Measurement of the *Moli‘ere radius* from the 2014 TB data

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motivation

- A major design aim of LumiCal is it's compact structure.
- One aspect is to limit the development of the electromagnetic showers in the transverse direction.
- It allow better separation and identification of the electromagnetic element.

Moli'ere radius definition

The transverse development of electromagnetic showers in different materials scales fairly accurately with the *Moli'ere radius* $R_{\mathcal{M}}$, given by

$$R_{\mathcal{M}} = X_0 \frac{E_s}{E_c} \tag{1}$$

where $E_s \approx 21$ MeV, and E_c is the critical energy. In a compound the *Moli'ere radius* is given by

$$\frac{1}{R_{\mathcal{M}}} = \frac{1}{E_s} \sum \frac{w_j E_{cj}}{X_{0j}} = \sum \frac{w_j}{R_{\mathcal{M}}} \tag{2}$$

On the average,only 10% of the energy lies outside the cylinder with radius of 1 *Moli'ere radius*. The distributions are characterized by a narrow core, and broaden as the shower develops, often represented as the sum of two Gaussians.

density considerations

- In order to take in to effect the density the $R_{\mathcal{M}}$ units are $[gr/cm^2]$.
- During discussion on a structures and compounds the $R_{\mathcal{M}}$ is corrected for the density, ρ like

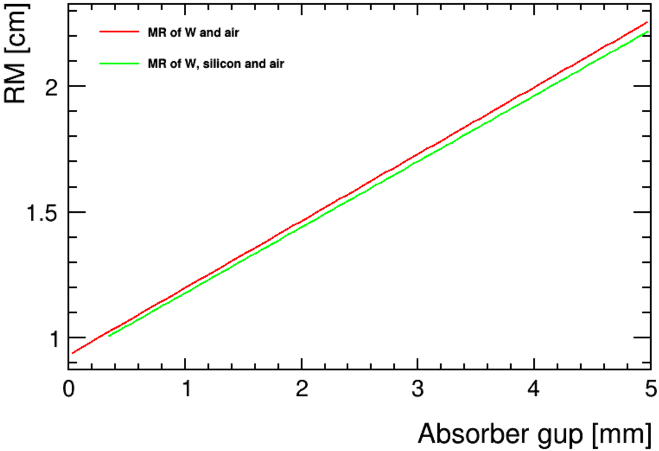
$$R_{\mathcal{M}}[cm] = \frac{R_{\mathcal{M}}[gr/cm^2]}{\rho[gr/cm^3]} \tag{3}$$

- We can calculate the $R_{\mathcal{M}}$ of the stack in different configuration using :

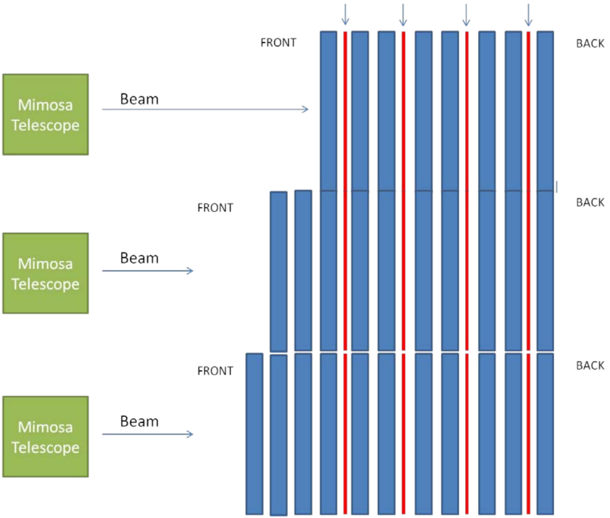
$$\frac{1}{R_{\mathcal{M}}[cm]} = \frac{\rho[gr/cm^3]}{R_{\mathcal{M}}[gr/cm^2]} = \rho \sum \frac{w_j}{R_{\mathcal{M}}} = \frac{W}{V} \sum \frac{W_j}{WR_{\mathcal{M}}} = \sum \frac{\rho_j \frac{Z_j}{Z}}{R_{\mathcal{M}}} \tag{4}$$

air gap

We can see the importance of the gap between absorbers in the calorimeter design on the the *Moli'ere radius* from the calculation :



2014 configuration



Moli'ere radius of 2014 configuration

Summery of all the material in our setup

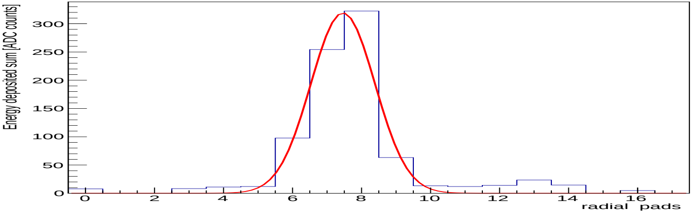
material	W	Cu	Ni	PL-95%	MGS-93%	air	Si	PCB
density	19.3	8.96	8.9	18.0*	17.8*	0.0012	2.33	1.7
$R_{\mathcal{M}}[gr/cm^2]$	18.0	14.0	13.4	17.7**	17.6**	8.8	11.5	10.3
$R_{\mathcal{M}}[cm]$	0.93	1.57	1.51	0.98	0.99	7330	4.94	6.06

Summery of calculated *Moli'ere radius*

	PL-95%	MGS-93%	air	Si	PCB	total	$R_{\mathcal{M}}[cm]$
general 93	0	0.7	0.37	0.032	0.25	1.35	1.79
general 95	0.7	0	0.37	0.032	0.25	1.35	1.78
CONF 1	1.05	1.75	1.57	0.128	1.0	5.5	1.81
CONF 2	1.75	1.75	1.77	0.128	1.0	6.4	1.71
CONF 3	2.1	1.75	1.87	0.128	1.0	6.85	1.67

1 event

- For each event from the data or simulation, we can look on the sum of the energy deposit, along the radial direction.
- The sum of energy deposit include both instrumented sectors in all 4 sensor layers.
- The hit position can be estimate by fitting or by calculating the center of gravity .



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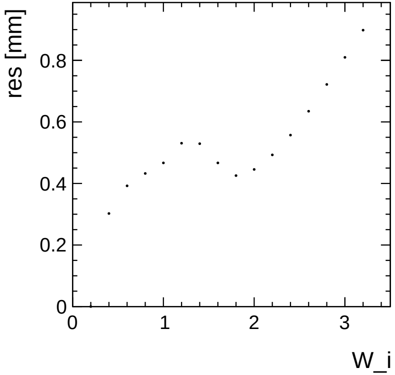
$$Y_s = \frac{\sum_n n w_n}{\sum_n w_n}, \tag{5}$$

where :

$$w_n = max\left\{0; W_0 + \ln \frac{E_n}{\sum_n E_n}\right\}, \tag{6}$$

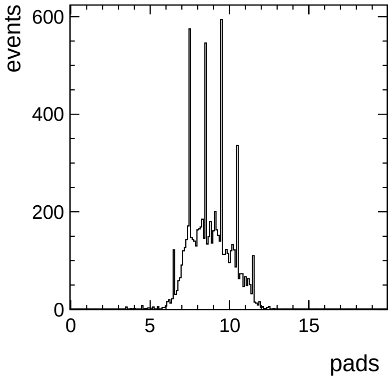
hit position

we can look for the W_0 that will give the best resolution:



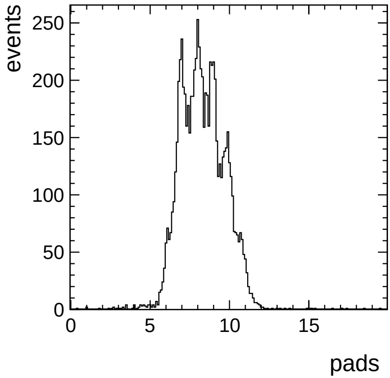
hit position

position reconstruction using $W_0 = 1.8$



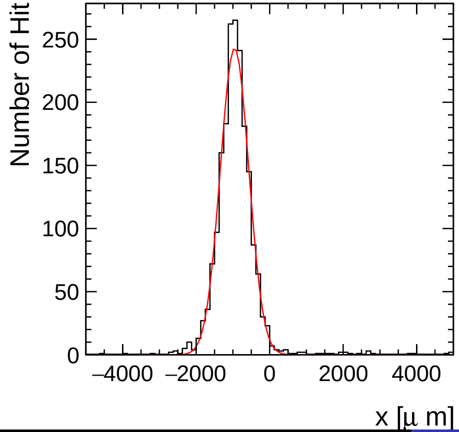
hit position

position reconstruction using fit results



hit position resolution

We can compare between the LumiCal reconstructed hit position and the extrapolated hit position from the beam Telescope to the LumiCal first layer. we can estimate the resolution to be around 0.4 mm.

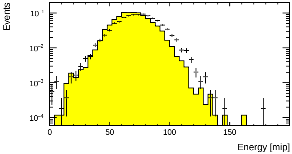


energy deposit

- By folding all events to start in a central pad the beam profile is canceled out.
- the mean radial energy deposit distribution can extracted from the single pad energy distribution.

energy deposit

The pad energy deposit distribution (with 0, 1, 2, 3 from shower center):



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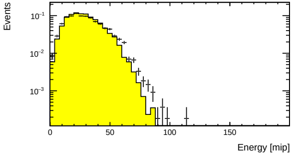

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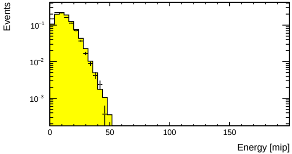

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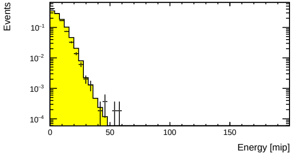

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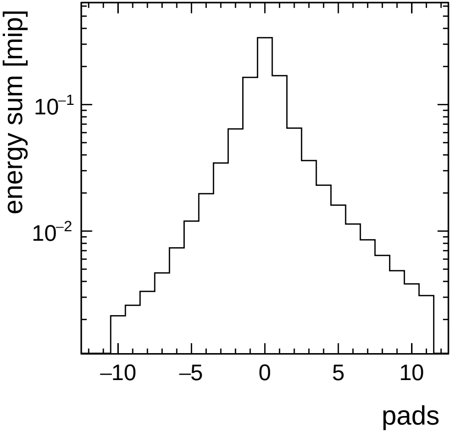
Radial energy distribution



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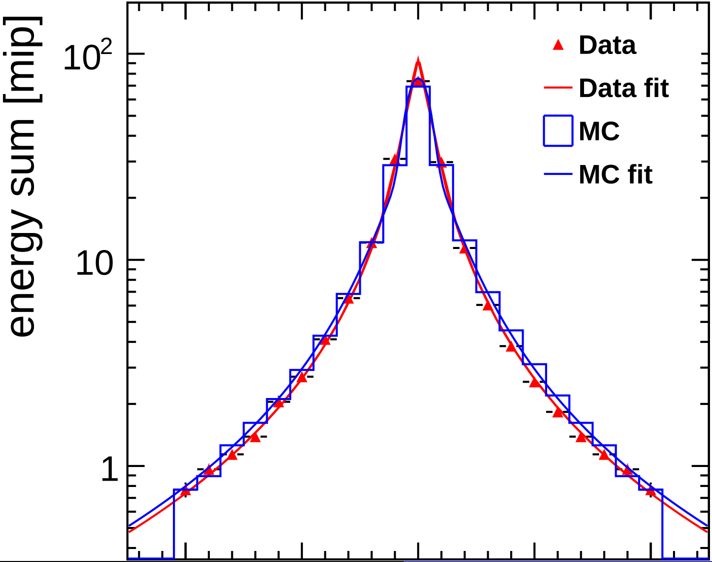

calculation

- The result is not direct from the fit we need to solve numericly :

$$0.9 = \int_0^{2\pi} d\varphi \int_0^{R_{\mathcal{M}}} F_E(r) r dr \ , \quad (7)$$

- LumiCal pads are long (strip like) and acts like 1 dimension integration, so we need to find:

$$G_E(y) = \int_{X_{min}}^{X_{max}} F_E(\sqrt{x^2 + y^2}) dx \ . \quad (8)$$

so we use :

$$F(r) = (A_C) e^{-(\frac{r}{R_C})^2} + (A_T) \frac{2 r R_T^2}{(r^2 + R_T^2)^2} \ . \quad (9)$$

Moli'ere radius of 2014 configuration

Summery of calculated *Moli'ere radius*

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fit model	bin center		integral		mo
data set	$R_{\mathcal{M}}$ [mm]	χ^2/NDF	$R_{\mathcal{M}}$ [mm]	χ^2/NDF	
CONF1 - data	13.03	14 /16	10.58	15.8 / 16	
CONF1 - MC	14.82	50 / 16	16.67	50.1 / 16	
CONF2 - data	14.06	9.4 /16	15.13	9.5 / 16	
CONF2 - MC	16.40	64.4 / 18	18.27	61.0 / 16	
CONF3 - data	13.42	6.4 /16	14.79	6.3 / 16	
CONF3 - MC	17.12	55 / 16	18.59	55.9 / 16	

Summary

- *Moli'ere radius* of 15 16 mm can by calculated from the 2014 TB data.
- MC simulation (Lucas) is giving similar results.
- *Moli'ere radius* error can by calculated from psodo experiment.
- error calculation is not complete and has big effect on measurement.