Higgs Self Coupling Systematic Error from HZZ & HHZZ Coupling Uncertainties

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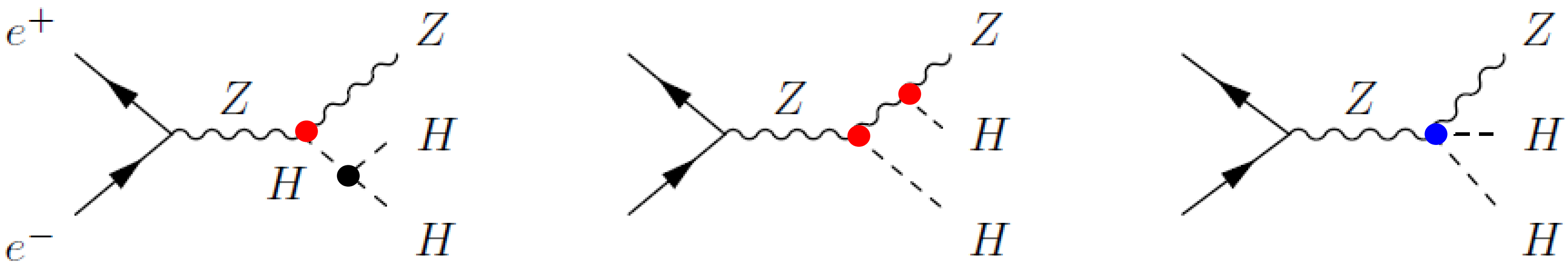
Jun 15, 2016

Higgs Self Coupling Systematic Error

Uncertainties for g_{ZZH} g_{ZZHH} in $\sigma(e^+e^- \to HHZ)$



We assume that $\sigma(e^+e^- \to HHZ)$ can be described by an effective field theory (EFT) containing a general $SU(2)\times U(1)$ gauge invariant Lagrangian with dimension-6 operators in addition to the SM.

Using the convention of arXiv:1310.5150 we have, before EWSB, the following dim-6 operators:

$$\mathcal{L}_{\rm SILH} = \frac{\bar{c}_H}{2v^2}\partial^\mu\left[\Phi^\dagger\Phi\right]\partial_\mu\left[\Phi^\dagger\Phi\right] + \frac{\bar{c}_T}{2v^2}\left[\Phi^\dagger \overleftrightarrow{D}^\mu \Phi\right]\left[\Phi^\dagger \overleftrightarrow{D}_\mu \Phi\right] - \frac{\bar{c}_6\lambda}{v^2}\left[\Phi^\dagger\Phi\right]^3$$
$$+ \frac{ig\ \bar{c}_W}{m_W^2}\left[\Phi^\dagger T_{2k}\overleftrightarrow{D}^\mu\Phi\right]D^\nu W^k_{\mu\nu} + \frac{ig'\ \bar{c}_B}{2m_W^2}\left[\Phi^\dagger\overleftrightarrow{D}^\mu\Phi\right]\partial^\nu B_{\mu\nu}$$
$$+ \frac{2ig\ \bar{c}_{HW}}{m_W^2}\left[D^\mu\Phi^\dagger T_{2k}D^\nu\Phi\right]W^k_{\mu\nu} + \frac{ig'\ \bar{c}_{HB}}{m_W^2}\left[D^\mu\Phi^\dagger D^\nu\Phi\right]B_{\mu\nu}$$
$$+ \frac{g'^2\ \bar{c}_\gamma}{m_W^2}\Phi^\dagger\Phi B_{\mu\nu}B^{\mu\nu} + \text{ other operators that are not relevant or that violate CP}$$

Note there are only 8 EFT parameters: $\bar{c}_H$ $\bar{c}_T$ $\bar{c}_6$ $\bar{c}_W$ $\bar{c}_B$ $\bar{c}_{HW}$ $\bar{c}_{HB}$ $\bar{c}_\gamma$

Now add the CP violating terms:

$$\mathcal{L}_{CP} = \frac{ig\ \tilde{c}_{HW}}{m_W^2} D^\mu \Phi^\dagger T_{2k} D^\nu \Phi \widetilde{W}^k_{\mu\nu} + \frac{ig'\ \tilde{c}_{HB}}{m_W^2} D^\mu \Phi^\dagger D^\nu \Phi \widetilde{B}_{\mu\nu} + \frac{g'^2\ \tilde{c}_\gamma}{m_W^2} \Phi^\dagger \Phi B_{\mu\nu} \widetilde{B}^{\mu\nu}$$

$$+ \frac{g^3\ \tilde{c}_{3W}}{m_W^2} \epsilon_{ijk} W^i_{\mu\nu} W^{\nu j}{}_\rho \widetilde{W}^{\rho\mu k}$$

where the dual field strength tensors are defined by

$$\widetilde{B}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} B^{\rho\sigma}\ , \quad \widetilde{W}^k_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} W^{\rho\sigma k}$$

This gives 4 additional EFT parameters: $\tilde{c}_{HW}$ $\tilde{c}_{HB}$ $\tilde{c}_\gamma$ $\tilde{c}_{3W}$

After EWSB we have $\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_{3V}$ where

$$\mathcal{L}_3 = -\frac{m_H^2}{2v} g^{(1)}_{hhh} h^3 + \frac{1}{2} g^{(2)}_{hhh} h \partial_\mu h \partial^\mu h$$

$$- \frac{1}{4} g^{(1)}_{hzz} Z_{\mu\nu} Z^{\mu\nu} h - g^{(2)}_{hzz} Z_\nu \partial_\mu Z^{\mu\nu} h + \frac{1}{2} g^{(3)}_{hzz} Z_\mu Z^\mu h$$

$$- \frac{1}{2} g^{(1)}_{hww} W^{\mu\nu} W^\dagger_{\mu\nu} h - \left[g^{(2)}_{hww} W^\nu \partial^\mu W^\dagger_{\mu\nu} h + \text{h.c.} \right] + g(1 - \frac{1}{2}\bar{c}_H) m_W W^\dagger_\mu W^\mu h$$

$$\mathcal{L}_4 = -\frac{1}{8} g^{(1)}_{hhzz} Z_{\mu\nu} Z^{\mu\nu} h^2 - \frac{1}{2} g^{(2)}_{hhzz} Z_\nu \partial_\mu Z^{\mu\nu} h^2 + \frac{1}{4} g^{(3)}_{hhzz} Z_\mu Z^\mu h^2$$

$$\mathcal{L}_{3V} = \left[i g^{(1)}_{aww} W^\dagger_{\mu\nu} A^\mu W^\nu + \text{ h.c.} \right] + i g^{(2)}_{aww} F_{\mu\nu} W^\mu W^{\nu\dagger}$$
$$+ \left[i g^{(1)}_{zww} W^\dagger_{\mu\nu} Z^\mu W^\nu + \text{ h.c.} \right] + i g^{(2)}_{zww} Z_{\mu\nu} W^\mu W^{\nu\dagger}$$

In the SM at tree level

$$g^{(1)}_{aww} = g^{(2)}_{aww} = e \quad g^{(1)}_{zww} = g^{(2)}_{zww} = g c_{\theta_W} \quad g^{(1)}_{aww} = g^{(1)}_{hhh} = 1 \quad g^{(3)}_{hzz} = \frac{g M_W}{c^2_{\theta_W}} \quad g^{(3)}_{hhzz} = \frac{g^2}{2 c^2_{\theta_W}}$$

and all other $g^{(j)}_{xxx}$, $g^{(j)}_{xxxx}$ =0

And the CP violating piece $\mathcal{L}_{CP} = \mathcal{L}_{3_{CP}} + \mathcal{L}_{4_{CP}} + \mathcal{L}_{3V_{CP}}$ where

$$\mathcal{L}_{3_{CP}} = -\frac{1}{4}\tilde{g}_{hzz} Z_{\mu\nu}\tilde{Z}^{\mu\nu} h - \frac{1}{2}\tilde{g}_{hww} W^{\mu\nu}\tilde{W}^{\dagger}_{\mu\nu} h$$

$$\mathcal{L}_{4_{CP}} = -\frac{1}{8}\tilde{g}_{hhzz} Z_{\mu\nu}\tilde{Z}^{\mu\nu} h^2$$

$$\mathcal{L}_{3V_{CP}} = i\tilde{\kappa}_{\gamma} W^{\dagger}_{\mu} W_{\nu} \tilde{F}^{\mu\nu} + i\frac{\tilde{\lambda}_{\gamma}}{M_W^2} W^{\dagger}_{\lambda\mu} W^{\mu}_{\nu} \tilde{F}^{\nu\lambda} + i\tilde{\kappa}_{z} W^{\dagger}_{\mu} W_{\nu} \tilde{F}^{\mu\nu} + i\frac{\tilde{\lambda}_{z}}{M_W^2} W^{\dagger}_{\lambda\mu} W^{\mu}_{\nu} \tilde{F}^{\nu\lambda}$$

In the SM at tree level

all $\tilde{g}^{(j)}_{xxx}$, $\tilde{g}^{(j)}_{xxxx}$, $\tilde{\kappa}_x$, $\tilde{\lambda}_x$ =0

The couplings $g^{(j)}_{xxx}$ and $g^{(j)}_{xxxx}$ take the following form in our EFT:

$g^{(1)}_{hhh}$ $\quad 1 + \frac{7}{8}\bar{c}_6 - \frac{1}{2}\bar{c}_H$

$g^{(2)}_{hhh}$ $\quad \frac{g}{m_W}\bar{c}_H$

$g^{(1)}_{hzz}$ $\quad \frac{2g}{c_W^2 m_W}\left[\bar{c}_{HB}s_W^2 - 4\bar{c}_\gamma s_W^4 + c_W^2\bar{c}_{HW}\right]$

$g^{(2)}_{hzz}$ $\quad \frac{g}{c_W^2 m_W}\left[(\bar{c}_{HW}+\bar{c}_W)c_W^2 + (\bar{c}_B+\bar{c}_{HB})s_W^2\right]$

$g^{(3)}_{hzz}$ $\quad \frac{g m_W}{c_W^2}\left[1 - \frac{1}{2}\bar{c}_H - 2\bar{c}_T + 8\bar{c}_\gamma\frac{s_W^4}{c_W^2}\right]$

$g^{(1)}_{hww}$ $\quad \frac{2g}{m_W}\bar{c}_{HW}$

$g^{(2)}_{hww}$ $\quad \frac{g}{m_W}\left[\bar{c}_W + \bar{c}_{HW}\right]$

$\left\{g^{(1)}_{hhzz}, g^{(2)}_{hhzz}, g^{(1)}_{hhaz}, g^{(2)}_{hhaz}, g^{(1)}_{hhww}, g^{(2)}_{hhww}\right\}$ $\quad \frac{g}{2m_W}\left\{g^{(1)}_{hzz}, g^{(2)}_{hzz}, g^{(1)}_{haz}, g^{(2)}_{haz}, g^{(1)}_{hww}, g^{(2)}_{hww}\right\}$

$g^{(3)}_{hhzz}$ $\quad \frac{g^2}{2c_W^2}\left[1 - 6\bar{c}_T - \bar{c}_H + 8\bar{c}_\gamma\frac{s_W^4}{c_W^2}\right]$

$g^{(1)}_{aww}$ $\quad e\left[1 - 2\bar{c}_W\right]$

$g^{(2)}_{aww}$ $\quad e\left[1 - 2\bar{c}_W - \bar{c}_{HB} - \bar{c}_{HW}\right]$

$g^{(1)}_{zww}$ $\quad \frac{g}{c_W}\left[c_W^2 - \bar{c}_{HW} + (2s_W^2 - 3)\bar{c}_W\right]$

$g^{(2)}_{zww}$ $\quad \frac{g}{c_W}\left[c_W^2(1 - \bar{c}_{HW}) + s_W^2\bar{c}_{HB} + (2s_W^2 - 3)\bar{c}_W\right]$

$\bar{c}_6$ is uniquely accessible through the Higgs self coupling measurement. The other 7 EFT parameters appear in several places.

Note the relationships between the *hzz* & *hhzz* couplings.

The TGC's depend on $\bar{c}_W$, $\bar{c}_{HW}$, $\bar{c}_{HB}$ through the Goldstone bosons that are eaten by the Z and W fields

The CP violating couplings $\tilde{g}_{hzz}$ $\tilde{g}_{hhzz}$ $\tilde{\kappa}_{\gamma}$ $\tilde{\lambda}_{\gamma}$ $\tilde{\kappa}_{z}$ $\tilde{\lambda}_{z}$ take the following form in our EFT

$$\tilde{g}_{hzz} = \frac{2g}{c_{\theta w}^2 M_W}\left[\tilde{c}_{HB} s_{\theta w}^2 - 4\tilde{c}_{\gamma} s_{\theta w}^4 + c_{\theta w}^2 \tilde{c}_{HW}\right]$$

$$\tilde{g}_{hhzz} = \frac{g}{2M_W}\tilde{g}_{hzz}$$

$$\tilde{\kappa}_{\gamma} = \tilde{c}_{HB} + \tilde{c}_{HW} - \frac{8M_W^2}{v^2}\tilde{c}_{3W}$$

$$\tilde{\lambda}_{\gamma} = -\frac{8M_W^2}{v^2}\tilde{c}_{3W}$$

$$\tilde{\kappa}_{Z} = -\frac{s_{\theta w}}{c_{\theta w}}\tilde{c}_{HB} + \frac{c_{\theta w}}{s_{\theta w}}\tilde{c}_{HW} - \frac{8M_W^2}{v^2 s_{\theta w}}\tilde{c}_{3W}$$

$$\tilde{\lambda}_{Z} = -\frac{8M_W^2}{v^2 s_{\theta w}}\tilde{c}_{3W}$$

The parameters $\bar{c}_W$, $\bar{c}_B$, $\bar{c}_T$ are constrained by electroweak precision tests.

From arXiv:1410.7703 :

Operator	Coefficient	LEP Constraints	
		Individual	Marginalized
$\mathcal{O}_W = \frac{ig}{2}\left(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H\right) D^\nu W^a_{\mu\nu}$ $\mathcal{O}_B = \frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right) \partial^\nu B_{\mu\nu}$	$\frac{m_W^2}{\Lambda^2}(c_W + c_B)$	$(-0.00055, 0.0005)$	$(-0.0033, 0.0018)$
$\mathcal{O}_T = \frac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$	$\frac{v^2}{\Lambda^2} c_T$	$(0, 0.001)$	$(-0.0043, 0.0033)$

By definition $\bar{c}_W \equiv \frac{M_W^2}{\Lambda^2} c_W$, $\bar{c}_T \equiv \frac{v^2}{\Lambda^2} c_T$, etc.

We will assume in the following that, at the 10^{-3} level,

$$\bar{c}_W = -\bar{c}_B \qquad \bar{c}_T = 0$$

The CP violating parameters $\tilde{c}_{HB}$, $\tilde{c}_{HW}$, $\tilde{c}_{3W}$ are constrained by the electric dipole moment of the neutron

$|d_{\text{neutron}}| < 1.2\times 10^{-25}\ e\ \text{cm} \quad \Rightarrow \quad \tilde{\lambda}_\gamma < 2.5\times 10^{-4}$ and $\tilde{\kappa}_\gamma$ even smaller (?)

We can assume $\tilde{c}_{3W} = 0$ and $\tilde{c}_{HB} = -\tilde{c}_{HW}$ at the 10^{-4} level which leads to

$\tilde{c}_{HW} = c_{\theta w} s_{\theta s} \tilde{\kappa}_Z$

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TABLE II. Contributions of the four anomalous $WW\gamma$ couplings λ_γ, $\tilde{\lambda}_\gamma$, $\Delta\kappa_\gamma$, and $\tilde{\kappa}_\gamma$ of Eq. (1.1) to the four electromagnetic form factors of a fermion. The unit, the meaning of signs, and the neglected pieces are the same as those in Table I.

	R_f	A_f	$\Delta\mu_f$	d_f
λ_γ	$\pm 6\lambda_\gamma \ln\frac{\Lambda^2}{m_W^2}$	$\pm\lambda_\gamma m_f^2 \ln\frac{\Lambda^2}{m_W^2}$	$\sim\pm\lambda_\gamma m_f$	
$\Delta\kappa_\gamma$	$\pm\frac{3}{2}\Delta\kappa_\gamma \frac{\Lambda^2}{m_W^2}$	$\pm\frac{1}{4}\Delta\kappa_\gamma m_f^2 \frac{\Lambda^2}{m_W^2}$	$\pm\Delta\kappa_\gamma m_f \ln\frac{\Lambda^2}{m_W^2}$	
$\tilde{\lambda}_\gamma$				$\sim\pm\tilde{\lambda}_\gamma m_f$
$\tilde{\kappa}_\gamma$				$\pm\tilde{\kappa}_\gamma m_f \ln\frac{\Lambda^2}{m_W^2}$

The TGC's $\Delta\kappa_\gamma$ and Δg_1^Z are related to EFT parameters by

$$\Delta\kappa_\gamma = \bar{c}_{HW} + \bar{c}_{HB}$$

$$\Delta g_1^Z = \bar{c}_W + \bar{c}_{HW}$$

At ILC with the full H-20 scenario the error on the TGC's are

$$\Delta(\Delta\kappa_\gamma) = 2\times 10^{-4}$$

$$\Delta(\Delta g_1^Z) = 8\times 10^{-4}$$

We can then assume at the 10^{-3} level that $\bar{c}_W = -\bar{c}_{HW} = \bar{c}_{HB}$

so that we can write $g_{hzz}^{(2)}$ and $g_{hww}^{(2)}$ as

$$g_{hzz}^{(2)} = \frac{g}{c_{\theta_W}^2 M_W}\left[\Delta g_1^Z c_{\theta_W}^2 + \Delta\kappa_\gamma s_{\theta_W}^2\right] \approx 0$$

$$g_{hww}^{(2)} = \frac{g}{M_W}\Delta g_1^Z \approx 0$$

Tesla TDR

coupling	error $\times 10^{-4}$	
	$\sqrt{s} = 500\,\mathrm{GeV}$	$\sqrt{s} = 800\,\mathrm{GeV}$
C,P-conserving, SU(2) × U(1) relations:		
Δg_1^Z	2.8	1.8
$\Delta\kappa_\gamma$	3.1	1.9
λ_γ	4.3	2.6
C,P-conserving, no relations:		
Δg_1^Z	15.5	12.6
$\Delta\kappa_\gamma$	3.3	1.9
λ_γ	5.9	3.3
$\Delta\kappa_Z$	3.2	1.9
λ_Z	6.7	3.0
not C or P conserving:		
g_5^Z	16.5	14.4
g_4^Z	45.9	18.3
$\tilde{\kappa}_Z$	39.0	14.3
$\tilde{\lambda}_Z$	7.5	3.0

Table 5.1.1: *Results of the single parameter fits (1σ) to the different triple gauge couplings. For $\sqrt{s} = 500$ GeV $\mathcal{L} = 500\,\mathrm{fb}^{-1}$ and for $\sqrt{s} = 800$ GeV $\mathcal{L} = 1000\,\mathrm{fb}^{-1}$ has been assumed. For both energies $\mathcal{P}_{e^-} = 80\%$ and $\mathcal{P}_{e^+} = 60\%$ has been used.*

For H20 $\Delta\tilde{\kappa}_Z = 0.0039 / \sqrt{8} \quad \Rightarrow \quad \Delta\tilde{c}_{HW} = c_{\theta w} s_{\theta s} \Delta\tilde{\kappa}_Z = 0.0011$

So we conclude $\tilde{c}_{HW} = 0$ at the 10^{-3} level ,

leaving us with one remaining CP violating parameter $\tilde{c}_\gamma$

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Through EWPT's and ILC measurements of TGC's the number of independent EFT parameters has been reduced from 12 to just 5: $\bar{c}_H$, $\bar{c}_\gamma$, $\bar{c}_{HW}$, $\bar{c}_6$, $\tilde{c}_\gamma$

The parameters $\bar{c}_H$, $\bar{c}_\gamma$, $\bar{c}_{HW}$, $\tilde{c}_\gamma$ are related to the measured g_{HWW} , g_{HZZ} , b_z , $\tilde{b}_z$ couplings via

$$\bar{c}_H = 2\left(1 - \frac{g_{HWW}}{gM_W}\right) \qquad \bar{c}_\gamma = \frac{c^2_{\theta_W}}{8gM_W s^4_{\theta_W}}\left[c^2_{\theta_W} g_{HZZ} - g_{HWW}\right]$$

$$\bar{c}_{HW} = \frac{1}{1 - t^2_{\theta_W}}\left[\frac{1}{2gM_W}\left(g_{HZZ} - \frac{1}{c^2_{\theta_W}} g_{HWW}\right) - \frac{b_z}{4}\right] \qquad \tilde{c}_\gamma = \frac{c^2_{\theta_W}}{16 s^4_{\theta_W}} \tilde{b}_z$$

g_{HWW} , g_{HZZ} are the usual couplings of the Higgs to W & Z

The "b" and "$\tilde{b}$" parameters are the anomalous HVV Lorentz structure coefficients studied by Fujii, Tian, Ogawa and others:

HWW arXiv:1011.5805

HZZ Talk by T. Ogawa at LCWS15 http://agenda.linearcollider.org/event/6662/

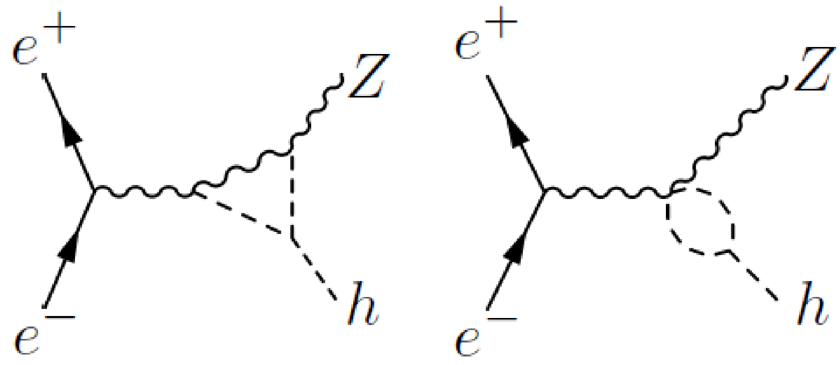
The full EFT Lagrangian can be written using the measured variables g_{HZZ} g_{HZZ} b_z $\tilde{b}_z$ and the one remaining unconstrained EFT parameter $\bar{c}_6$

$$\mathcal{L} = -\frac{M_H^2}{2v}\left(\frac{g_{HWW}}{gM_W}+\frac{7}{8}\bar{c}_6\right)h^3+\frac{g}{M_W}\left(1-\frac{g_{HWW}}{gM_W}\right)h\partial_\mu h\partial^\mu h+\frac{b_z}{4v}Z_{\mu\nu}Z^{\mu\nu}h+\frac{1}{2}g_{HZZ}Z_\mu Z^\mu h$$
$$+\frac{\tilde{b}_z}{4v}Z_{\mu\nu}\tilde{Z}^{\mu\nu}h+\frac{b_z}{8v^2}Z_{\mu\nu}Z^{\mu\nu}h^2+\frac{g^2}{8c_{\theta_W}^2}\left(\frac{g_{HWW}}{gM_W}+c_{\theta_W}^2\frac{g_{HZZ}}{gM_W}-1\right)Z_\mu Z^\mu h^2+\frac{\tilde{b}_z}{8v^2}Z_{\mu\nu}\tilde{Z}^{\mu\nu}h^2$$

In this EFT approach all of the couplings in the calculation of $\sigma(e^+e^-\to HHZ)$ are tightly constrained by the other Higgs coupling measurements, TGC measurements, and EWPT's. The only unconstrained parameter is the anomalous Higgs self coupling $\bar{c}_6$, which is uniquely accessed through the measurement of $\sigma(e^+e^-\to HHZ)$.

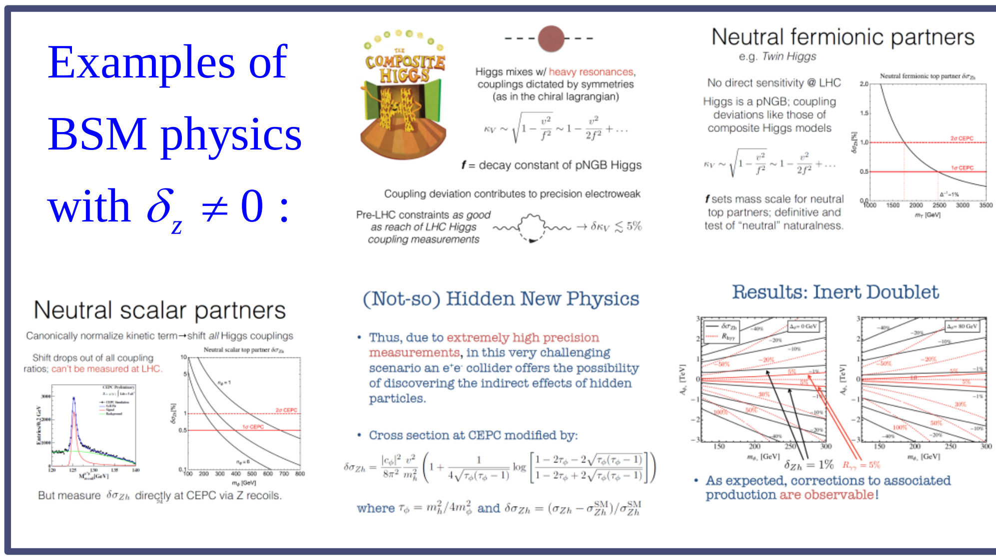
The unmeasured ZZHH quartic coupling is related to the HZZ and HWW couplings, which are measured to 0.3% and 0.4%, respectively, at the ILC in the H-20 scenario. The systematic error due to the unmeasured quartic coupling is therefore very small.

FCC-ee Higgs Self Coupling Measurement at Ecm=240 GeV



M. McCullough, arXiv:1312.3322

$$\delta_\sigma^{240} = 100\,(2\delta_Z + 0.014\delta_h)\,\%$$

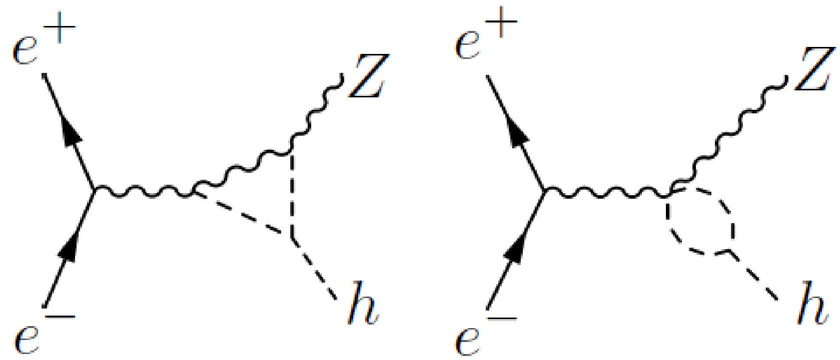


g_{hZZ} fixed to SM value $(\delta_z = 0)$

g_{hhZZ} fixed to SM value

$$\Rightarrow \delta_H = \frac{\delta_\sigma^{240}}{0.014} = \frac{0.004}{0.014} = 29\%$$

CEPC Higgs Self Coupling Measurement at Ecm=240 GeV



M. McCullough, arXiv:1312.3322

$$\delta_\sigma^{240} = 100\,(2\delta_Z + 0.014\delta_h)\,\%$$

g_{hZZ} fixed to SM value $(\delta_z = 0)$

g_{hhZZ} fixed to SM value

$$\Rightarrow \delta_H = \frac{\delta_\sigma^{240}}{0.014} = \frac{0.0051}{0.014} = 36\%$$

Note : Oft quoted 30% error comes from combining CEPC with 50% HL-LHC meas.

M. McCullough,
arXiv:1312.3322

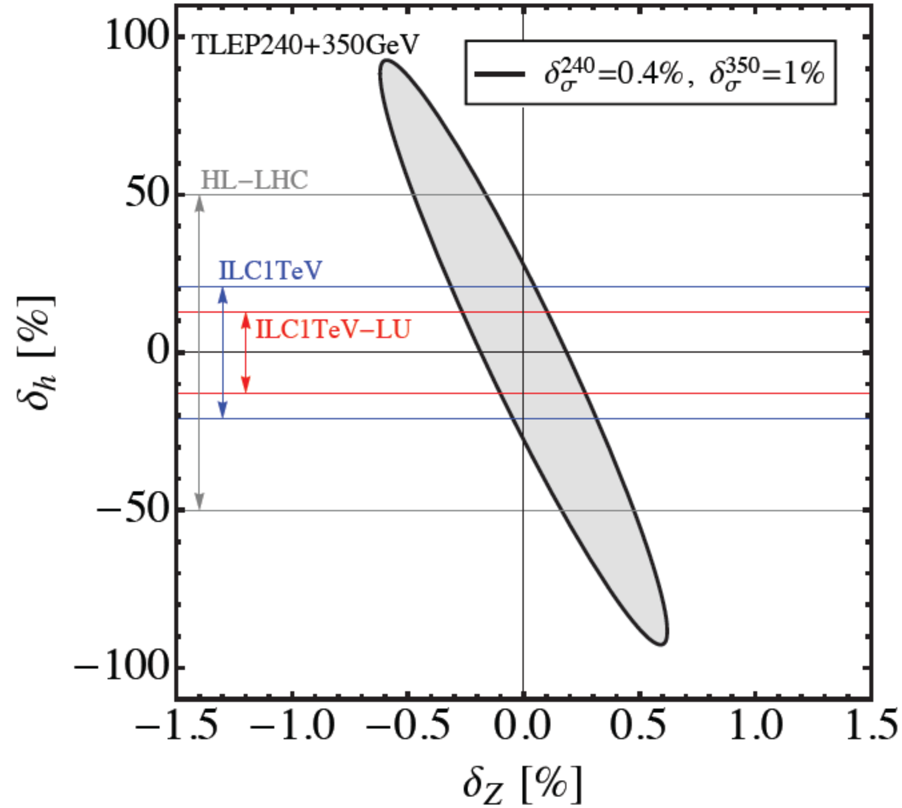


FIG. 3: Indirect 1σ constraints possible in $\delta_Z - \delta_h$ parameter space by combining associated production cross section measurements of 0.4% (1%-estimated) precision at $\sqrt{s} = 240$ GeV, (350 GeV) in solid black. It should be kept in mind that for large values of $|\delta_h|$ this ellipse can only be considered qualitatively as the calculation is only valid to lowest order in δ_h. The different axes scales should also be noted. Direct constraints possible at the high luminosity LHC and 1 TeV ILC (with LU denoting luminosity upgrade) are also shown for comparison. Lines are drawn to emphasize that direct constraints do not suffer from uncertainty in the hZZ coupling.