Model-Independent Determination of the Triple Higgs Coupling at e^+e^- Colliders

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Abstract

I. ERROR ESTIMATE FOR c_6 IN THE CP-CONSERVING CASE

A. Parameterization for the cross section of $e^+e^- \to ZHH$

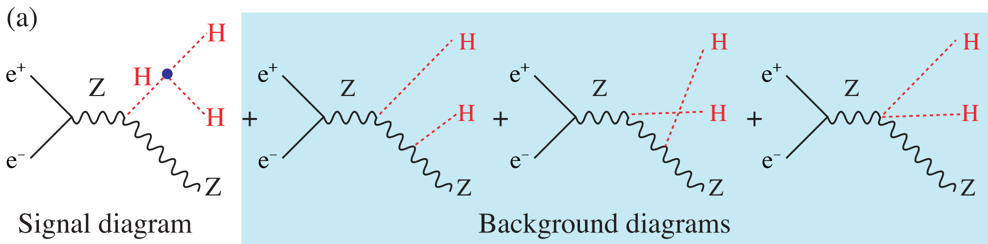

FIG. 1. Diagrams contributing to $e^+e^- \to ZHH$.

We start with a general form of CP-conserving Lagrangian relevant for double Higgs production process $e^+e^- \to ZHH$, of which Feynman diagrams are shown in Fig. 1. The Lagrangian can be written as follows,

$$L = \kappa_\lambda \lambda_{hhh} HHH + \kappa_Z g_{zzh} Z_\mu Z^\mu H + \kappa_Q g_{zzh} Z_\mu Z^\mu HH + \frac{d_h}{\Lambda} H\partial_\mu H \partial^\mu H + \frac{b_Z}{\Lambda} Z_{\mu\nu} Z^{\mu\nu} H + \frac{q_Z}{\Lambda} Z_{\mu\nu} Z^{\mu\nu} HH. \quad (1)$$

The total amplitude (A_0) can be formed by three components, $A_0 = A_1 + A_2 + A_3$, where A_1, A_2 and A_3 are respectively the amplitude of the left, right and middle two diagrams in Fig. 1. Each of those amplitudes can be parameterized in terms of the six couplings, κ_λ, κ_Q, κ_Z, d_h, q_Z and b_Z, as follows

$$A_1 = \kappa_Z \kappa_\lambda A_{11} + \kappa_Z d_h A_{12} + \kappa_\lambda b_Z A_{13} + b_Z d_h A_{14}, \quad (2)$$

$$A_2 = \kappa_Q A_{21} + q_Z A_{22}, \quad (3)$$

$$A_3 = \kappa_Z^2 A_{31} + \kappa_Z b A_{32} + b^2 A_{33}, \quad (4)$$

where A_{xx} can be computed and correspond to all couplings equal to one.

The total cross section (σ_{ZHH}) then can be calculated based on $|A_0|^2$, and, up to the first order of anomalous couplings, can be parameterized as

$$\sigma_{ZHH} = \kappa_Z^2 \kappa_\lambda^2 I_{SS} + \kappa_Q^2 I_{QQ} + \kappa_Z^4 I_{BB} + \kappa_Z \kappa_\lambda \kappa_Q I_{SQ} + \kappa_Z^3 \kappa_\lambda I_{SB} + \kappa_Q \kappa_Z^2 I_{QB} + \kappa_Z^2 \kappa_\lambda d_h I_{Sd} + \kappa_Z \kappa_Q d_h I_{Qd} + \kappa_Z^3 d_h I_{Bd} + \kappa_Z \kappa_\lambda q_Z I_{Sq} + \kappa_Q q_Z I_{Qq} + \kappa_Z^2 q_Z I_{Bq}$$

$$+\kappa_Z\kappa_\lambda^2 b_Z I_{Sb_1} + \kappa_\lambda\kappa_Q b_Z I_{Qb_1} + \kappa_Z^2\kappa_\lambda b_Z IBb_1$$
$$+\kappa_Z^3\kappa_\lambda b_Z I_{Sb_2} + \kappa_Q\kappa_Z b_Z I_{Qb_2} + \kappa_Z^3 b_Z I_{Bb_2}, \qquad (5)$$

where coefficients I_{xx} are computed numerically as shown in Table I. Figure 2 gives the values of σ_{ZHH} at $\sqrt{s} = 500$ GeV for $P(e^-, e^+) = (-0.8, +0.3)$ at the ILC, as a function of individual coupling.

I_{xx}	S	Q	B	d	q	b1	b2
S	0.097	0.31	0.067	0.24	0.10	0.26	0.65
Q	0.31	0.27	0.16	0.42	0.18	0.42	1.16
B	0.067	0.16	0.10	0.12	0.080	0.16	0.45

TABLE I. Coefficients for parameterization of the total cross section as in Eqn. 5, in units of σ_{ZHH} value in the SM (~0.2 fb).

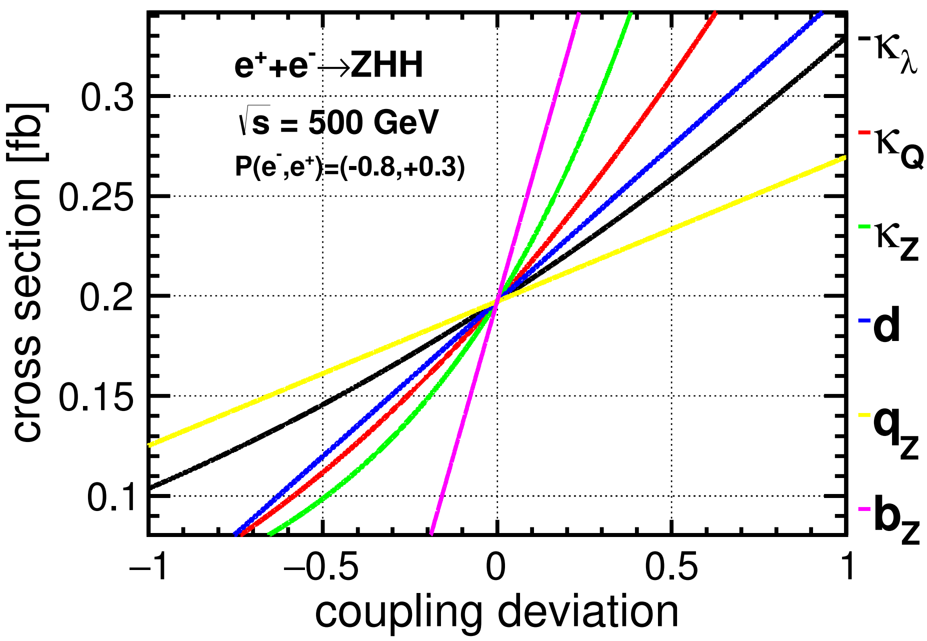

FIG. 2. The cross section of $e^+e^- \to ZHH$ as a function of individual coupling in Eqn. 5, for which its SM value is subtracted.

B. Method to extract κ_λ, and its error from σ_{ZHH} measurement and other couplings

Experimentally, σ_{ZHH} can be directly measured, e.g. with $\sigma_{meas.} \pm \Delta\sigma_{meas.}$. In order to extract κ_λ using Eqn. 5, one has to either know values of other couplings, reply on differential cross sections, or measure other independent double Higgs production cross sections. Here we investigate the first case, using only one measurement of σ_{ZHH} at $\sqrt{s} = 500$ GeV and knowing other couplings. Nevertheless the uncertainties on other couplings with be propagated to κ_λ. As one way of approaching the error progagation, a χ^2 is constructed as follows,

$$\chi^2 = (\frac{\sigma_{meas.} - \sigma_{ZHH}}{\Delta\sigma_{meas.}})^2 + \sum_i (\frac{\kappa_i - \kappa_i|_{SM}}{\delta\kappa_i})^2, \tag{6}$$

where κ_i goes over κ_Q, κ_Z, d_h, q_Z and b_Z, $\delta\kappa_i$ is the uncertainty on κ_i from either other direct measurements or theoretical constraints, and $\Delta\sigma_{meas.}/\sigma_{meas.}$ is estimated to 19% for ILC H20 scenario. By minimizing the χ^2, κ_λ together with its error $\delta\kappa_\lambda$ can be obtained.

[to understand the effect one by one with others fixed in Figure 3]

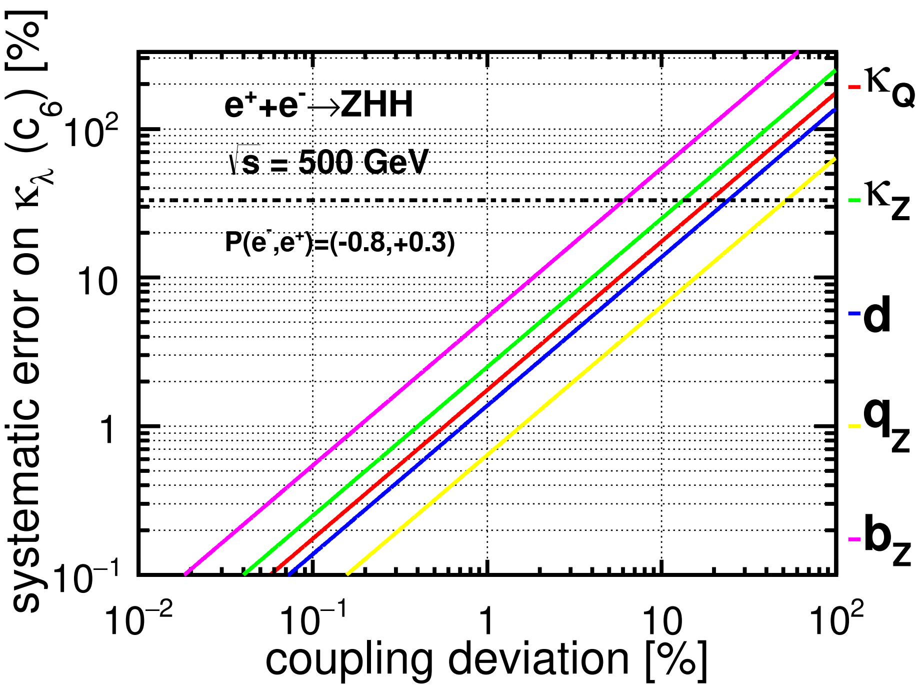

FIG. 3. $\delta\kappa_\lambda$ as a function of $\delta\kappa_i$, when all $\kappa_j (j \neq i)$ are fixed, statistical error from $\Delta\sigma_{meas.}$ is subtracted.

C. results

[input uncertainties]

$$\delta\kappa_Z = 0.003 \tag{7}$$

$$\delta\kappa_W = 0.004 \tag{8}$$

$$\delta\kappa_Q = 0.005 \tag{9}$$

$$\delta d_h = 0.004 \tag{10}$$

$$\delta b_Z = 0.007 \tag{11}$$

$$\delta q_Z = 0.007 \tag{12}$$

[result of systematic error from other couplings one by one with others constrained realistically in Figure 4]

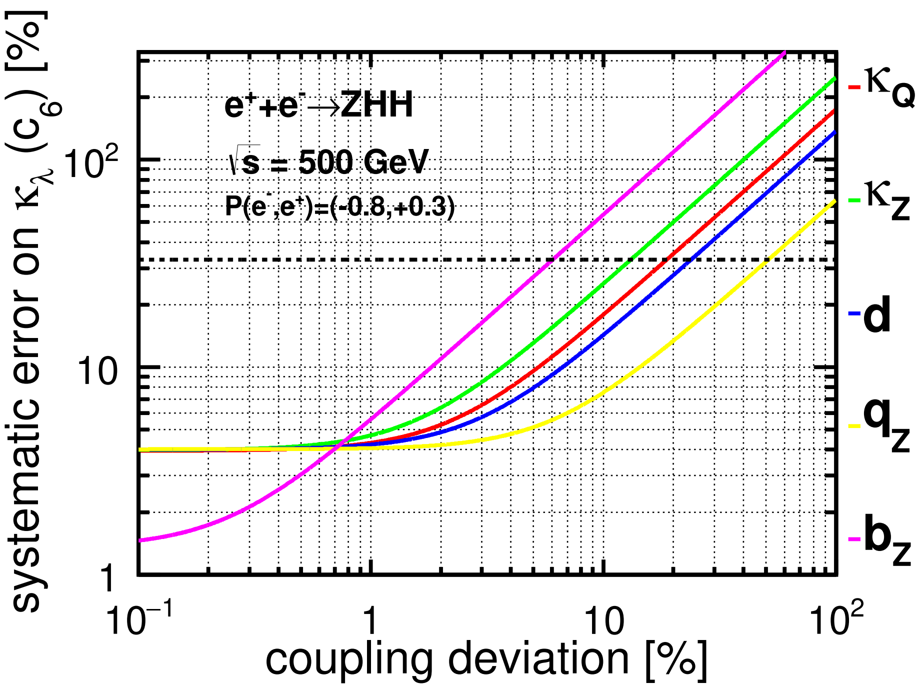

FIG. 4. $\delta\kappa_\lambda$ as a function of $\delta\kappa_i$, when all $\kappa_j (j \neq i)$ are constrained based on realistic estimation from above sections, statistical error from $\Delta\sigma_{meas.}$ is subtracted.

[add relations of those couplings from EFT analysis, number of parameters get reduced; give the similar figure but with only κ_Z, κ_W, b_Z.]

[result of systematic error: $\delta\kappa_\lambda = 4\%$]