Testing dark matter in the inert doublet model by precision data of Higgs boson couplings

S. kanemura, M. Kikuchi, K. Sakurai, arXiv:1605.08520

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Introduction

- The Higgs boson was established at the LHC in 2012.
  - The Standard Model (SM) established as low energy effective theory.
  - The Higgs sector is adopted the minimal one :

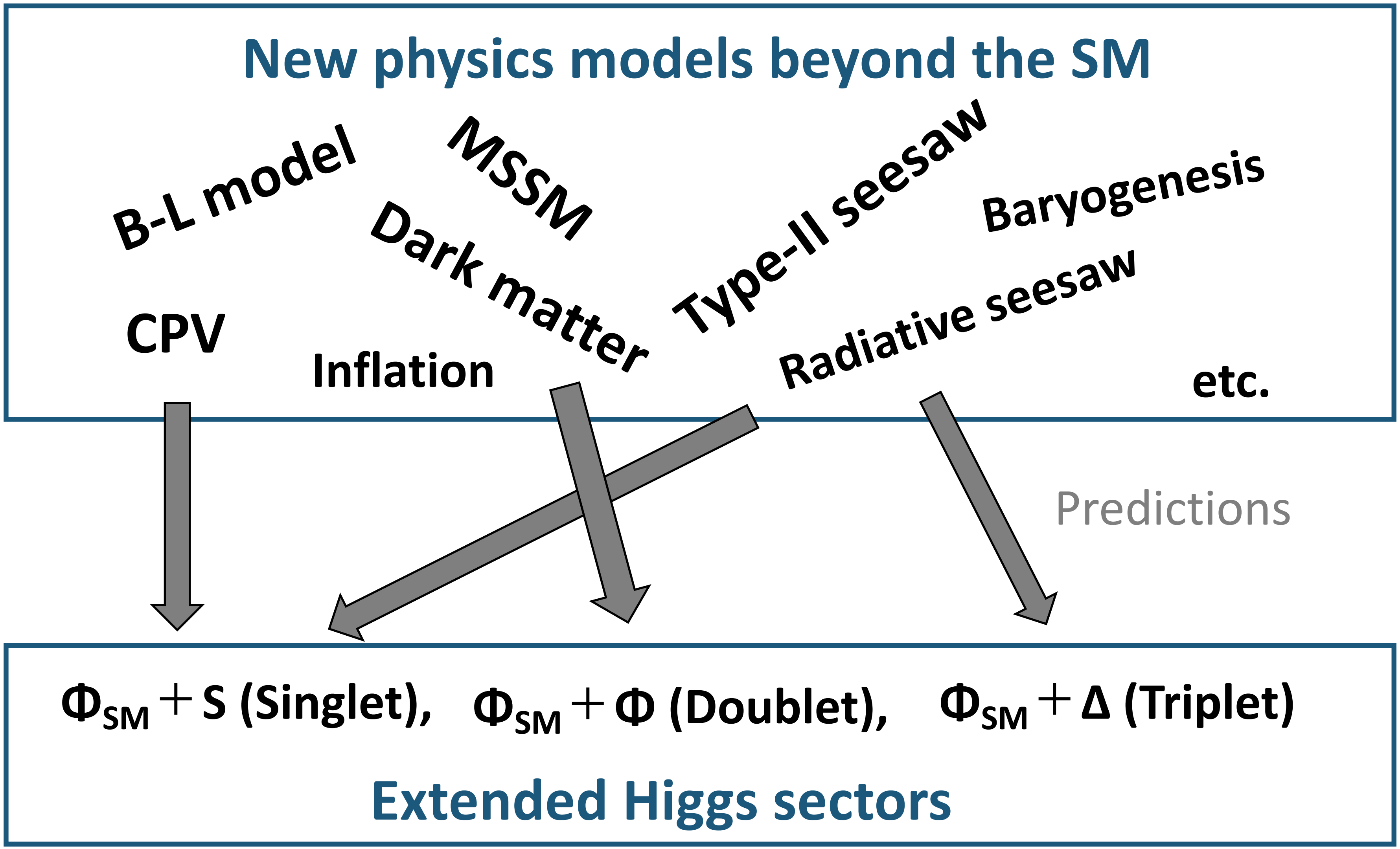
$$V(\Phi) = \mu^2|\Phi|^2 + \lambda|\Phi|^4$$

This is assumption. There is no theoretical reason why the Higgs sector must be the simplest.

- In the end, the structure of the Higgs sector is still unknown.
  - Numbers of scalar multiplets, their representations
  - Symmetries
  - Essence of Higgs field (elementary? or composite?)
  - Origin of $\mu^2 < 0$

The Structure of the Higgs sector and New Physics

If we can clarify the structure of the Higgs sector, we could approach new physics beyond the standard model.



The Structure of the Higgs sector and New Physics

If we can clarify the structure of the Higgs sector, we could approach new physics beyond the standard model.

New physics models beyond the SM

B-L model
MSSM
Dark matter
Type-II seesaw
Baryogenesis
Radiative seesaw
CPV
Inflation
etc.

Determination of the direction

$\Phi_{SM} + S$ (Singlet), $\Phi_{SM} + \Phi$ (Doublet), $\Phi_{SM} + \Delta$ (Triplet)

Extended Higgs sectors

In This Talk

- We focus on the Inert Doublet Model (IDM).
  - One of the natural extended Higgs models
  - Containing a dark matter candidate
- We discuss testability of the IDM by utilizing precision data of the Higgs boson couplings at future experiments.
- In particular, we pay attention to benchmark scenarios which explain data from dark matter experiments.

Inert Doublet Model(IDM)

- Symmetries : $SU(3)_C \times SU(2)_I \times U(1)_Y \times Z_2$ (Unbroken)
- The model contains two scalar doublets; $\boldsymbol{\Phi_1}$ and $\boldsymbol{\Phi_2}$
  - $\boldsymbol{\Phi_1} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix}$: Z_2 even, trigger EWSB
  - $\boldsymbol{\Phi_2} = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix}$: Z_2 odd, New field.
- Physical states : $h, \underbrace{H, A, H^\pm}_{\text{Additional Higgs}}$

h : SM Higgs
H : CP-even Higgs
A : CP-odd Higgs
$H^\pm$: Charged Higgs

- A lighter one of H or A can be dark matter candidate.

- Scalar potential :

$$V(\Phi_1,\Phi_2) = -\mu_1^2|\Phi_1|^2 - \mu_2^2|\Phi_2|^2 + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4$$
$$+\lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4\left|\Phi_1\Phi_2^\dagger\right|^2 + \frac{\lambda_5}{2}\left[\left(\Phi_1\Phi_2^\dagger\right)^2 + h.c.\right]$$

- Mass formulae :

$$m_h^2 = \lambda_1 v^2$$
$$m_\Phi^2 = \mu_2^2 + \frac{1}{2}\lambda_\Phi v^2 \quad (\Phi = H^\pm, H, A)$$

$$\lambda_{H^\pm} = \lambda_3$$
$$\lambda_H = \lambda_3 + \lambda_4 + \lambda_5$$
$$\lambda_A = \lambda_3 + \lambda_4 - \lambda_5$$

- Parameters :

Original ones

$$\{\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\}$$

$\Rightarrow$

Physical ones

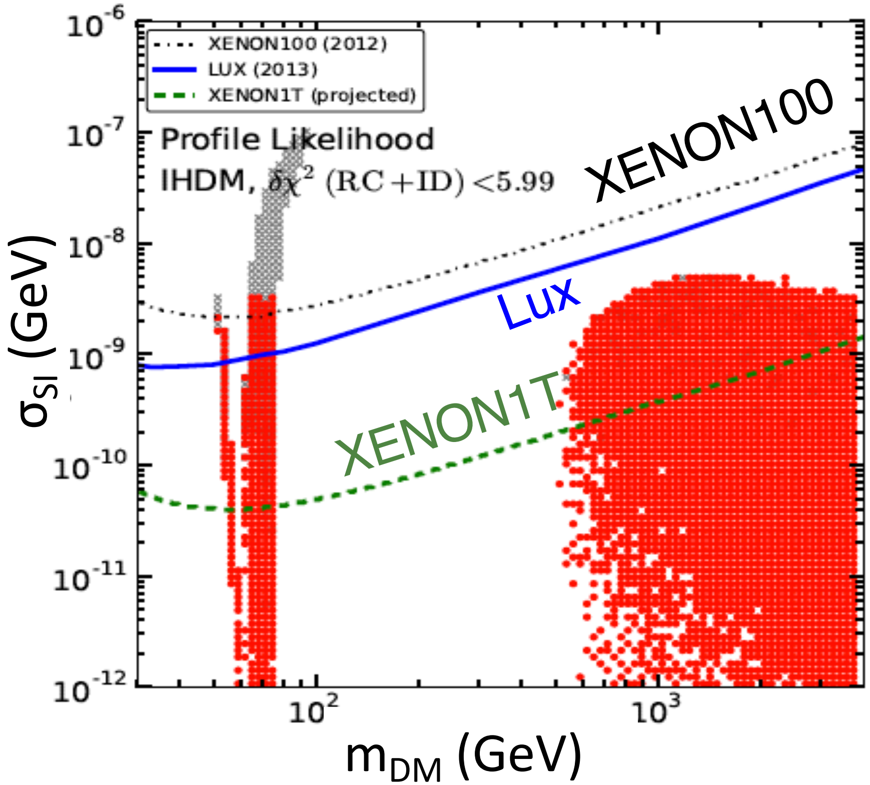
$$\{v, m_h, \mu_2^2, \lambda_2, m_{H^\pm}, m_H, m_A\}$$

These parameters are used as input ones.

Direct detection of dark matter

As the direct search experiments of dark matter, a scattering of a dark matter with a nucleon have been investigated.

A. Arhrib, Y. L. S. Tsai, Q. Yuan,T. C. Yuan, JCAP06(2014)030.



➡ The mass of the dark matter is limited:

(I): $50\mathrm{GeV} \lesssim m_{DM} \lesssim 80\mathrm{GeV}$, (II): $500\mathrm{GeV} \lesssim m_{DM}$

Higgs resonance region Heavy mass region

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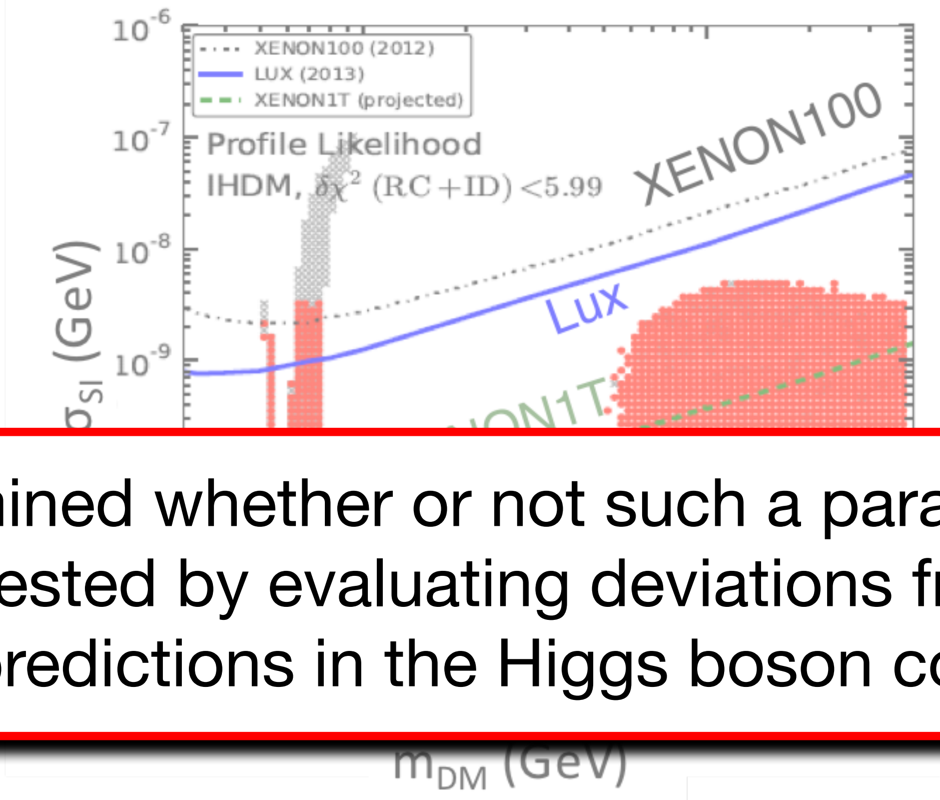

We examined whether or not such a parameter region can be tested by evaluating deviations from the SM predictions in the Higgs boson couplings.

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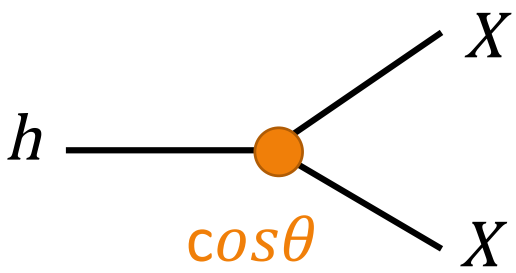
(I): $50\text{GeV} \lesssim m_{DM} \lesssim 80\text{GeV}$, (II): $500\text{GeV} \lesssim m_{DM}$

Higgs resonance region — Heavy mass region

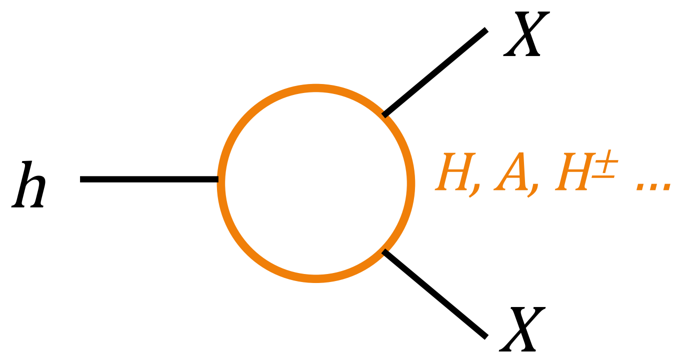
Indirect search with the Higgs boson couplings

- In extended Higgs models, predictions of the Higgs boson couplings can deviate from the SM.

The effect of field mixing



Loop effects of new particles



- In general, a pattern of the deviations is different at each model.

Model	htt	hbb	$h\tau\tau$	hVV	$h\gamma\gamma$	hhh
X	↑	↑	↓	↑	↑	↑
Y	↓	↑	↑	↓	↓	↑
Z	↓	↑	↓	↓	↑	↑
...						

↑ : more than SM prediction

↓ : less than SM prediction

- The Higgs boson couplings are measured at accuracy of dozen % by LHC Run I, but done at better accuracy by HL-LHC and ILC.

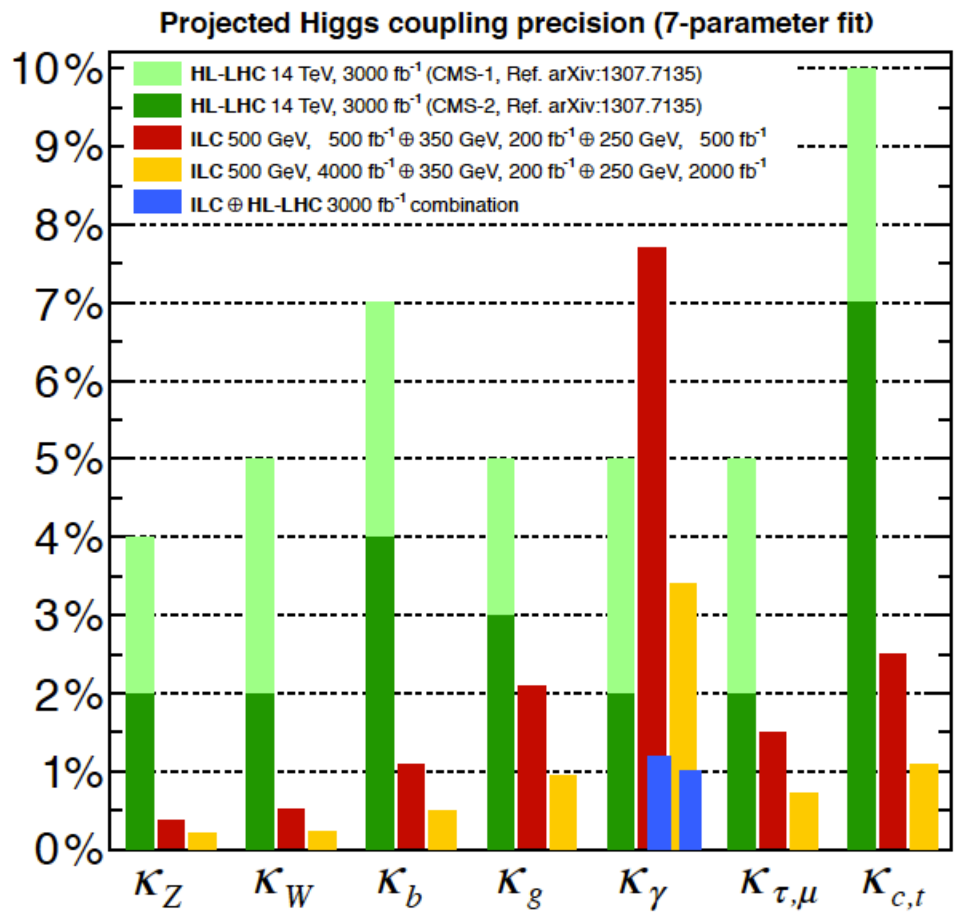
Current data (LHC Run I)

ATLAS and CMS, JHEP08(2016)045

Parameter	ATLAS+CMS Measured
κ_Z	1.00 $[-1.05, -0.86] \cup$ $[0.90, 1.11]$
κ_W	$0.91^{+0.10}_{-0.12}$
κ_t	$0.87^{+0.15}_{-0.15}$
$\lvert\kappa_\tau\rvert$	$0.90^{+0.14}_{-0.16}$
κ_b	0.67 $[-0.73, -0.47] \cup$ $[0.40, 0.89]$
$\lvert\kappa_\mu\rvert$	$0.2^{+1.2}$

Future prospect (HL-LHC and ILC)

arXiv:1506.05992[hep-ph]



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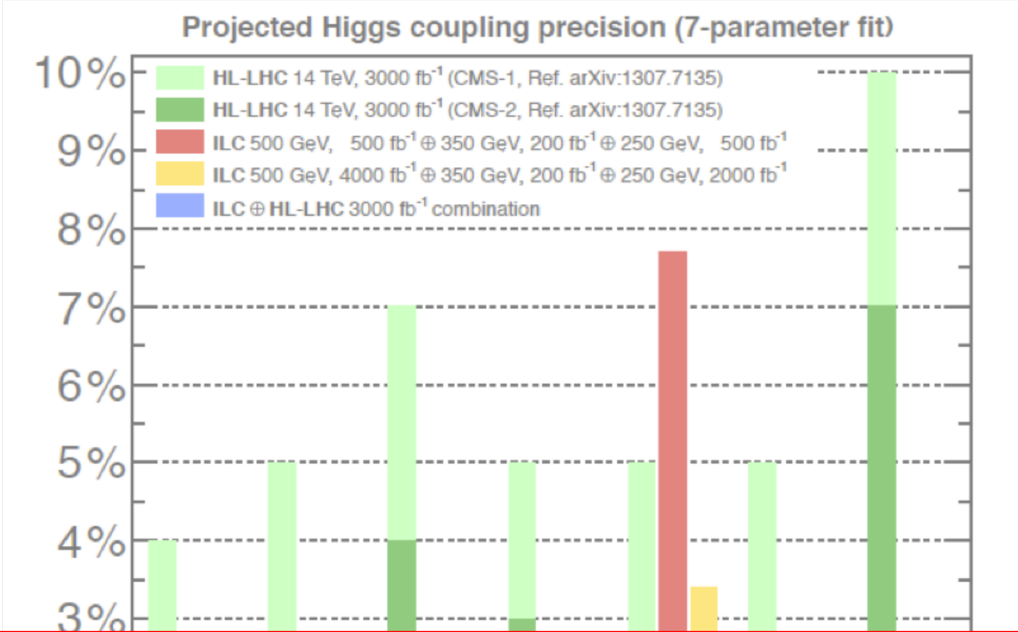


We can test and distinguish various models by comparing theoretical predictions of deviations and precision data at future collider experiments.

Our analysis

- We calculated Higgs boson couplings *hZZ, hWW, htt, hbb,* $h\tau\tau$, *hhh* at the one-loop level, using on-shell scheme.

- We numerically evaluated deviations of the SM predictions in the Higgs couplings, and examined whether or not characteristic pattern of deviations are obtained.

Benchmark scenario

- CP-Odd Higgs A is assumed a Dark matter.

- We set three benchmark scenarios as follows,

  A . DM Scenario (Higgs resonance region) : m_A = 65 GeV, $\lambda_A \sim 10^{-3}$

  B . DM Scenario（Heavy mass region）: m_A = 500 GeV, $\lambda_A \sim 10^{-3}$

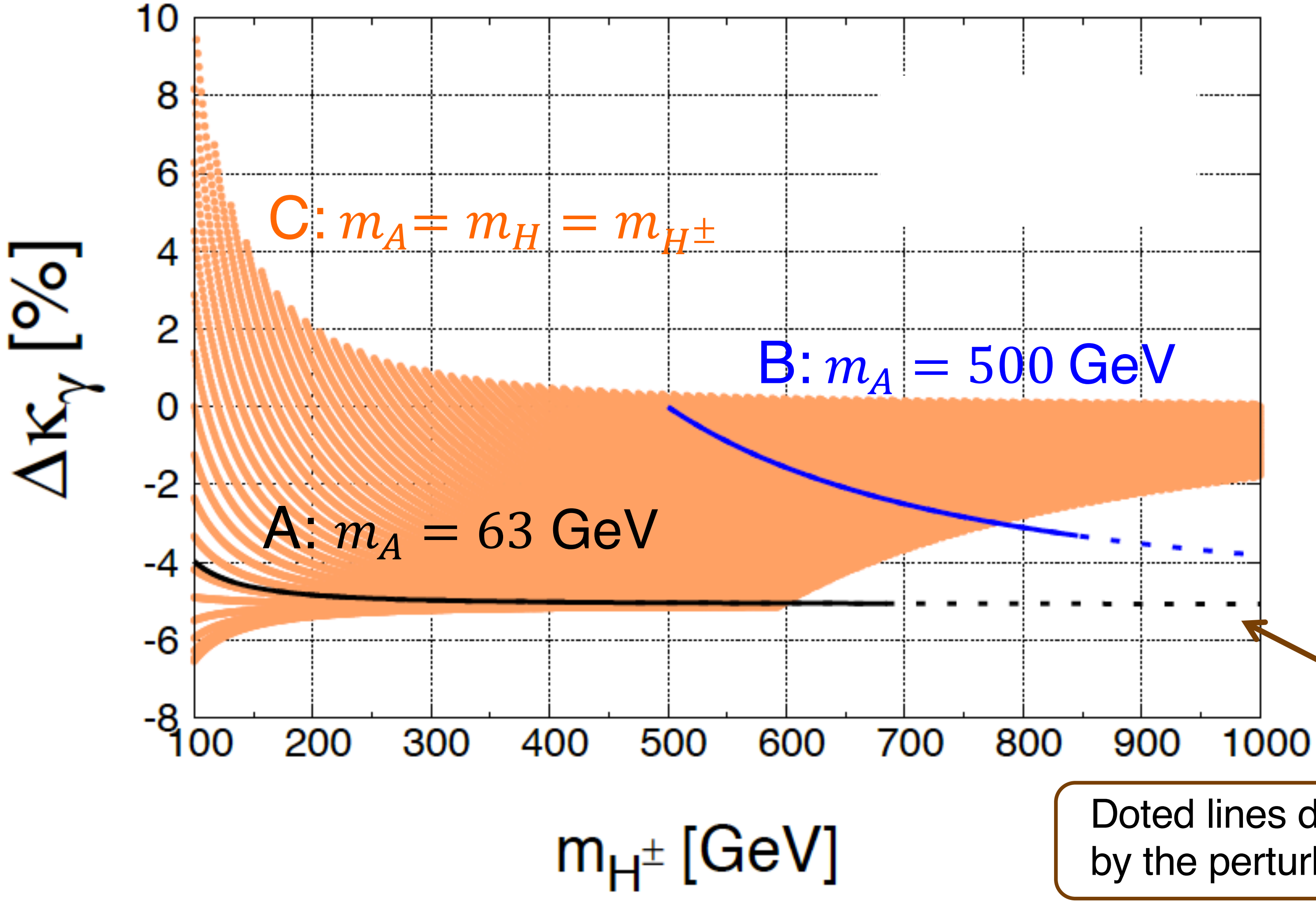
  C . Unrelated to DM : $m_A = m_H = m_{H^\pm}$, $\lambda_A < 4\pi$

- We take into account the following constraints :
  - Theoretical ones : Perturbative unitarity, Vacuum stability
  - Experimental ones: S, T, U prameters, LEP experiments

Deviation of the h$\gamma\gamma$

$$\Delta\kappa_\gamma \equiv \sqrt{\frac{\Gamma^{\rm IDM}_{h\to\gamma\gamma}}{\Gamma^{\rm SM}_{h\to\gamma\gamma}}} - 1$$

S. Kanemura, M. Kikuchi, K. Sakurai, arXiv:1605.08520.



Doted lines denote excluded regions by the perturbative unitarity.

- In Scenario-A, the model predicts always 4-5% deviation.
- In Scenario-B, the model predicts deviation less than 4%.

Deviation of the $h\gamma\gamma$

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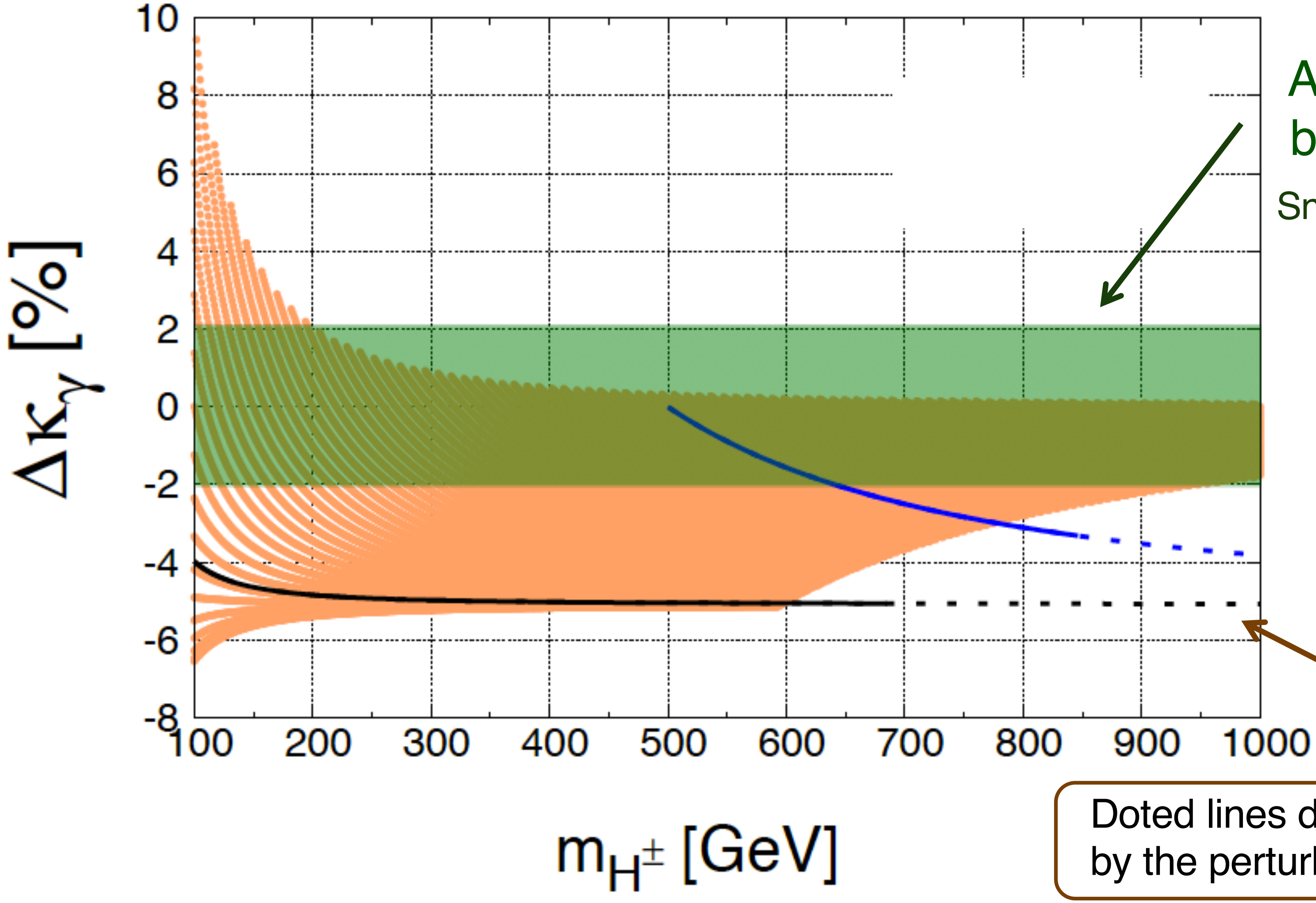

An accuracy of 2% by HL-LHC

Snowmass, arXiv:1310.8361

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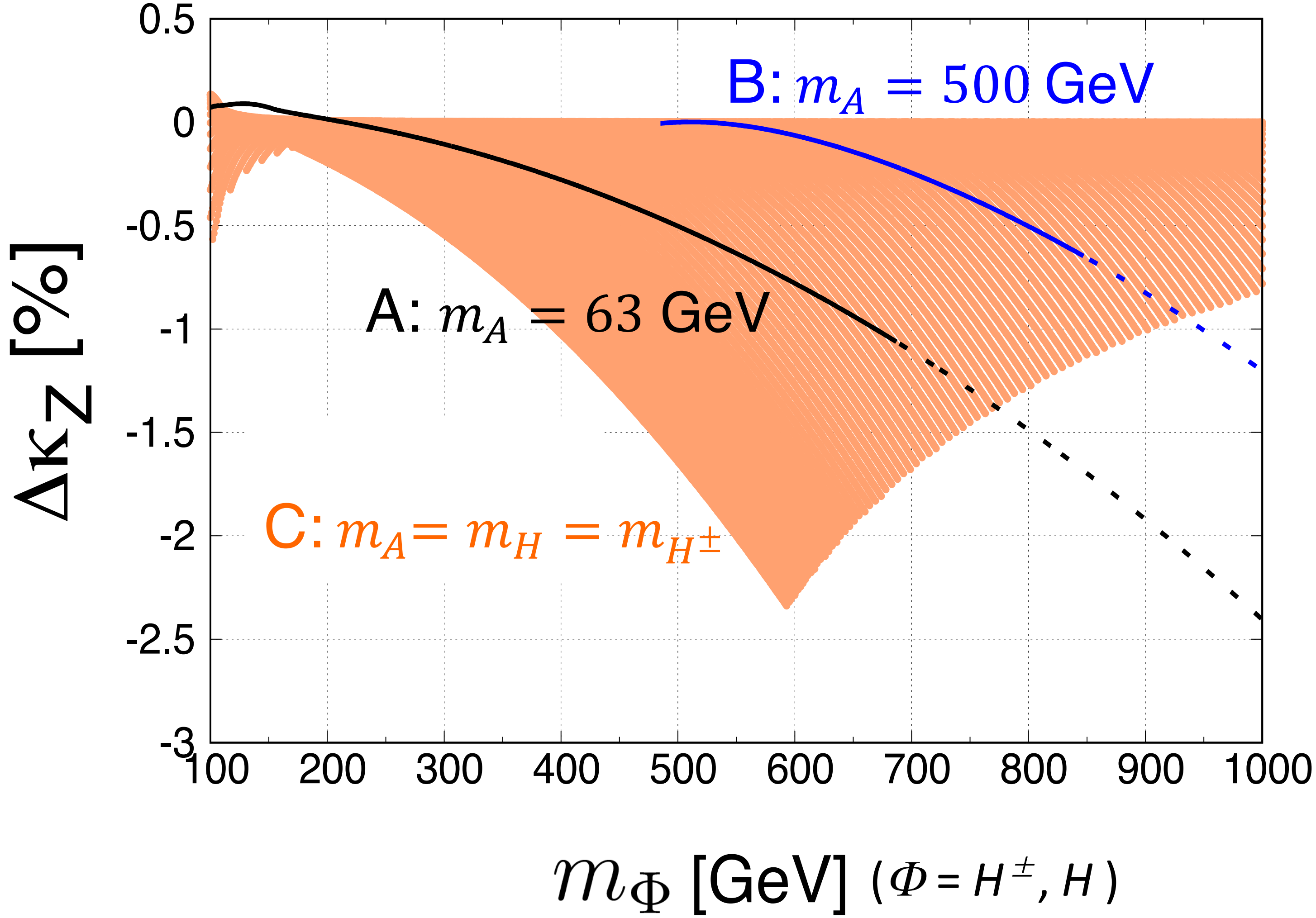
Deviation of the *hZZ*

$$\Delta\kappa_Z \equiv \frac{g_{hZZ}^{IDM}}{g_{hZZ}^{SM}} - 1$$

S. Kanemura, M. Kikuchi, K. Sakurai, arXiv:1605.08520.



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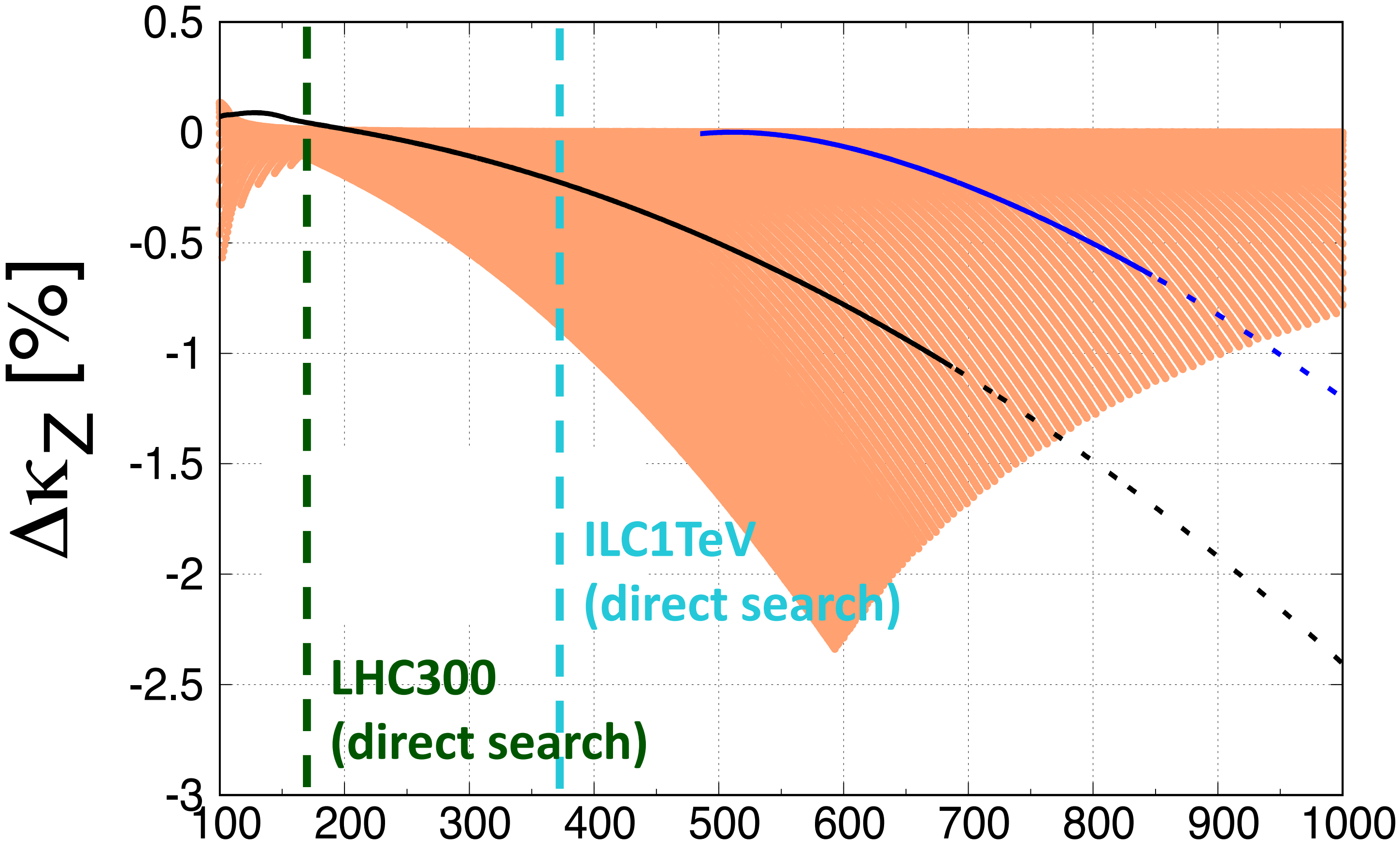

G. Belanger, et. al,
PRD91(2015)115011

m_Φ [GeV] ($\Phi = H^\pm, H$)

the light mass region is investigated by direct searches, and the heavy mass region is done by indirect searches.

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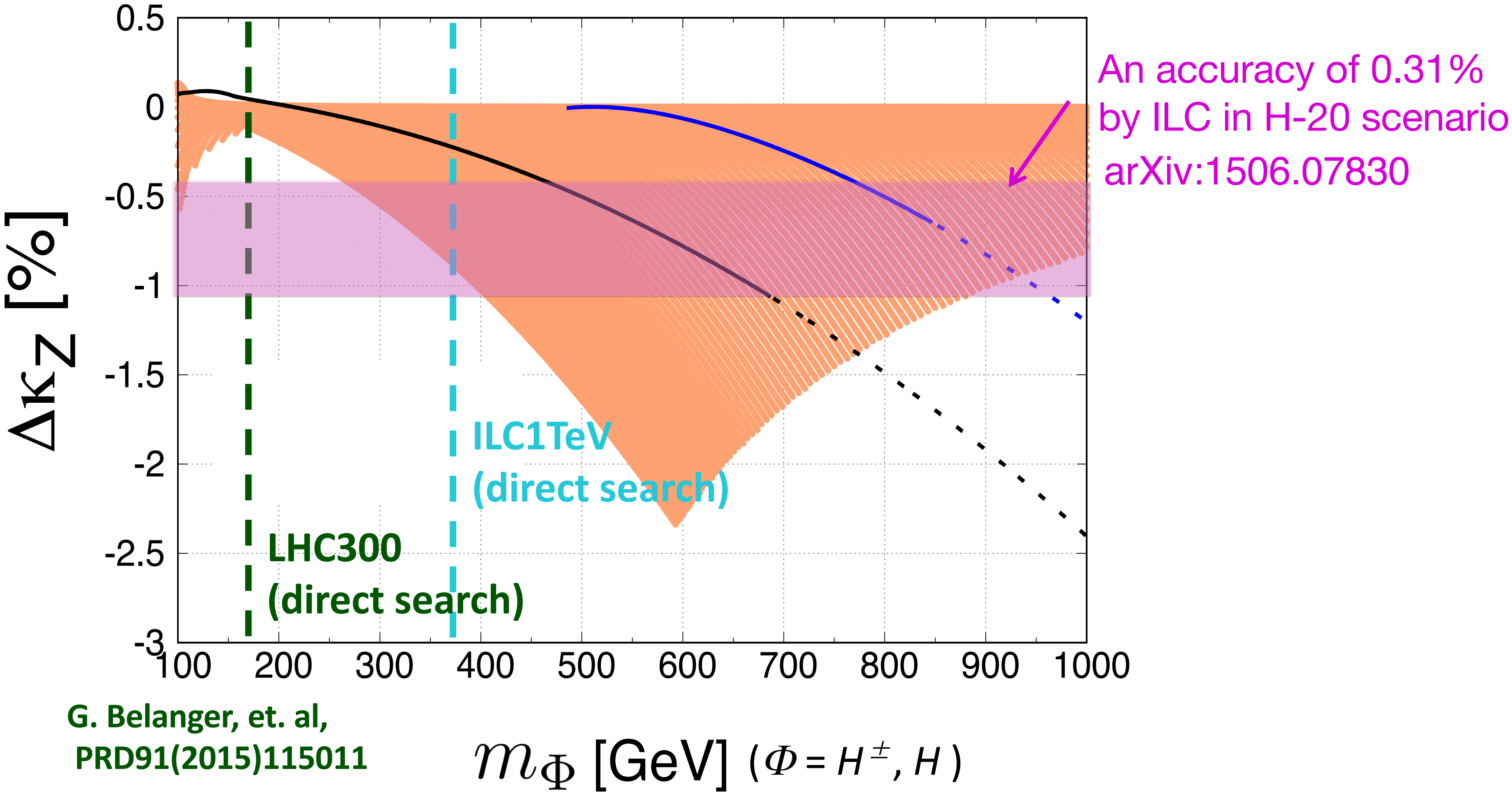

G. Belanger, et. al, PRD91(2015)115011

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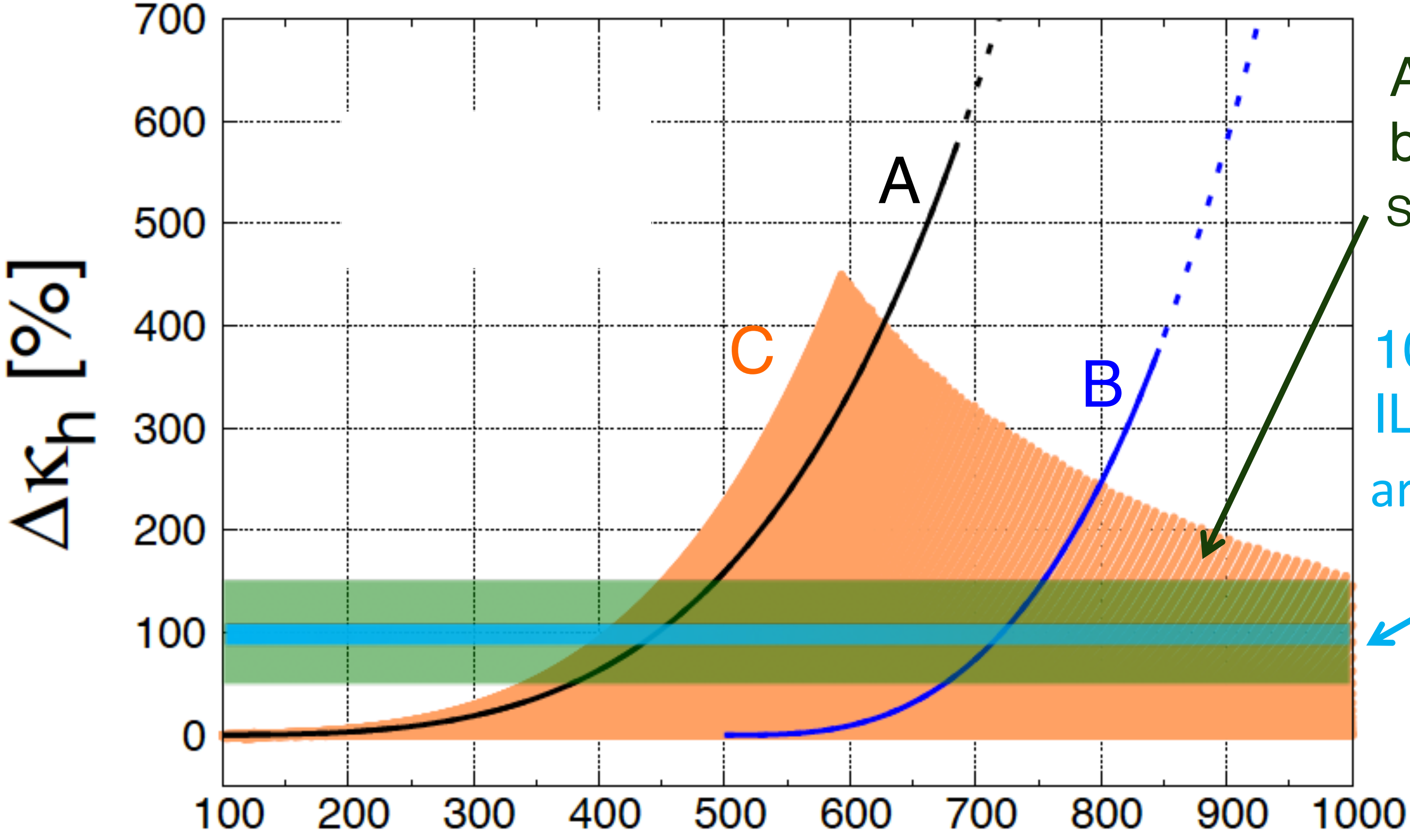
Deviation of the *hhh*

$$\Delta\kappa_h \equiv \frac{g_{hhh}^{IDM}}{g_{hhh}^{SM}} - 1$$

S. Kanemura, M. Kikuchi, K. Sakurai, arXiv:1605.08520.



About 50% accuracy by HL-LHC

Snowmass, arXiv:1310.8361

10% accuracy by ILC1TeV

arXiv: 1506.05992

m_Φ [GeV] ($\Phi = H^\pm, H$)

The *hhh* coupling can deviate 100% or more. If such a significant deviations are found, we could extract the information of m_Φ.

Summary

- We discussed testability of the Inert Doublet Model (IDM) by utilizing precision measurements of the Higgs boson couplings at future collider experiments.

- We focused on the following scenarios,

Scenario-A : m_{DM} = 65 GeV
Scenario-B : m_{DM} = 500 GeV

which are difficult to be excluded by direct searches for DM.

- We confirmed that such scenarios could be tested by collider experiments such as the ILC.

Back up slides

Our input palameters

Scenario-A $\quad 100\ \text{GeV} \leq m_{H^\pm} \leq 1000\ \text{GeV},$

$$m_A = 63\text{GeV}, \mu_2^2 = (61.50\text{GeV})^2, m_H = m_{H^\pm},$$

Scenario-B $\quad 500\ \text{GeV} \leq m_{H^\pm} \leq 1000\ \text{GeV},$

$$m_A = 500\text{GeV}, \mu_2^2 = (499.9\text{GeV})^2, m_H = m_{H^\pm},$$

Scenario-C $\quad 100\ \text{GeV} \leq m_\Phi \leq 1000\ \text{GeV},\ 0 \leq \mu_2^2 \leq (2000\ \text{GeV})^2$

$$m_H = m_A = m_{H^\pm}$$

h-Φ-Φ coupling

$$\lambda_{hhh} = -\frac{m_h^2}{2v}, \quad \lambda_{hHH} = -\frac{m_H^2 - \mu_2^2}{v}, \quad \lambda_{hAA} = -\frac{m_A^2 - \mu_2^2}{v},$$

$$\lambda_{hH^+H^-} = -2\frac{m_{H^+}^2 - \mu_2^2}{v}, \quad \lambda_{hG^0G^0} = -\frac{m_h^2}{2v}, \quad \lambda_{hG^+G^-} = -\frac{m_h^2}{v}$$

A pattern of deviations of the Higgs boson couplings

Deviations from the SM : $\kappa_X = g_{hXX} / g_{hXX}^{SM}$

Models	κ_f	κ_V
Φ(SM)	1	1
$\Phi + S$ (Singlet)	$\cos\alpha$	$\cos\alpha$
$\Phi + \Phi$(Doublet)	$\cos\alpha/\sin\beta$	$\sin\beta\cos\alpha\text{-}\cos\alpha\sin\beta$
$\Phi + \Delta$(Triplet)	$\cos\alpha/\sin\beta$	$\sin\beta\cos\alpha\text{-}1.6\cos\alpha\sin\beta$
$\Phi + \phi_7$(Septet)	$\cos\alpha/\sin\beta$	$\sin\beta\cos\alpha\text{-}4\cos\alpha\sin\beta$

Kanemura, Tsumura, Yagyu, Yokoya, PRD90(2014)075001

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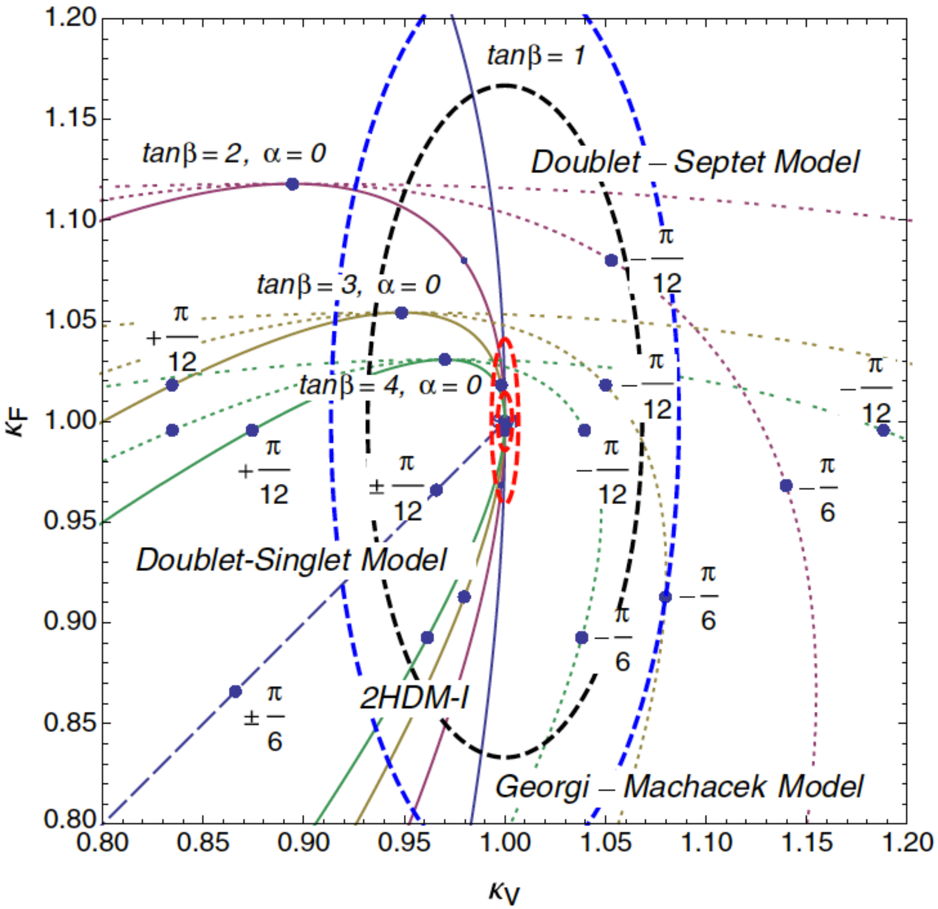


Kanemura, Tsumura, Yagyu, Yokoya, PRD90(2014)075001

Approximate fomulae (1)

$m_H, m_A, m_{H^\pm} >> q^2 ; m_{H^\pm} - m_{H,A} << 1$

$$\Delta\kappa_Z \simeq -\sum_{\Phi=A,H,H^\pm}\frac{1}{16\pi^2}\frac{1}{6}c_\Phi\frac{m_\Phi^2}{v^2}\left(1-\frac{\mu^2}{m_\Phi^2}\right)^2$$
$$-\frac{1}{16\pi^2}\frac{1}{v^2}\left\{\frac{2}{3}(m_H-m_A)^2-\frac{1}{30}\frac{(m_H-m_A)^4}{m_H^2}\right\}$$
$$+\frac{1}{16\pi^2}\frac{m_Z^2}{v^2}\left\{-\frac{1}{3}\left(1-\frac{m_A}{m_H}\right)-\frac{1}{30}\left(1-\frac{m_A}{m_H}\right)^2\right\}$$
$$+\frac{1}{16\pi^2}\frac{1}{v^2}\frac{1}{2}\left\{\frac{2}{3}(m_H^\pm-m_H)^2-\frac{1}{30}\frac{(m_H^\pm-m_H)^4}{m_{H^\pm}^2}\right.$$
$$\left.+\frac{2}{3}(m_H^\pm-m_A)^2-\frac{1}{30}\frac{(m_H^\pm-m_A)^4}{m_{H^\pm}^2}\right\}$$
$$+\frac{1}{16\pi^2}\frac{m_A^2}{v^2}\left(1-\frac{\mu^2}{m_A^2}\right)\left\{\frac{4}{6}(1-\frac{m_A}{m_H})+\frac{4}{6}(1-\frac{m_A}{m_H})^2\right.$$
$$\left.-2\left(1-\frac{m_A}{m_H}\right)-\left(1-\frac{m_A}{m_H}\right)^2\right\}$$
$$+\frac{1}{16\pi^2}\frac{m_H^2}{v^2}4\left(1-\frac{\mu^2}{m_H^2}\right)\left\{\frac{1}{6}(1-\frac{m_A}{m_H})-\frac{1}{60}(1-\frac{m_A}{m_H})^3\right\}$$

$$\Delta\kappa_b=\Delta\kappa_t=\Delta\kappa_\tau\simeq-\sum_{\Phi=A,H,H^\pm}\frac{1}{16\pi^2}\frac{1}{6}c_\Phi\frac{m_\Phi^2}{v^2}\left(1-\frac{\mu^2}{m_\Phi^2}\right)^2$$
$$+\frac{1}{16\pi^2}\frac{1}{v^2}\frac{1}{2}\left\{\frac{2}{3}(m_H^\pm-m_H)^2-\frac{1}{30}\frac{(m_H^\pm-m_H)^4}{m_{H^\pm}^2}\right.$$
$$\left.+\frac{2}{3}(m_H^\pm-m_A)^2-\frac{1}{30}\frac{(m_H^\pm-m_A)^4}{m_{H^\pm}^2}\right\}$$

Approximate fomulae (2)

m_H, m_A, $m_{H^\pm}$, >> q^2 ; $m_\Phi = m_H = m_A = m_{H^\pm}$

$$\Delta\kappa_V = \Delta\kappa_f \simeq -\sum_{\Phi=A,H,H^\pm} \frac{1}{16\pi^2}\frac{1}{6}c_\Phi \frac{m_\Phi^2}{v^2}\left(1-\frac{\mu_2^2}{m_\Phi^2}\right)^2$$

$$\Delta\kappa_h \simeq \sum_{\Phi=A,H,H^\pm} \frac{1}{16\pi^2}\frac{4}{3}c_\Phi \frac{m_\Phi^4}{m_h^2 v^2}\left(1-\frac{\mu_2^2}{m_\Phi^2}\right)^3$$

$$\Gamma_{h\to\gamma\gamma}^{IDM} \simeq \frac{\sqrt{2}G_F\alpha_{\rm em}^2 m_h^3}{256\pi^3}\left|-\frac{1}{3}\left(1-\frac{\mu_2^2}{m_{H^\pm}^2}\right)+C_V+C_F\right|^2$$

C_F : contributions of fermion loop

C_F : Contribution of W boson loop

Measurement accuracy of the Higgs boson couplings(future prospects)

Deviations of the SM: $\kappa_X = g_{hXX} / g_{hXX}^{SM}$

Snowmass Higgs Working Group Report (1310. 8361)

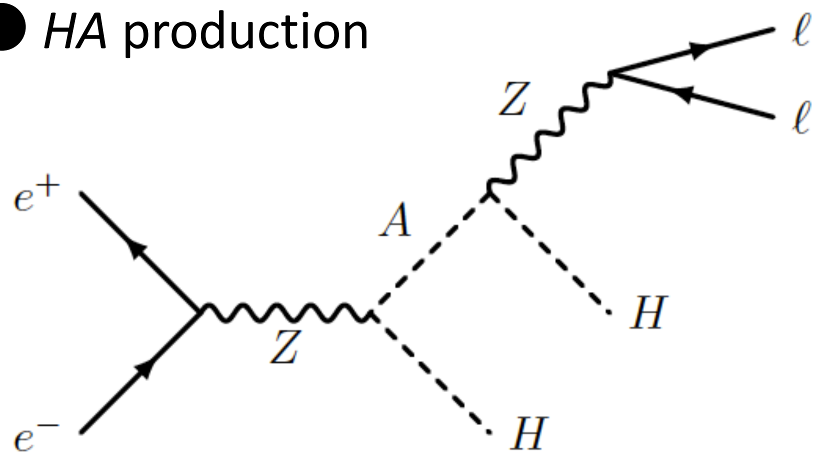
Facility	LHC	HL-LHC	ILC500	ILC500-up	ILC1000
$\sqrt{s}$ (GeV)	14,000	14,000	250/500	250/500	250/500/1000
$\int \mathcal{L}dt$ (fb^{-1})	300/expt	3000/expt	250+500	1150+1600	250+500+1000
κ_γ	$5-7\%$	$2-5\%$	8.3%	4.4%	3.8%
κ_g	$6-8\%$	$3-5\%$	2.0%	1.1%	1.1%
κ_W	$4-6\%$	$2-5\%$	0.39%	0.21%	0.21%
κ_Z	$4-6\%$	$2-4\%$	0.49%	0.24%	0.50%
κ_ℓ	$6-8\%$	$2-5\%$	1.9%	0.98%	1.3%
$\kappa_d = \kappa_b$	$10-13\%$	$4-7\%$	0.93%	0.60%	0.51%
$\kappa_u = \kappa_t$	$14-15\%$	$7-10\%$	2.5%	1.3%	1.3%

The Higgs couplings will be measured about a few % by the HL-LHC and about 1% and or less by the ILC

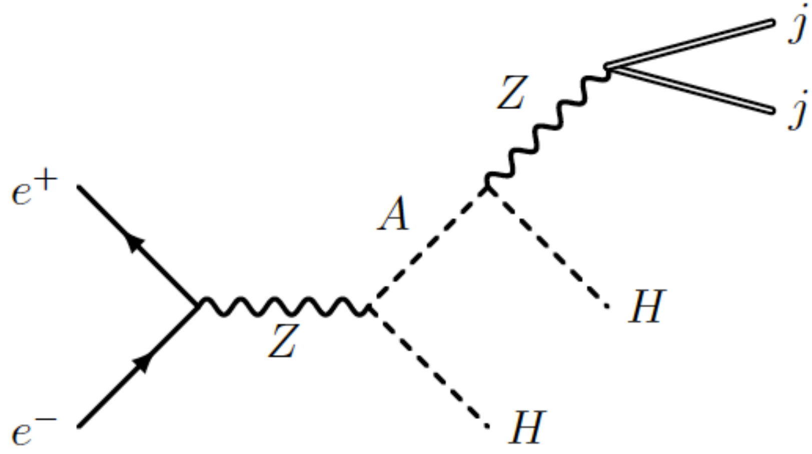
Direct search by the ILC

M. Hashemi, M. Krawczyk, S. Najjari, A. F. Zarnecki, arXiv:1512.01175

- *HA* production

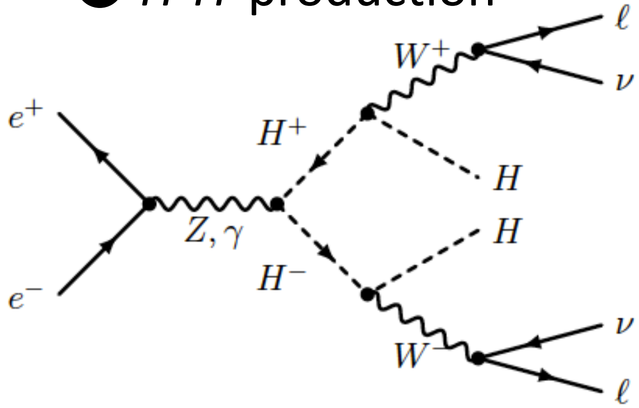


$e^+e^- \rightarrow HA \rightarrow \ell^+\ell^-HH$

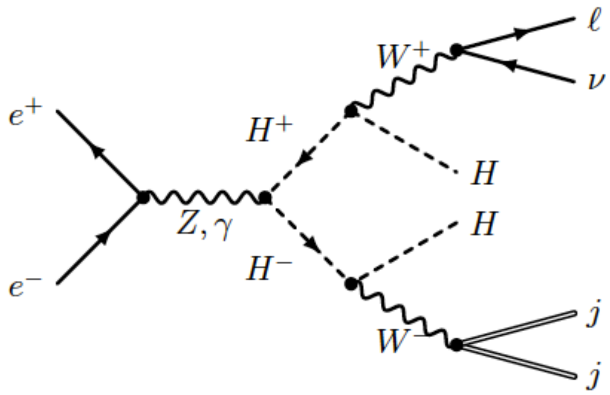


$e^+e^- \rightarrow HA \rightarrow jjHH$

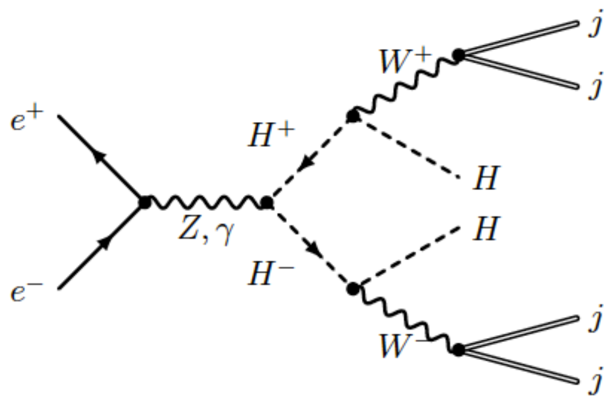
- H^+H^- production



$e^+e^- \rightarrow H^+H^- \rightarrow \ell^+\nu\ell^-\nu HH$



$e^+e^- \rightarrow H^+H^- \rightarrow \ell^+\nu jjHH$



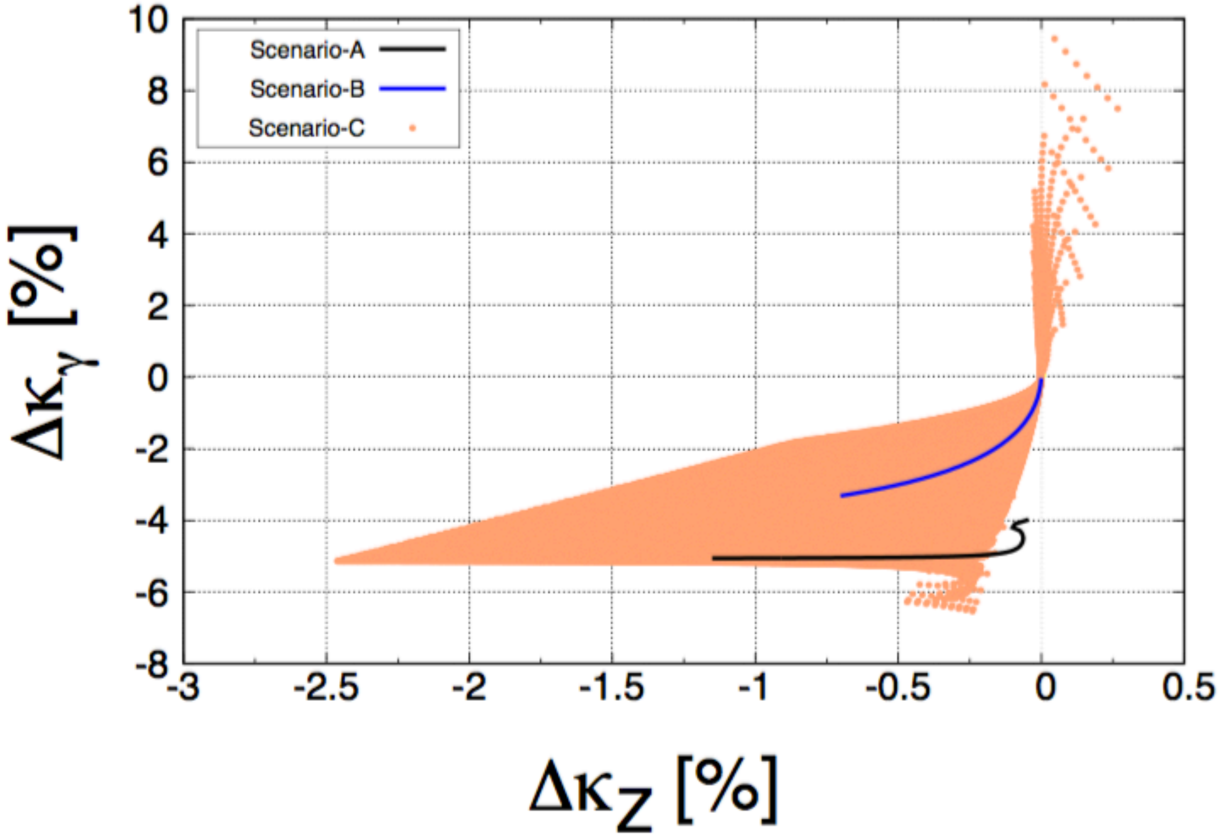
$e^+e^- \rightarrow H^+H^- \rightarrow jjjjHH$

$\Delta\kappa_\gamma$ vs $\Delta\kappa_Z$

Scenario-A : m_A = 65 GeV

Scenario-B : m_A = 500 GeV

Scenario-C : $m_A = m_H = m_{H^\pm}$



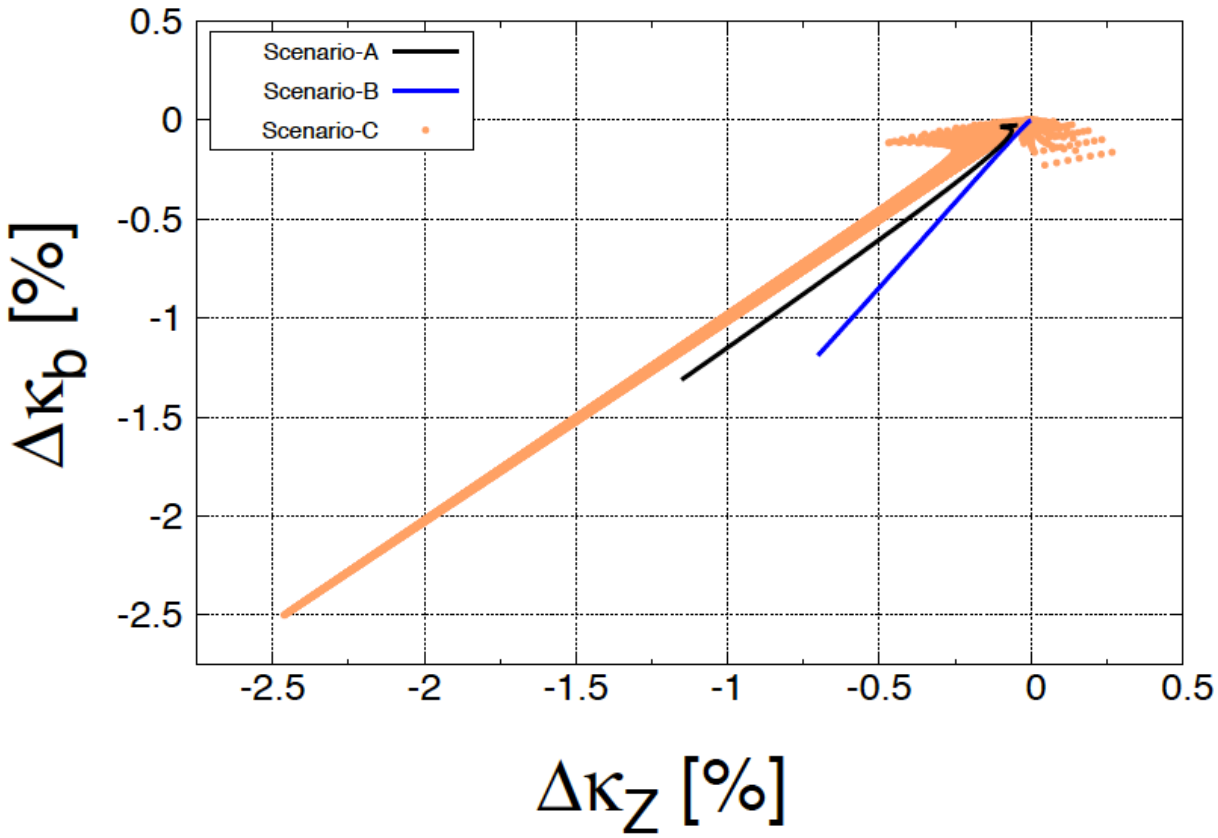
If we we obtained value of $\Delta\kappa_Z$ more than 0.7 %
we could exclude scenario-B.

$\Delta\kappa_b$ vs $\Delta\kappa_Z$

Scenario-A : m_A = 65 GeV

Scenario-B : m_A = 500 GeV

Scenario-C : $m_A = m_H = m_{H^\pm}$



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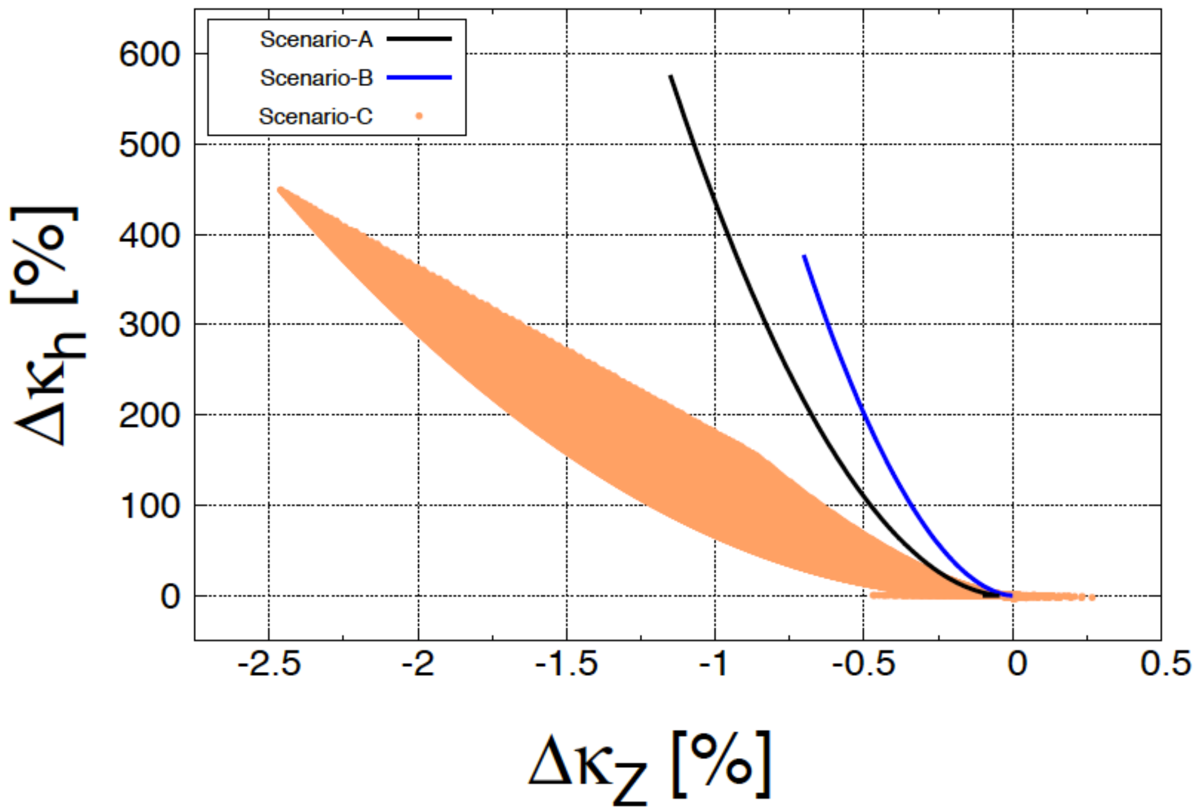
$\Delta\kappa_h$ VS $\Delta\kappa_Z$

Scenario-A : m_A = 65 GeV

Scenario-B : m_A = 500 GeV

Scenario-C : $m_A = m_H = m_{H^\pm}$



If we we obtained value of $\Delta\kappa_Z$ more than 0.7 % we could exclude scenario-B.

By correlation between *hZZ* and *hhh,* scenario-B and c can be discriminated

Procedure of prescription of renormalization

1 . Count number of parameters and fields in Lagrangian.

$$\mathcal{L} = \mathcal{L}(\mu_1^B, \mu_2^B, \ldots \lambda_1^B, \lambda_2^B, \ldots; \phi_1, \phi_2, \ldots)$$

2. Shift parameters and fields to introduce counterterms as same number these.

$$\mu_i^B \to \mu_i^R + \delta\mu_i \ , \ \phi_i \to Z_{\phi_i}\phi_i^R$$

$$\lambda_i^B \to \lambda_i^R + \delta\lambda_i \qquad (\,i = 1,2,3, \ldots\,)$$

3. Impose only as many renormalization conditions as number of counterterms to determine these counterterms.

→ Any observables can be renormalized.

$$\mathcal{O} = \mathcal{O}(\mu_1^R, \mu_2^R, \ldots \ \lambda_1^R, \lambda_2^R, \ldots) + \mathcal{O}^{1PI} + \delta\mathcal{O}(\delta\mu_1^R, \mu_2^R, \ldots \ \delta\lambda_1^R, \delta\lambda_2, \ldots)$$

Introduction of counterterms

- Parameters of Higgs potential ：7

$$T_h \;\; m_h \;\; m_H \;\; m_A \;\; m_{H^\pm} \;\; \mu_2 \;\; \lambda_2$$

- Fields of Higgs sector ：4

$$h \;\; H^\pm \;\; H \;\; A$$

- Shift of parameters : ($\Phi = h, H^{\pm}, H, A$)

$$m_\Phi \to m_\Phi + \delta m_\Phi \qquad \mu_2 \to \mu_2 + \delta\mu_2 \qquad T_h \to 0 + \delta T_h$$

$$\Phi \to \Phi + Z_\Phi \Phi/2 \qquad \lambda_2 \to \lambda_2 + \delta\lambda_2$$

- Counter terms : 11

$$\delta T_h \;\; \delta m_h \;\; \delta m_H \;\; \delta m_A \;\; \delta m_{H^\pm} \;\; \delta\mu_2 \;\; \delta\lambda_2 \;\; \delta Z_h \;\; \delta Z_H \;\; \delta Z_A \;\; \delta Z_{H^+}$$

(On-shell renormalization)

Renormalization conditions

δT_h : $\Gamma_h^R \equiv T_h^{1PI} + \delta T_h = 0$ $\Rightarrow$ $\delta T_h = -T_h^{1PI}$

δm_h : $Re\Gamma_{hh}^R[m_h^2] = 0$ $\Rightarrow$ $\delta m_h^2 = Re\Pi_{hh}^{1PI}(m_h^2) - \frac{1}{v} ReT_h^{1PI}$

δZ_h : $\frac{\partial}{\partial p^2} Re\Gamma_{hh}^R(p^2)\Big|_{p^2=m_h^2} = 0$ $\Rightarrow$ $\delta Z_h = -\frac{\partial}{\partial p^2} Re\Pi_{hh}^{1PI}(p^2)\Big|_{p^2=m_h^2}$

(at Φ =H , A , $H^{\pm}$)

δm_Φ : $Re\Gamma_{\Phi\Phi}^R[m_h^2] = 0$ $\Rightarrow$ $\delta m_\Phi^2 = Re\Pi_{\Phi\Phi}^{1PI}(m_h^2)$

δZ_Φ : $\frac{\partial}{\partial p^2} Re\Gamma_{\Phi\Phi}^R(p^2)\Big|_{p^2=m_\Phi^2} = 0$ $\Rightarrow$ $\delta Z_\Phi = -\frac{\partial}{\partial p^2} Re\Pi_\Phi^{1PI}(p^2)\Big|_{p^2=m_\Phi^2}$

$\delta\mu_2$: It is determined to cancel a divergence at a scalar triinear coupling such as *hHH.*

$\delta\lambda_2$: It is determined to cancel a divergence at a scalar quartic coupling such as *HHHH.*

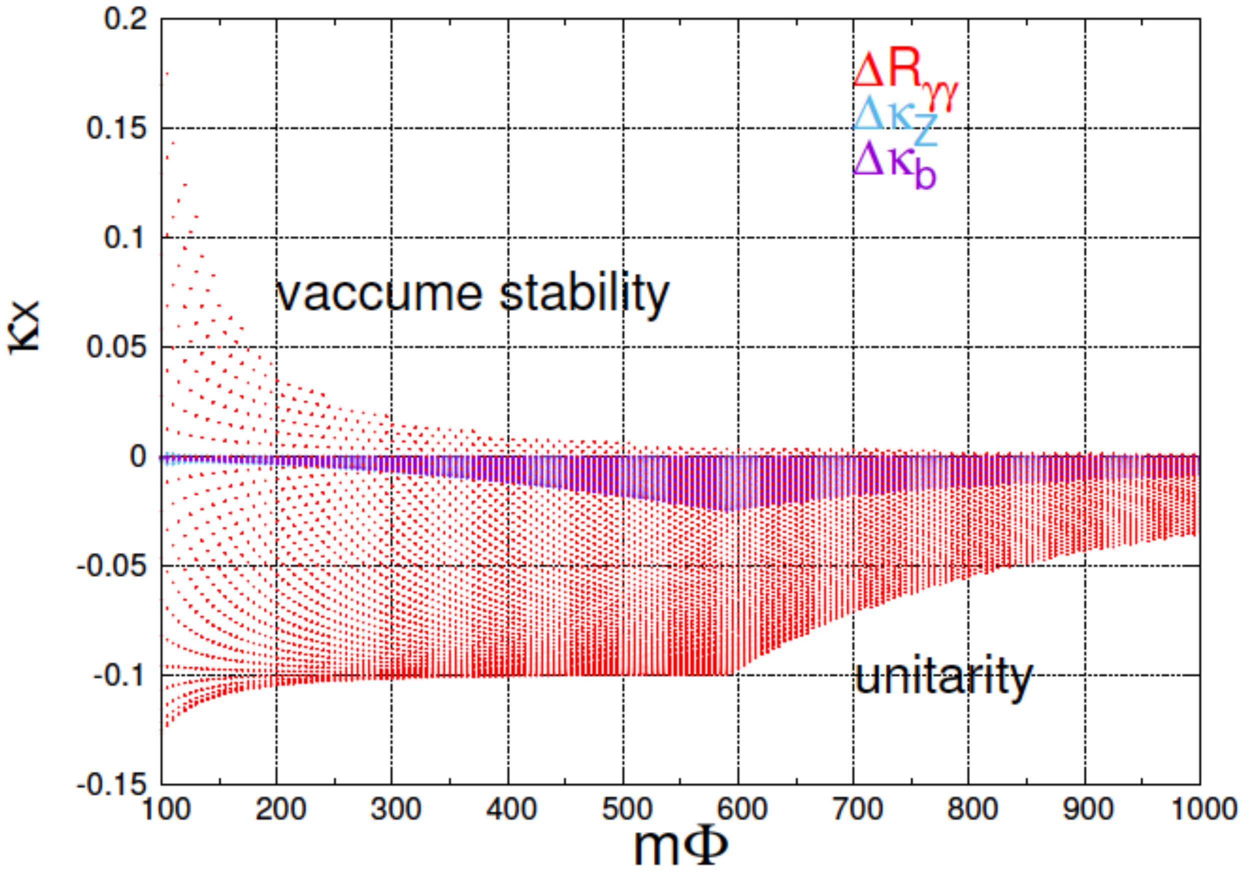
Renormalizzed hVV,hff,hhh

$$\Gamma_{\mathrm{hff}}[p_1^2, p_2^2, q^2] = \overbrace{-\frac{m_f}{v}}^{\text{tree}} + \overbrace{\Gamma^{\mathrm{1PI}}_{\mathrm{hff}}[p_1^2, p_2^2, q^2]}^{\text{1PI}} \overbrace{- \frac{m_f}{v}\left\{\frac{\delta m_f}{m_f} - \frac{\delta v}{v} + \frac{1}{2}\delta Z_h + \delta Z_V^f\right\}}^{\text{counter terms}}$$

$$\Gamma_{\mathrm{hVV}}[p_1^2, p_2^2, q^2] = \frac{2m_V^2}{v} + \Gamma^{\mathrm{1PI}}_{\mathrm{hVV}}[p_1^2, p_2^2, q^2] + \frac{2m_Z^2}{v}\left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta v}{v} + \delta Z_Z + \frac{1}{2}\delta Z_h\right)$$

$$\Gamma_{\mathrm{hhh}}[p_1^2, p_2^2, q^2] = -\frac{3m_h^2}{v} + \Gamma^{\mathrm{1PI}}_{\mathrm{hhh}}[p_1^2, p_2^2, q^2] - \left\{\frac{3\delta m_h^2}{v} + \frac{3m_h^2}{v}\left(-\frac{\delta v}{v} + \frac{3}{2}\delta Z_h\right)\right\}$$

Constraints of vaccum stability and perturbative unitarity



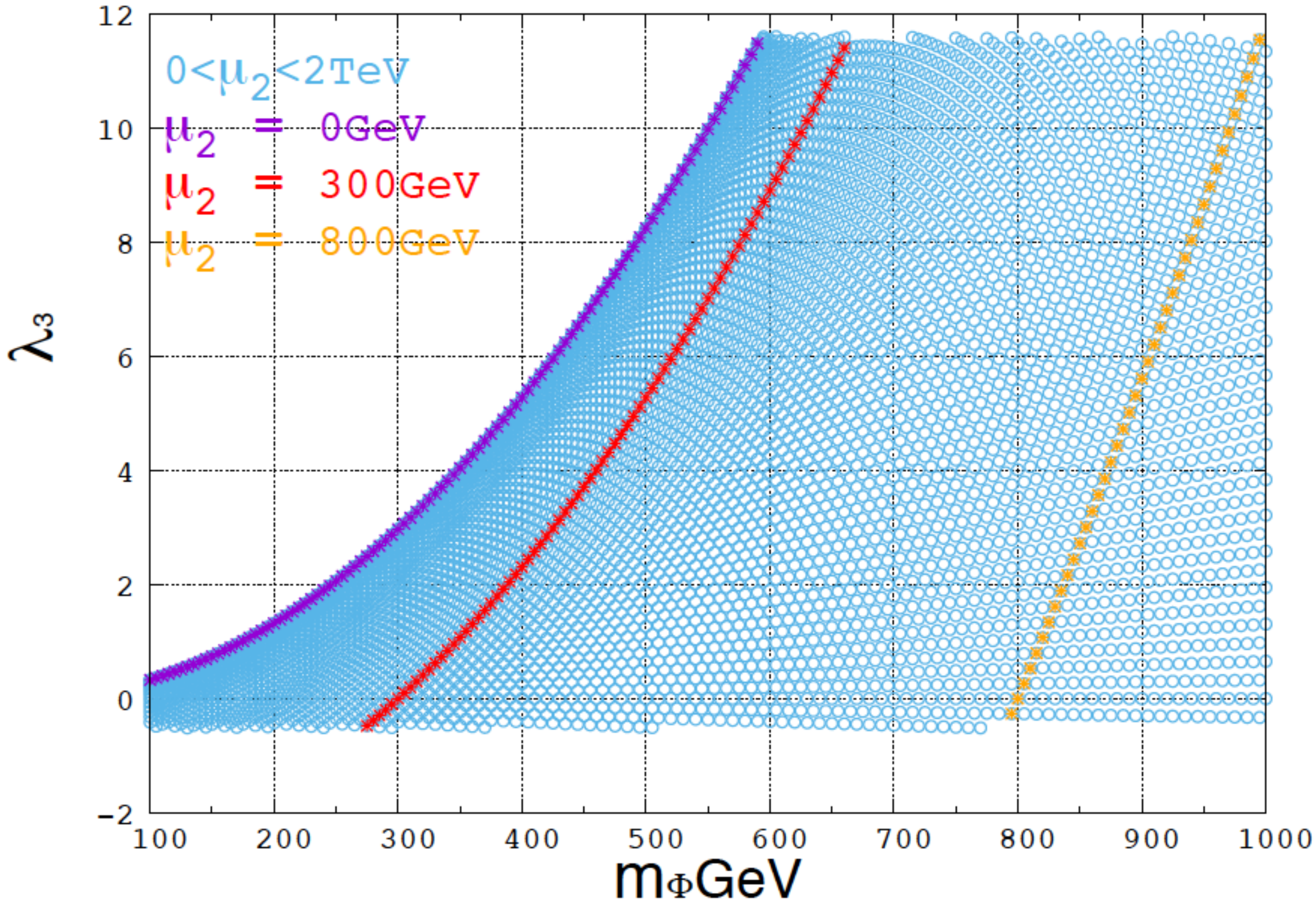
$m_{\Phi}=m_H=m_A=m_{H^+}$

IDM

$100 \leq m_{\Phi} \leq 1000\text{GeV}$

$0 \leq \mu_2^2 \leq 2\text{TeV}$

λ_3 vs mΦ



λ_3 vs mΦ (DM scenario A)