HOMEWORK

1) Design a bunch compressor for an electron beam with the following characteristics:
   - $W_{in} = 300\ MeV; W_{out} = 1.2\ GeV$
   - $chicane\ R_{56} = -40\ mm$
   - $compressor\ factor = 4$
   - $f_{RF} = 1300\ MHz$

   Calculate V_{RF}, φ_0, and the approximate length of the linac portion dedicated to the bunch compression assuming @ $E_{acc} = 30\ MV/m$

2) Considering an observation time of $100\ ms$ (i.e. $f_L = 10\ Hz$) the arrival time jitter of the bunches entering the compressor is $\sigma_{t_{in}} = 300\ fs$, while the RF in the compressor shows a SSB phase noise power spectrum of the f^{-2} type with $\mathcal{L}(f = 1\ kHz) \approx -110\ dB_c/Hz$.

   Calculate the maximum allowed RF amplitude jitter $\sigma_{(\Delta V_{RF}/V_{RF})}\big|_{max}$ to limit the bunch arrival time jitter at the compressor output to $\sigma_{t_{out}} = 180\ fs$.

   For the calculation assume all jitter contributions uncorrelated and neglect the intra-bunch energy spread and the bunch-to-bunch energy scatter of the incoming beam.

HOMEWORKS: Solutions

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$$h = \frac{-qV_{RF}\ sin\varphi_0}{W_0}\frac{\omega_{RF}}{c}$$

Compression factor = 4 ⇨ $1 + hR_{56} = 0.25$ ⇨ $hR_{56} = -0.75$ ⇨ $h = 18.75\ m^{-1}$

$$\begin{cases} qV_{RF}\ cos\varphi_0 = W_0 - W_{in} \\ qV_{RF}\ sin\varphi_0 = -h\dfrac{c}{\omega_{RF}}W_0 \end{cases} \Rightarrow \begin{cases} tan\varphi_0 = -\dfrac{hc}{\omega_{RF}}\dfrac{W_0}{W_0 - W_{in}} \\ V_{RF} = \sqrt{\left(\dfrac{W_0 - W_{in}}{q}\right)^2 + \left(\dfrac{hc}{\omega_{RF}}\dfrac{W_0}{q}\right)^2} \end{cases} \Rightarrow \begin{cases} \varphi_0 = -42.56\,° \\ V_{RF} = 1.22\ GV \end{cases}$$

$L_{linac} = V_{RF}/E_{acc} = 40.7\ m$ ⇨ active length, ≈ 40 ILC 9-cells cavities

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$$S_\varphi(f) = b_{-2}/f^2 \quad \Rightarrow \quad 10\ Log\left[S_\varphi(f = 1kHz)\right] = \mathcal{L}(f = 1\ kHz) + 3\ dB \approx -107\ dB_c/Hz$$

$$b_{-2} = (1\ kHz)^2 \cdot 10^{-(107/10)} = \\ = 2 \cdot 10^{-5} rad^2\ \ Hz \quad \Rightarrow \quad \sigma_{t_{RF}} = \frac{1}{\omega_{RF}}\sqrt{\int_{f_L}^{\infty}\frac{b_{-2}}{f^2}\,df} = \frac{1}{\omega_{RF}}\sqrt{\frac{b_{-2}}{f_L}} = 173\ fs$$

$$\underbrace{\boxed{\sigma_{t_0}^2}}_{(180\ \mathrm{fs})^2} = \underbrace{\boxed{(1 + hR_{56})^2\sigma_{t_{in}}^2}}_{(75\ \mathrm{fs})^2} + \underbrace{\boxed{(hR_{56})^2\sigma_{t_{RF}}^2}}_{(130\ \mathrm{fs})^2} + \underbrace{\boxed{\left(\frac{R_{56}}{c}\frac{W_0 - W_{in}}{W_0}\right)^2\left(\frac{\sigma_{V_{RF}}}{V_{RF}}\right)^2}}_{\sigma_{t_{V_{RF}}}^2 \approx (100\ \mathrm{fs})^2}$$

$$\left(\frac{\sigma_{V_{RF}}}{V_{RF}}\right) = \sigma_{t_{V_{RF}}} \Big/ \left(\frac{R_{56}}{c}\frac{W_0 - W_{in}}{W_0}\right) \approx 1\ ‰$$