Model-Independent Determination of the Triple Higgs Coupling at the ILC

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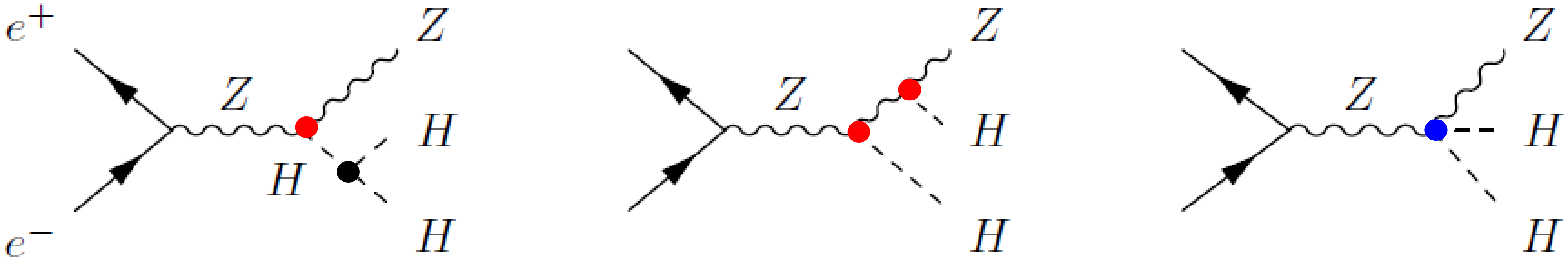
Higgs Self Coupling Systematic Error Uncertainties for

g_{ZZH} g_{ZZHH} (and other, BSM, couplings) in $\sigma(e^+e^- \to HHZ)$



We assume that $\sigma(e^+e^- \to HHZ)$ can be described by an effective field theory (EFT) containing a general $SU(2) \times U(1)$ gauge invariant Lagrangian with dimension-6 operators in addition to the SM.

Using the "Warsaw" basis, with the pure Higgs operators in the "SILH" basis, these are the 10 CP-conserving dim-6 operators relevant to this analysis:

$$
\begin{aligned}
\Delta\mathcal{L} = {} & \frac{c_H}{2v^2}\partial^\mu(\Phi^\dagger\Phi)\partial_\mu(\Phi^\dagger\Phi) + \frac{c_T}{2v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) - \frac{c_6\lambda}{v^2}(\Phi^\dagger\Phi)^3 \\
& + \frac{g^2 c_{WW}}{m_W^2}\Phi^\dagger\Phi W^a_{\mu\nu}W^{a\mu\nu} + \frac{4gg' c_{WB}}{m_W^2}\Phi^\dagger t^a \Phi W^a_{\mu\nu}B^{\mu\nu} \\
& + \frac{g'^2 c_{BB}}{m_W^2}\Phi^\dagger\Phi B_{\mu\nu}B^{\mu\nu} + \frac{g^3 c_{3W}}{m_W^2}\epsilon_{abc}W^a_{\mu\nu}W^{b\nu}{}_\rho W^{c\rho\mu} \\
& + i\frac{c_{HL}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\overline{L}\gamma_\mu L) + +4i\frac{c'_{HL}}{v^2}(\Phi^\dagger t^a \overleftrightarrow{D}^\mu \Phi)(\overline{L}\gamma_\mu t^a L) \\
& + i\frac{c_{HE}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\overline{e}\gamma_\mu e) \ .
\end{aligned}
$$

In addition there are 4 CP violating terms:

$$\Delta\mathcal{L}_{CP} = +\frac{g^2\tilde{c}_{WW}}{m_W^2}\Phi^\dagger\Phi W^a_{\mu\nu}\widetilde{W}^{a\mu\nu} + \frac{4gg'\tilde{c}_{WB}}{m_W^2}\Phi^\dagger t^a\Phi W^a_{\mu\nu}\widetilde{B}^{\mu\nu}$$
$$+\frac{g'^2\tilde{c}_{BB}}{m_W^2}\Phi^\dagger\Phi B_{\mu\nu}\widetilde{B}^{\mu\nu} + \frac{g^3\tilde{c}_{3W}}{m_W^2}\epsilon_{abc}W^a_{\mu\nu}W^{b\nu}{}_\rho\widetilde{W}^{c\rho\mu}$$

where the dual field strength tensors are defined by

$$\widetilde{B}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}B^{\rho\sigma}\ , \quad \widetilde{W}^k_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}W^{\rho\sigma k}$$

In summary there are 10 CP-conserving coefficients:

c_{HL} c'_{HL} c_{HE} c_T c_{WB} c_{3W}

c_H c_{WW} c_{BB}

c_6

and 4 CP-violating coefficients:

$\tilde{c}_{WW}$ $\tilde{c}_{WB}$ $\tilde{c}_{BB}$ $\tilde{c}_{3W}$

After EWSB we have, $\Delta\mathcal{L} = \Delta\mathcal{L}_h + \Delta\mathcal{L}_{eehZ} + \Delta\mathcal{L}_{TGC}$ where

$$\Delta\mathcal{L}_h = -\eta_h \lambda_0 v_0 h^3 + \frac{\theta_h}{v_0} h \partial_\mu h \partial^\mu h + \eta_Z \frac{m_Z^2}{v_0} Z_\mu Z^\mu h + \frac{1}{2}\eta_{2Z}\frac{m_Z^2}{v_0^2} Z_\mu Z^\mu h^2$$
$$+\eta_W \frac{2m_W^2}{v_0} W_\mu^+ W^{-\mu} h + \eta_{2W}\frac{m_W^2}{v_0^2} W_\mu^+ W^{-\mu} h^2$$
$$+\frac{1}{2}\left(\zeta_Z \frac{h}{v_0} + \frac{1}{2}\zeta_{2Z}\frac{h^2}{v_0^2}\right)\hat{Z}_{\mu\nu}\hat{Z}^{\mu\nu} + \left(\zeta_W \frac{h}{v_0} + \frac{1}{2}\zeta_{2W}\frac{h^2}{v_0^2}\right)\hat{W}_{\mu\nu}^+ \hat{W}^{-\mu\nu}$$
$$+\frac{1}{2}\left(\zeta_A \frac{h}{v_0} + \frac{1}{2}\zeta_{2A}\frac{h^2}{v_0^2}\right)\hat{A}_{\mu\nu}\hat{A}^{\mu\nu} + \left(\zeta_{AZ} \frac{h}{v_0} + \zeta_{2AZ}\frac{h^2}{v_0^2}\right)\hat{A}_{\mu\nu}\hat{Z}^{\mu\nu} \ .$$

$$\Delta\mathcal{L}_{eehZ} = g_{LZh}(\bar{e}_L \gamma_\mu e_L) Z^\mu \left(\frac{h}{v_0} + \frac{1}{2}\frac{h^2}{v_0^2}\right) + g_{RZh}(\bar{e}_R \gamma_\mu e_R) Z^\mu \left(\frac{h}{v_0} + \frac{1}{2}\frac{h^2}{v_0^2}\right)$$

$$\Delta\mathcal{L}_{TGC} = i g_V \Big\{ g_{1V} V^\mu (\hat{W}_{\mu\nu}^- W^{+\nu} - \hat{W}_{\mu\nu}^+ W^{-\nu}) + \kappa_V W_\mu^+ W_\nu^- \hat{V}^{\mu\nu}$$
$$+ \frac{\lambda_V}{m_W^2} \hat{W}_\mu^{-\rho} \hat{W}_{\rho\nu}^+ \hat{V}^{\mu\nu} \Big\} , \quad V = A, Z \qquad g_{1A} = 1 \qquad g_{1Z} = 1 + \Delta_g$$

$$\kappa_A = 1 + \Delta_\kappa \qquad \kappa_Z = 1 + \Delta_g - \frac{s_0^2}{c_0^2}\Delta_\kappa$$

$$\lambda_A = \Delta_\lambda \qquad \lambda_Z = \Delta_\lambda \ ,$$

In the SM at tree level $g_A = e$ $g_Z = g c_0$ $g_{1V} = \kappa_V = \eta_H = \eta_Z = \eta_{2Z} = \eta_W = \eta_{2W} = 1$, and all others =0

And the CP violating piece $\Delta\mathcal{L}_{CP} = \Delta\mathcal{L}_{hCP} + \Delta\mathcal{L}_{3V_{CP}}$ where

$$\Delta\mathcal{L}_{hCP} = \frac{\tilde{\zeta}_Z}{v_0}\hat{Z}_{\mu\nu}\hat{\tilde{Z}}^{\mu\nu} h + \frac{1}{2}\frac{\tilde{\zeta}_{ZZ}}{v_0^2}\hat{Z}_{\mu\nu}\hat{\tilde{Z}}^{\mu\nu} h^2 + 2\frac{\tilde{\zeta}_W}{v_0}\hat{\tilde{W}}^{+}_{\mu\nu}\hat{W}^{-\mu\nu} h + \frac{\tilde{\zeta}_{WW}}{v_0^2}\hat{W}^{+}_{\mu\nu}\hat{\tilde{W}}^{-\mu\nu} h^2$$

$$\Delta\mathcal{L}_{3V_{CP}} = i\tilde{\kappa}_\gamma W^\dagger_\mu W_\nu \tilde{F}^{\mu\nu} + i\frac{\tilde{\lambda}_\gamma}{M_W^2} W^\dagger_{\lambda\mu} W^\mu_\nu \tilde{F}^{\nu\lambda} + i\tilde{\kappa}_z W^\dagger_\mu W_\nu \tilde{F}^{\mu\nu} + i\frac{\tilde{\lambda}_z}{M_W^2} W^\dagger_{\lambda\mu} W^\mu_\nu \tilde{F}^{\nu\lambda}$$

In the SM at tree level all $\tilde{\zeta}_x$, $\tilde{\zeta}_{xx}$, $\tilde{\kappa}_x$, $\tilde{\lambda}_x$ =0

The couplings θ_h, η_x ζ_x g_{xZH}, TGC's & EWPO's take the following form in our EFT:

$$\eta_h = (1 - c'_{HL} - \frac{1}{2}c_H + c_6)$$
$$\theta_h = c_H$$
$$\eta_Z = (1 - c_T - \frac{1}{2}c_H - c'_{HL})$$
$$\eta_{2Z} = (1 - 5c_T - c_H - 2c'_{HL})$$
$$\eta_W = (1 - \frac{1}{2}c_H - c'_{HL})$$
$$\eta_{2W} = (1 - c_H - c'_{HL}) \; .$$
$$\zeta_W = \zeta_{2W} = 8(c_{WW})$$
$$\zeta_Z = \zeta_{2Z} = 8(c_0^2 c_{WW} + 2s_0^2 c_{WB} + \frac{s_0^4}{c_0^2}c_{BB})$$
$$\zeta_{AZ} = \zeta_{2AZ} = 8(s_0 c_0 c_{WW} - s_0 c_0 (1 - \frac{s_0^2}{c_0^2})c_{WB} - \frac{s_0^3}{c_0}c_{BB})$$
$$\zeta_A = \zeta_{2A} = 8s_0^2(c_{WW} - 2c_{WB} + c_{BB}) \; .$$

c_6 is uniquely accessible through the Higgs self coupling measurement. The other 9 EFT parameters appear in several places.

Note the relationship between the SM *hzz* & *hhzz* couplings η_Z & η_{2Z}

Precise measurement of $\Gamma(H \to \gamma\gamma)$ from LHC+ILC will be used to constrain $c_{WW} + c_{BB}$

$$g_{LZh} = -\frac{e_0}{c_0 s_0}(c_{HL} + c'_{HL})$$
$$g_{RZh} = -\frac{e_0}{c_0 s_0}(c_{HE})$$

$$\Delta_g = \frac{1}{c_0^2 - s_0^2}(\frac{1}{2}c_T - c'_{HL} - 8\frac{s_0^2}{c_0^2}c_{WB})$$
$$\Delta_\kappa = +8c_{WB}$$
$$\Delta_\lambda = -6\frac{e_0^2}{s_0^2}c_{3W}$$

The TGC's depend on c_T, c'_{HL}, c_{WB} through the Goldstone bosons that are eaten by the Z and W fields

$$m_W^2/m_Z^2 = c_0^2 + \frac{c_0^2}{c_0^2 - s_0^2}(c_0^2 c_T - 2s_0^2(c'_{HL} + 8c_{WB})$$
$$s_*^2 = s_0^2 + \frac{s_0^2}{c_0^2 - s_0^2}(c'_{HL} + 8c_{WB} - c_0^2 c_T) - \frac{1}{2}c_{HE} - s_0^2(c_{HL} - c_{HE})$$
$$g_L - g_R = \frac{1}{2}\frac{e_0}{s_0 c_0}(1 + c_{HL} - c_{HE} + \frac{1}{2}c_T)$$

EWPO's also depend on many of the Higgs-related operator coefficients, again through EWSB.

The coefficients c_{HL} c'_{HL} c_{HE} c_{T} c_{WB} c_{3W}

are determined by the 3 EWPO's

$$m_W^2/m_Z^2 = c_0^2 + \frac{c_0^2}{c_0^2 - s_0^2}(c_0^2 c_T - 2s_0^2(c'_{HL} + 8c_{WB})$$

$$s_*^2 = s_0^2 + \frac{s_0^2}{c_0^2 - s_0^2}(c'_{HL} + 8c_{WB} - c_0^2 c_T) - \frac{1}{2}c_{HE} - s_0^2(c_{HL} - c_{HE})$$

$$g_L - g_R = \frac{1}{2}\frac{e_0}{s_0 c_0}(1 + c_{HL} - c_{HE} + \frac{1}{2}c_T)$$

and the 3 TGC's

$$\Delta_g = \frac{1}{c_0^2 - s_0^2}(\frac{1}{2}c_T - c'_{HL} - 8\frac{s_0^2}{c_0^2}c_{WB})$$

$$\Delta_\kappa = +8c_{WB}$$

$$\Delta_\lambda = -6\frac{e_0^2}{s_0^2}c_{3W}$$

At ILC with the full H-20 scenario the error on the TGC's are

$$\Delta(\Delta_\kappa) = 2\times 10^{-4}$$

$$\Delta(\Delta_g) = 8\times 10^{-4}$$

Through EWPOs and ILC measurements of TGC's the number of independent EFT parameters has been reduced from 10 to just 4: c_H c_{WW} c_{BB} c_6

With c_T and c'_{HL} tightly constrained by EWPO's &TGC's , c_H is obtained through the σ_{ZH} measurement via

$$\eta_Z = (1 - c_T - \frac{1}{2}c_H - c'_{HL})$$

With c_{WB} tightly constrained by EWPO's &TGC's. c_{WW} & c_{BB} are obtained through measurements of $\Gamma(\mathrm{H} \to \gamma\gamma)$ (from LHC+ILC) and the HZZ Lorentz structure parameter ζ_Z measured at the ILC with an angular analysis of $e^+e^- \to ZH$. The relationship between these two measurements and the coefficients c_{WW} & c_{BB} is given by

$$\zeta_A = \zeta_{2A} = 8s_0^2(c_{WW} - 2c_{WB} + c_{BB})$$

$$\zeta_Z = \zeta_{2Z} = 8(c_0^2 c_{WW} + 2s_0^2 c_{WB} + \frac{s_0^4}{c_0^2} c_{BB})$$

Combining LHC and ILC gives $\Delta g_{H\gamma\gamma} = 0.01$

At LCWS15 T. Ogawa obtained $\Delta\zeta_Z = 0.004$ (http://agenda.linearcollider.org/event/6662/)

Let's now rewrite the Lagrangian using our measured variables

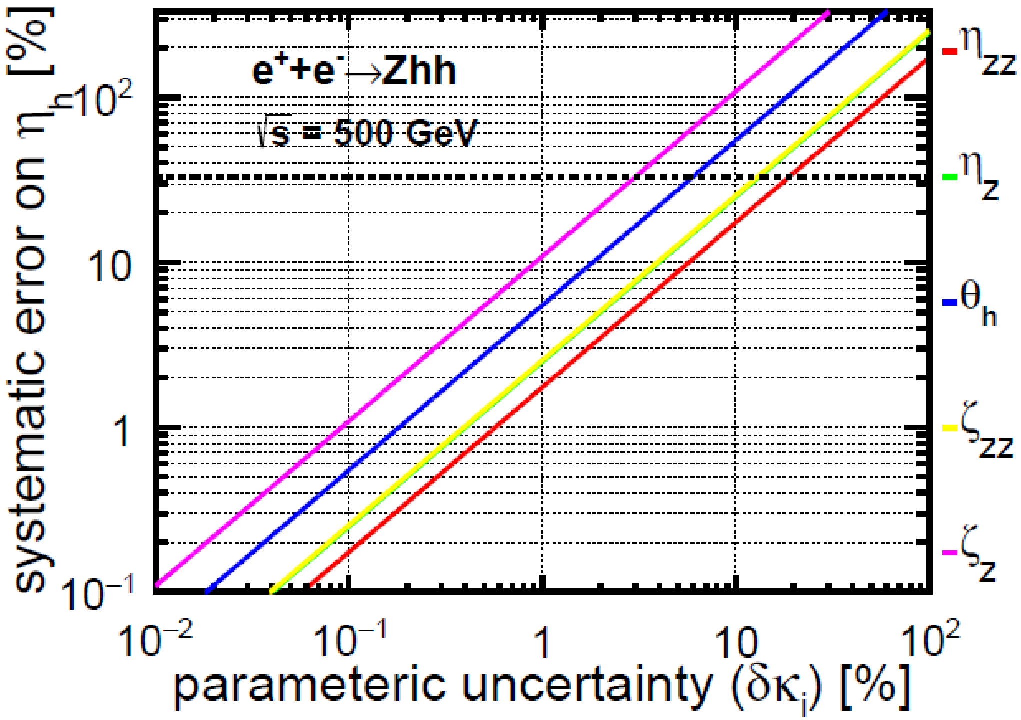
η_Z ζ_Z and the one remaining unconstrained EFT parameter c_6

$$\mathcal{L} = -\lambda v \left(\eta_Z - c_6\right) h^3 + \frac{2}{v}\left(1 - \eta_Z\right) h \partial_\mu h \partial^\mu h + \eta_Z \frac{M_Z^2}{v} Z_\mu Z^\mu h + (2\eta_Z - 1) \frac{M_Z^2}{2v^2} Z_\mu Z^\mu h^2$$
$$+ \frac{\zeta_Z}{2v} Z_{\mu\nu} Z^{\mu\nu} h + \frac{\zeta_Z}{4v^2} Z_{\mu\nu} Z^{\mu\nu} h^2$$

In this EFT approach all of the couplings in the calculation of $\sigma(e^+e^- \to HHZ)$ are tightly constrained by the other Higgs coupling measurements, TGC measurements, and EWPT's. The only unconstrained parameter is c_6. If the best match to the measured $\sigma(e^+e^- \to HHZ)$ is $c_6 = 0$ within sys+stat errors then we have observed SM Higgs self coupling.

At this point we are left with the problem of propagating the errors from the EWPO , TGC, σ_{ZH} $\Gamma(h \to \gamma\gamma)$ ζ_Z and σ_{ZHH} measurements to the error on c_6 . This is a work in progress.

Here I only show a figure giving the systematic error on η_h as the uncertainties of individual parameters are varied one at a time. (Horizontal dashed line is the statistical error.



Let's include operator coefficients constrained by TGC's & EWPO's.

$$\mathcal{L} = -\lambda v\left(\eta_Z + c_T - c_6\right)h^3 + \frac{2}{v}\left(1-\eta_Z - c_T - c'_{HL}\right)h\partial_\mu h\partial^\mu h + \eta_Z \frac{M_Z^2}{v} Z_\mu Z^\mu h$$

$$+ (2\eta_Z - 1 - 3c_T)\frac{M_Z^2}{2v^2} Z_\mu Z^\mu h^2 + \frac{\zeta_Z}{2v} Z_{\mu\nu} Z^{\mu\nu} h + \frac{\zeta_Z}{4v^2} Z_{\mu\nu} Z^{\mu\nu} h^2$$

$$m_W^2/m_Z^2 = c_0^2 + \frac{c_0^2}{c_0^2 - s_0^2}(c_0^2 c_T - 2s_0^2(c'_{HL} + 8c_{WB})$$

$$\Delta_g = \frac{1}{c_0^2 - s_0^2}(\frac{1}{2}c_T - c'_{HL} - 8\frac{s_0^2}{c_0^2}c_{WB})$$

$$\Delta_\kappa = +8c_{WB}$$

c_T & c'_{HL} are linear combinations of M_W^2 / M_Z^2 , Δ_g , Δ_κ

$$\frac{\Delta(M_W^2)}{M_Z^2} = \frac{2M_W(\Delta M_W)}{M_Z^2} = \frac{2*80.385*0.015}{(91.1876)^2} = 0.00029$$

At ILC with the full H-20 scenario

$$\Delta(\Delta_g) = 0.0008$$

$$\Delta(\Delta_\kappa) = 0.0002$$

$$\Delta\eta_Z = 0.003$$

$$\Delta\zeta_Z = 0.004$$

Summary

- In an EFT approach all but one of the couplings in the calculation of $\sigma(e^+e^- \to HHZ)$ are tightly constrained by the other Higgs coupling measurements, TGC measurements, and EWPT's.

- The unmeasured ZZHH quartic coupling is related to the HZZ coupling, which is measured to 0.3% at the ILC in the H-20 scenario. The systematic error due to the unmeasured quartic coupling is therefore very small. A full error analysis is underway

- Imagine trying to perform this type of analysis with a loop level measurement of the Higgs self coupling. The number of terms to be considered would be much greater.