New Results for M_{W} and $\sin^2\theta_{\mathrm{eff}}$ in the MSSM

Georg Weiglein

IPPP Durham

Wien, 11/2005

Based on: J. Haestier, S. Heinemeyer, D. Stöckinger, G. W., hep-ph/0508139,

S. Heinemeyer, W. Hollik, D. Stöckinger, A.M. Weber, G. W., in preparation

1. Introduction
2. Two-loop Yukawa corrections to electroweak precision observables in the MSSM
3. Numerical results for M_{W} and $\sin^2\theta_{\mathrm{eff}}$
4. Estimate of remaining uncertainties from unknown higher-orders
5. Conclusions

1. Introduction

Electroweak precision observables:
current situation vs. ILC $+$ GigaZ

- M_W: $34\ \mathrm{MeV} \xrightarrow{\text{ILC + GigaZ}} 7\ \mathrm{MeV}$
- $\sin^2\theta_{\mathrm{eff}}$: $16 \times 10^{-5} \xrightarrow{\text{ILC + GigaZ}} 1 \times 10^{-5}$
- m_t: $2.9\ \mathrm{GeV} \xrightarrow{\text{ILC}} 0.1\ \mathrm{GeV}$

$\Rightarrow$ High sensitivity to indirect effects of new physics

Theoretical predictions for M_W, $\sin^2\theta_\mathrm{eff}$:

Comparison of prediction for muon decay with experiment (Fermi constant G_μ)

$$\Rightarrow \quad M_\mathrm{W}^2\left(1-\frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_\mu}\left(1+\Delta r\right),$$

$\Updownarrow$

loop corrections

$\Rightarrow$ Theo. prediction for M_W in terms of M_Z, α, G_μ, $\Delta r(m_\mathrm{t}, m_{\tilde{\mathrm{t}}}, \ldots)$

Effective couplings at the Z resonance:

$$\Rightarrow \quad \sin^2\theta_\mathrm{eff}=\frac{1}{4}\left(1-\mathrm{Re}\,\frac{g_V}{g_A}\right)=\left(1-\frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right)\mathrm{Re}\,\kappa_l(s=M_\mathrm{Z}^2)$$

Leading contributions to precison observables

SM result for M_W, one-loop: [*A. Sirlin '80*], [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\text{loop}} = \quad \Delta\alpha \quad - \quad \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho \quad + \quad \Delta r_\text{rem}(M_\mathrm{H})$$

$$\sim \log\frac{M_\mathrm{Z}}{m_f} \qquad \sim m_\mathrm{t}^2$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

Leading contributions to $M_\mathrm{W}, \sin^2\theta_\mathrm{eff}, \ldots$ from mass splitting between isospin doublet fields enter via

$$\Delta\rho = \frac{\Sigma_\mathrm{Z}(0)}{M_\mathrm{Z}^2} - \frac{\Sigma_\mathrm{W}(0)}{M_\mathrm{W}^2}$$

$$\Rightarrow \quad \Delta M_\mathrm{W} \approx \frac{M_\mathrm{W}}{2}\frac{c_\mathrm{W}^2}{c_\mathrm{W}^2 - s_\mathrm{W}^2}\Delta\rho, \qquad \Delta\sin^2\theta_\mathrm{eff} \approx -\frac{c_\mathrm{W}^2 s_\mathrm{W}^2}{c_\mathrm{W}^2 - s_\mathrm{W}^2}\Delta\rho$$

Theoretical uncertainties: current status

- From experimental errors of input parameters

$$\delta m_{\rm t} = 2.9 \text{ GeV} \;\Rightarrow\; \Delta M_{\rm W}^{\rm para} \approx 18 \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff}^{\rm para} \approx 9 \times 10^{-5}$$

Theoretical uncertainties: current status

- From experimental errors of input parameters

$$\delta m_{\rm t} = 2.9 \text{ GeV} \;\Rightarrow\; \Delta M_{\rm W}^{\rm para} \approx 18 \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff}^{\rm para} \approx 9 \times 10^{-5}$$

- From unknown higher-order corrections (“intrinsic”)

  SM: Complete 2-loop result $+$ leading higher-order corrections known for $M_{\rm W}$, complete 2-loop fermionic corr. $+$ leading higher-order corrections known for $\sin^2\theta_{\rm eff}$

  $\Rightarrow$ Remaining uncertainties:
  [*M. Awramik, M. Czakon, A. Freitas, G.W. ’03, ’04*]

$$\Delta M_{\rm W}^{\rm intr} \approx 4 \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff}^{\rm intr} \approx 5 \times 10^{-5}$$

Higher-order corrections in the MSSM

Only known higher-order SUSY corrections to M_W, $\sin^2\theta_{\rm eff}$:

- $\mathcal{O}(\alpha\alpha_s)$ corrections to $\Delta\rho$ ($+$ gluonic corrections to Δr)
  [*A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97*]
  [*S. Heinemeyer, W. Hollik, G. W. '98*]

- $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$ Yukawa corrections to $\Delta\rho$ in limit $M_{\rm SUSY} \to \infty$ (SUSY loop contributions decouple)
  [*S. Heinemeyer, G. W. '02*]

  $\Rightarrow$ well approximated by SM contribution

  $\Rightarrow$ SUSY loop contribution potentially larger (no SM counterpart)

$\Rightarrow$ Intrinsic theoretical uncertainties can be much larger than in the SM

Aim

- Improve prediction for two-loop Yukawa corrections to electroweak precision observables in the MSSM

  [*J. Haestier, S. Heinemeyer, D. Stöckinger, G. W. '05*]

- Provide new result for $M_{\rm W}$ (later also for $\sin^2\theta_{\rm eff}$ and other Z-observables) that contains all available corrections in the SM and the state of the art of the MSSM corrections:

  $\Rightarrow$ Write MSSM prediction for observable O ($O = M_{\rm W}, \sin^2\theta_{\rm eff}, \ldots$) as

$$O_{\rm MSSM} = O_{\rm SM} + O_{\rm MSSM-SM}$$

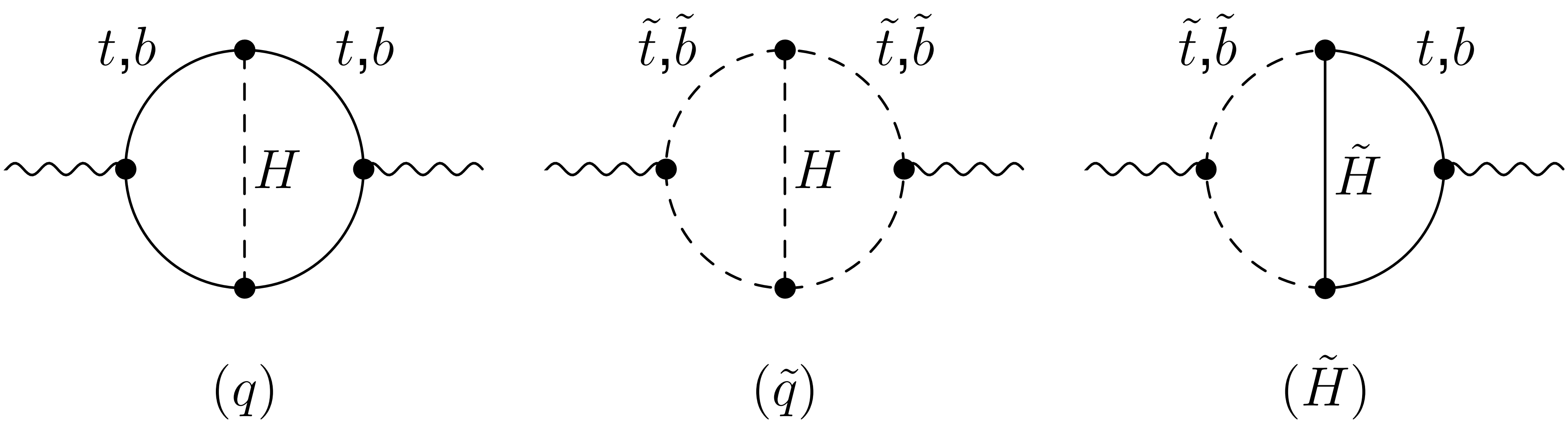
[*S. Heinemeyer, W. Hollik, D. Stöckinger, A.M. Weber, G. W. '05*]

2. Two-loop Yukawa corrections to electroweak precision observables in the MSSM

Calculation of complete $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$ Yukawa corrections to $\Delta\rho$ in the MSSM:

[*J. Haestier, S. Heinemeyer, D. Stöckinger, G. W. '05*]

– quark loops with Higgs exchange,
– squark loops with Higgs exchange,
– quark/squark loops with Higgsino exchange



The gauge-less limit

Yukawa couplings of top and bottom quarks:

$$y_t = \frac{\sqrt{2}\, m_\mathrm{t}}{v\ \sin\beta}, \qquad y_b = \frac{\sqrt{2}\, m_\mathrm{b}}{v\ \cos\beta}$$

$\Rightarrow$ leading Yukawa corrections are obtained in the gauge-less limit:

$$g_{1,2} \to 0, \quad M_\mathrm{W}^2 = \frac{1}{2} g_2^2 v^2 \to 0, \quad M_\mathrm{Z}^2 = \frac{1}{2}(g_1^2 + g_2^2) v^2 \to 0,$$

$$c_\mathrm{w} \equiv \frac{M_\mathrm{W}}{M_\mathrm{Z}} : \ \text{fixed}, \quad v : \ \text{fixed}$$

The gauge-less limit

Yukawa couplings of top and bottom quarks:

$$y_t = \frac{\sqrt{2}\, m_{\rm t}}{v\ \sin\beta}, \qquad y_b = \frac{\sqrt{2}\, m_{\rm b}}{v\ \cos\beta}$$

⇒ leading Yukawa corrections are obtained in the gauge-less limit:

$$g_{1,2} \to 0, \quad M_{\rm W}^2 = \frac{1}{2} g_2^2 v^2 \to 0, \quad M_{\rm Z}^2 = \frac{1}{2}(g_1^2 + g_2^2) v^2 \to 0,$$

$$c_{\rm w} \equiv \frac{M_{\rm W}}{M_{\rm Z}} : \ \text{fixed}, \qquad v : \ \text{fixed}$$

⇒ in MSSM, 1-loop: only fermion/sfermion contributions

Higgs sector of general 2HDM contributes
contribution in MSSM vanishes due to symmetry relations

Two-loop Yukawa corrections: $\mathcal{O}(\alpha_f^2)$

Gauge-less limit at 2-loop yields 2-loop Yukawa corr. $\mathcal{O}(\alpha_f^2)$

$\mathcal{O}(\alpha_f^2)$ contributions to $\Delta\rho$ are the only corrections at this order to M_W, $\sin^2\theta_\mathrm{eff}$

Two-loop Yukawa corrections: $\mathcal{O}(\alpha_f^2)$

Gauge-less limit at 2-loop yields 2-loop Yukawa corr. $\mathcal{O}(\alpha_f^2)$

$\mathcal{O}(\alpha_f^2)$ contributions to $\Delta\rho$ are the only corrections at this order to M_W, $\sin^2\theta_\mathrm{eff}$

In the SM $\mathcal{O}(\alpha_t^2)$ corrections were first obtained for special case $M_\mathrm{H} = 0$: [*J. van der Bij, F. Hoogeveen '87*]

$$\Delta\rho^{\mathrm{SM},\alpha_t^2}_{2-\mathrm{loop}|M_\mathrm{H}=0} = 3\,\frac{G_\mu^2}{128\pi^4}\,m_\mathrm{t}^4\,\left(19 - 2\pi^2\right)$$

$\Rightarrow$ very small correction because of accidental cancellation of terms in the bracket

Two-loop Yukawa corrections: $\mathcal{O}(\alpha_f^2)$

Gauge-less limit at 2-loop yields 2-loop Yukawa corr. $\mathcal{O}(\alpha_f^2)$

$\mathcal{O}(\alpha_f^2)$ contributions to $\Delta\rho$ are the only corrections at this order to M_{W}, $\sin^2\theta_{\mathrm{eff}}$

In the SM $\mathcal{O}(\alpha_t^2)$ corrections were first obtained for special case $M_{\mathrm{H}} = 0$: [*J. van der Bij, F. Hoogeveen '87*]

$$\Delta\rho^{\mathrm{SM},\alpha_t^2}_{2-\mathrm{loop}|M_{\mathrm{H}}=0} = 3\,\frac{G_\mu^2}{128\pi^4}\,m_{\mathrm{t}}^4\,\left(19 - 2\pi^2\right)$$

$\Rightarrow$ very small correction because of accidental cancellation of terms in the bracket

SM $\mathcal{O}(\alpha_t^2)$ result for arbitrary M_{H} [*R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere '92*], [*J. Fleischer, O. Tarasov, F. Jegerlehner '93*]

$\Rightarrow$ much bigger correction for realistic values of $M_{\mathrm{H}}^{\mathrm{SM}}$

Consequences of the gauge-less limit

- Higgs sector:

$$M_{\mathrm{H}^\pm}^2 = M_{\mathrm{H}}^2 = M_{\mathrm{A}}^2, \quad \sin\alpha = -\cos\beta, \quad \cos\alpha = \sin\beta$$

Consequences of the gauge-less limit

- Higgs sector:

$$M^2_{\mathrm{H}^\pm} = M^2_{\mathrm{H}} = M^2_{\mathrm{A}}, \quad \sin\alpha = -\cos\beta, \quad \cos\alpha = \sin\beta$$

$$M_{\mathrm{h}} = 0$$

Consequences of the gauge-less limit

- Higgs sector:

$$M^2_{\mathrm{H}^\pm} = M^2_{\mathrm{H}} = M^2_{\mathrm{A}}, \quad \sin\alpha = -\cos\beta, \quad \cos\alpha = \sin\beta$$

$$M_{\mathrm{h}} = 0$$

$\Rightarrow$ investigate in how far dependence on M_{h} can be kept in a consistent way

Consequences of the gauge-less limit

- Higgs sector:

$$M^2_{\mathrm{H}^\pm} = M^2_{\mathrm{H}} = M^2_{\mathrm{A}}, \quad \sin\alpha = -\cos\beta, \quad \cos\alpha = \sin\beta$$

$$M_{\mathrm{h}} = 0$$

$\Rightarrow$ investigate in how far dependence on M_{h} can be kept in a consistent way

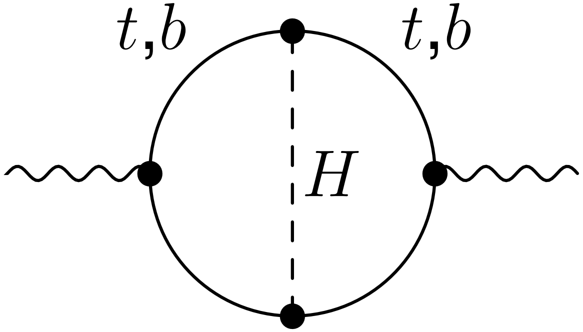
- Higgsino sector:

Diagonal mass matrices of charginos, neutralinos simplify to

$$m_{\tilde{\chi}^\pm_i} = (0, +\mu), \quad m_{\tilde{\chi}^0_i} = (0, 0, +\mu, -\mu)$$

M_h *dependence: pure fermion contributions*

Full M_h dependence can be kept (i.e. can use true MSSM value for M_h) in pure fermion contributions of class (q):



(q)

t/b loops with Higgs and Goldstone boson exchange

[*S. Heinemeyer, G. W. '02*]

Reason: diagrams + counterterm contribution of class (q) correspond to a special case of a general 2HDM (where M_h is a free parameter)

[*J. Haestier, S. Heinemeyer, D. Stöckinger, G. W. '05*]

Renormalisation in the $\tilde{\mathrm{t}}/\tilde{\mathrm{b}}$ *sector*

Mass matrices in the stop/sbottom sector:

$$\mathcal{M}^2_{\tilde{\mathrm{t}}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m^2_{\mathrm{t}} + \cos 2\beta \, (\frac{1}{2} - \frac{2}{3} s^2_{\mathrm{W}}) M^2_{\mathrm{Z}} & m_{\mathrm{t}} X_{\mathrm{t}} \\ m_{\mathrm{t}} X_{\mathrm{t}} & M^2_{\tilde{t}_R} + m^2_{\mathrm{t}} + \frac{2}{3} \cos 2\beta \; s^2_{\mathrm{W}} M^2_{\mathrm{Z}} \end{pmatrix},$$

$$\mathcal{M}^2_{\tilde{\mathrm{b}}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m^2_{\mathrm{b}} + \cos 2\beta \, (-\frac{1}{2} + \frac{1}{3} s^2_{\mathrm{W}}) M^2_{\mathrm{Z}} & m_{\mathrm{b}} X_{\mathrm{b}} \\ m_{\mathrm{b}} X_{\mathrm{b}} & M^2_{\tilde{b}_R} + m^2_{\mathrm{b}} - \frac{1}{3} \cos 2\beta \; s^2_{\mathrm{W}} M^2_{\mathrm{Z}} \end{pmatrix}$$

SU(2) gauge invariance requires: $M^2_{\tilde{t}_L} = M^2_{\tilde{b}_L}$

Renormalisation in the $\tilde{\mathrm{t}}/\tilde{\mathrm{b}}$ *sector*

Mass matrices in the stop/sbottom sector:

$$\mathcal{M}^2_{\tilde{\mathrm{t}}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m^2_{\mathrm{t}} + \cos 2\beta\,(\frac{1}{2} - \frac{2}{3}s^2_{\mathrm{W}})M^2_{\mathrm{Z}} & m_{\mathrm{t}} X_{\mathrm{t}} \\ m_{\mathrm{t}} X_{\mathrm{t}} & M^2_{\tilde{t}_R} + m^2_{\mathrm{t}} + \frac{2}{3}\cos 2\beta\; s^2_{\mathrm{W}} M^2_{\mathrm{Z}} \end{pmatrix},$$

$$\mathcal{M}^2_{\tilde{\mathrm{b}}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m^2_{\mathrm{b}} + \cos 2\beta\,(-\frac{1}{2} + \frac{1}{3}s^2_{\mathrm{W}})M^2_{\mathrm{Z}} & m_{\mathrm{b}} X_{\mathrm{b}} \\ m_{\mathrm{b}} X_{\mathrm{b}} & M^2_{\tilde{b}_R} + m^2_{\mathrm{b}} - \frac{1}{3}\cos 2\beta\; s^2_{\mathrm{W}} M^2_{\mathrm{Z}} \end{pmatrix}$$

SU(2) gauge invariance requires: $M^2_{\tilde{t}_L} = M^2_{\tilde{b}_L}$

$\Rightarrow$ relation between $m_{\tilde{\mathrm{t}}_1}$, $m_{\tilde{\mathrm{t}}_2}$, $\theta_{\tilde{\mathrm{t}}}$, $m_{\tilde{\mathrm{b}}_1}$, $m_{\tilde{\mathrm{b}}_2}$, $\theta_{\tilde{\mathrm{b}}}$

Renormalisation in the $\tilde{\mathrm{t}}/\tilde{\mathrm{b}}$ *sector*

Mass matrices in the stop/sbottom sector:

$$\mathcal{M}^2_{\tilde{\mathrm{t}}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m^2_{\mathrm{t}} + \cos 2\beta\,(\frac{1}{2} - \frac{2}{3} s^2_{\mathrm{W}}) M^2_{\mathrm{Z}} & m_{\mathrm{t}} X_{\mathrm{t}} \\ m_{\mathrm{t}} X_{\mathrm{t}} & M^2_{\tilde{t}_R} + m^2_{\mathrm{t}} + \frac{2}{3} \cos 2\beta\; s^2_{\mathrm{W}} M^2_{\mathrm{Z}} \end{pmatrix},$$

$$\mathcal{M}^2_{\tilde{\mathrm{b}}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m^2_{\mathrm{b}} + \cos 2\beta\,(-\frac{1}{2} + \frac{1}{3} s^2_{\mathrm{W}}) M^2_{\mathrm{Z}} & m_{\mathrm{b}} X_{\mathrm{b}} \\ m_{\mathrm{b}} X_{\mathrm{b}} & M^2_{\tilde{b}_R} + m^2_{\mathrm{b}} - \frac{1}{3} \cos 2\beta\; s^2_{\mathrm{W}} M^2_{\mathrm{Z}} \end{pmatrix}$$

SU(2) gauge invariance requires: $M^2_{\tilde{t}_L} = M^2_{\tilde{b}_L}$

$\Rightarrow$ relation between $m_{\tilde{\mathrm{t}}_1}$, $m_{\tilde{\mathrm{t}}_2}$, $\theta_{\tilde{\mathrm{t}}}$, $m_{\tilde{\mathrm{b}}_1}$, $m_{\tilde{\mathrm{b}}_2}$, $\theta_{\tilde{\mathrm{b}}}$

$\Rightarrow$ not all parameters can be renormalised independently

Renormalisation in the $\tilde{\mathrm{t}}/\tilde{\mathrm{b}}$ sector

Mass matrices in the stop/sbottom sector:

$$\mathcal{M}^2_{\tilde{\mathrm{t}}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m^2_{\mathrm{t}} + \cos 2\beta \, (\frac{1}{2} - \frac{2}{3} s^2_{\mathrm{W}}) M^2_{\mathrm{Z}} & m_{\mathrm{t}} X_{\mathrm{t}} \\ m_{\mathrm{t}} X_{\mathrm{t}} & M^2_{\tilde{t}_R} + m^2_{\mathrm{t}} + \frac{2}{3} \cos 2\beta \; s^2_{\mathrm{W}} M^2_{\mathrm{Z}} \end{pmatrix},$$

$$\mathcal{M}^2_{\tilde{\mathrm{b}}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m^2_{\mathrm{b}} + \cos 2\beta \, (-\frac{1}{2} + \frac{1}{3} s^2_{\mathrm{W}}) M^2_{\mathrm{Z}} & m_{\mathrm{b}} X_{\mathrm{b}} \\ m_{\mathrm{b}} X_{\mathrm{b}} & M^2_{\tilde{b}_R} + m^2_{\mathrm{b}} - \frac{1}{3} \cos 2\beta \; s^2_{\mathrm{W}} M^2_{\mathrm{Z}} \end{pmatrix}$$

SU(2) gauge invariance requires: $M^2_{\tilde{t}_L} = M^2_{\tilde{b}_L}$

$\Rightarrow$ relation between $m_{\tilde{\mathrm{t}}_1}$, $m_{\tilde{\mathrm{t}}_2}$, $\theta_{\tilde{\mathrm{t}}}$, $m_{\tilde{\mathrm{b}}_1}$, $m_{\tilde{\mathrm{b}}_2}$, $\theta_{\tilde{\mathrm{b}}}$

$\Rightarrow$ not all parameters can be renormalised independently

$\Rightarrow$ choose $\delta m^2_{\tilde{\mathrm{b}}_1}$ as dependent counterterm

$\Rightarrow$ $\delta m^2_{\tilde{\mathrm{b}}_1}\big|_{\mathrm{symm}} = f(\delta m_{\tilde{\mathrm{t}}_1}, \delta m_{\tilde{\mathrm{t}}_2}, \delta\theta_{\tilde{\mathrm{t}}}, \delta m_{\tilde{\mathrm{b}}_2}, \delta\theta_{\tilde{\mathrm{b}}}, \delta m_{\mathrm{t}}, \delta m_{\mathrm{t}})$

Total result for $\Delta\rho$

On-shell renormalisation of the other parameters:

$$\delta m^2_{\tilde{f}_i} = \mathrm{Re}\,\Sigma_{\tilde{f}_i}(m^2_{\tilde{f}_i}) \qquad \text{for } \tilde{f}_i = \tilde{t}_{1,2}, \tilde{b}_2$$

$$\delta\theta_{\tilde{f}} = \frac{\mathrm{Re}\,\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1}) + \mathrm{Re}\,\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_2})}{2(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2})} \qquad \text{for } \tilde{f} = \tilde{t}, \tilde{b}$$

Total result for $\Delta\rho$

On-shell renormalisation of the other parameters:

$$\delta m^2_{\tilde{f}_i} = \mathrm{Re}\,\Sigma_{\tilde{f}_i}(m^2_{\tilde{f}_i}) \qquad \text{for } \tilde{f}_i = \tilde{t}_{1,2}, \tilde{b}_2$$

$$\delta\theta_{\tilde{f}} = \frac{\mathrm{Re}\,\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_1}) + \mathrm{Re}\,\Sigma_{\tilde{f}_1\tilde{f}_2}(m^2_{\tilde{f}_2})}{2(m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2})} \qquad \text{for } \tilde{f} = \tilde{t}, \tilde{b}$$

$\Rightarrow$ total result for $\Delta\rho$ can be written as

$$\Delta\rho^{(q,\tilde{q},\tilde{H})} = \Delta\rho^{(q)}_{\mathrm{MSSM}} + \Delta\rho^{(\tilde{q},\tilde{H})}_{\mathrm{MSSM,\ full\ OS}} + \Delta m^2_{\tilde{\mathrm{b}}_1}\,\partial_{m^2_{\tilde{\mathrm{b}}_1}}\,\Delta\rho^{\mathrm{SUSY}}_{1-\mathrm{loop}},$$

$$\text{where } \Delta m^2_{\tilde{\mathrm{b}}_1} = \delta m^2_{\tilde{\mathrm{b}}_1}\big|_{\mathrm{symm}} - \delta m^2_{\tilde{\mathrm{b}}_1}\big|_{\mathrm{OS}}, \qquad \delta m^2_{\tilde{\mathrm{b}}_1}\big|_{\mathrm{OS}} = \mathrm{Re}\,\Sigma_{\tilde{b}_1}(m^2_{\tilde{\mathrm{b}}_1})$$

Alternative renormalisation scheme: $\overline{\mathrm{DR}}$ ren. for soft SUSY-breaking parameters in stop/sbottom sector

M_h *dependence: sfermion and higgsino contributions,* $\Delta\rho^{(\tilde{q},\tilde{H})}$

$\Delta\rho^{(\tilde{q},\tilde{H})}_{\text{MSSM, full OS}}$: full M_h dependence can be kept

$\Delta m^2_{\tilde{\mathrm{b}}_1}\, \partial_{m^2_{\tilde{\mathrm{b}}_1}} \Delta\rho^{\mathrm{SUSY}}_{1-\mathrm{loop}}$: needs to be evaluated in full gauge-less limit, i.e. for $M_\mathrm{h} = 0$

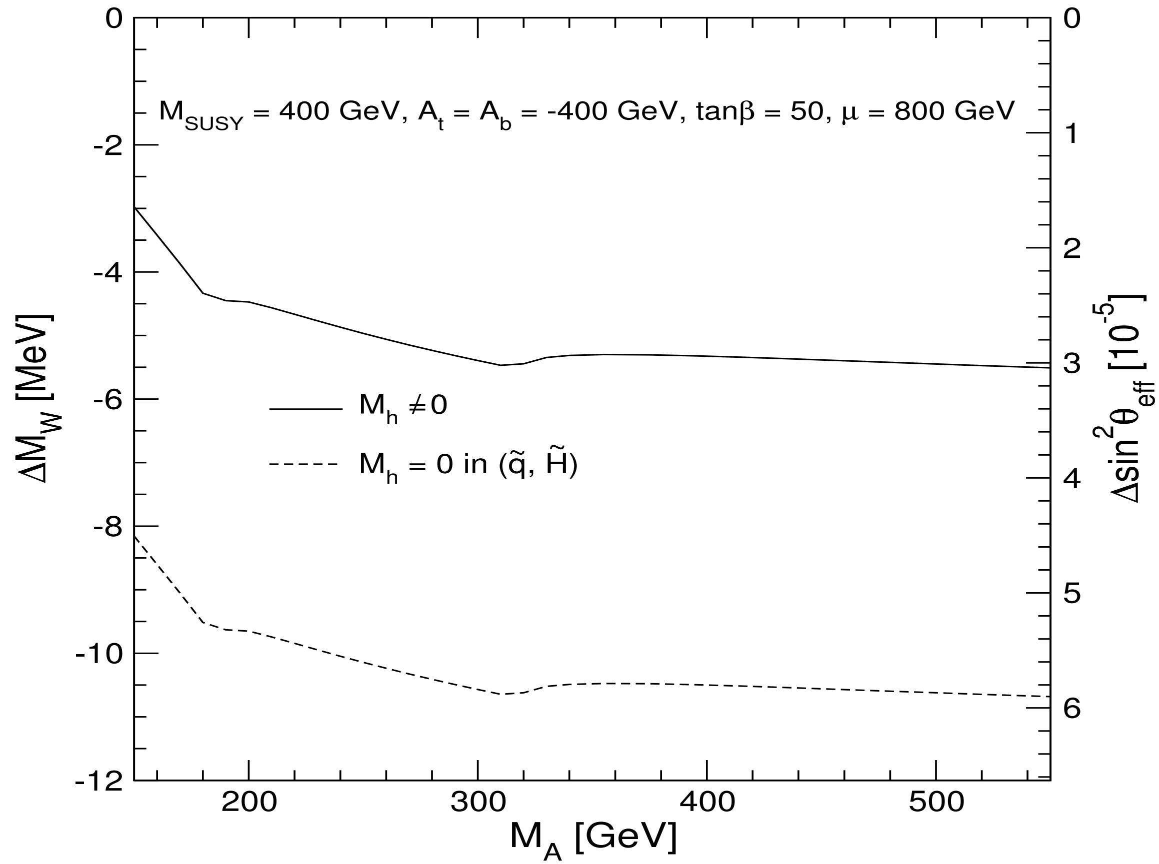
$\Rightarrow$ treat M_h-dependence of $\Delta\rho^{(\tilde{q},\tilde{H})}$ as theoretical uncertainty in the following

3. Numerical results for M_W and $\sin^2\theta_\mathrm{eff}$

M_h-dependence of $\Delta\rho^{(\tilde{q},\tilde{H})}$:

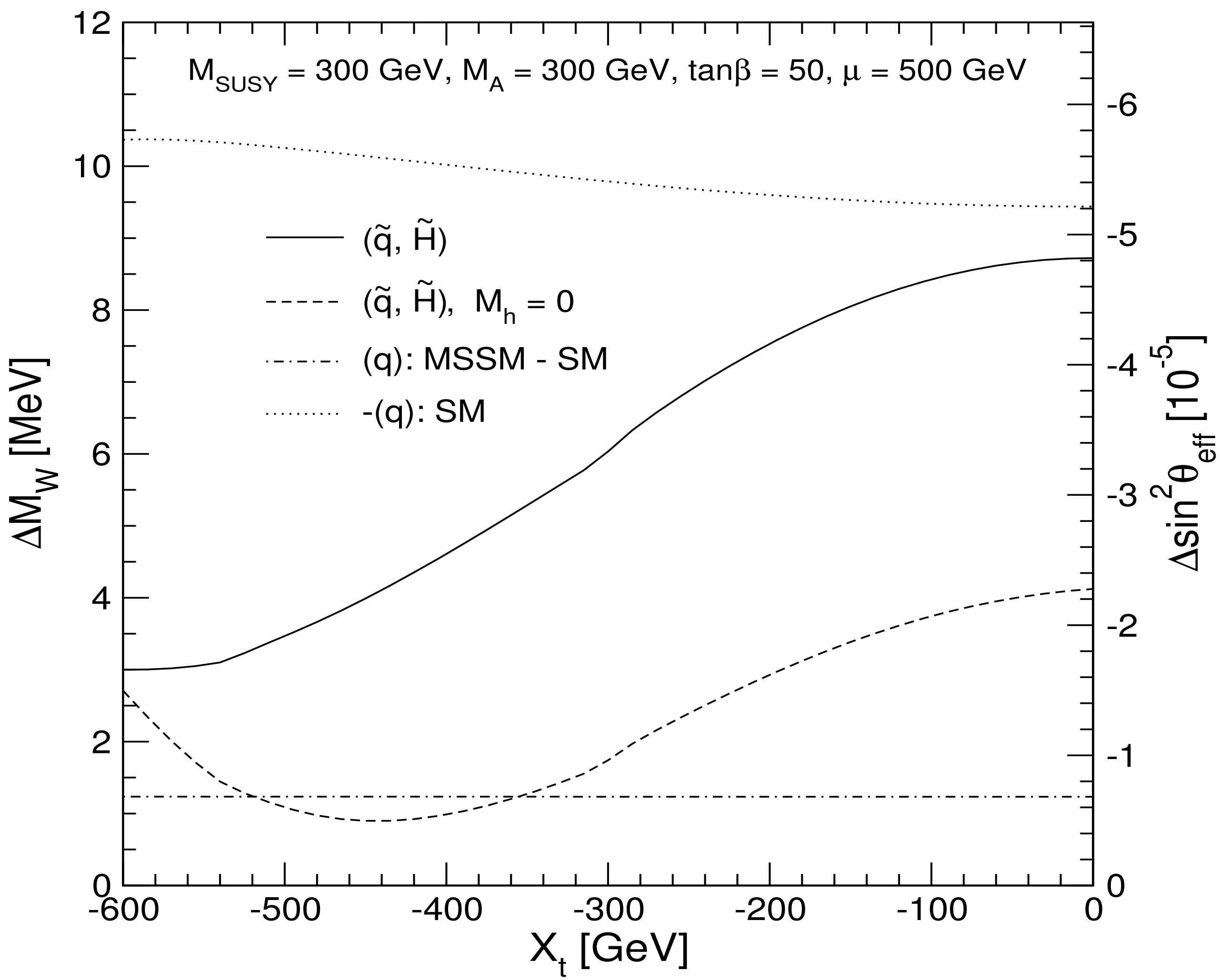
Shift induced by $\Delta\rho^{(\tilde{q},\tilde{H})}$ in M_W, $\sin^2\theta_\mathrm{eff}$:

"extreme scenario", where M_h dependence of $\Delta\rho^{(\tilde{q},\tilde{H})}$ is particularly large



$\Rightarrow$ M_h-dependence of squark and higgsino contributions induces shift of up to $+5$ MeV in M_W and -3×10^{-5} to $\sin^2\theta_\mathrm{eff}$

Shifts induced in $M_{\rm W}$ and $\sin^2\theta_{\rm eff}$ by two-loop Yukawa corrections as function of mixing in scalar top sector

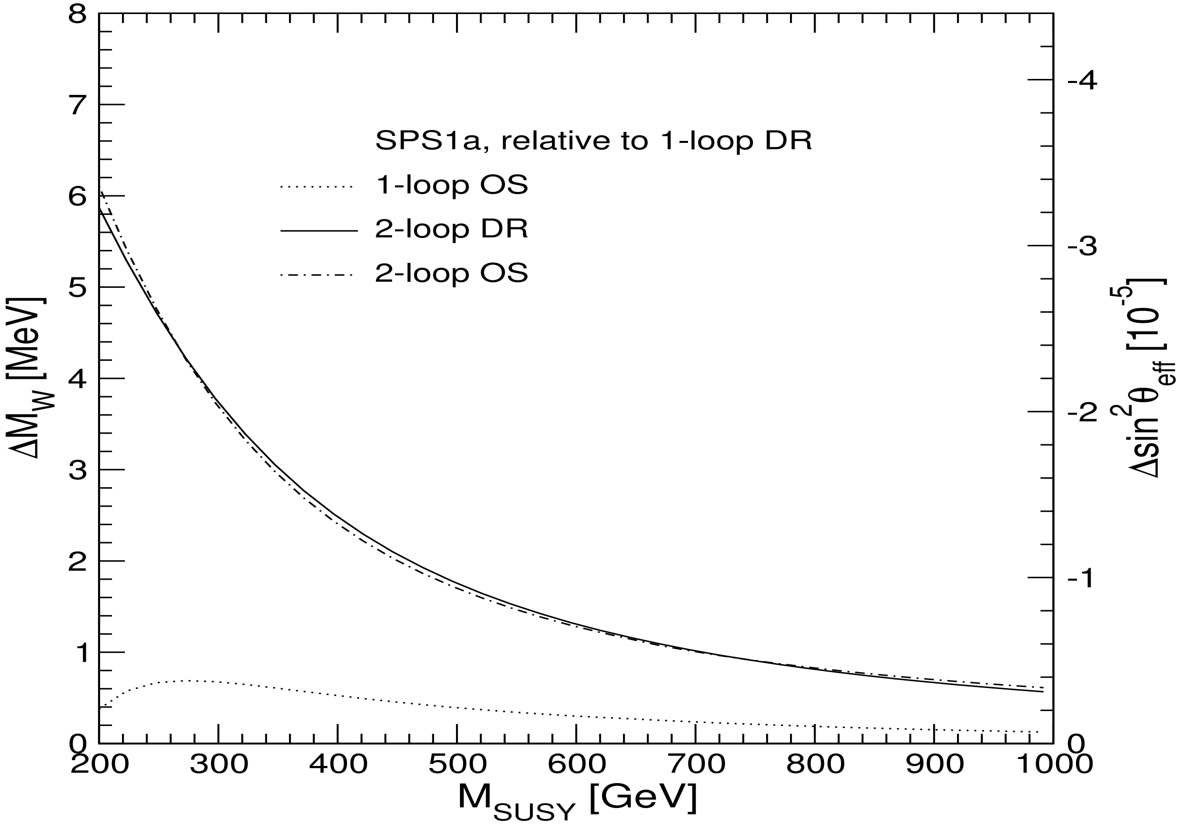


⇒ Corrections up to $\Delta M_{\rm W} \approx +8~{\rm MeV}$, $\Delta\sin^2\theta_{\rm eff} \approx -4\times10^{-5}$ from SUSY loops, can be as large as SM quark loops

Result in SPS 1a scenario: 2-loop on-shell and 2-loop $\overline{\mathrm{DR}}$ result relative to 1-loop result with $\overline{\mathrm{DR}}$ parameters

Shifts induced by squark and higgsino corrections

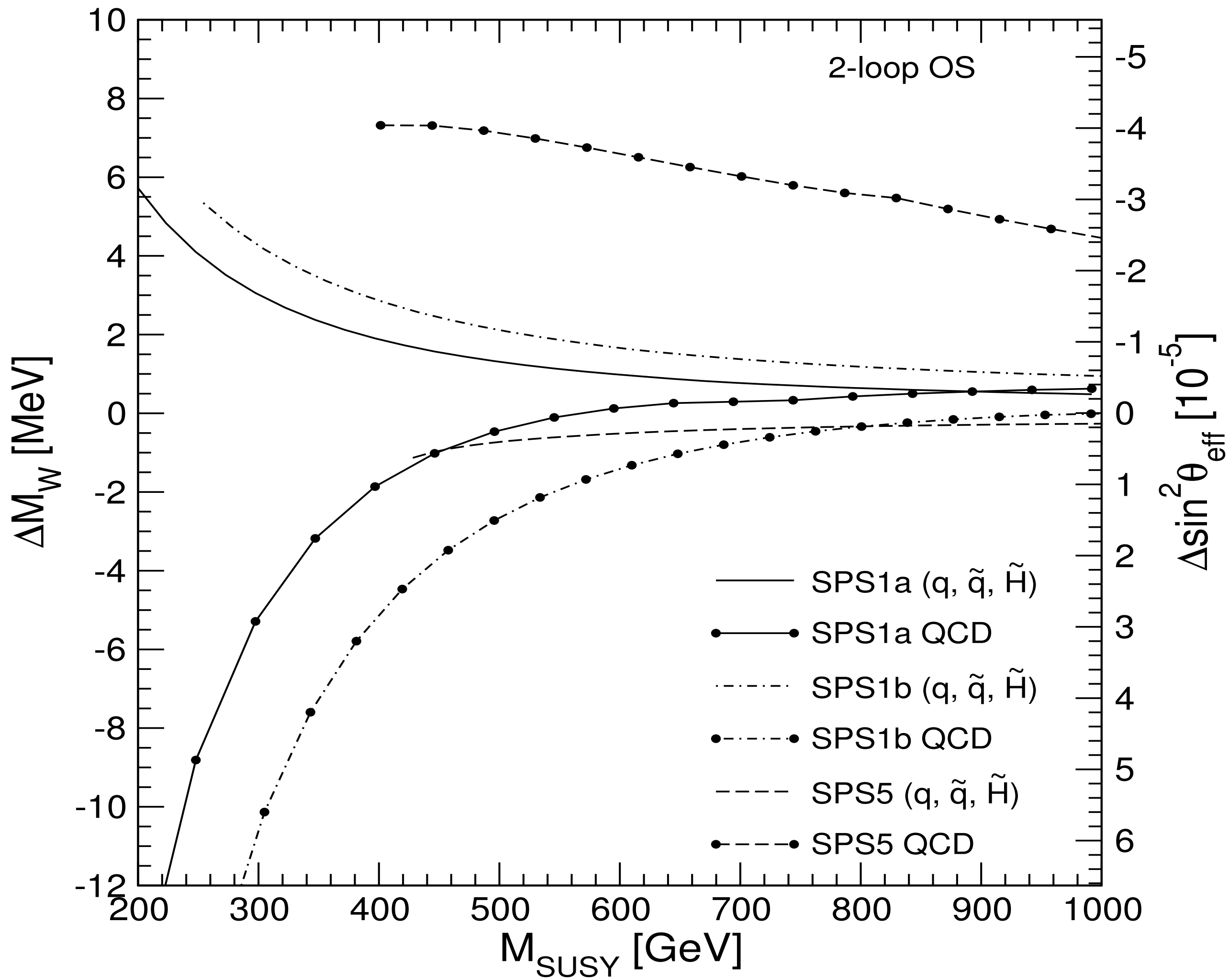
M_{SUSY}, A_{t}, A_{b}, μ, $\mu^{\overline{\mathrm{DR}}}$ varied using common scale factor



$\Rightarrow$ Corrections up to $\Delta M_{\mathrm{W}} \approx +6\ \mathrm{MeV}$, $\Delta \sin^2\theta_{\mathrm{eff}} \approx -3 \times 10^{-5}$

large reduction of scheme dependence

Yukawa corrections vs. $\mathcal{O}(\alpha\alpha_s)$ *corrections to* M_W, $\sin^2\theta_{\rm eff}$ *as function of* $M_{\rm SUSY}$ *for three SPS scenarios*

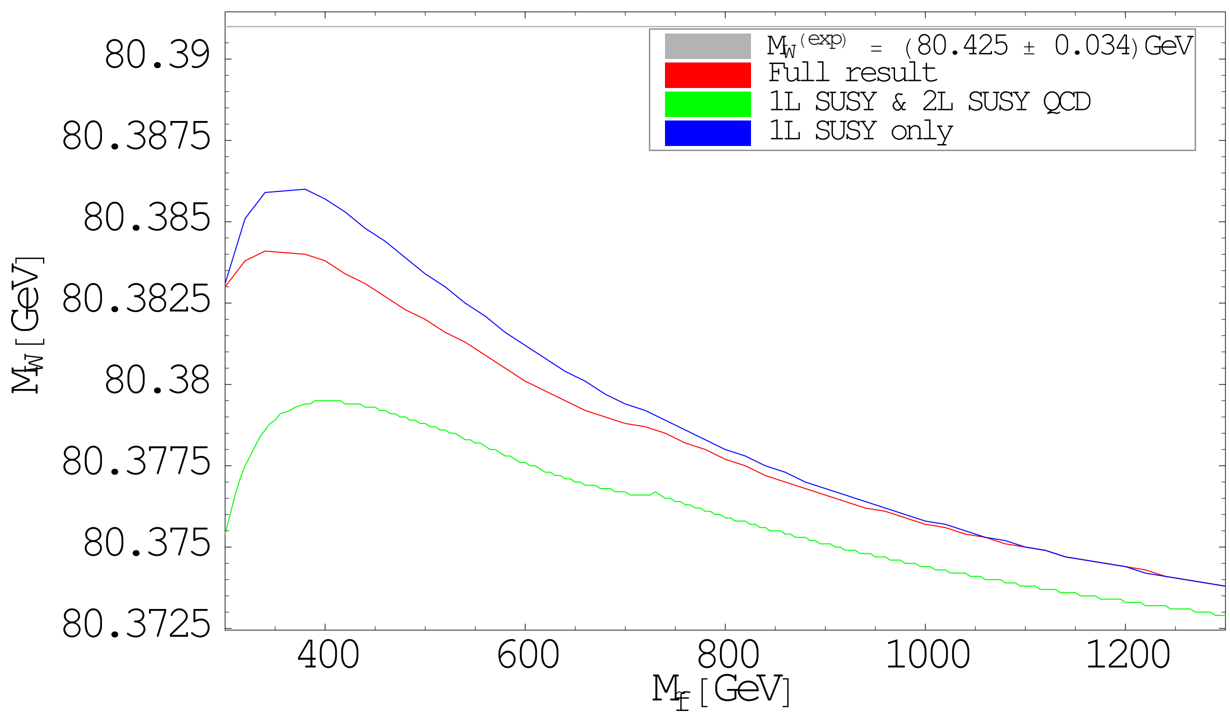


⇒ Corrections have similar size, large compensations for small $M_{\rm SUSY}$

Result for M_W: Yukawa vs. $\mathcal{O}(\alpha\alpha_s)$ corrections

Prediction for M_W as function of M_{SUSY} for $\tan\beta = 50$,
$A_t = A_b = -600$ GeV, $\mu = 300$ GeV, $m_{\tilde{g}} = 500$ GeV

[*S. Heinemeyer, W. Hollik, D. Stöckinger, A.M. Weber, G. W. '05*]



$\Rightarrow$ Large compensations for small M_{SUSY}

The two corrections add up if sign of A_t, A_b is reversed

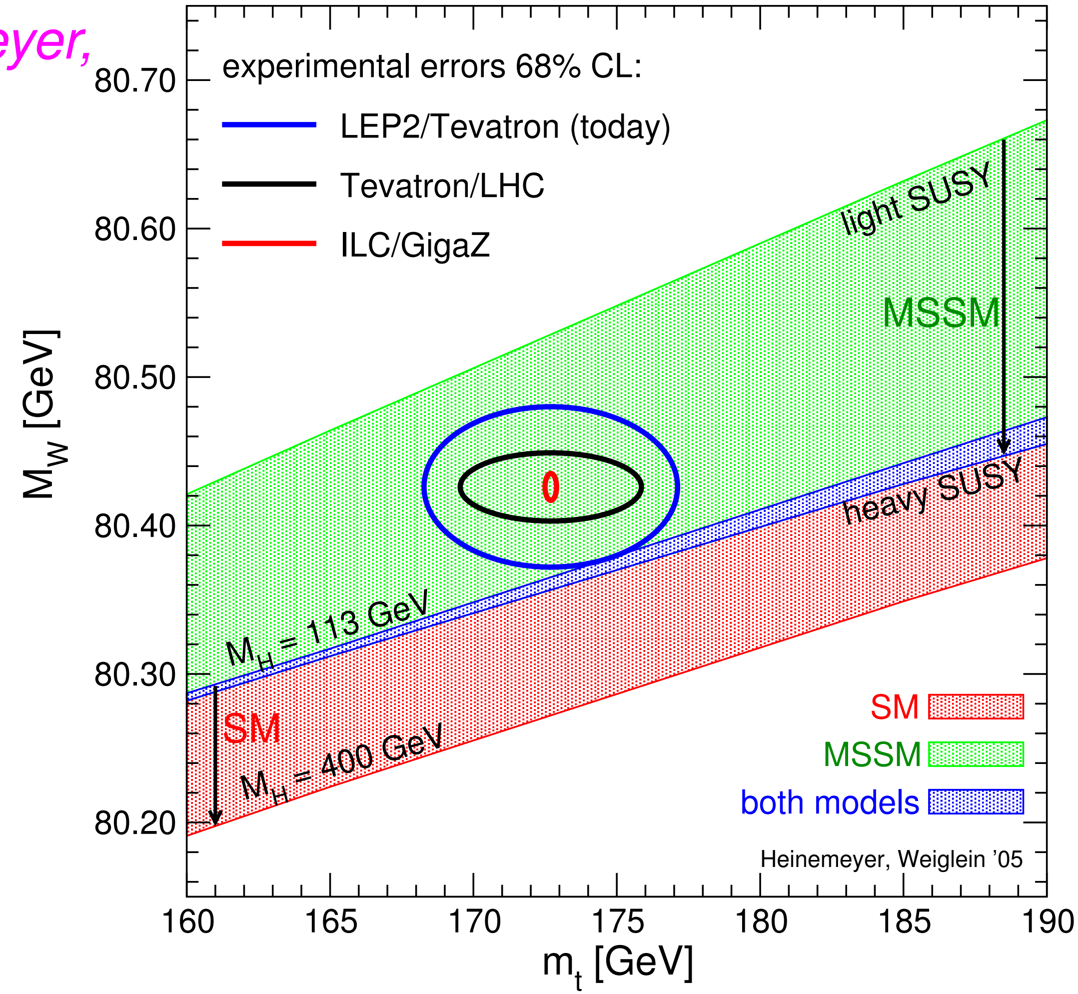
Electroweak precision tests: SM vs. MSSM

Prediction for M_W in the SM and the MSSM ($m_\mathrm{t} = 172.7 \pm 2.9$ GeV):

[A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97]

[S. Heinemeyer, W. Hollik, G. W. '98]

[S. Heinemeyer, G. W. '02]



SM: M_H varied

MSSM: SUSY parameters varied

4. Estimate of remaining uncertainties from unknown higher-orders

Theoretical evaluation of precision observables is more advanced in the SM than in the MSSM

$\Rightarrow$ Writing MSSM prediction for observable O ($O = M_{\rm W}, \sin^2\theta_{\rm eff}, \ldots$) as

$$O_{\rm MSSM} = O_{\rm SM} + O_{\rm MSSM-SM}$$

takes all known SM corrections into account

4. Estimate of remaining uncertainties from unknown higher-orders

Theoretical evaluation of precision observables is more advanced in the SM than in the MSSM

⇒ Writing MSSM prediction for observable O ($O = M_{\rm W}, \sin^2\theta_{\rm eff}, \ldots$) as

$$O_{\rm MSSM} = O_{\rm SM} + O_{\rm MSSM-SM}$$

takes all known SM corrections into account

⇒ higher-order uncertainties reduce to SM uncertainties in decoupling limit (where $O_{\rm MSSM-SM} \to 0$)

4. Estimate of remaining uncertainties from unknown higher-orders

Theoretical evaluation of precision observables is more advanced in the SM than in the MSSM

⇒ Writing MSSM prediction for observable O ($O = M_{\rm W}, \sin^2\theta_{\rm eff}, \ldots$) as

$$O_{\rm MSSM} = O_{\rm SM} + O_{\rm MSSM-SM}$$

takes all known SM corrections into account

⇒ higher-order uncertainties reduce to SM uncertainties in decoupling limit (where $O_{\rm MSSM-SM} \to 0$)

⇒ Need estimate of further uncertainties from SUSY loop corrections as a function of the SUSY scale

Estimate of uncertainties from SUSY loop corrections depending on M_{SUSY}

- Electroweak 2-loop corrections beyond the leading Yukawa corrections:
  - assume ratio of subleading ew 2-loop corr. to 2-loop Yukawa corr. is the same in the MSSM as in the SM
  - M_{h} dependence of squark and higgsino contributions

- $\mathcal{O}(\alpha\alpha_s)$ corrections beyond the $\Delta\rho$ approximation:

  assume ratio of contribution entering via $\Delta\rho$ to full result is the same as in the SM

Estimate of uncertainties from SUSY loop corrections depending on M_{SUSY}

- $\mathcal{O}(\alpha\alpha_s^2)$ corrections:
  - assume ratio of supersymmetric $\mathcal{O}(\alpha\alpha_s^2)$ contributions to $\mathcal{O}(\alpha\alpha_s)$ supersymmetric contributions is the same as for corresponding corrections in the SM
  - geometric progression from lower orders: assume ratio is the same as of $\mathcal{O}(\alpha\alpha_s)$ supersymmetric contributions to $\mathcal{O}(\alpha)$ supersymmetric contributions
  - variation of renormalisation scale of $\alpha_s(\mu^{\overline{\text{DR}}})$ entering the $\mathcal{O}(\alpha\alpha_s)$ result according to $m_{\text{t}}/2 \leq \mu^{\overline{\text{DR}}} \leq 2m_{\text{t}}$

Estimate of uncertainties from SUSY loop corrections depending on $M_{\rm SUSY}$

- $\mathcal{O}(\alpha^2\alpha_s)$ corrections:
  - assume ratio of supersymmetric $\mathcal{O}(\alpha^2\alpha_s)$ contributions to $\mathcal{O}(\alpha^2)$ leading Yukawa supersymmetric contributions is the same as for corresponding corrections in the SM
  - geometric progression from lower orders: assume ratio is the same as of $\mathcal{O}(\alpha\alpha_s)$ supersymmetric contributions to $\mathcal{O}(\alpha)$ supersymmetric contributions
  - change value of $m_{\rm t}$ in result for 2-loop supersymmetric Yukawa corrections from $m_{\rm t}^{\rm OS}$ to $m_{\rm t}(m_{\rm t}) = m_{\rm t}^{\rm OS}/(1 + 4/(3\pi)\,\alpha_s(m_{\rm t}))$

- Electroweak three-loop corrections:

  renormalisation scheme dependence of result for 2-loop supersymmetric Yukawa corrections

Resulting estimates for uncertainty in $M_{\rm W}$ (in $\rm MeV$) for different values of $M_{\rm SUSY}$

Values obtained for SPS 1a, SPS 1b, SPS 5 (largest value taken as estimate):

$M_{\rm SUSY}$	<500 GeV	500 GeV	1000 GeV
$\mathcal{O}(\alpha^2)$ sublead.	6.0	2.0	0.8
$\mathcal{O}(\alpha\alpha_s)$ sublead.	1.8	0.9	0.5
$\mathcal{O}(\alpha\alpha_s^2)$	3.0, 5.3, 1.5	1.4, 1.1, 0.7	0.9, 2.2, 0.5
$\mathcal{O}(\alpha^2\alpha_s)$	1.5, 2.2, 1.4	0.6, 0.8, 0.4	0.2, 0.2, 0.2
$\mathcal{O}(\alpha^3)$	0.3	0.3	0.3

Estimates for uncertainties in M_W and $\sin^2\theta_\mathrm{eff}$ from unknown higher-order SUSY contrib.

$$
\begin{aligned}
\delta M_\mathrm{W} &= 8.5\ \mathrm{MeV}\ \text{for}\ M_\mathrm{SUSY} = 200\ \mathrm{GeV} \\
\delta M_\mathrm{W} &= 2.7\ \mathrm{MeV}\ \text{for}\ M_\mathrm{SUSY} = 500\ \mathrm{GeV} \\
\delta M_\mathrm{W} &= 2.4\ \mathrm{MeV}\ \text{for}\ M_\mathrm{SUSY} = 1000\ \mathrm{GeV}
\end{aligned}
$$

$$
\begin{aligned}
\delta\sin^2\theta_\mathrm{eff} &= 4.7\times 10^{-5}\ \text{for}\ M_\mathrm{SUSY} = 200\ \mathrm{GeV} \\
\delta\sin^2\theta_\mathrm{eff} &= 1.5\times 10^{-5}\ \text{for}\ M_\mathrm{SUSY} = 500\ \mathrm{GeV} \\
\delta\sin^2\theta_\mathrm{eff} &= 1.3\times 10^{-5}\ \text{for}\ M_\mathrm{SUSY} = 1000\ \mathrm{GeV}
\end{aligned}
$$

Estimates for uncertainties in M_W and $\sin^2\theta_{\rm eff}$ from unknown higher-order SUSY contrib.

$$\delta M_W = 8.5 \text{ MeV for } M_{\rm SUSY} = 200 \text{ GeV}$$
$$\delta M_W = 2.7 \text{ MeV for } M_{\rm SUSY} = 500 \text{ GeV}$$
$$\delta M_W = 2.4 \text{ MeV for } M_{\rm SUSY} = 1000 \text{ GeV}$$

$$\delta \sin^2\theta_{\rm eff} = 4.7 \times 10^{-5} \text{ for } M_{\rm SUSY} = 200 \text{ GeV}$$
$$\delta \sin^2\theta_{\rm eff} = 1.5 \times 10^{-5} \text{ for } M_{\rm SUSY} = 500 \text{ GeV}$$
$$\delta \sin^2\theta_{\rm eff} = 1.3 \times 10^{-5} \text{ for } M_{\rm SUSY} = 1000 \text{ GeV}$$

$\Rightarrow$ Total uncertainty including higher-order SM corrections:

$$\delta M_W = (5 - 9) \text{ MeV}, \quad \delta \sin^2\theta_{\rm eff} = (5 - 7) \times 10^{-5}$$

5. Conclusions

- New result for 2-loop Yukawa corr. to $\Delta\rho$ in the MSSM:
  $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$ contributions from SM fermions, sfermions, Higgs bosons and higgsinos

5. Conclusions

- New result for 2-loop Yukawa corr. to $\Delta\rho$ in the MSSM:
  $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$ contributions from SM fermions, sfermions, Higgs bosons and higgsinos
- Corrections up to $\Delta M_W = +8\ \mathrm{MeV}$, $\Delta\sin^2\theta_{\mathrm{eff}} = -4 \times 10^{-5}$
  can be as large as SM quark loop and SUSY $\mathcal{O}(\alpha\alpha_s)$ corrections

5. Conclusions

- New result for 2-loop Yukawa corr. to $\Delta\rho$ in the MSSM:
  $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$ contributions from SM fermions, sfermions, Higgs bosons and higgsinos
- Corrections up to $\Delta M_W = +8$ MeV, $\Delta \sin^2\theta_{\mathrm{eff}} = -4 \times 10^{-5}$
  can be as large as SM quark loop and SUSY $\mathcal{O}(\alpha\alpha_s)$ corrections
- Detailed estimates of remaining uncertainties of M_W and $\sin^2\theta_{\mathrm{eff}}$ from unknown higher-order contributions for different values of M_{SUSY}:
  $\delta M_W = (5 - 9)$ MeV, $\delta \sin^2\theta_{\mathrm{eff}} = (5 - 7) \times 10^{-5}$

5. Conclusions

- New result for 2-loop Yukawa corr. to $\Delta\rho$ in the MSSM:
  $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$ contributions from SM fermions, sfermions, Higgs bosons and higgsinos

- Corrections up to $\Delta M_W = +8$ MeV, $\Delta \sin^2\theta_{\text{eff}} = -4 \times 10^{-5}$
  can be as large as SM quark loop and SUSY $\mathcal{O}(\alpha\alpha_s)$ corrections

- Detailed estimates of remaining uncertainties of M_W and $\sin^2\theta_{\text{eff}}$ from unknown higher-order contributions for different values of M_{SUSY}:
  $\delta M_W = (5-9)$ MeV, $\delta \sin^2\theta_{\text{eff}} = (5-7) \times 10^{-5}$

- New result for M_W (will appear as public program soon), contains all known corrections in the SM and the MSSM