

Problems A1-1: Lattice Design

- 1) A transport lattice with no acceleration consists of FODO cells with quadrupole spacing $L = 10 \text{ m}$ and focal length of $f = 7 \text{ m}$. How large is the phase advance per cell?
- 2) The first quadrupole is focusing. The quadrupole is moved up by $1 \mu\text{m}$. How much does the beam trajectory move in the third quadrupole?
- 3) Calculate the average beta-function in a thin lens FODO lattice as a function of $\hat{\beta}$, $\check{\beta}$ and L/f

Solutions

1) We use

$$\begin{aligned}\cos \mu &= 1 - \frac{L^2}{2f^2} \\ \Rightarrow \cos \mu &= 1 - \frac{100}{98} \\ \Rightarrow \mu &= \arccos\left(\frac{1}{2}\right) = 91.17^\circ\end{aligned}$$

2) The angular deflection is given by the offset δ and the focal strength f

$$y' = \frac{\delta}{f}$$

we transform into normalised phase space

$$y'_N = \sqrt{\beta(s_1)} \frac{\delta}{f}$$

450° downstream this is

$$y_N = \sqrt{\beta(s_1)} \frac{\delta}{f}$$

which translates into

$$y = \sqrt{\beta(s_1)\beta(s_2)} \frac{\delta}{f}$$

inserting number we find

$$y \approx 4.9\delta$$

hence the RMS jitter $\sigma_{y,jitt} = 4.9\mu\text{m}$.

Solutions

3) We will integrate from the centre of a defocusing quadrupole (at $s = 0$) to the centre of the next focusing quadrupole (at $s = L$). In the centre of the defocusing quadrupole we have $\beta = \check{\beta}$ and $\alpha = 0$. We calculate the Twiss parameters immediately after the quadrupole (at $\epsilon \rightarrow 0$):

$$\begin{pmatrix} \beta(\epsilon) & -\alpha(\epsilon) \\ -\alpha(\epsilon) & \gamma(\epsilon) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1/(2f) & 1 \end{pmatrix} \begin{pmatrix} \check{\beta} & 0 \\ 0 & 1/\check{\beta} \end{pmatrix} \begin{pmatrix} 1 & 1/(2f) \\ 0 & 1 \end{pmatrix}$$
$$\Rightarrow \begin{pmatrix} \beta(\epsilon) & -\alpha(\epsilon) \\ -\alpha(\epsilon) & \gamma(\epsilon) \end{pmatrix} = \begin{pmatrix} \check{\beta} & \check{\beta}/(2f) \\ \check{\beta}/(2f) & 1/\check{\beta} + \check{\beta}/(2f)^2 \end{pmatrix}$$

now we calculate beta along a drift using

$$\begin{pmatrix} \beta(s) & -\alpha(s) \\ -\alpha(s) & \gamma(s) \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \check{\beta} & \check{\beta}/(2f) \\ \check{\beta}/(2f) & 1/\check{\beta} + \check{\beta}/(2f)^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ s & 1 \end{pmatrix}$$

$$\beta(s) = \check{\beta} + \frac{\check{\beta}}{f}s + \left(\frac{1}{\check{\beta}} + \frac{\check{\beta}}{4f^2} \right) s^2$$

$$\langle \beta \rangle = \frac{1}{L} \int_0^L \beta(s) ds = \check{\beta} + \frac{\check{\beta}}{2f}L + \frac{L^2}{3} \left(\frac{1}{\check{\beta}} + \frac{\check{\beta}}{4f^2} \right)$$

to avoid to much calculation we exploit

$$\beta(L) = \hat{\beta} = \check{\beta} + \frac{\check{\beta}}{f}L + \left(\frac{1}{\check{\beta}} + \frac{\check{\beta}}{4f^2} \right) L^2$$

hence

$$\langle \beta \rangle = \frac{2}{3}\check{\beta} + \frac{1}{3}\hat{\beta} + \frac{L}{6f}\check{\beta}$$