Homework at 12/14

Homework #1

Calculate the Transfer Matrix for skew quadrupole magnet from Hamiltonian

$$H = \frac{1}{2}\left(p_x^2 + p_y^2\right) + k_{1S}\, xy \quad (k_{1S} > 0)$$

(1) Calculate the equation of motion (second-order differential equation) for $(x+y)$ and $(x-y)$.

(2) The beam is transported in a constant skew quadrupole field. When the initial condition is $(x_0, p_{x0}, y_0, p_{y0})$ at $s = s_0$, derive the particle position $x(s)$ and $y(s)$.

(3) Derive a Transfer Matrix for skew quadrupole field from s_0 to $s_1 = s_0 + L$.

Homework #3

The vector potential for skew sextupole magnet is $-\frac{q}{p_0} A_{s,2S} = \frac{k_{2S} r^3}{3!} \sin 3\theta$.

(1) Show the vector potential in Frenet-Serret coordinate (x, y, s).

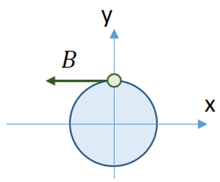
(2) When the skew sextupole is moved by Δx in horizontal direction, the beam position is changed by $-\Delta x$ with respect to the magnet. What kind of field is generated ?

(3) When the skew sextupole is moved by Δy in vertical direction, what kind of field is generated ?

Homework #2

Assume the particle is pass through the beams with uniform distribution. The bunch population of the crossing beam are $N = 2 \times 10^{10}$.

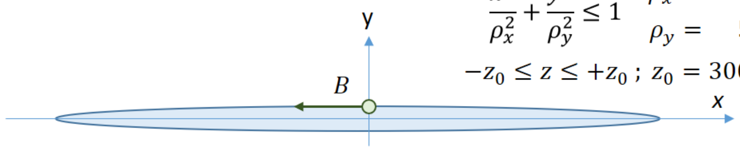
(1) Calculate the magnetic field by following round beam at $y = +r_0$. Hence, $\mu_0 = 4\pi \times 10^{-7}$.



$$\sqrt{x^2 + y^2} \le r_0\ ;\ r_0 = 50\ \text{nm}$$
$$-z_0 \le z \le +z_0\ ;\ z_0 = 300\ \mu\text{m}$$

(2) Calculate the magnetic field by following flat beam at $y = +\rho_y$. Hence, you can simplify the circumference of flat beam as $4\rho_x$.

$$\frac{x^2}{\rho_x^2} + \frac{y^2}{\rho_y^2} \le 1 \qquad \rho_x = 500\ \text{nm} \qquad \rho_y = 5\ \text{nm}$$
$$-z_0 \le z \le +z_0\ ;\ z_0 = 300\ \mu\text{m}$$



(3) For the beam with uniform distribution, the magnetic field along the y-axis (outside of beam) is changing as

$$(\textit{Round beam})\ B_x \propto \frac{1}{y} \qquad (\textit{Flat beam})\ B_x \propto \frac{2}{\pi} \arctan\frac{\rho_x}{y}$$

Calculate the magnetic field at $y = 500$ nm both for round and flat beam.

#1 and #3 are beam dynamics.

#2 is beam-beam interaction.