Complete electroweak $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to 4$ fermions

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work in collaboration with S. Dittmaier, M. Roth and L.H. Wieders

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- Motivation
- Some details of the calculation (complex-mass scheme, . . .)
- Numerical results

Introduction

Many interesting processes at ILC and LHC have more than four external particles: $\mathrm{e^+e^- \to \nu\bar{\nu}H, e^+e^- \to t\bar{t}H, e^+e^- \to 4}f, \ldots, \mathrm{pp \to t\bar{t}H, pp \to t\bar{t}b\bar{b}}, \ldots$

- experimental accuracy typically at the level of some per cent to some per mille at ILC (e.g. $\mathrm{e^+e^- \to W^+W^- \to 4}f$)
- electroweak (EW) radiative corrections grow with energy e.g. leading logarithmic corrections $\propto \alpha \ln^2(E/M_\mathrm{W})$ (EW Sudakov logarithms)
- radiative corrections grow with number of external particles

$\Rightarrow$ need electroweak radiative corrections for $2 \to 3$ and $2 \to 4$ processes

Problems in corrections to $2 \to 3$ and $2 \to 4$ processes

- amount of algebra ($\mathcal{O}(1000)$ Feynman diagrams, many complicated ones)
- numerical stability (5-point functions, 6-point functions, phase space, ...)
- treatment of unstable particles

W-pair production at LEP2

- cross-section measurement with $\Delta\sigma_{WW}/\sigma_{WW} \sim 1\%$

  $\hookrightarrow$ significance of non-universal electroweak corrections

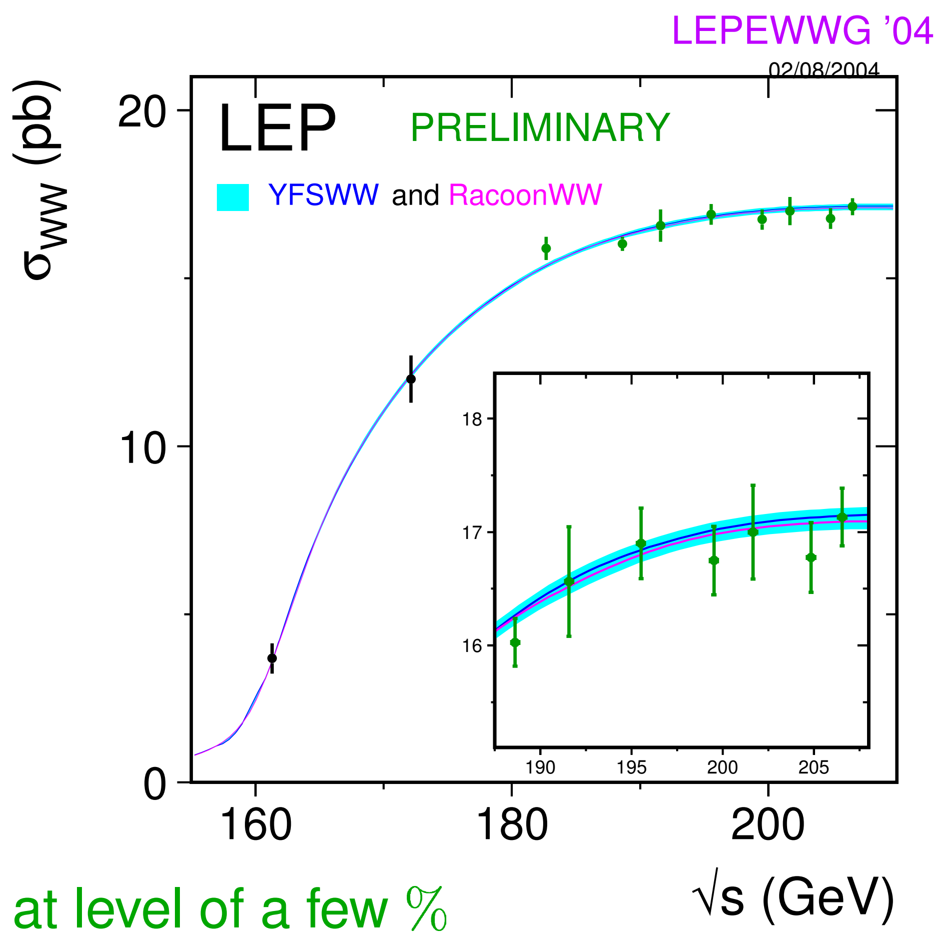
- M_W from threshold cross section with $\Delta M_W \sim 200\,\mathrm{MeV}$

- M_W from direct reconstruction with $\Delta M_W \sim 40\,\mathrm{MeV}$

  $\hookrightarrow$ strengthening of M_H bounds

- constraints on anomalous triple gauge-boson couplings (TGC) at level of a few $\%$

  $\hookrightarrow$ verification of gauge structure



Predictions for $e^+e^- \to WW \to 4f(+\gamma)$ at LEP2

- lowest-order predictions based on full $e^+e^- \to 4f(+\gamma)$ matrix elements
- universal radiative corrections $\to$ "improved Born approximations" (IBA)
- non-universal radiative corrections in "double-pole approximation" (DPA)

Beenakker, Berends, Chapovsky '98
Jadach et al. '99–'01
Denner, Dittmaier, Roth, Wackeroth '99–'01
Kurihara, Kuroda, Schildknecht '01

$\Rightarrow$ corresponding generators:
KoralW $\oplus$ *YFSWW* (Jadach,Płaczek,Skrzypek,Ward) and
RacoonWW (Denner, Dittmaier,Roth,Wackeroth)

Estimates of theoretical uncertainties (TU) for

- total cross section (Denner et al., Jadach et al.)

$$\Delta\sigma_{WW}/\sigma_{WW} \lesssim \begin{cases} 2\% & \text{for} \quad \sqrt{s} < 170\,\text{GeV} & \text{(IBA)} \\ 0.7\% & \text{for} \quad 170\,\text{GeV} < \sqrt{s} < 180\,\text{GeV} & \text{(DPA)} \\ 0.5\% & \text{for} \quad 180\,\text{GeV} < \sqrt{s} < 500\,\text{GeV} & \text{(DPA)} \end{cases}$$

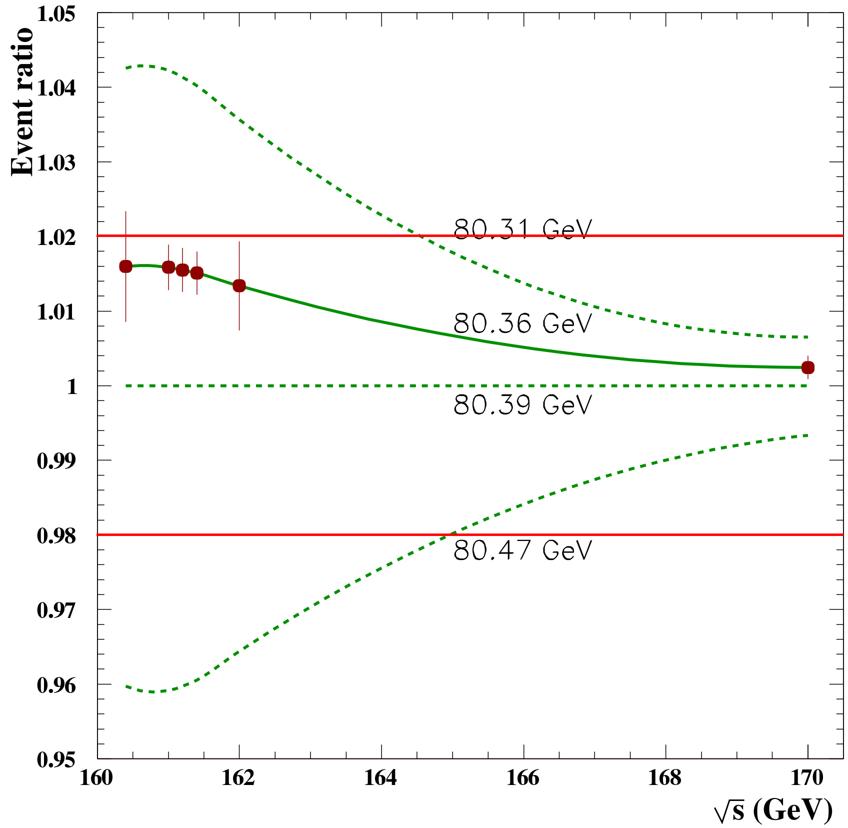
- direct M_W reconstruction: $\Delta M_W \lesssim 5\,\text{MeV}$ (Jadach et al. '01) $- 10\,\text{MeV}$ (Cossutti '04)
- bounds on anomalous TGC λ: $\Delta\lambda \lesssim 0.005$ (Brunelière et al. '02)

W-pair production at ILC

- cross-section measurement
  with $\Delta\sigma_{\mathrm{WW}}/\sigma_{\mathrm{WW}} \lesssim 0.5\%$
- M_{W} from threshold cross section
  with $\Delta M_{\mathrm{W}} \sim 7\,\mathrm{MeV}$

  $\hookrightarrow$ IBA totally insufficient $\Longrightarrow$
- M_{W} from direct reconstruction
  with $\Delta M_{\mathrm{W}} \sim 10\,\mathrm{MeV}$
- constraints on anomalous
  TGC at level of 0.1%

(see e.g. TESLA-TDR '01)



Theoretical requirements for ILC:

- full $\mathcal{O}(\alpha)$ correction for $\mathrm{e^+e^-} \to 4f$

  $\hookrightarrow$ subject of this talk !
- leading corrections beyond $\mathcal{O}(\alpha)$

Processes and Feynman diagrams

Complete $\mathcal{O}(\alpha)$ corrections to $\mathrm{e^+e^-} \to$ $\nu_\tau\tau^+\mu^-\bar{\nu}_\mu$ leptonic

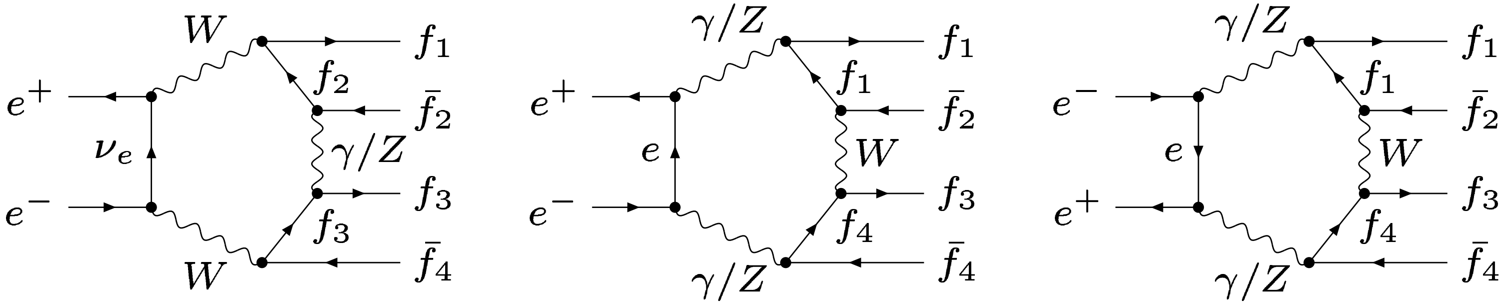
(CC11 class) $\mathrm{u\bar{d}}\mu^-\bar{\nu}_\mu$ semileptonic

$\mathrm{u\bar{d}s\bar{c}}$ hadronic final state

~ 1200 one-loop diagrams: generation with FEYNARTS versions 1 and 3

Küblbeck, Böhm, Denner '90 Hahn '00

- 40 hexagons



+ graphs with reversed fermion-number flow in final state

- 112 pentagons
- 227 boxes ('t Hooft–Feynman gauge)
- many vertex corrections and self-energy diagrams

Approach for calculation of virtual corrections

- External fermion masses neglected whenever possible
  (everywhere but in mass-singular logarithms)
- algebraic simplifications using two independent in-house programs implemented in *Mathematica*, one builds upon FORMCALC,
  special reduction algorithms for spinorial structures

  automatic translation into *Fortran* code
- finite width via complex-mass scheme
- (complex) on-shell renormalization scheme
- numerically stable reduction of tensor integrals to master integrals
  (scalar 1-, 2-, 3-, 4-point integrals and others in exceptional cases)
- scalar integrals: evaluated with standard techniques
  and analytic continuation for complex masses

details given in the following and in talk of S. Dittmaier

Algebraic reduction of spinor chains

Feynman amplitude contains $\mathcal{O}(10^3)$ different spinorial structures of the form

$$\bar{v}_1(p_1)A\omega_\rho u_2(p_2) \;\times\; \bar{v}_3(p_3)B\omega_\sigma u_4(p_4) \;\times\; \bar{v}_5(p_5)C\omega_\tau u_6(p_6)$$

$$\omega_\pm = \tfrac{1}{2}(1 \pm \gamma_5)$$

example

$$\bar{v}_1(p_1)\gamma^\mu\gamma^\nu \not{p}_i\omega_\rho u_2(p_2) \;\times\; \bar{v}_3(p_3)\gamma_\nu\gamma^\rho \not{p}_j\omega_\sigma u_4(p_4) \;\times\; \bar{v}_5(p_5)\gamma_\mu\gamma_\rho \not{p}_k\omega_\tau u_6(p_6)$$

using 4-dimensional and D-dimensional relations
(Dirac algebra, Chisholm identity, decomposition of metric tensor, . . .)

$\Rightarrow$ reduction to $\mathcal{O}(10)$ standard structures with well-behaved coefficients

two different completely independent reduction algorithms

for details see NPB724 (2005) 247 [hep-ph/0505042]

Treatment of finite width

Need scheme that works well in one-loop calculation, in particular also in threshold region, where doubly resonant diagrams do not dominate!

Pole expansion: Stuart '91, Aeppli et al. '93, Aeppli et al. '94

consistent and gauge invariant, not reliable near threshold

Effective field theory approach Beneke et al. '04

equivalent to pole expansion

Naive fixed width scheme: (mildly) breaks gauge invariance,
inclusion of finite width in loop diagrams not unique,
cancellation of singularities not automatic

desired:
simple uniform description that is valid in the complete phase space without any matching (resonant and non-resonant regions, threshold region and continuum)
$\Rightarrow$ complex-mass scheme

Complex-mass scheme (CMS) at tree level

Denner, Dittmaier, Roth, Wackeroth '99

Define masses of unstable particles from propagator poles in complex plane

replace real masses by complex masses everywhere in tree-level expressions:

$$M_{\mathrm{W}}^2 \to \mu_{\mathrm{W}}^2 = M_{\mathrm{W}}^2 - \mathrm{i} M_{\mathrm{W}} \Gamma_{\mathrm{W}}, \qquad M_{\mathrm{Z}}^2 \to \mu_{\mathrm{Z}}^2 = M_{\mathrm{Z}}^2 - \mathrm{i} M_{\mathrm{Z}} \Gamma_{\mathrm{Z}}$$

in particular in definition of weak mixing angle

$$\cos^2\theta_{\mathrm{W}} \equiv c_{\mathrm{w}}^2 = 1 - s_{\mathrm{w}}^2 = \frac{\mu_{\mathrm{W}}^2}{\mu_{\mathrm{Z}}^2}$$

virtues:
all algebraic relations remain valid: Ward identities, Slavnov–Taylor identities
$\hookrightarrow$ gauge-parameter independence, unitarity cancellations

drawback:
spurious $\mathcal{O}(\Gamma/M) = \mathcal{O}(\alpha)$ terms in tree-level amplitudes
from terms proportional to Γ in t-channel propagators and in mixing angle
are beyond accuracy of tree-level approximation

Complex-mass scheme (CMS) at one-loop level

Split bare masses into complex masses and complex counterterms

$$M^2_{\mathrm{W},0} = \mu^2_{\mathrm{W}} + \delta\mu^2_{\mathrm{W}}, \qquad M^2_{\mathrm{Z},0} = \mu^2_{\mathrm{Z}} + \delta\mu^2_{\mathrm{Z}}$$

at level of Lagrangian
$\hookrightarrow$ Feynman rules with complex masses and counterterms

virtues

- perturbative calculations can be performed as usual
- no double counting of contributions (bare Lagrangian not changed!)

drawbacks

- need loop integrals with complex masses
- spurious $\mathcal{O}(\alpha^2)$ terms in one-loop amplitudes
- unitarity of S matrix only up to higher-order terms

Complex renormalization: W-boson as example

Direct generalization of on-shell renormalization scheme

Aoki et al. '81; Denner '93; Denner, Dittmaier, Weiglein '94

$\Rightarrow$ need complex field renormalization besides complex mass renormalization

$$W_0^\pm = \left(1 + \tfrac{1}{2}\delta\mathcal{Z}_W\right) W^\pm$$

complex $\delta\mathcal{Z}_W$ applies to both W^+ and $W^- \Rightarrow (W^+)^\dagger \neq W^-$

$\delta\mathcal{Z}_W$ drops out in S-matrix elements without external W-bosons

on-shell renormalization conditions for W-boson self-energy

$$\hat\Sigma_\mathrm{T}^W(\mu_\mathrm{W}^2) = 0, \qquad \hat\Sigma_\mathrm{T}'^W(\mu_\mathrm{W}^2) = 0$$

$\Rightarrow$ renormalized mass is equal to pole of propagator

solutions of renormalization conditions

$$\delta\mu_\mathrm{W}^2 = \Sigma_\mathrm{T}^W(\mu_\mathrm{W}^2), \qquad \delta\mathcal{Z}_W = -\Sigma_\mathrm{T}'^W(\mu_\mathrm{W}^2)$$

require self-energy for complex squared momenta ($p^2 = \mu_\mathrm{W}^2$)

$\hookrightarrow$ analytic continuation of the 2-point functions to unphysical Riemann sheet

Expansion of counterterms about real momentum arguments

Way around: appropriate expansions about real arguments

$$\Sigma_{\mathrm{T}}^{W}(\mu_{\mathrm{W}}^2) = \Sigma_{\mathrm{T}}^{W}(M_{\mathrm{W}}^2) + (\mu_{\mathrm{W}}^2 - M_{\mathrm{W}}^2)\Sigma_{\mathrm{T}}^{\prime W}(M_{\mathrm{W}}^2) + \mathcal{O}(\alpha^3)$$

modified counterterms

$$\delta\mu_{\mathrm{W}}^2 = \Sigma_{\mathrm{T}}^{W}(M_{\mathrm{W}}^2) + (\mu_{\mathrm{W}}^2 - M_{\mathrm{W}}^2)\Sigma_{\mathrm{T}}^{\prime W}(M_{\mathrm{W}}^2), \qquad \delta\mathcal{Z}_W = -\Sigma_{\mathrm{T}}^{\prime W}(M_{\mathrm{W}}^2)$$

neglected terms are beyond $\mathcal{O}(\alpha)$ and UV-finite by construction

$\Rightarrow$ renormalized self-energy

$$\hat{\Sigma}_{\mathrm{T}}^{W}(k^2) = \Sigma_{\mathrm{T}}^{W}(k^2) - \delta M_{\mathrm{W}}^2 + (k^2 - M_{\mathrm{W}}^2)\delta Z_W$$

with

$$\delta M_{\mathrm{W}}^2 = \Sigma_{\mathrm{T}}^{W}(M_{\mathrm{W}}^2), \qquad \delta Z_W = -\Sigma_{\mathrm{T}}^{\prime W}(M_{\mathrm{W}}^2)$$

exactly the form of the renormalized self-energies in usual on-shell scheme

but
- no real parts are taken
- self-energies depend on complex masses and complex mixing angle

Algebraic reduction of tensor integrals

For details see talk of S. Dittmaier and hep-ph/0509141

- 6-point integrals $\to$ six 5-point integrals — Melrose '65; Denner '93
- 5-point integrals $\to$ five 4-point integrals — Melrose '65; Denner, Dittmaier '02
- 3-point and 4-point integrals: Passarino–Veltman reduction
  $\hookrightarrow$ inverse Gram determinants of up to three momenta
  $\hookrightarrow$ serious numerical instabilities where $\det Z \to 0$
  (at phase-space boundary, but also within phase space !)

  two alternative "rescue systems"

  variant 1: appropriate expansions of tensor coefficients in small Gram determinants

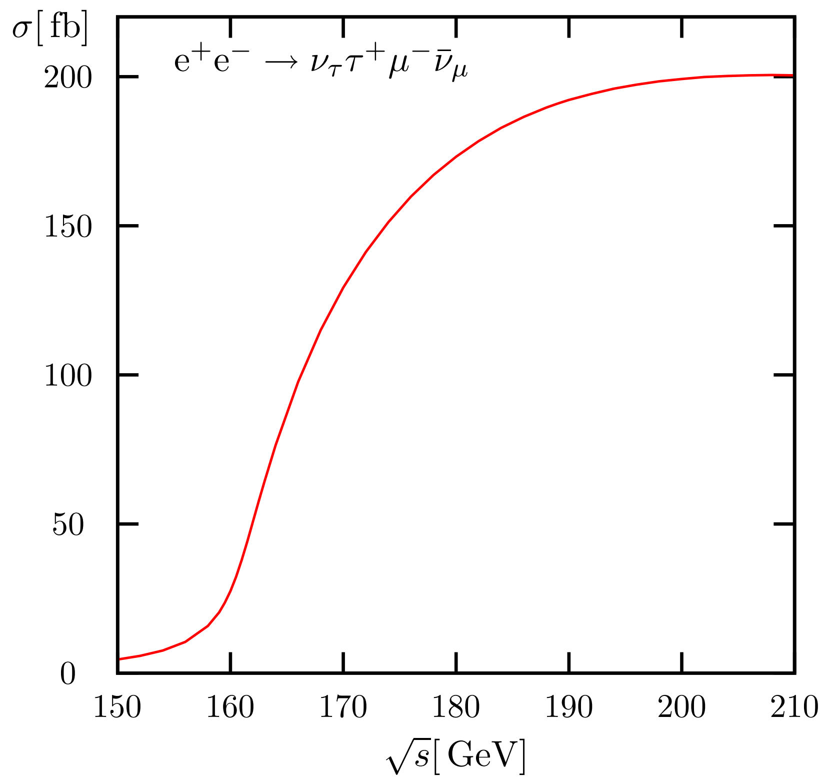
  variant 2: numerical evaluation of one appropriate tensor coefficient (logarithmic Feynman-parameter integral) and algebraic reduction to this basis integral
- 2-point integrals: numerically stable direct calculation

Checks of the calculation

- UV structure of virtual corrections
  $\hookrightarrow$ independence of reference mass μ of dimensional regularization
- IR structure of virtual + soft-photonic corrections
  $\hookrightarrow$ independence of $\ln m_\gamma$ (m_γ = infinitesimal photon mass)
- mass singularities of virtual + related collinear photonic corrections
  $\hookrightarrow$ independence of $\ln m_{f_i}$ (m_{f_i} = small masses of external fermions)
- gauge invariance of amplitudes with $\Gamma_{\mathrm{W}}, \Gamma_{\mathrm{Z}} \neq 0$
  $\hookrightarrow$ identical results in 't Hooft–Feynman and background-field gauge
  Denner, Dittmaier, Weiglein '94
- real corrections
  $\hookrightarrow$ taken from RacoonWW Denner, Dittmaier, Roth, Wackeroth '99–'01
- combination of virtual and real corrections
  $\hookrightarrow$ identical results with two-cutoff slicing and dipole subtraction
  Dittmaier '99; Roth '00
- two completely independent calculations of all ingredients !

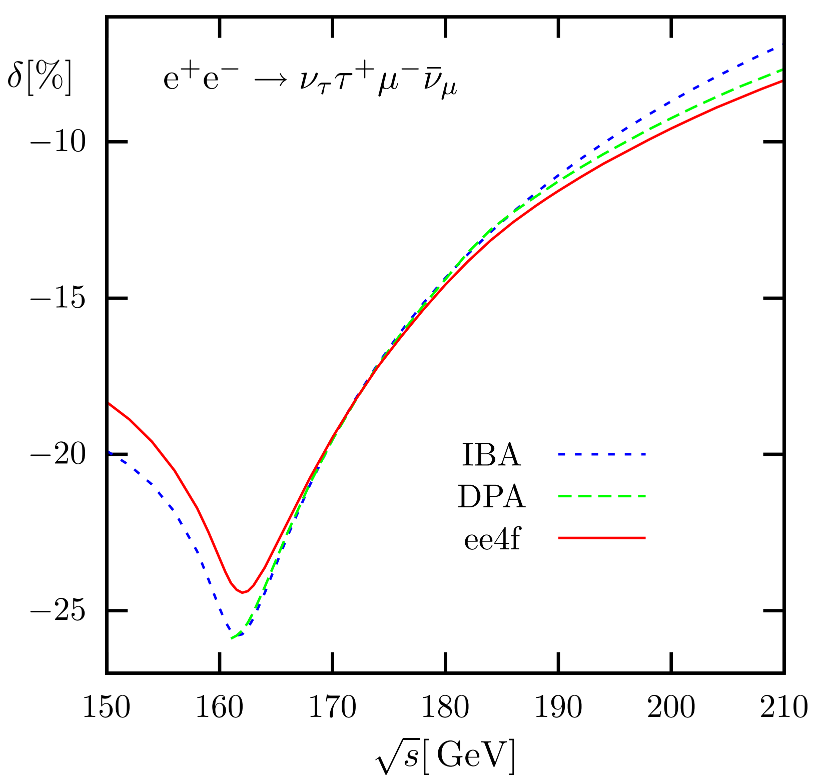
Complete $\mathcal{O}(\alpha)$ corrections to total cross section – LEP2 energies

Corrected cross section:



relative corrections (G_μ-scheme):

Denner, Dittmaier, Roth, Wieders '05

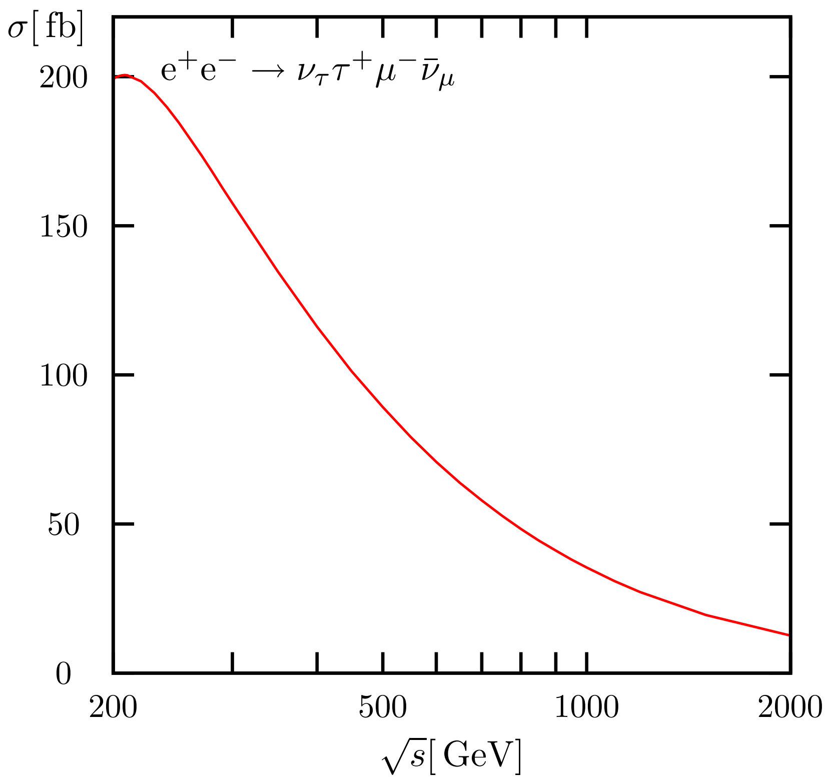


- $|\mathrm{ee4f} - \mathrm{DPA}| \sim 0.5\%$ for $170\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 210\,\mathrm{GeV}$
- $|\mathrm{ee4f} - \mathrm{IBA}| \sim 2\%$ for $\sqrt{s} \lesssim 170\,\mathrm{GeV}$

$\hookrightarrow$ agreement with error estimates of DPA and IBA

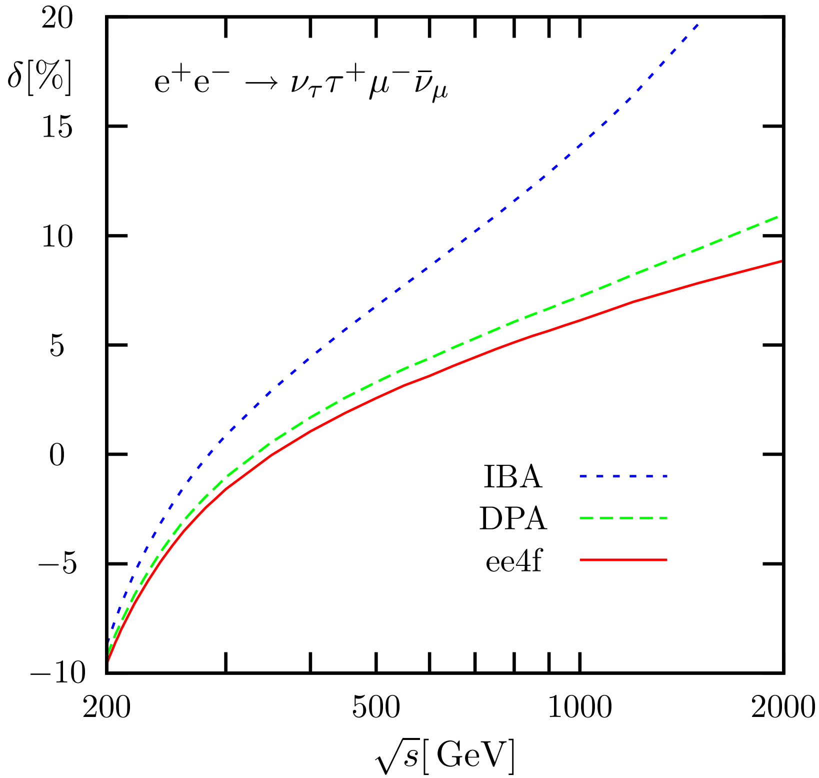
Complete $\mathcal{O}(\alpha)$ corrections to total cross section – ILC energies

Corrected cross section:



relative corrections (G_μ-scheme):

Denner, Dittmaier, Roth, Wieders '05



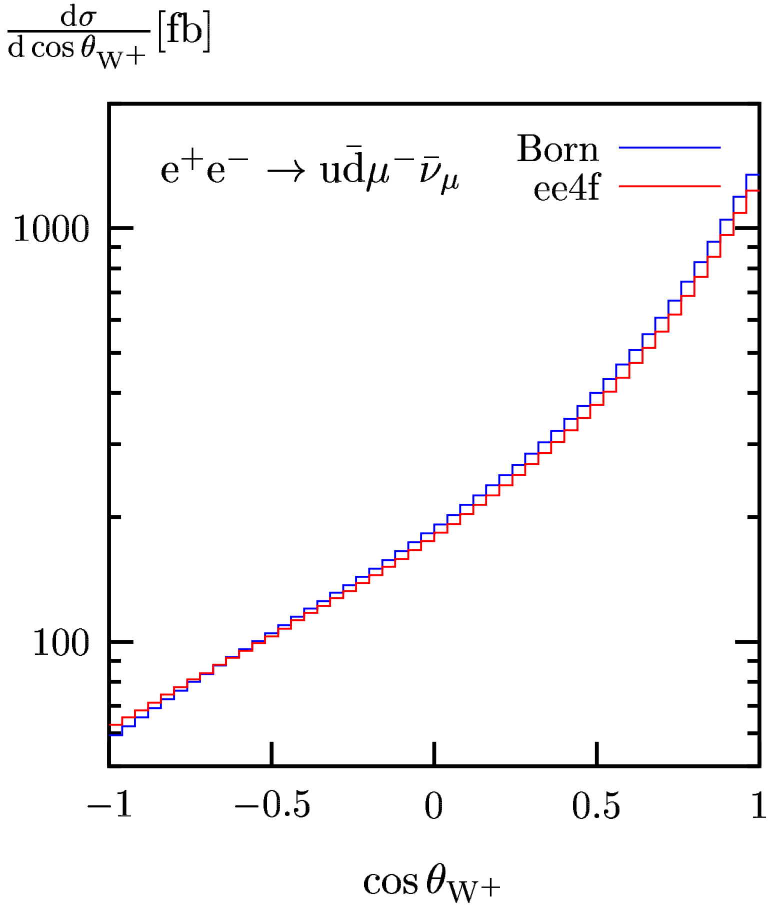
- $|\mathrm{ee4f} - \mathrm{DPA}| \sim 0.7\%$ for $200\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 500\,\mathrm{GeV}$

  $\hookrightarrow$ agreement with error estimate of DPA

- $|\mathrm{ee4f} - \mathrm{DPA}| \sim 1\text{–}2\%$ for $500\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 1\text{–}2\,\mathrm{TeV}$

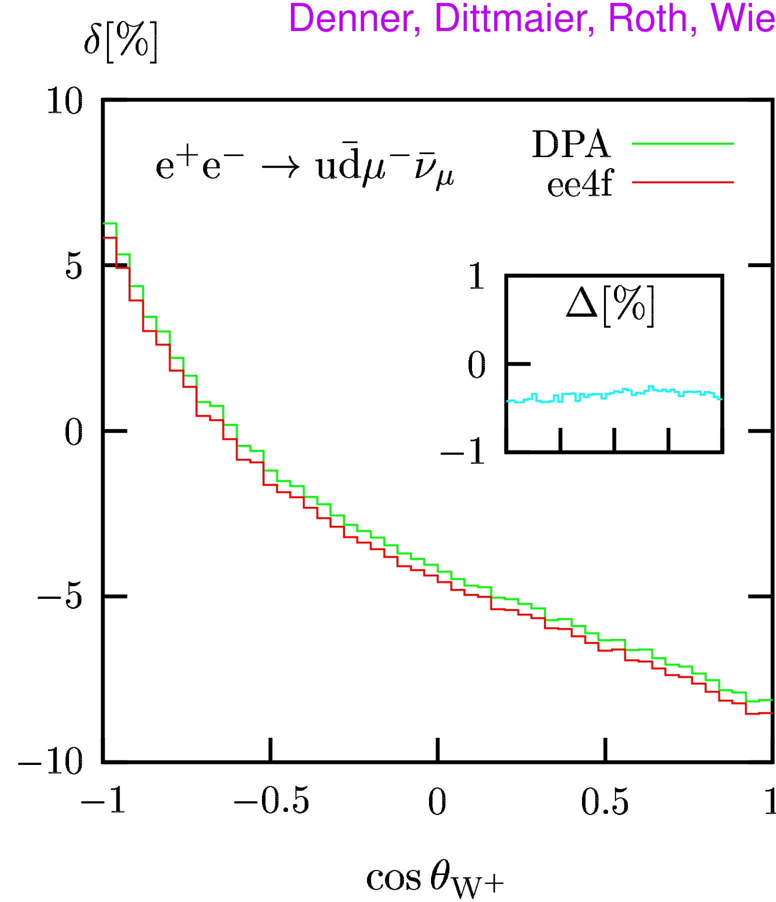
W-production angle distribution at $\sqrt{s} = 200$ GeV

Differential cross section:



relative corrections (G_μ-scheme):

Denner, Dittmaier, Roth, Wieders '05

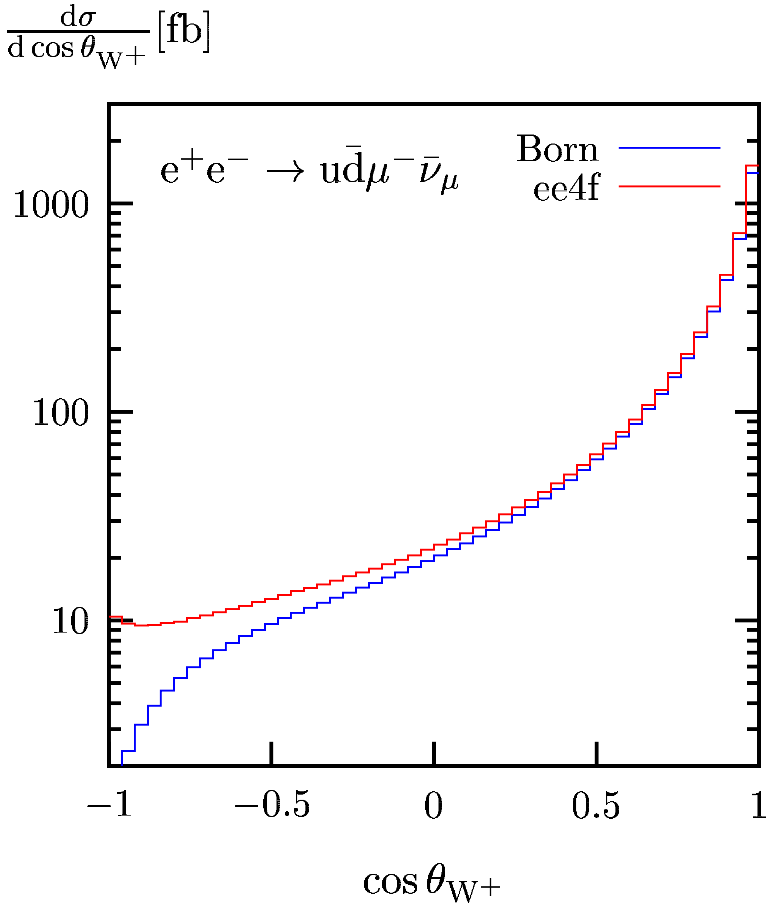


$$\Delta = \frac{d\sigma_{ee4f}}{d\sigma_{DPA}} - 1$$

no visible distortion of shape w.r.t. DPA at LEP2 energies

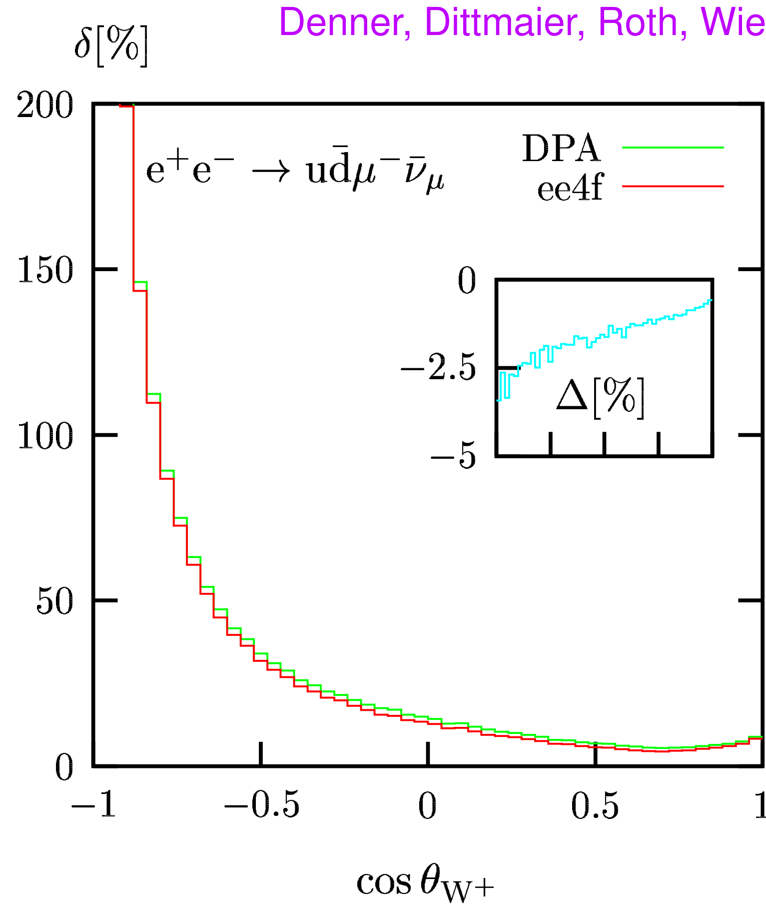
W-production angle distribution at $\sqrt{s} = 500\,\text{GeV}$

Differential cross section:



relative corrections (G_μ-scheme):

Denner, Dittmaier, Roth, Wieders '05



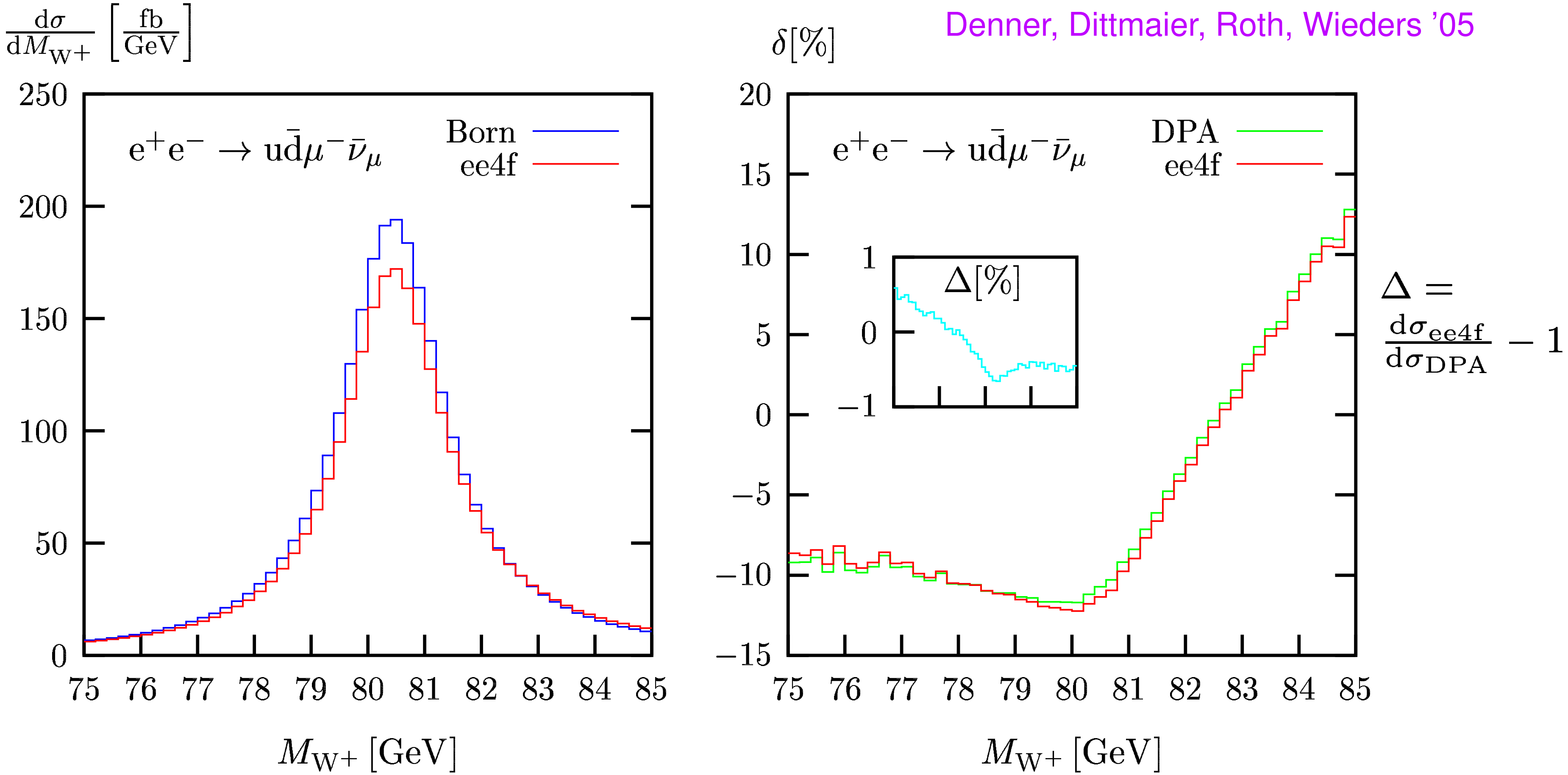
significant distortion of shape w.r.t. DPA at ILC energies

$\hookrightarrow$ important for TGC studies at ILC

W-invariant-mass distribution at $\sqrt{s} = 200$ GeV

Differential cross section:
(photon recombination applied)

relative corrections (G_μ-scheme):



small distortion of shape w.r.t. DPA at LEP2 energies

$\hookrightarrow$ shift in M_W in direct reconstruction ?

Conclusions

Complete $\mathcal{O}(\alpha)$ correction for $\mathrm{e^+e^-} \to \nu_\tau \tau^+ \mu^- \bar{\nu}_\mu, \mathrm{u\bar{d}}\mu^-\bar{\nu}_\mu, \mathrm{u\bar{d}s\bar{c}}$ calculated

- calculation required new techniques
  - complex-mass scheme for finite width at one loop
  - new reduction algorithms for matrix elements
  - new tensor-integral reductions
- theoretical uncertainty at threshold reduced from $\sim 2\%$ to a few 0.1%

remaining theoretical uncertainties dominated by

- electroweak effects beyond $\mathcal{O}(\alpha)$, e.g. $(\frac{\alpha}{\pi})^2 \ln\left(\frac{m_\mathrm{e}^2}{s}\right) \sim 0.1\%$
- QCD effects

first established calculation of $\mathcal{O}(\alpha)$ corrections for $2 \to 4$ process

other progress in $2 \to 4$ processes by GRACE-loop

- progress report on calculation for $\mathrm{e^+e^-} \to \mathrm{u\bar{d}}\mu^-\bar{\nu}_\mu$ Boudjema et al. '04
- electroweak corrections to $\mathrm{e^+e^-} \to \nu\bar{\nu}HH$ Boudjema et al., hep-ph/0510184

Other renormalization constants

Complex renormalization of photon and Z boson similar to W boson

charge renormalization constant

$$\frac{\delta e}{e} = \frac{1}{2}\Sigma'^{AA}(0) - \frac{s_\mathrm{w}}{c_\mathrm{w}}\frac{\Sigma_\mathrm{T}^{AZ}(0)}{\mu_\mathrm{Z}^2}$$

becomes complex via $\mu_\mathrm{Z}^2, \mu_\mathrm{W}^2$ and $s_\mathrm{w}, c_\mathrm{w}$ $\Rightarrow$ complex renormalized charge
at one loop: imaginary part of renormalized charge drops out
beyond one loop: imaginary part of renormalized charge enters

renormalization of massless fermions:

$$\delta\mathcal{Z}_{f,\sigma} = -\Sigma^{f,\sigma}(m_f^2) - m_f^2\left[\Sigma'^{f,\mathrm{R}}(m_f^2) + \Sigma'^{f,\mathrm{L}}(m_f^2) + 2\Sigma'^{f,\mathrm{S}}(m_f^2)\right]$$

for both fermions and anti-fermions
Σ'^{f} involves no absorptive parts but is complex owing to complex parameters

complex-mass scheme can be generalized to

- background-field formalism
- unstable Higgs boson and unstable fermions (top quark)

Real corrections

Matrix elements

- evaluated with Weyl-van der Waerden spinor technique — Dittmaier '99
  $\hookrightarrow$ compact expressions
- checked numerically against MadGraph (for $\Gamma = 0$) — Stelzer '94

soft and collinear singularities: treated with two methods

- dipole subtraction formalism — Dittmaier '99, Roth '00
- phase-space slicing

numerical agreement within 0.03%

leading-log ISR beyond $\mathcal{O}(\alpha)$

- included using structure functions

phase-space integration

- Monte Carlo integration $\Rightarrow$ distributions available

Numerical results

Total cross section without cuts (based on 10^7 weighted events)

Differential cross sections with cuts (based on 10^8 weighted events)

cut and recombination procedure

1. all bremsstrahlung photons within a cone of 5 degrees around the beams are treated as invisible.
2. the invariant masses $M_{f\gamma}$ of the photon with each of the charged final-state fermions are calculated. If the smallest $M_{f\gamma}$ is smaller than $M_{\mathrm{rec}} = 25\,\mathrm{GeV}$ or if the energy of the photon is smaller than $1\,\mathrm{GeV}$, the photon is combined with the charged final-state fermion that leads to the smallest $M_{f\gamma}$.
3. all events are discarded in which one of the charged final-state fermions is within a cone of 10 degrees around the beams (after a possible recombination with a photon).

Total cross section without cuts

process $e^+e^- \to \nu_\tau \tau^+ \mu^- \bar{\nu}_\mu$ — Denner, Dittmaier, Roth, Wieders ’05

$\sqrt{s}$/GeV	Born(FW)	Born(CMS)	IBA	DPA	ee4f
161	50.04(2)	50.01(2)	37.18(2)	37.08(2)	37.95(2)
		[−0.06%]	[−25.67(6)%]	[−25.90(3)%]	[−24.12(4)%]
170	160.53(6)	160.44(6)	129.12(6)	129.17(6)	129.23(6)
		[−0.06%]	[−19.52(5)%]	[−19.53(3)%]	[−19.45(3)%]
200	220.41(9)	220.29(9)	201.13(9)	200.04(10)	199.21(10)
		[−0.06%]	[−8.70(6)%]	[−9.24(2)%]	[−9.57(3)%]
500	86.95(5)	86.90(5)	92.79(5)	89.81(6)	89.13(6)
		[−0.06%]	[+6.78(9)%]	[+3.29(3)%]	[+2.57(4)%]
1000	33.35(2)	33.33(2)	38.04(4)	35.76(3)	35.37(3)
		[−0.06%]	[+14.12(14)%]	[+7.21(5)%]	[+6.12(6)%]

- $|\text{ee4f} - \text{IBA}| \sim 2\%$ for $\sqrt{s} \lesssim 170\,\text{GeV}$
- $|\text{ee4f} - \text{DPA}| \sim 0.5\%$ for $170\,\text{GeV} \lesssim \sqrt{s} \lesssim 210\,\text{GeV}$
- $|\text{ee4f} - \text{DPA}| \sim 0.7\%$ for $\sqrt{s} \sim 500\,\text{GeV}$

$\hookrightarrow$ agreement with error estimates of DPA and IBA