

# Guariguanchi: Analytical Calculation Tool for Vertex Detector Studies

Alejandro Pérez  
IPHC – CNRS Strasbourg

# Outline

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- **Motivation**
- **Introduction**
- **Some Studies for ILD VXD**
- **Summary and Outlook**

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# Introduction

# Motivation

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- **Track parameters resolution and efficiency main aspects for tracker design**
- **Those are mainly dependent on**
  - Detector geometry  $\Rightarrow$  material distribution
  - Sensitive elements performances  $\Rightarrow \sigma_{sp}, t_{R.O.}, \epsilon_{det}$
  - Hit density  $\Rightarrow$  Physics environment
- **For given momentum  $(p, \theta, \phi)$  it is possible to calculate the track parameter's average resolution**
- **Calculation ignores effects only accounted for with full simulation, but it has some advantages**
  - Quick calculation of tracking performances for several geometries
  - Can be used to identify the crucial parameters in detector optimization process
  - Identify subset of detector geometries for full simulation

# Introduction: Guariguanchi's Work-flow

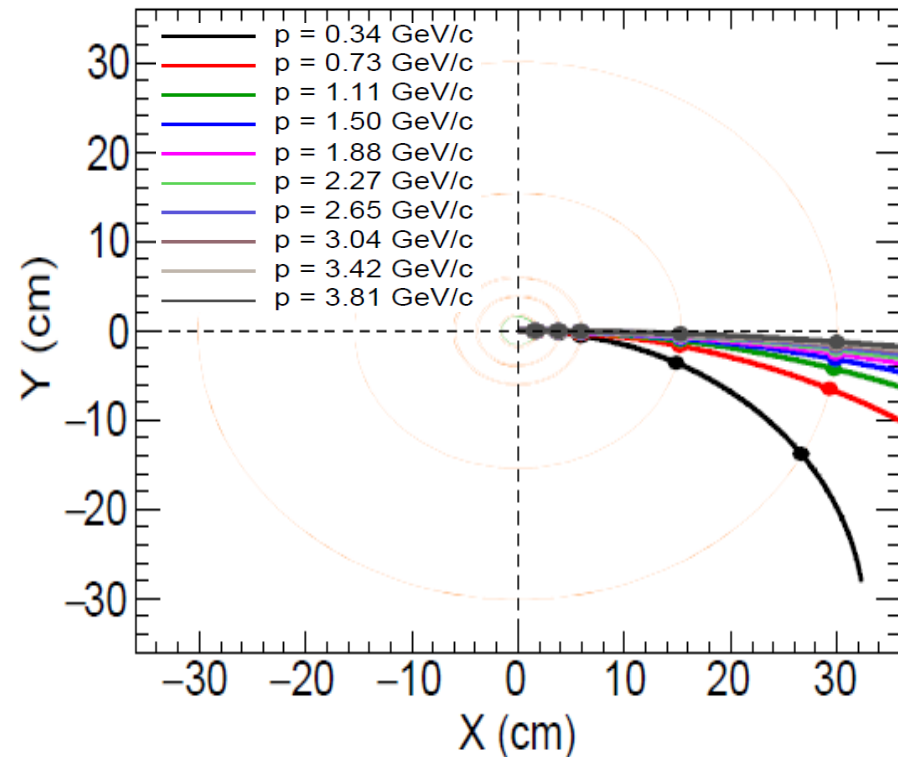
## Work-flow

- From initial conditions  $(x_{\text{init}}, p_{\text{init}}) \Rightarrow$  particle's intersections with geometry
- Multiple-scattering (MS) cov-matrix of intersection coordinates  $\Rightarrow V_{\text{MS}}$
- Total cov-matrix  $\Rightarrow V_{\text{tot}} = V_{\text{SP}} + V_{\text{MS}}$
- Track parameter's covariance matrix

## Different analyses

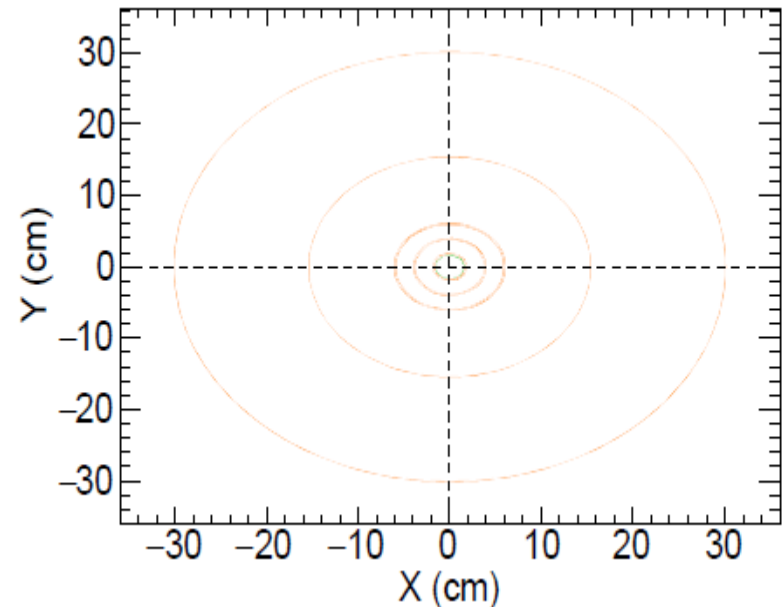
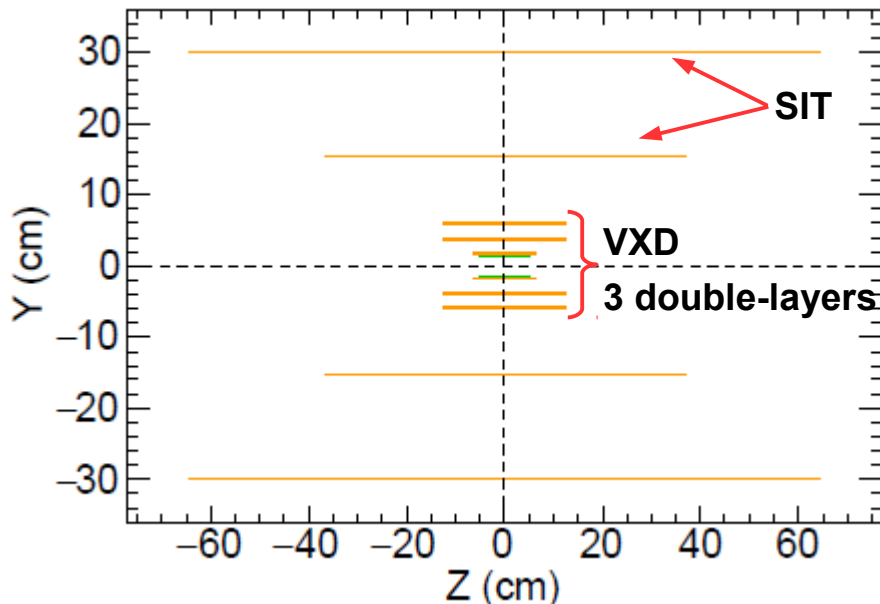
- Material budget seen vs  $(p, \theta, \phi)$
- Track parameter's resolutions vs  $(p, \theta, \phi)$
- Tracking pseudo-efficiency vs  $(p, \theta, \phi)$

## Different geometries can be compared for each analysis



# Introduction: Geometry

- **Full geometry is a combination of simple elemental objects**
  - Cylinder, Plane, Disk
- **Elements attributes**
  - Position, orientation, sensitive/insensitive dimensions
  - Material budget
  - Performances (if sensitive):  $\epsilon_{\text{det}}$ ,  $\sigma_{\text{sp}}$ ,  $t_{\text{R.O.}}$
- **Full geometries also have its own (constant) B-field**

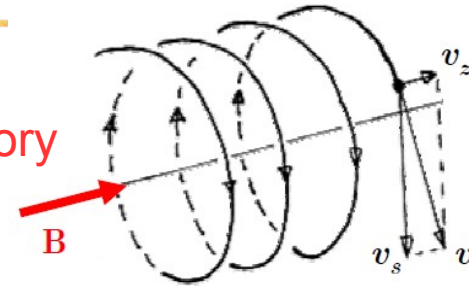


# Introduction: Navigation

- Solve motion's equation

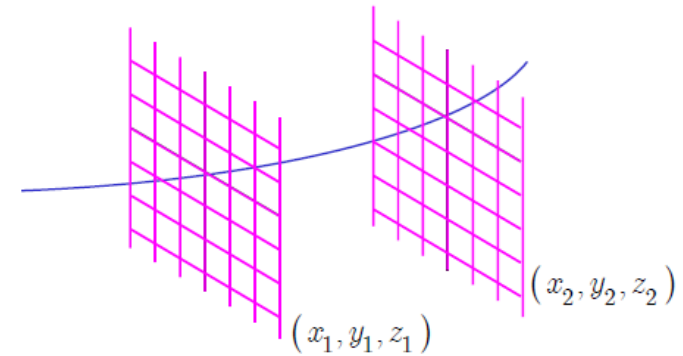
$$\frac{d^2 \mathbf{r}}{ds^2} = \frac{e}{p} \frac{d\mathbf{r}}{ds} \times \mathbf{B}$$

⇒ nominal trajectory

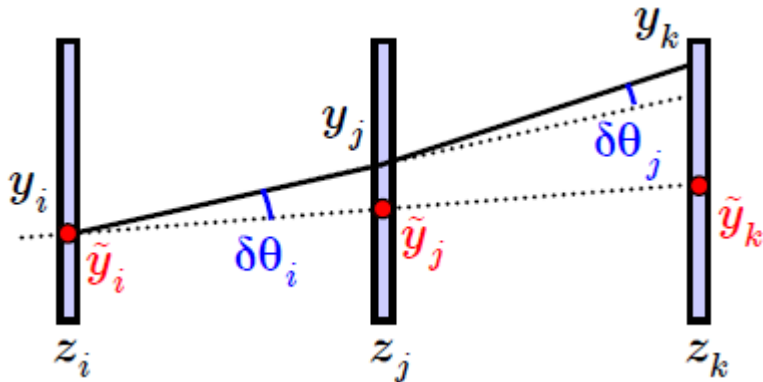


- Navigate geometry

⇒ trajectory intersection with geometry elements



- Calculate intersection coordinates correlations due to MS



$$p(\delta\theta_i) = \frac{1}{\sqrt{2\pi}} e^{-\delta\theta_i/2\theta_0}$$

$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta p c} z \sqrt{x/X_0} [1 + 0.038 \ln(x/X_0)]$$

# Introduction: Track parameters resolution

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## Track parametrization

$$\begin{cases} x = x_0 + d_\rho \cos \phi_0 & + \frac{\alpha}{\kappa} \{ \cos \phi_0 - \cos(\phi_0 + \phi) \} \\ y = y_0 + d_\rho \sin \phi_0 & + \frac{\alpha}{\kappa} \{ \sin \phi_0 - \sin(\phi_0 + \phi) \} \\ z = z_0 + d_z & - \frac{\alpha}{\kappa} \tan \lambda \cdot \phi, \end{cases}$$

$$\vec{\alpha} = \{ d_\rho, \phi_0, d_z, \tan \lambda, \rho = \alpha / \kappa \} \quad P_t (GeV/c) = 0.3 B(T) \rho (m)$$

## Track fitting

$$\chi^2 = \sum_{i,j} \{ \Gamma_i(\vec{\alpha}) - \Gamma_i^{meas} \} (V_{sp} + V_{MS})_{ij}^{-1} \{ \Gamma_j(\vec{\alpha}) - \Gamma_j^{meas} \}$$

## Track parameters cov-matrix

$$(V_{\vec{\alpha}}^{-1})_{km} = \frac{\partial^2 \chi^2}{\partial \alpha_k \partial \alpha_m} = \frac{\partial \Gamma_i(\vec{\alpha})}{\partial \alpha_k} (V_{sp} + V_{MS})_{ij}^{-1} \frac{\partial \Gamma_j(\vec{\alpha})}{\partial \alpha_m}$$



# Introduction: Track parameters resolution

## Track parametrization

$$\begin{cases} x = x_0 + d_\rho \cos \phi_0 & + \frac{\alpha}{\kappa} \{ \cos \phi_0 - \cos(\phi_0 + \phi) \} \\ y = y_0 + d_\rho \sin \phi_0 & + \frac{\alpha}{\kappa} \{ \sin \phi_0 - \sin(\phi_0 + \phi) \} \\ z = z_0 + d_z & - \frac{\alpha}{\kappa} \tan \lambda \cdot \phi, \end{cases}$$

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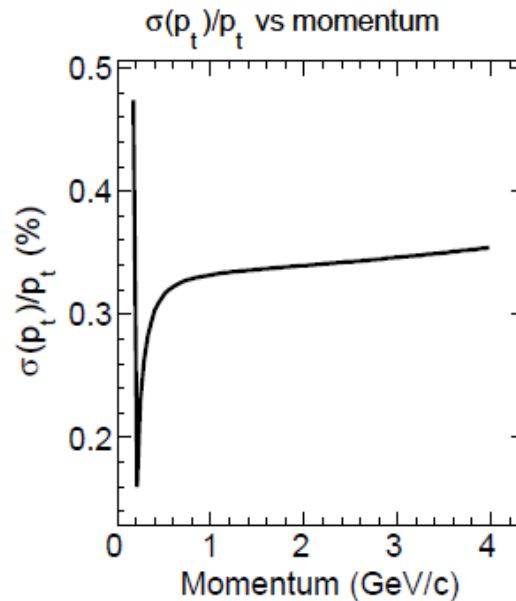
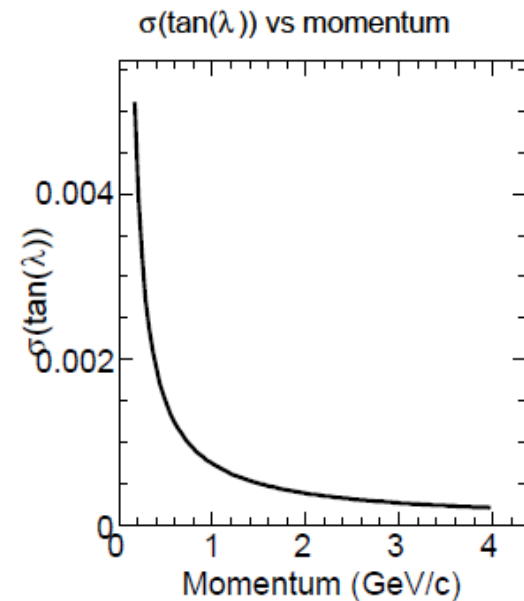
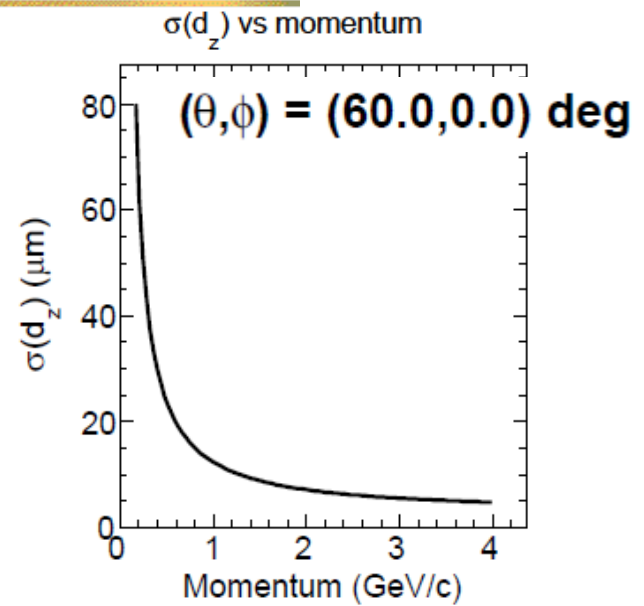
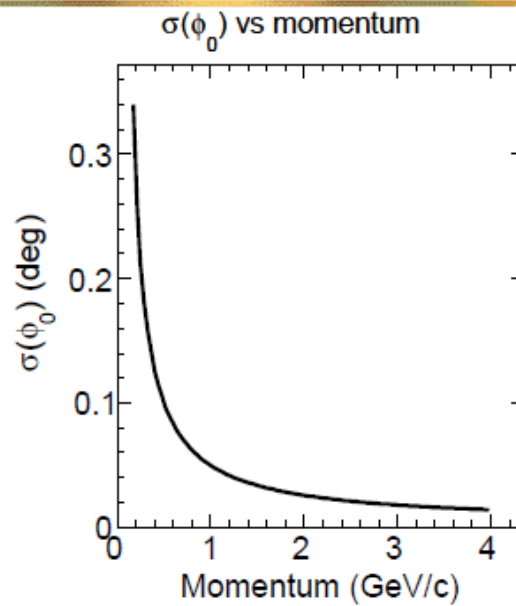
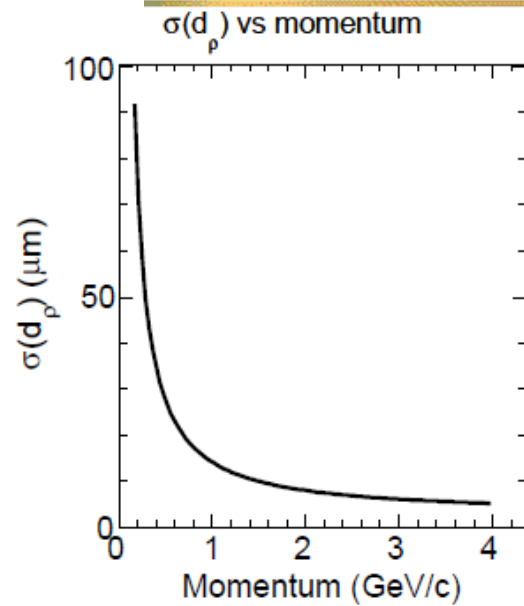
## Track fitting

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## Track parameters cov-matrix

$$(V_{\vec{\alpha}}^{-1})_{km} = \frac{\partial^2 \chi^2}{\partial \alpha_k \partial \alpha_m} = \frac{\partial \Gamma_i(\vec{\alpha})}{\partial \alpha_k} (V_{sp} + V_{MS})_{ij}^{-1} \frac{\partial \Gamma_j(\vec{\alpha})}{\partial \alpha_m}$$

# Introduction: Track parameters resolution (Example)

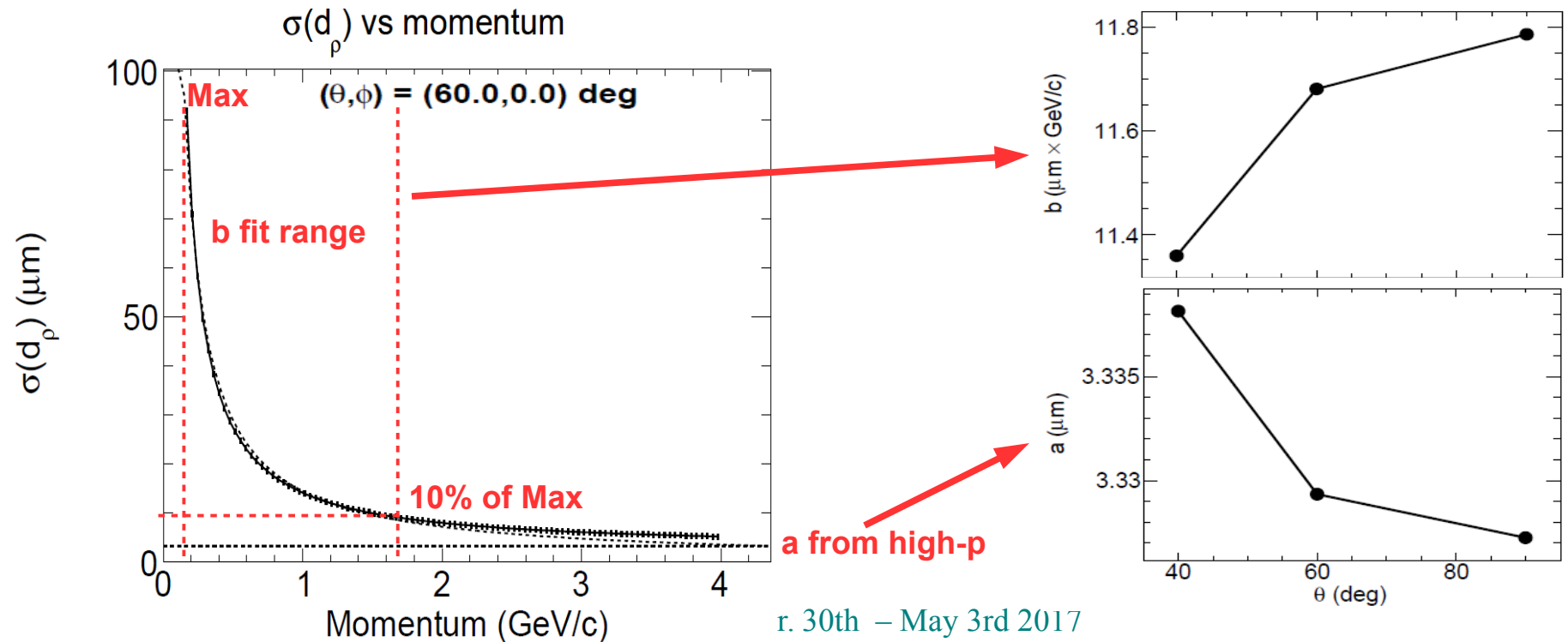


## DBD Geometry: (B = 3.5 T)

- VXD: 0.3%  $X/X_0$  /double-layer
  - L1: 2.8  $\mu\text{m}$  / 50  $\mu\text{s}$
  - L2: 6.0  $\mu\text{m}$  / 10  $\mu\text{s}$
  - L3-6: 4.0  $\mu\text{m}$  / 100  $\mu\text{s}$
- SIT: 0.65%  $X/X_0$  /double-layer
  - L1-2: (7.0,50.0)  $\mu\text{m}$  / 308 ns

# Introduction: Impact parameter parametrization

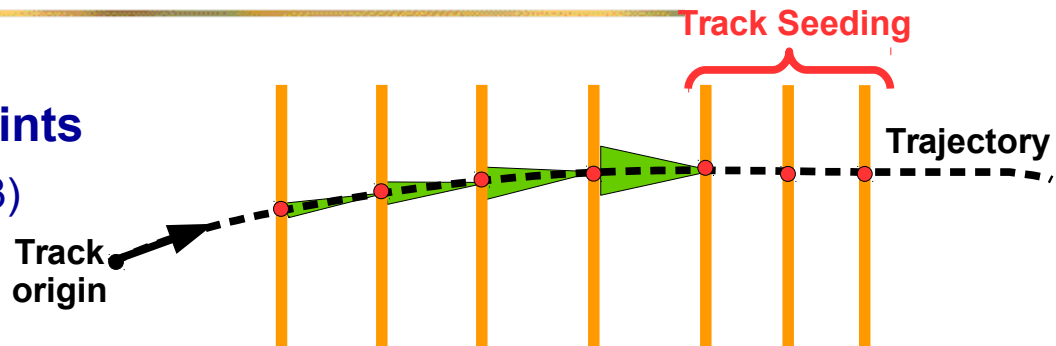
- $\sigma(d_\rho)$  empirical parametrization vs  $(p, \theta)$ :  $a \oplus b / p\beta \sin^{3/2}\theta$ 
  - ILD bench-mark values:  $a = 5 \mu\text{m}$  &  $b = 10 \mu\text{m} \times \text{GeV}/c$
- How  $a$  and  $b$  are calculated?
  - $a$ :  $\sigma(d_\rho)$  at high- $p$  (e.g. 300 GeV/c)
  - $b$ : fit  $\sigma(d_\rho)$  curve using empirical formula in low- $p$



# Introduction: Pseudo-Efficiency

## Track seeding with 3 outermost points

- $\epsilon_{seed} = \text{Prob}(\chi^2/\text{ndf} < \text{cut})$  (usually cut = 3)



## Pointing to next layer $l$ th. Hit-association probability depends on

- Track-pointing & hit resolution  $(\sigma_{r\phi}^{p,hit}, \sigma_z^{p,hit})$ ,
- Hit density in layer  $\rho(r,\phi,z)$ ,
- $\chi^2/\text{ndf}$  cut  $\Rightarrow \gamma = \text{frac. of hits lost}$   
(e.g.  $\gamma \sim 5\%$ ,  $\chi^2/\text{ndf} < 3$ , for  $\text{ndf} = 2$ )

$$\left\{ \begin{aligned} P_{corr} &= \frac{\epsilon_{d,l}(1 - \gamma^{1+2\pi\rho\sigma_{r\phi}\sigma_z})}{1 + 2\pi\rho\sigma_{r\phi}\sigma_z} \\ P_{null} &= (1 - \epsilon_{d,l} + \epsilon_{d,l}\gamma) \cdot \gamma^{(2\pi\rho\sigma_{r\phi}\sigma_z)} \\ P_{fake} &= 1 - P_{null} - P_{corr} \end{aligned} \right.$$

$$\sigma_{r\phi,z} = \sqrt{(\sigma_{r\phi,z}^p)^2 + (\sigma_{r\phi,z}^{hit})^2}$$

## Simplified picture: $\epsilon_{tot} = \epsilon_{seed} \prod_l P_l$

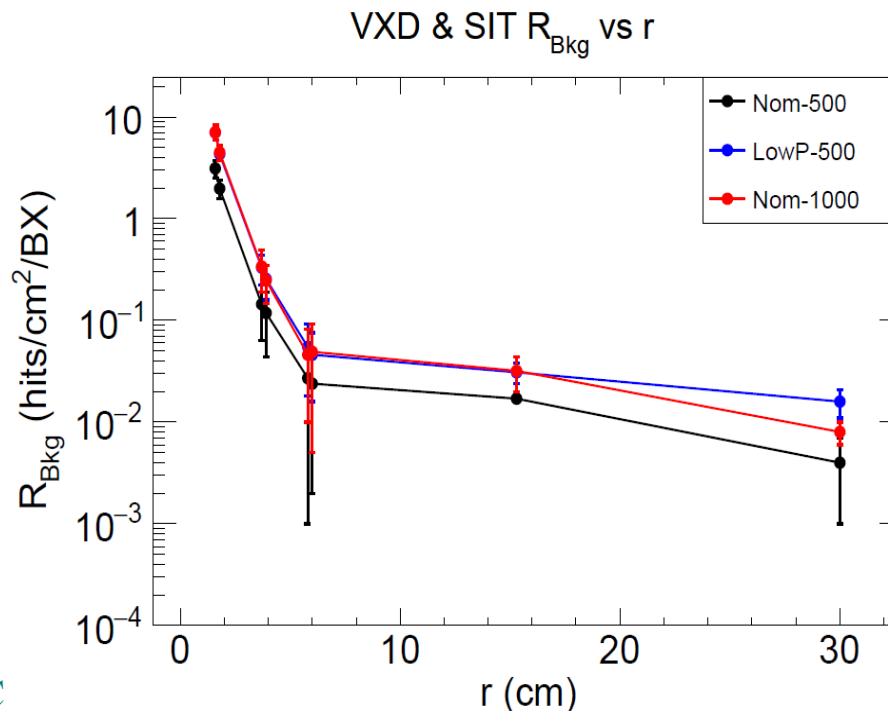
## Calculate all combinations $P_l = P_{corr}, P_{null}, P_{fake}$ consistent with requirements

- Min # hits,  $\chi^2/\text{ndf} < \text{cut}$

## Total efficiency is sum of all combinations

# Introduction: Pseudo-Efficiency (hit-rates)

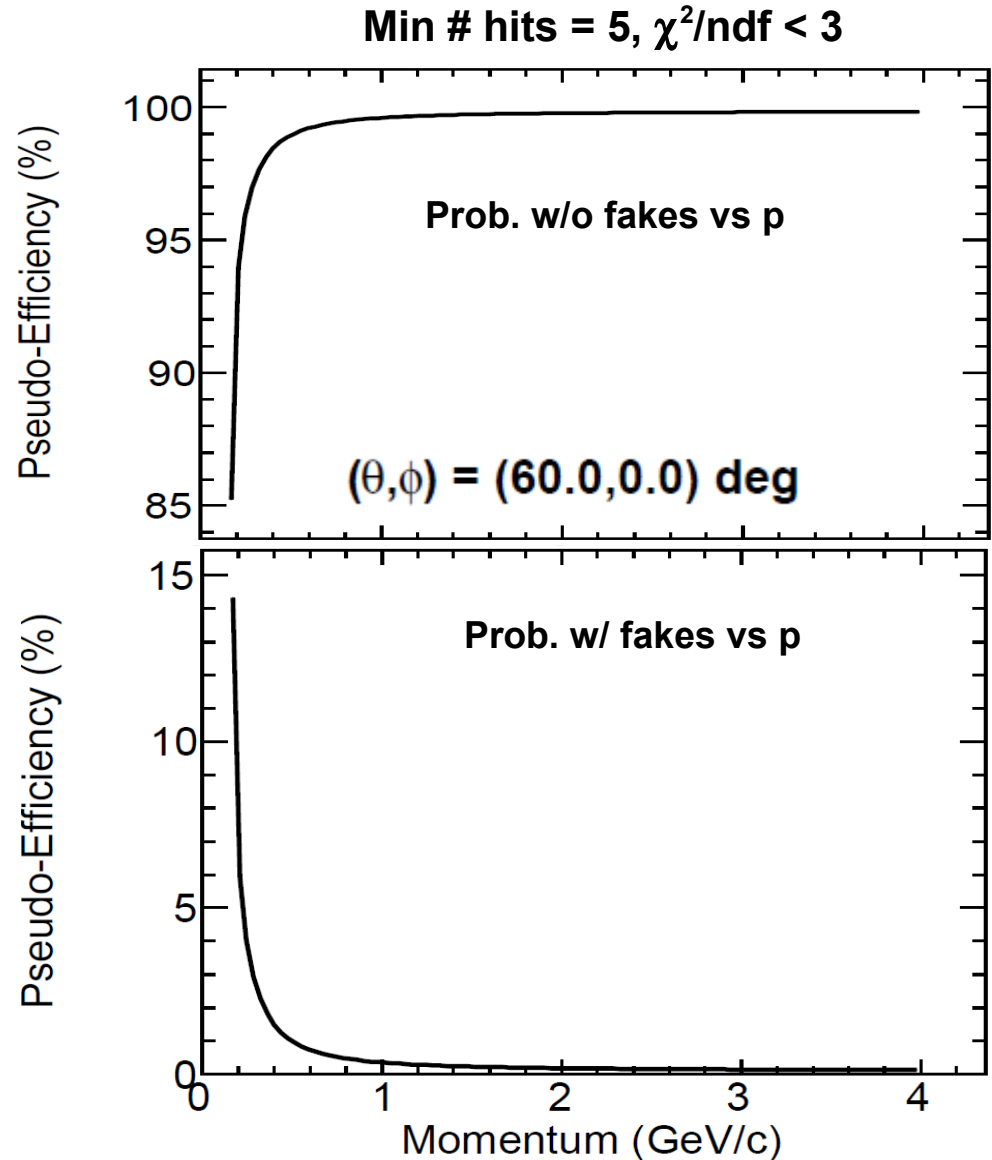
- Hit-rates from machine background full-simulation
- $R_{\text{bkg}}(r)$  vs radius shape independent of  $E_{\text{beam}}$
- Rates parametrization (salt & pepper model):  $R_{\text{bkg}}(r) = R_0 \times [R_{\text{bkg}}(r)/R_{\text{bkg}}(r_{\text{min}})]$ 
  - $R_{\text{bkg}}(r)/R_{\text{bkg}}(r_{\text{min}})$  from full-sim
  - $R_0$  is a parameter in the code (e.g. 10 MHz/cm<sup>2</sup> for  $E_{\text{beam}} = 250$  GeV)
  - For the moment no ( $\phi, z$ ) dependence



# Introduction: Pseudo-Efficiency (Example)

## DBD Geometry: (B = 3.5 T)

- VXD: 0.3%  $X/X_0$  /double-layer
  - L1: 2.8  $\mu\text{m}$  / 50  $\mu\text{s}$
  - L2: 6.0  $\mu\text{m}$  / 10  $\mu\text{s}$
  - L3-6: 4.0  $\mu\text{m}$  / 100  $\mu\text{s}$
- SIT: 0.65%  $X/X_0$  /double-layer
  - L1-2: (7.0,50.0)  $\mu\text{m}$  / 308 ns



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# **Some Studies for ILD VXD**

# ILD VXD Studies: Introduction

- Tracker: VXD + SIT with characteristics as in DBD (baseline)

- Cylindrical Geometries Studies (layers are cylinders)

- Sensors  $\sigma_{sp}$  and  $t_{R.O.}$
- Ladders spacing
- Material budget
- Innermost layer radius

- More realistic geometries

- Material budget Studies

VXD characteristics

	$R$ (mm)	$ z $ (mm)	$ \cos\theta $	$\sigma$ ( $\mu\text{m}$ )	Readout time ( $\mu\text{s}$ )
Layer 1	16	62.5	0.97	2.8	50
Layer 2	18	62.5	0.96	6	10
Layer 3	37	125	0.96	4	100
Layer 4	39	125	0.95	4	100
Layer 5	58	125	0.91	4	100
Layer 6	60	125	0.9	4	100

SIT characteristics (current baseline = false double-sided Si microstrips)					
Geometry			Characteristics		Material
R[mm]	Z[mm]	$\cos\theta$	Resolution R- $\phi$ [ $\mu\text{m}$ ]	Time [ns]	RL[%]
153	368	0.910	R: $\sigma=7.0$ ,	307.7 (153.8)	0.65
300	644	0.902	z: $\sigma=50.0$	$\sigma=80.0$	0.65

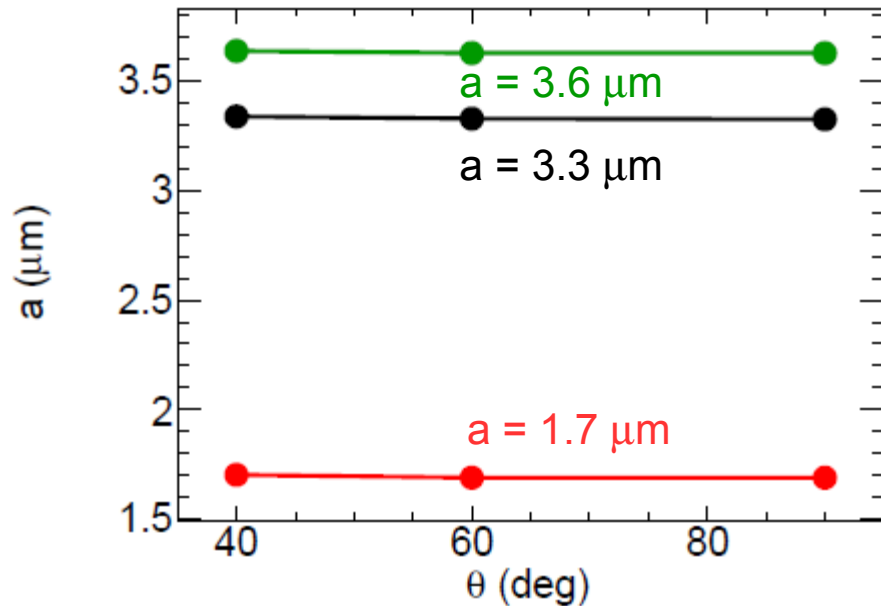


# ILD VXD Studies: Cylindrical Geometries (1/5)

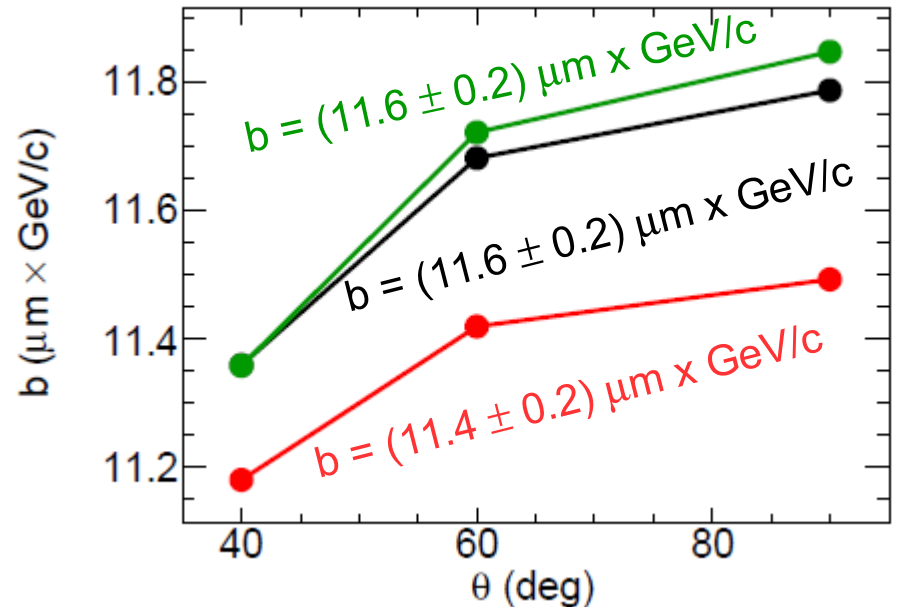
## Varying $\sigma_{sp}$ of VXD layers

- L1: 2.8  $\mu\text{m}$ / 50  $\mu\text{s}$ , L2: 6  $\mu\text{m}$ /10  $\mu\text{s}$ , L3-6: 4  $\mu\text{m}$ / 100  $\mu\text{s}$  (DBD)
- L1: 1.0  $\mu\text{m}$ /800  $\mu\text{s}$ , L2: 6  $\mu\text{m}$ /0.5  $\mu\text{s}$ , L3-6: 4  $\mu\text{m}$ / 4  $\mu\text{s}$
- L1-6: 4  $\mu\text{m}$ / 4  $\mu\text{s}$

a parameters vs  $\theta$



b parameters vs  $\theta$



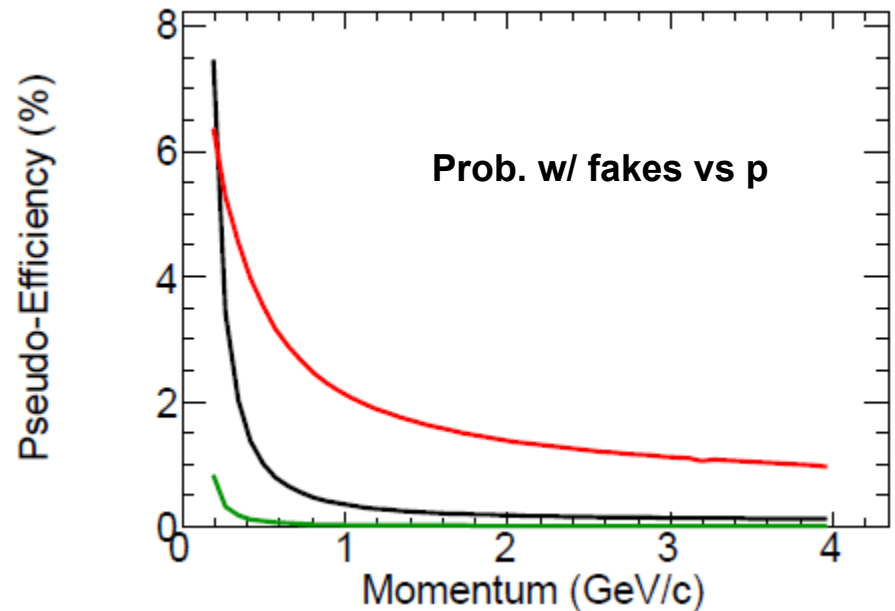
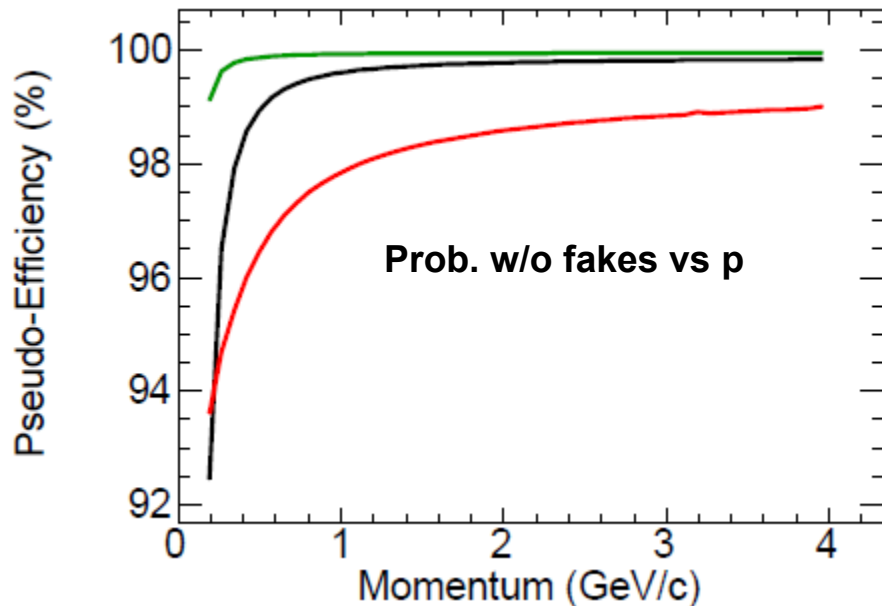
# ILD VXD Studies: Cylindrical Geometries (2/5)

## ■ Varying $t_{R.O.}$ of VXD layers

- L1: 2.8  $\mu\text{m}/$  50  $\mu\text{s}$ , L2: 6  $\mu\text{m}/$ 10  $\mu\text{s}$ , L3-6: 4  $\mu\text{m}/$  100  $\mu\text{s}$  (DBD)
- L1: 1.0  $\mu\text{m}/$ 800  $\mu\text{s}$ , L2: 6  $\mu\text{m}/$ 0.5  $\mu\text{s}$ , L3-6: 4  $\mu\text{m}/$  4  $\mu\text{s}$
- L1-6: 4  $\mu\text{m}/$  4  $\mu\text{s}$

$(\theta, \phi) = (60.0, 0.0)$  deg

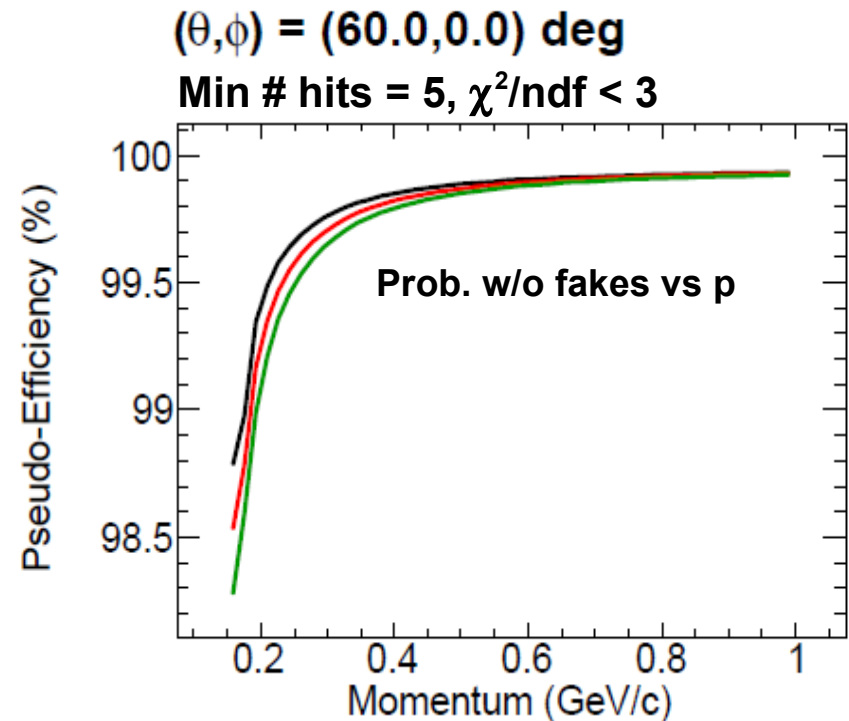
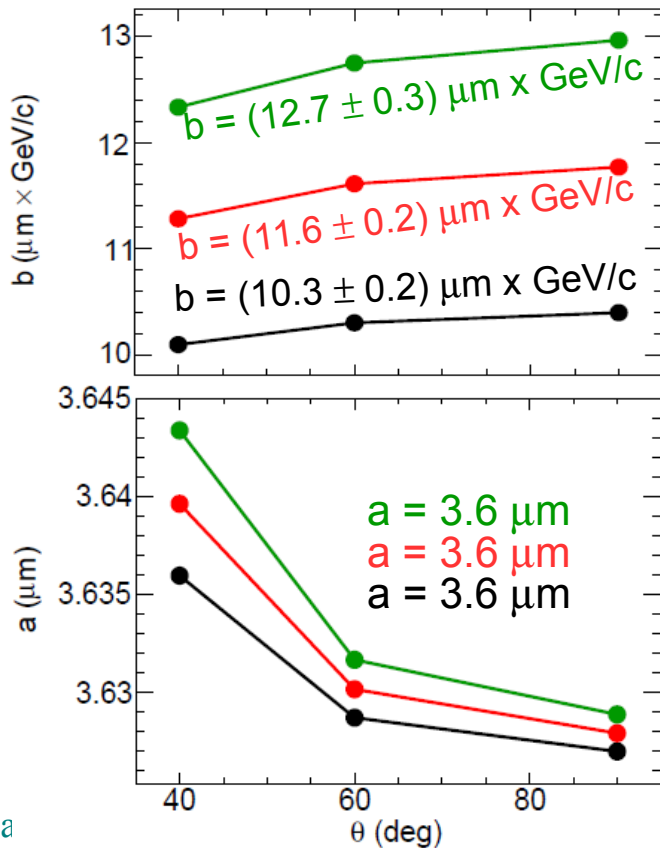
Min # hits = 5,  $\chi^2/\text{ndf} < 3$



# ILD VXD Studies: Cylindrical Geometries (3/5)

## Varying VXD Material Budget

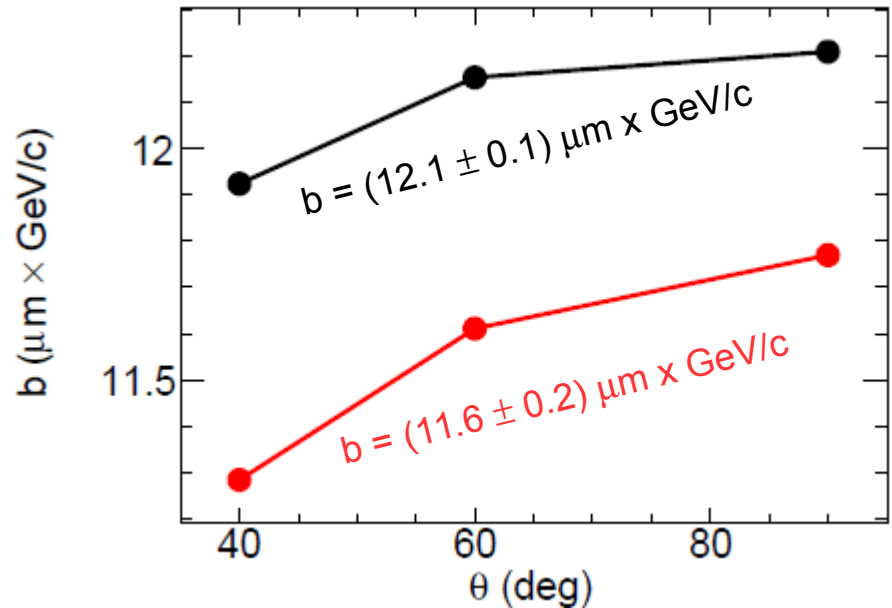
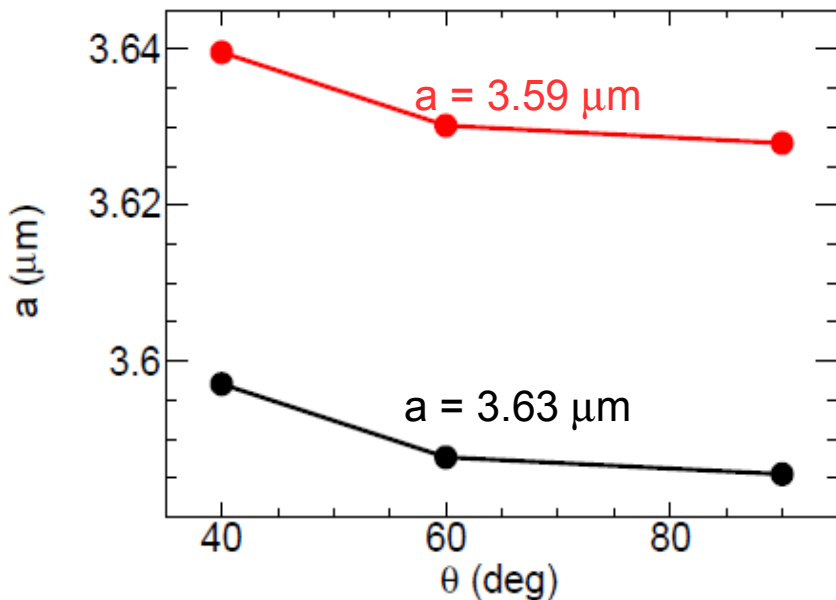
- L1-6: 4  $\mu\text{m}$ / 4  $\mu\text{s}$ ,  $X/X_0 = 0.2$  % per double-layer
- L1-6: 4  $\mu\text{m}$ / 4  $\mu\text{s}$ ,  $X/X_0 = 0.3$  % per double-layer
- L1-6: 4  $\mu\text{m}$ / 4  $\mu\text{s}$ ,  $X/X_0 = 0.4$  % per double-layer



# ILD VXD Studies: Cylindrical Geometries (4/5)

## Varying Ladder Spacing

- L1-6: 4  $\mu\text{m}$ / 4  $\mu\text{s}$ , spacing = 1.0 mm
- L1-6: 4  $\mu\text{m}$ / 4  $\mu\text{s}$ , spacing = 2.0 mm (nominal)

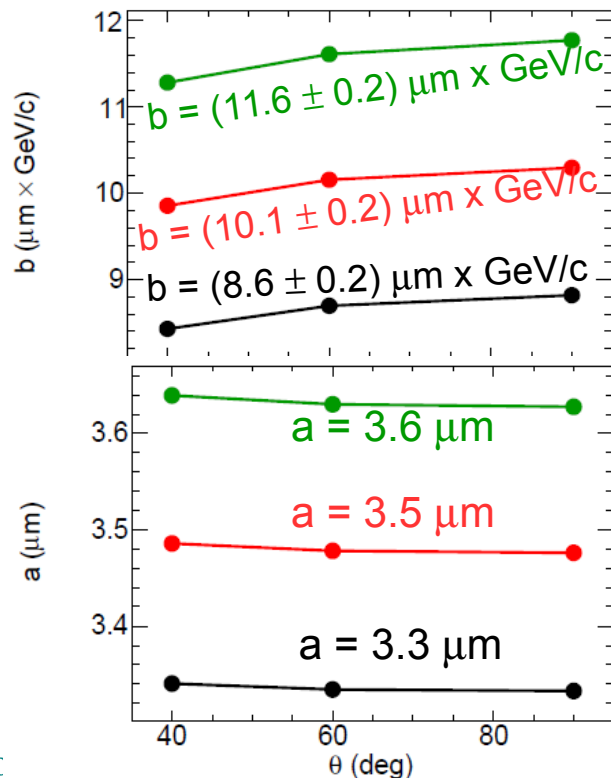


# ILD VXD Studies: Cylindrical Geometries (5/5)

## Varying L1 radius

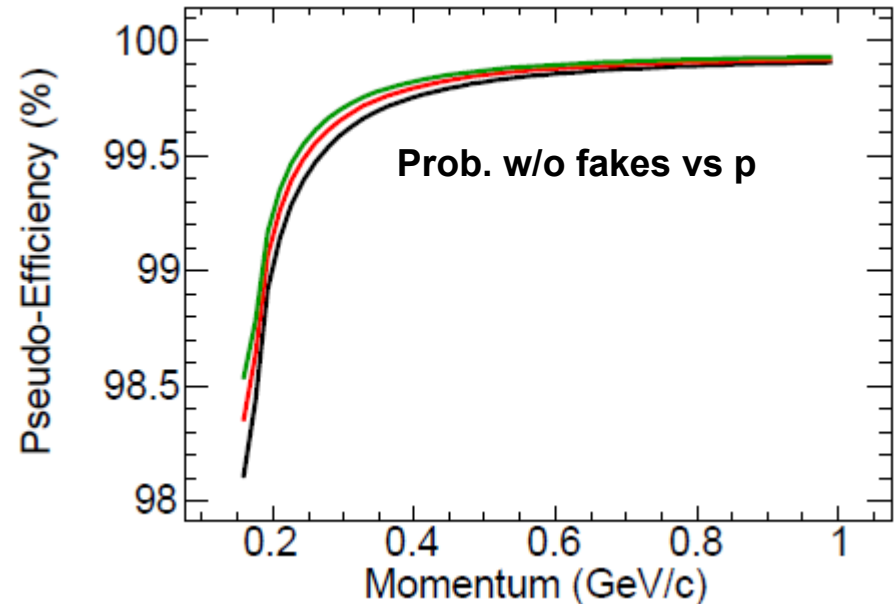
- L1-6: 4  $\mu\text{m}$ / 4  $\mu\text{s}$ , RL1 = 12 mm,  $R_{\text{pipe}} = 11$  mm
- L1-6: 4  $\mu\text{m}$ / 4  $\mu\text{s}$ , RL1 = 14 mm,  $R_{\text{pipe}} = 13$  mm
- L1-6: 4  $\mu\text{m}$ / 4  $\mu\text{s}$ , RL1 = 16 mm,  $R_{\text{pipe}} = 15$  mm (Nominal)

Beam-pipe  
thickness = 500  $\mu\text{m}$



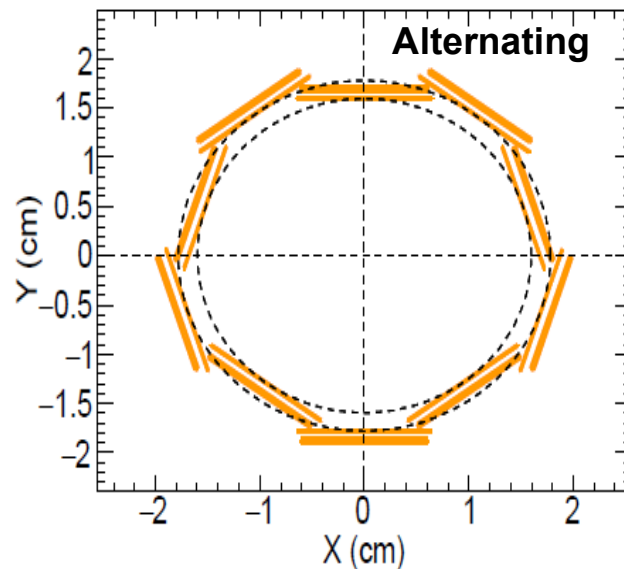
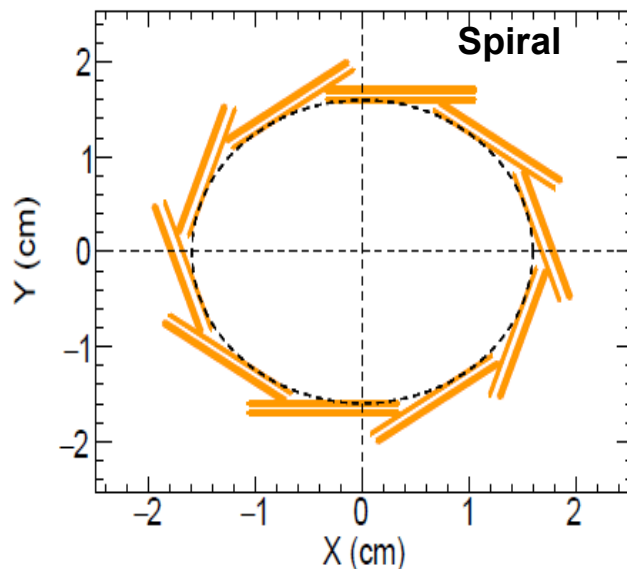
$(\theta, \phi) = (60.0, 0.0)$  deg

Min # hits = 5,  $\chi^2/\text{ndf} < 3$



# ILD VXD Studies: More Realistic Geometries (1/3)

- Real life detector not cylinders but mosaic of ladders
- Ladders arrangement depends several factor
  - Mechanical constrains
  - Ladder characteristics: sensor size, sensitive/insensitive surface, thickness, ...
- Start point for more realistic geometry
  - Innermost double layer: tight region
  - Alternating geometry: easier mechanical implementation
  - Material budget studies



# ILD VXD Studies: More Realistic Geometries (2/3)

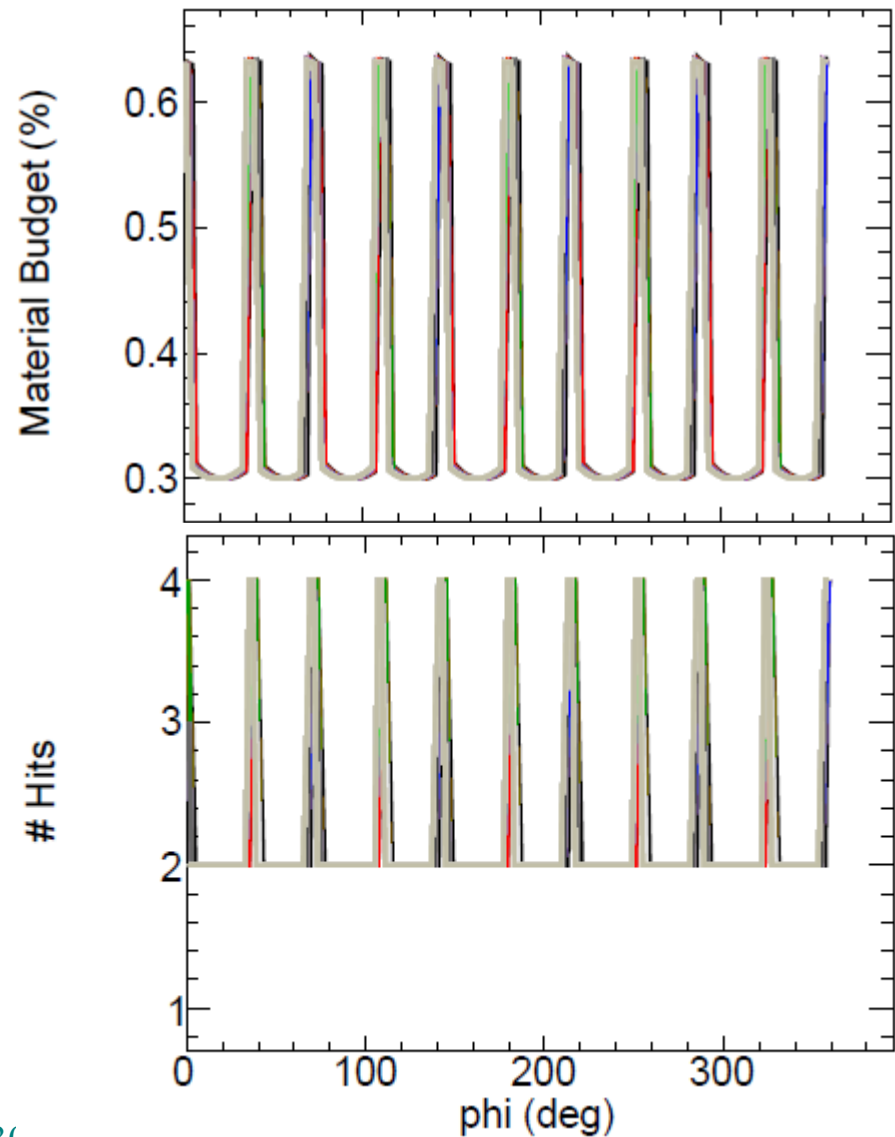
$W_s = 13$  mm

## ■ Preliminary material budget studies

- L1 radius = 16 mm
- Spacing = 1 mm
- #ladders = 10
- Insensitive width = 1 mm
- Material budget vs sensitive width ( $W_s$ )

## ■ The method

- Shot pions with different  $(p, \phi)$ :  
(0.18, 1.5) GeV/c & (0, 360) deg
- Material budget average w.r.t  $p$  &  $\phi$



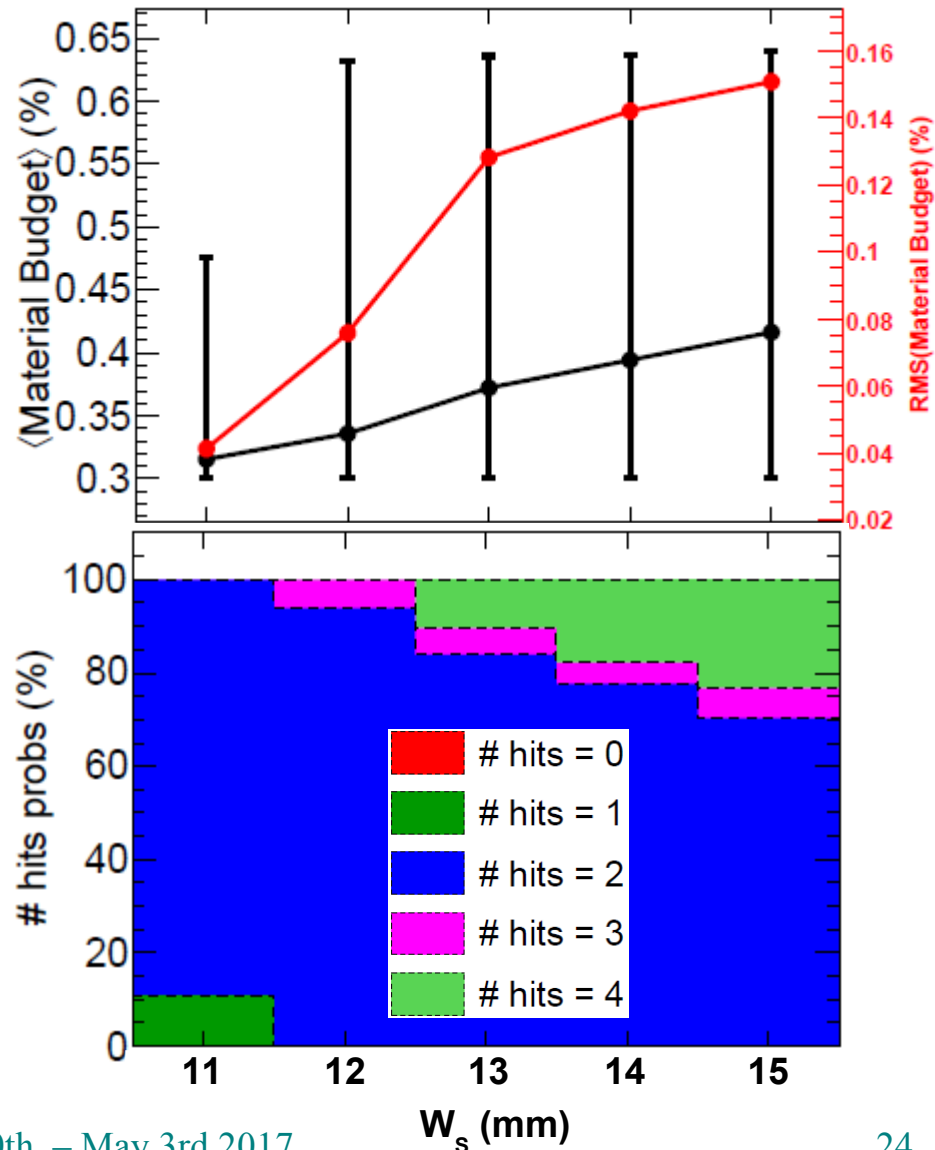
# ILD VXD Studies: More Realistic Geometries (3/3)

## Preliminary material budget studies

- L1 radius = 16 mm
- Spacing = 1 mm
- #ladders = 10
- Insensitive width = 1 mm
- Material budget vs sensitive width ( $W_s$ )

## The method

- Shot pions with different  $(p, \phi)$ :  
(0.18, 1.5) GeV/c & (0, 360) deg
- Material budget average w.r.t  $p$  &  $\phi$





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# Summary and Outlook

# Summary and Outlook

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## Guariguanchi Tool

- Tracking performances: track parameters resolution & pseudo-efficiency
- Can quickly compare several geometries

## Preliminary VXD studies: cylindrical geometry

- $\sigma_{sp}$ : critical to achieve bench-mark track parameter performances
- $t_{R.O.}$ : too large values in L1 deteriorate significantly  $\epsilon_{det}$
- $\sigma_{sp}/t_{R.O.} = 4\mu\text{m}/4\mu\text{s}$  achievable with current technologies gives good performances
- Layers material budget, Ladder spacing, L1 radius  $\Rightarrow$  not critical

## Preliminary VXD studies: realistic geometry

- Material budget studies started on L1 with alternating geometry
- Need to specify constrains on ladder construction for optimization studies

## Outlook

- Would like to calibrate this tool by comparing with full simulation
- Continue studies of more realistic geometries

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# Back up Slides