}-\frac{2\zeta(3)}{9}\right.$$
$$+\left(-\frac{3}{32}+\frac{3V^\epsilon}{32}+\left(\frac{\ln 3}{16}-\frac{79}{3456}\right)\pi^2+\left(\frac{27\ln 3}{32}-\frac{315}{128}\right)S_2-\frac{3\sqrt{3}}{16}\mathrm{Ls}_3\left(\frac{2\pi}{3}\right)-\frac{7\sqrt{3}}{384}\pi^3+\left(+-\frac{5\ln 3}{192}-\frac{\ln^2 3}{64}+\frac{9S_2}{32}+\frac{85}{576}\right)\pi\sqrt{3}-\frac{\zeta(3)}{2}\right)y_{H,1}$$
$$+\left(\frac{115}{192}+\frac{V^\epsilon}{32}+\left(\frac{\ln 3}{48}+\frac{1417}{20736}\right)\pi^2+\left(\frac{9\ln 3}{32}-\frac{1029}{256}\right)S_2-\frac{\sqrt{3}}{16}\mathrm{Ls}_3\left(\frac{2\pi}{3}\right)-\frac{7\sqrt{3}}{1152}\pi^3+\left(\frac{41\ln 3}{1152}-\frac{\ln^2 3}{192}+\frac{3S_2}{32}-\frac{295}{3456}\right)\pi\sqrt{3}-\frac{\zeta(3)}{6}\right)y_{H,1}^2$$
$$+\left(\frac{1397}{3456}+\frac{7867\pi^2}{186624}-\frac{1441S_2}{768}+\left(\frac{17\ln 3}{2592}-\frac{797}{15552}\right)\pi\sqrt{3}\right)y_{H,1}^3$$
$$+\left(\frac{2435}{6912}+\frac{4267\pi^2}{186624}-\frac{871S_2}{768}+\left(\frac{31\ln 3}{5184}-\frac{679}{15552}\right)\pi\sqrt{3}\right)y_{H,1}^4$$
$$\left.+\left(\frac{12487}{41472}+\frac{98903\pi^2}{5598720}-\frac{20263S_2}{23040}+\left(\frac{347\ln 3}{77760}-\frac{3311}{93312}\right)\pi\sqrt{3}\right)y_{H,1}^5\right],$$

$$z_{2,1',\infty}^{H,\mathrm{mix}} = y_W^2\left[-\frac{643}{256}+\frac{9L_H^2}{64}+\left(-\frac{13}{128}+\frac{\ln 2}{2}\right)\pi^2-\frac{3\zeta(3)}{4}+\left(\frac{9}{16}-\frac{27L_H}{64}-\frac{27\ln 2}{32}\right)\pi\frac{1}{y_H}\right.$$
$$+\left(\frac{3637}{1152}-\frac{77L_H}{384}-\frac{25L_H^2}{128}+\left(\frac{7}{192}-\frac{15\ln 2}{32}\right)\pi^2+\frac{45\zeta(3)}{64}\right)\frac{1}{y_H^2}+\left(\frac{217}{2304}+\frac{229L_H}{1536}+\frac{53\ln 2}{768}\right)\pi\frac{1}{y_H^3}$$
$$\left.+\left(-\frac{97429}{115200}+\frac{181L_H}{1920}+\frac{25L_H^2}{512}+\left(+\frac{13}{384}+\frac{3\ln 2}{32}\right)\pi^2-\frac{9\zeta(3)}{64}\right)\frac{1}{y_H^4}+\left(-\frac{26839}{153600}+\frac{303L_H}{40960}+\frac{1551\ln 2}{20480}\right)\pi\frac{1}{y_H^5}\right],$$

$$Z_2^{L,\mathrm{OS}} = -\Sigma_L(m^2)-2\left(\Sigma_S'(m^2)+\frac{1}{2}\left(\Sigma_L'(m^2)+\Sigma_R'(m^2)\right)\right)$$
$$Z_2^{R,\mathrm{OS}} = -\Sigma_R(m^2)-2\left(\Sigma_S'(m^2)+\frac{1}{2}\left(\Sigma_L'(m^2)+\Sigma_R'(m^2)\right)\right)$$

$$z_{2,1',\infty}^{W,\mathrm{mix}} = y_W^2\left[-\frac{15}{256}-\frac{\pi^2}{32}-\frac{3\zeta(3)}{32}+\left(\frac{339}{256}+\frac{21L_W}{64}-\frac{9L_W^2}{128}-\frac{27\pi^2}{128}+\frac{3\zeta(3)}{16}\right)\frac{1}{y_W^2}\right.$$
$$\left.+\left(\frac{325}{512}-\frac{3L_W}{256}-\frac{9L_W^2}{256}-\frac{11\pi^2}{96}+\frac{15\zeta(3)}{32}\right)\frac{1}{y_W^4}+\left(\frac{917}{768}-\frac{21L_W}{32}+\frac{27L_W^2}{128}+\frac{137\pi^2}{576}-\frac{9\zeta(3)}{16}\right)\frac{1}{y_W^6}\right],$$

$$z_{2,1',\infty}^{Z,\mathrm{mix}} = y_W^2\left[\frac{13}{256}+\frac{3\pi^2}{128}+\left(\frac{113}{128}+\frac{27L_Z}{128}-\frac{9L_Z^2}{128}+\left(\frac{5}{64}-\frac{5\ln 2}{32}\right)\pi^2+\frac{15\zeta(3)}{64}\right)\frac{1}{y_Z^2}+\left(-\frac{1}{2}+\frac{45L_Z}{256}+\frac{45\ln 2}{128}\right)\pi\frac{1}{y_Z^3}\right.$$
$$+\left(-\frac{1909}{4608}-\frac{5L_Z}{192}+\frac{25L_Z^2}{512}+\left(-\frac{1}{768}+\frac{3\ln 2}{32}\right)\pi^2-\frac{9\zeta(3)}{64}\right)\frac{1}{y_Z^4}+\left(-\frac{5857}{46080}+\frac{19L_Z}{6144}+\frac{227\ln 2}{3072}\right)\pi\frac{1}{y_Z^5}$$
$$\left.+\left(\frac{7909}{76800}-\frac{41L_Z}{1280}-\frac{21\pi^2}{2048}\right)\frac{1}{y_Z^6}\right]$$
$$+a_t^2 s_W^2\left[-\frac{593}{256}-\frac{3L_Z}{16}+\frac{27L_Z^2}{64}+\left(-\frac{121}{128}+\frac{3\ln 2}{2}\right)\pi^2-\frac{9\zeta(3)}{4}+\left(\frac{81}{32}-\frac{81L_Z}{32}-\frac{81\ln 2}{32}\right)\pi\frac{1}{y_Z}\right.$$
$$+\left(\frac{1145}{288}-\frac{19L_Z}{96}-\frac{5L_Z^2}{8}+\left(\frac{59}{192}-\frac{5\ln 2}{4}\right)\pi^2+\frac{15\zeta(3)}{8}\right)\frac{1}{y_Z^2}+\left(\frac{145}{192}+\frac{139L_Z}{512}-\frac{37\ln 2}{256}\right)\pi\frac{1}{y_Z^3}$$
$$\left.+\left(-\frac{91423}{57600}+\frac{29L_Z}{120}+\frac{25L_Z^2}{256}+\left(\frac{43}{768}+\frac{3\ln 2}{16}\right)\pi^2-\frac{9\zeta(3)}{32}\right)\frac{1}{y_Z^4}\right]$$
$$+v_t^2 s_W^2\left[\frac{399}{256}+\frac{3L_Z}{8}-\frac{9L_Z^2}{64}\left(\frac{55}{128}-\frac{\ln 2}{2}\right)\pi^2+\frac{3\zeta(3)}{4}+\left(-\frac{21}{16}+\frac{27L_Z}{64}+\frac{27\ln 2}{32}\right)\pi\frac{1}{y_Z}+\left(-\frac{109}{144}+\frac{19L_Z}{96}-\frac{L_Z^2}{32}+\frac{49\pi^2}{192}\right)\frac{1}{y_Z^2}\right.$$
$$\left.+\left(-\frac{2527}{2304}+\frac{381L_Z}{1536}+\frac{757\ln 2}{768}\right)\pi\frac{1}{y_Z^3}+\left(-\frac{18127}{57600}-\frac{23L_Z}{240}+\frac{25L_Z^2}{256}+\left(-\frac{65}{768}+\frac{3\ln 2}{16}\right)\pi^2-\frac{9\zeta(3)}{32}\right)\frac{1}{y_Z^4}\right],$$

$$z_{2,1',\infty}^{A,\mathrm{mix}} = \frac{433}{144}+\left(\frac{8\ln 2}{9}-\frac{49}{72}\right)\pi^2-\frac{4\zeta(3)}{3},$$

$$z_{2,A,\infty}^{W,\mathrm{mix}} = y_W^2\left[\frac{31}{256}+\frac{3\xi}{32}+\frac{\pi^2}{32}-\frac{\xi\pi^2}{256}-\frac{3\zeta(3)}{32}+\left(-\frac{13}{256}-\frac{9\xi}{64}+\frac{3L_W}{64}-\frac{3\xi L_W}{64}-\frac{L_W^2}{128}+\frac{\xi L_W^2}{128}-\frac{13\pi^2}{384}+\frac{7\xi\pi^2}{384}+\frac{3\zeta(3)}{8}\right)\frac{1}{y_W^2}\right.$$
$$+\left(\frac{129}{512}+\frac{3\xi}{512}-\frac{21L_W}{256}+\frac{21\xi L_W}{256}+\frac{5L_W^2}{256}-\frac{5\xi L_W^2}{256}-\frac{11\pi^2}{192}-\frac{5\xi\pi^2}{192}-\frac{15\zeta(3)}{32}\right)\frac{1}{y_W^4}$$
$$\left.+\left(-\frac{193}{2304}+\frac{29\xi}{1152}+\frac{5L_W}{96}-\frac{\xi L_W}{48}-\frac{L_W^2}{128}+\frac{L_W^2}{128}+\frac{11\pi^2}{144}+\frac{\pi^2\xi}{96}+\frac{3\zeta(3)}{16}\right)\frac{1}{y_W^6}\right],$$

$$z_{2,A,\infty}^{Z,\mathrm{mix}} = a_t v_t s_W^2\left[-\frac{9}{128}-\frac{3\xi}{8}+\left(\frac{37}{192}+\frac{\ln 2}{4}\right)\pi^2-\frac{\pi^2\xi}{192}+\frac{3\zeta(3)}{8}+\left(-\frac{3}{16}+\frac{L_Z}{16}+\frac{\ln 2}{8}\right)\pi\frac{1-\xi}{y_Z}\right.$$
$$+\left(-\frac{1}{8}+\frac{3\xi}{16}+\frac{3L_Z}{32}+\frac{3\xi L_Z}{32}+\frac{(1-\xi)L_Z^2}{32}+\left(-\frac{5}{32}+\frac{\ln 2}{2}\right)\pi^2-\frac{3\zeta(3)}{4}\right)\frac{1}{y_Z^2}+\left(-\frac{595}{1152}-\frac{13\xi}{128}+\frac{49L_Z}{384}+\frac{5\xi L_Z}{128}+\frac{113\ln 2}{192}+\frac{5\xi\ln 2}{64}\right)\pi\frac{1}{y_Z^3}$$
$$\left.+\left(\frac{1}{32}-\frac{31\xi}{576}-\frac{5L_Z}{192}-\frac{\xi L_Z}{96}+\frac{(1+\xi)L_Z^2}{128}+\left(-\frac{17}{768}-\frac{\ln 2}{8}\right)\pi^2+\frac{3\zeta(3)}{16}\right)\frac{1}{y_Z^4}\right].$$

Conclusions

We present a factible solution to the problem of the extraction of the $\mathcal{O}(\alpha\alpha_s)$-correction to the vertex matching coefficient based on asymptotic expansion around three limits $(mh>>mt,\ mh<<mt,\ mh\sim mt)$.

The validity of the approach is proved by reproducing the correction to Zm^{OS} by means of a set of compact expansions.

A participating piece in the vertex correction, namely $Z2^{OS}$ is presented by first time in the literature in its exact, analytic form, as well as in the expanded version. The last is the one that opens the path to the calculation of the whole vertex correction.

Work in progress is the calculation of the whole higgs contribution to the vertex correction (no boxes are present).

Integrals for parametrizing the two-loop graphs

Neutral

$$[\mathcal{B}\pm](x_1,x_2,x_3,x_4,x_5) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{(l^2 \pm l\cdot Q)^{x_2}} \frac{1}{(k^2)^{x_3}} \frac{1}{((k-l)^2)^{x_4}} \frac{1}{(l^2)^{x_5}} .$$

$$[\mathcal{H}\pm](x_1,x_2,x_3,x_4,x_5) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{(l^2 \pm l\cdot Q)^{x_2}} \frac{1}{(k^2)^{x_3}} \frac{1}{((k-l)^2 - m_H^2)^{x_4}} \frac{1}{(l^2)^{x_5}} .$$

$$[\mathcal{J}\pm](x_1,x_2,x_3,x_4,x_5) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{(l^2 \pm l\cdot Q)^{x_2}} \frac{1}{(k^2)^{x_3}} \frac{1}{((k-l)^2)^{x_4}} \frac{1}{(l^2 - m_H^2)^{x_5}} .$$

$$[\mathcal{W}\pm](x_1,x_2,x_3,x_4,x_5) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{(l^2 \pm l\cdot Q)^{x_2}} \frac{1}{((k-l)^2 \mp k\cdot Q \pm l\cdot Q)^{x_3}} \frac{1}{(k^2)^{x_4}} \frac{1}{(l^2)^{x_5}} .$$

$$[\mathcal{Y}\pm](x_1,x_2,x_3,x_4,x_5) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{(l^2 \pm l\cdot Q)^{x_2}} \frac{1}{((k-l)^2 \mp k\cdot Q \pm l\cdot Q)^{x_3}} \frac{1}{(k^2)^{x_4}} \frac{1}{(l^2 - m_H^2)^{x_5}} .$$

$$[\mathcal{Z}\pm](x_1,x_2,x_3,x_4,x_5) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{(l^2 \pm l\cdot Q)^{x_2}} \frac{1}{((k-l)^2 \mp k\cdot Q \pm l\cdot Q)^{x_3}} \frac{1}{((k-l)^2)^{x_4}} \frac{1}{(l^2 - m_H^2)^{x_5}} .$$

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$$[\mathbb{B}\pm](x_1,x_2,x_3,x_4,x_5,mx^2) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{((l \pm Q/2)^2 - mx^2)^{x_2}} \frac{1}{(k^2)^{x_3}} \frac{1}{((k-l)^2)^{x_4}} \frac{1}{(l^2 + mt^2 - mx^2)^{x_5}} .$$

$$[\mathbb{B}\pm](mx^2,x_1,x_2,x_3,x_4,x_5) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{((l \pm Q/2)^2 - mx^2)^{x_2}} \frac{1}{(k^2)^{x_3}} \frac{1}{((k-l)^2)^{x_4}} \frac{1}{(l^2)^{x_5}} .$$

$$[\mathbb{H}\pm](x_1,x_2,x_3,x_4,x_5,mx^2) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{((l \pm Q/2)^2)^{x_2}} \frac{1}{(k^2)^{x_3}} \frac{1}{((k-l)^2 - mx^2)^{x_4}} \frac{1}{(l^2 + mt^2)^{x_5}} .$$

$$[\mathbb{H}\pm](x_1,x_2,x_3,x_4,x_5) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{((l \pm Q/2)^2)^{x_2}} \frac{1}{(k^2)^{x_3}} \frac{1}{((k-l)^2 + mt^2)^{x_4}} \frac{1}{(l^2 + mt^2)^{x_5}} .$$

$$[\mathbb{J}\pm](x_1,x_2,x_3,x_4,x_5,mx^2) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{((k+Q/2)^2)^{x_1}} \frac{1}{((l \pm Q/2)^2)^{x_2}} \frac{1}{(k^2 - mx^2)^{x_3}} \frac{1}{((k-l)^2)^{x_4}} \frac{1}{(l^2 + mt^2)^{x_5}} .$$

$$[\mathbb{J}\pm](x_1,x_2,x_3,x_4,x_5) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{((k+Q/2)^2)^{x_1}} \frac{1}{((l \pm Q/2)^2)^{x_2}} \frac{1}{(k^2 + mt^2)^{x_3}} \frac{1}{((k-l)^2)^{x_4}} \frac{1}{(l^2 + mt^2)^{x_5}} .$$

$$[\mathbb{W}\pm](x_1,x_2,x_3,x_4,x_5,mx^2) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{((l \pm Q/2)^2)^{x_2}} \frac{1}{((k - l \mp Q/2)^2)^{x_3}} \frac{1}{(k^2)^{x_4}} \frac{1}{(l^2 - mx^2)^{x_5}} .$$

$$[\mathbb{W}\pm](x_1,x_2,x_3,x_4,x_5) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{((l \pm Q/2)^2)^{x_2}} \frac{1}{((k - l \mp Q/2)^2)^{x_3}} \frac{1}{(k^2)^{x_4}} \frac{1}{(l^2 + mt^2)^{x_5}} .$$

$$[\mathbb{Y}\pm](x_1,x_2,x_3,x_4,x_5,mx^2) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{((l \pm Q/2)^2)^{x_2}} \frac{1}{((k - l \pm Q/2)^2)^{x_3}} \frac{1}{(k^2)^{x_4}} \frac{1}{(l^2 - mx^2)^{x_5}} .$$

$$[\mathbb{Y}\pm](x_1,x_2,x_3,x_4,x_5) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{((l \pm Q/2)^2)^{x_2}} \frac{1}{((k - l \pm Q/2)^2)^{x_3}} \frac{1}{(k^2)^{x_4}} \frac{1}{(l^2 + mt^2)^{x_5}} .$$

$$[\mathbb{Z}\pm](x_1,x_2,x_3,x_4,x_5,mx^2) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{((l + Q/2)^2)^{x_2}} \frac{1}{((k - l \mp Q/2)^2)^{x_3}} \frac{1}{(k^2 + mt^2)^{x_4}} \frac{1}{(l^2 - mx^2)^{x_5}} .$$

$$[\mathbb{Z}\pm](x_1,x_2,x_3,x_4,x_5) = \int \frac{d^d k}{(2\pi)^d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(k^2+k\cdot Q)^{x_1}} \frac{1}{((l + Q/2)^2)^{x_2}} \frac{1}{((k - l \mp Q/2)^2)^{x_3}} \frac{1}{(k^2 + mt^2)^{x_4}} \frac{1}{(l^2 + mt^2)^{x_5}} .$$