Lecture B2:
Superconductive RF fundamental
Home work

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London equations

Proposed a 2-fluid model with a normal fluid and superfluid components

***n_s* : density of the superfluid component of velocity v_s**
***n_n* : density of the normal component of velocity v_n**

$$m\frac{\partial \vec{\upsilon}}{\partial t} = -e\vec{E}$$ superelectrons are accelerated by E

$$\vec{J}_s = -en_s\,\vec{\upsilon}$$

$$\frac{\partial \vec{J}_s}{\partial t} = \frac{n_s e^2}{m}\vec{E}$$ superelectrons (equation-0)

$$\vec{J}_n = \sigma_n \vec{E}$$ normal electrons

London equations

$$\frac{\partial \vec{J}_s}{\partial t} = \frac{n_s e^2}{m} \vec{E}$$

Maxwell: $$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t}\left(\frac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s + \vec{B} \right) = 0 \qquad \Rightarrow \frac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s + \vec{B} = \text{Constant}$$

F&H London postulated: $$\boxed{\frac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s + \vec{B} = 0}$$ (equation-1)

London equations

combine with $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_s$ (equation-2)

$$\nabla^2 \vec{B} - \frac{\mu_0 n_s e^2}{m} \vec{B} = 0$$ (equation-3)

$$B(x) = B_A \exp\left[-x/\lambda_L\right]$$ (equation-4)

$$\lambda_L = \left[\frac{m}{\mu_0 n_s e^2}\right]^{\frac{1}{2}}$$ London penetration depth

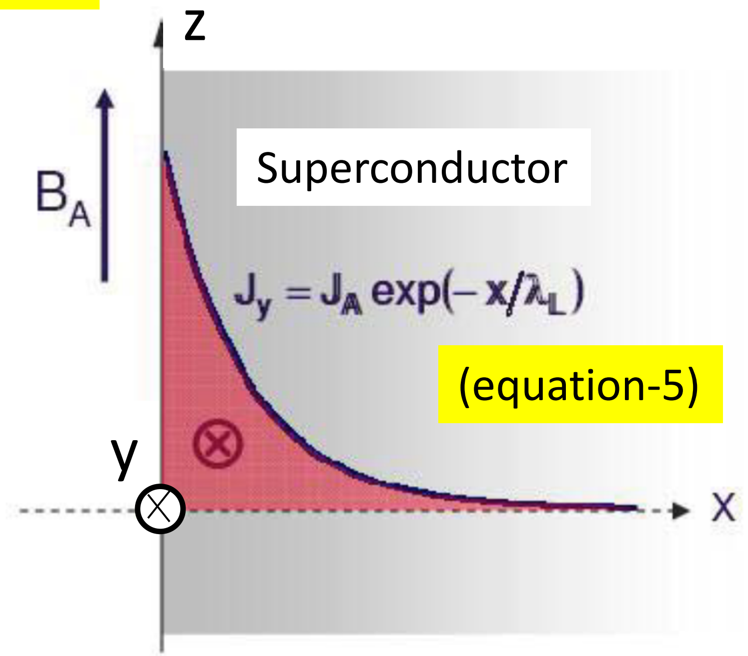

Figure-1

The magnetic field, and the current, decay exponentially over a distance λ (a few 10s of nm)

Homework

- Derive (equation-0).
- By using (equation-1) and (equation-2), derive (equation-3).
- In figure-1, there is a superconductor in the region of X>0. Outside the superconductor (x<0), there is constant magnetic field B = (0, 0, B_A) as shown in Figure-1. If the magnetic field inside the superconductor is described as B = (0, 0, Bz(x)), apply B to (equation-3) and derive (equation-4).
- Show that the electric current density J = (0, Jy, 0) inside the superconductor is described with (equation-5), and describe J_A by B_A, λL, and μ0.