Top electroweak couplings study with the Matrix Element Method using di-muonic state at $\sqrt{s} = 500$ GeV, ILC

Annual ILC detector meeting

Yo Sato[A]

Akimasa Ishikawa[A], Emi Kou[B], Francois Le Diberder[B], Hitoshi Yamamoto[A],

Junping Tian[C], Keisuke Fujii[D],

Tohoku University[A], LAL[B], University of Tokyo[C], KEK[D]

Top EW couplings at the ILC

- Top quark is the heaviest particle in the SM. Its large mass implies that this is strongly coupled to the mechanism of electroweak symmetry breaking (EWSB)

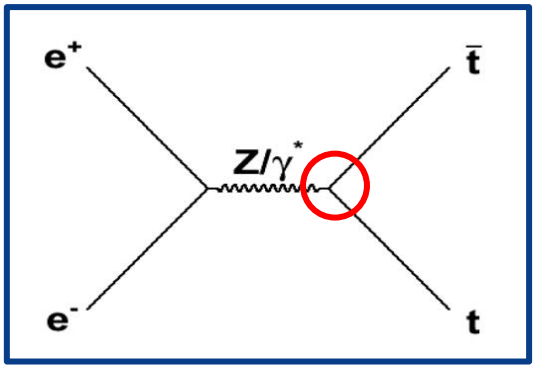
  → **Top EW couplings are good proves for New physics behind EWSB**

- **The ILC is advanced in the $t\bar{t}Z^0$ and $t\bar{t}\gamma$ couplings study**
  - Top pair production process, $e^+e^- \rightarrow t\bar{t}$, goes directly through the $t\bar{t}Z^0$ and $t\bar{t}\gamma$

The general Lagrangian

$$\mathcal{L}_{\text{int}} = \sum_{v=\gamma,Z} g^v \left[V_l{}^v \bar{t}\gamma^l (F_{1V}^v + F_{1A}^v \gamma_5) t + \frac{i}{2m_t} \partial_\nu V_l \bar{t}\sigma^{l\nu} (F_{2V}^v + F_{2A}^v \gamma_5) t \right]$$

F_{2A}^v can be a complex number → **10 real form factors**



eg)

- Composite models yields typically 10% deviation of ${g_{L,R}}^Z (= F_{1V}^Z \pm F_{1A}^Z)$
- In the 2HDM, F_{2A}^γ which is a CP-violating parameter can be non-zero

Previous research : Semi-leptonic state

Top pair production has three different final states:

1. **Fully-hadronic state** $(e^+e^- \to t\bar{t} \to bq\bar{q}bq\bar{q})$ **46.2 %**
2. **Semi-leptonic state** $(e^+e^- \to t\bar{t} \to bq\bar{q}bl\nu)$ **43.5%**
3. **Di-leptonic state** $(e^+e^- \to t\bar{t} \to bl\nu bl\nu)$ **10.3%**

- **Observables :** Cross-section, Forward-backward asymmetry

$$\sigma_I = 2\left(\frac{4\pi\alpha^2}{3s}\right) N_c\beta\left[(1+0.5\gamma^{-2})(\mathcal{F}^I_{1V})^2 + (\beta\mathcal{F}^I_{1A})^2 + 3\mathcal{F}^I_{1V}\mathcal{F}^I_{2V}\right]$$

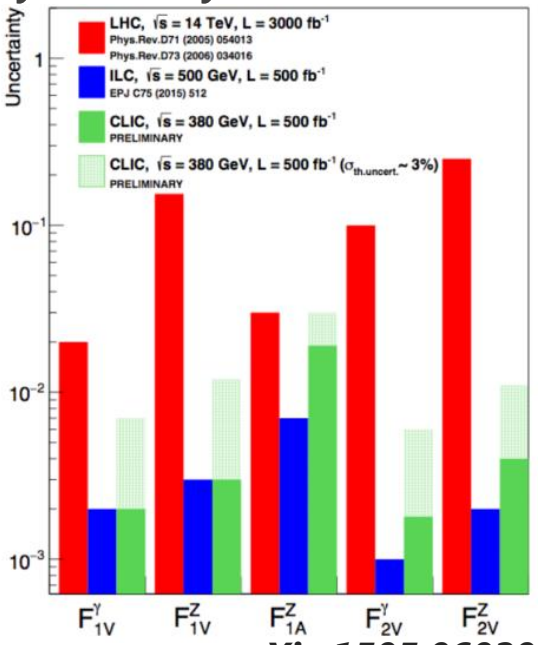
$$(A^t_{FB})_I = \frac{-3\beta\mathcal{F}^I_{1A}(\mathcal{F}^I_{1V}+\mathcal{F}^I_{2V})}{2\left[(1+0.5\gamma^{-2})(\mathcal{F}^I_{1V})^2 + (\beta\mathcal{F}^I_{1A})^2 + 3\mathcal{F}^I_{1V}\mathcal{F}^I_{2V}\right]}$$

$$\begin{pmatrix}\sigma(L), A_{FB}(L)\\ \sigma(R), A_{FB}(R)\end{pmatrix} \Longrightarrow \begin{pmatrix}F^\gamma_{1V}, -\\ F^Z_{1V}, F^Z_{1A}\end{pmatrix}, \begin{pmatrix}g^\gamma_L, g^\gamma_R\\ g^Z_L, g^Z_R\end{pmatrix}, \begin{pmatrix}F^\gamma_{2V}\\ F^Z_{2V}\end{pmatrix}$$

2 observables × 2 polarization

But, not sensitive to F^ν_{2A} because observables are limited

(CPV study is ongoing (see talk by M. Vos @LCWS2016))



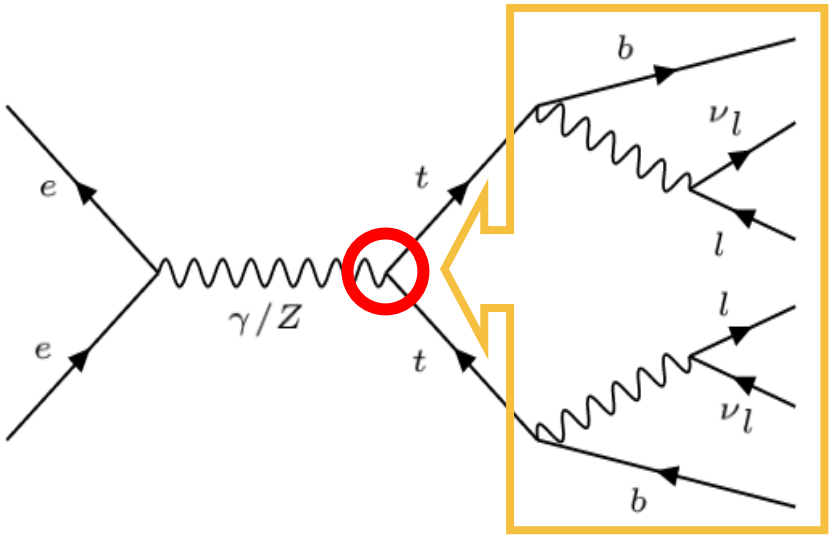
arXiv:1505.06020

Alternative : Di-leptonic state

Idea : Top quark decays before hadronization because of its large width

→ **Angles of the final state have the information of $t\bar{t}Z^0/\gamma$ vertex**

→ Use the **di-leptonic state** to obtain more angles



At most, there are 9 angles related to ttZ/γ vertex

$(\cos\theta_t, \cos\theta_{W^+}, \phi_{W^+}, \cos\theta_{l^+}, \phi_{l^+}, \cos\theta_{W^-}, \phi_{W^-}, \cos\theta_{l^-}, \phi_{l^-})$

Alternative : The Matrix Element Method

The Matrix Element Method (MEM)

The most efficient method when all the kinematics can be reconstructed

MEM with the 9 angles and the cross-section

- **All 10 form factors can be fitted simultaneously**

→ ~0.01 precision is obtained at parton level study with di-leptonic state

Statistical uncertainties and correlation with the SM LO as normalization

Kheim, E.K. Kurihara, Le Diberder: arXiv:1503:04247

$\mathcal{R}\mathrm{e}\ \delta\tilde{F}^{\gamma}_{1V}$	$\mathcal{R}\mathrm{e}\ \delta\tilde{F}^{Z}_{1V}$	$\mathcal{R}\mathrm{e}\ \delta\tilde{F}^{\gamma}_{1A}$	$\mathcal{R}\mathrm{e}\ \delta\tilde{F}^{Z}_{1A}$	$\mathcal{R}\mathrm{e}\ \delta\tilde{F}^{\gamma}_{2V}$	$\mathcal{R}\mathrm{e}\ \delta\tilde{F}^{Z}_{2V}$	$\mathcal{R}\mathrm{e}\ \delta\tilde{F}^{\gamma}_{2A}$	$\mathcal{R}\mathrm{e}\ \delta\tilde{F}^{Z}_{2A}$	$\mathcal{I}\mathrm{m}\ \delta\tilde{F}^{\gamma}_{2A}$	$\mathcal{I}\mathrm{m}\ \delta\tilde{F}^{Z}_{2A}$
0.0037	−0.18	−0.09	+0.14	+0.62	−0.15	0	0	0	0
	0.0063	+.14	−0.06	−0.13	+0.61	0	0	0	0
		0.0053	−0.15	−0.05	+0.09	0	0	0	0
			0.0083	+0.06	−0.04	0	0	0	0
				0.0105	−0.19	0	0	0	0
					0.0169	0	0	0	0
						0.0068	−0.15	0	0
							0.0118	0	0
								0.0069	−0.17
									0.0100

500 GeV&500 fb^{-1} Polarization 50/50 between ±80% and ±30%

→ Estimate the ultimate precision considering all effects

Setup

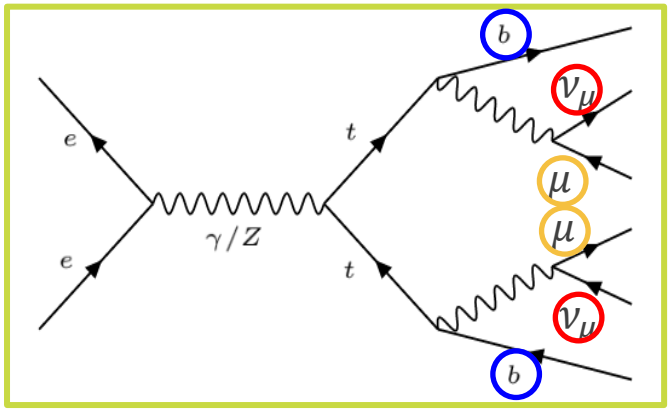
Situation

- ☑ The hadronization of b, $\bar{b}$
- ☐ ISR, beamsstrahlung, beam energy spread
- ☐ γγ → hadrons
- ☑ The detector effects — **Today's talk**
- ☐ Gluon emission from $t, \bar{t}$
- ☐ Background events — Switch off (Ongoing)

Sample (**Only signal**)	Di-muonic state (SM-LO) $e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b}\mu^+\nu\mu^-\bar{\nu}$
$\sqrt{s}$	500 GeV
Polarization (P_{e^-}, P_{e^+})	(-0.8, +0.3) / (+0.8, -0.3) (In this talk, I call them "Left" and "Right")
Integrated luminosity	500 fb^{-1} (50/50 between Left and Right)
Generator	Whizard
Detector	ILD_01_v05 (DBD ver.)

Kinematical Reconstruction : Strategy

Di-muonic state : $e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b}\mu^+\nu\mu^-\bar{\nu}$



Measurable
- muon's : $E_{\mu^+}, \theta_{\mu^+}, \phi_{\mu^+}, E_{\mu^-}, \theta_{\mu^-}, \phi_{\mu^-}$
- b-jet's : $(E_b), \theta_b, \phi_b, (E_{\bar{b}}), \theta_{\bar{b}}, \phi_{\bar{b}}$

Missing
- b-jet's : $(E_b), (E_{\bar{b}})$
- neutrino's : $E_\nu, \theta_\nu, \phi_\nu, E_{\bar{\nu}}, \theta_{\bar{\nu}}, \phi_{\bar{\nu}}$

=> 6 (8) unknowns

Strategy

① Recover them from 8 kinematical constraints
- initial state : $(\sqrt{s}, \vec{P}_{\text{init.}}) = (500, \vec{0})$
- mass : $m_t, m_{\bar{t}}, m_{W^+}, m_{W^-}$

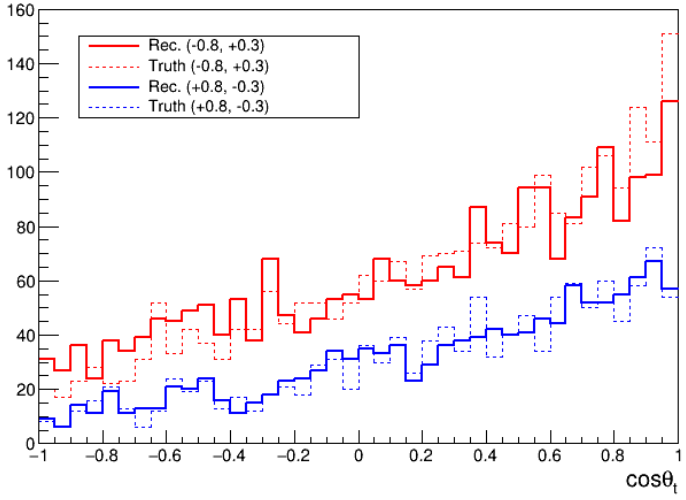
→ But the equation is non-linear and the b-charge ambiguity remains.

→ Typically 4 solutions per an event

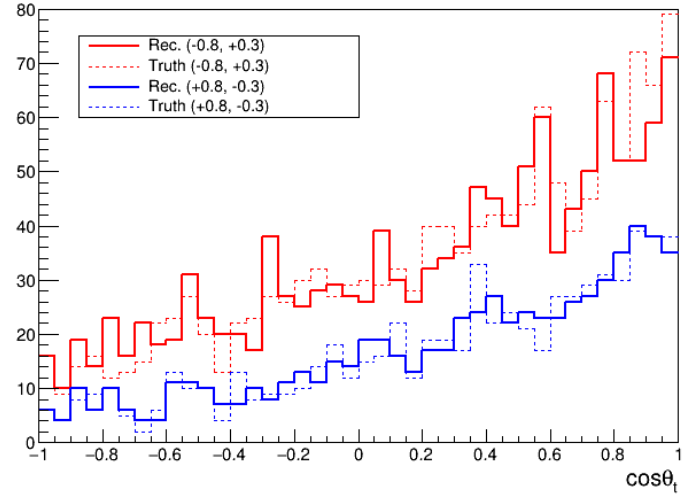
② Select the optimal one by comparing $E_b, E_{\bar{b}}$ between recovered and measured

Results : Polar angle distribution of top, $\cos\theta_t$

(Before cut)



(After cut)



- In the case of **Left,** migrations of events passing from forward to backward are observed.

  ← The miss pairing of W and b

- After selection of reliable events by the quality of the kinematical reconstruction, migrations become smaller.

Ratio of the miss pairing	Left	Right
Before cut (efficiency = ~92 %)	**8.9 %**	**6.0 %**
After cut (efficiency = ~50 %)	**5.5 %**	**3.0 %**

Results : All 10 form factors fit with MEM

Fit of all 10 form factors at same time

- Precision is typically ~0.03

eg) $\tilde{F}_{1V}^{\gamma}$

	Precision of $\tilde{F}_{1V}^{\gamma}$	$N/N_{\mathrm{di-muonic}}$
This result	**0.011**	**1**
Parton level	**0.004**	**~4**
Semi-leptonic study	**0.002**	**~25**

→ Comparable with the previous study

- Biases are thought to results from the detector effects and the miss pairing of Wb

→ One can reduce them by convoluting $|M|^2$ with the detector effects and applying relevant cuts.

Preliminary

(efficiency = ~50 %)

$$\begin{bmatrix} \mathcal{R}e\ \delta\tilde{F}_{1V}^{\gamma} & -0.0015 \pm 0.0108 \\ \mathcal{R}e\ \delta\tilde{F}_{1V}^{Z} & -0.0271 \pm 0.0187 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{\gamma} & -0.0314 \pm 0.0156 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{Z} & +0.0277 \pm 0.0246 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{\gamma} & -0.0266 \pm 0.0317 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{Z} & -0.0702 \pm 0.0504 \\ \mathcal{R}e\ \delta\tilde{F}_{2A}^{\gamma} & -0.0082 \pm 0.0211 \\ \mathcal{R}e\ \delta\tilde{F}_{2A}^{Z} & -0.0164 \pm 0.0360 \\ \mathcal{I}m\ \delta\tilde{F}_{2A}^{\gamma} & -0.0427 \pm 0.0206 \\ \mathcal{I}m\ \delta\tilde{F}_{2A}^{Z} & +0.0220 \pm 0.0297 \end{bmatrix}$$

$$F_{1V}^{\tilde{v}} = -(F_{1V}^{v} + F_{2V}^{v}), \quad F_{2V}^{\tilde{v}} = F_{2V}^{v}$$

$$F_{1A}^{\tilde{v}} = -F_{1A}^{v}, \quad F_{2A}^{\tilde{v}} = -iF_{2A}^{v}$$

Summery and Plan

Top EW couplings are good proves for New Physics behind EWSB

→ Alternative analysis by the MEM using the di-leptonic state

- **Kinematical Reconstruction**
  - Obtained the 9 angles by the measurement of isolated leptons and b-jets and the kinematical constraints
- **Analysis with MEM**
  - Fit all 10 form factors using the 9 angles
  - Obtained the comparable results (typically ~0.03) with the previous studies
- **Plan**
  - Optimize the $|M|^2$ and apply cuts to reduce the biases
  - Estimate the ultimate precision considering all effects which are being ignored
  - Investigate a possibility of the MEM with semi-leptonic state.

Back up

Thrust axis method

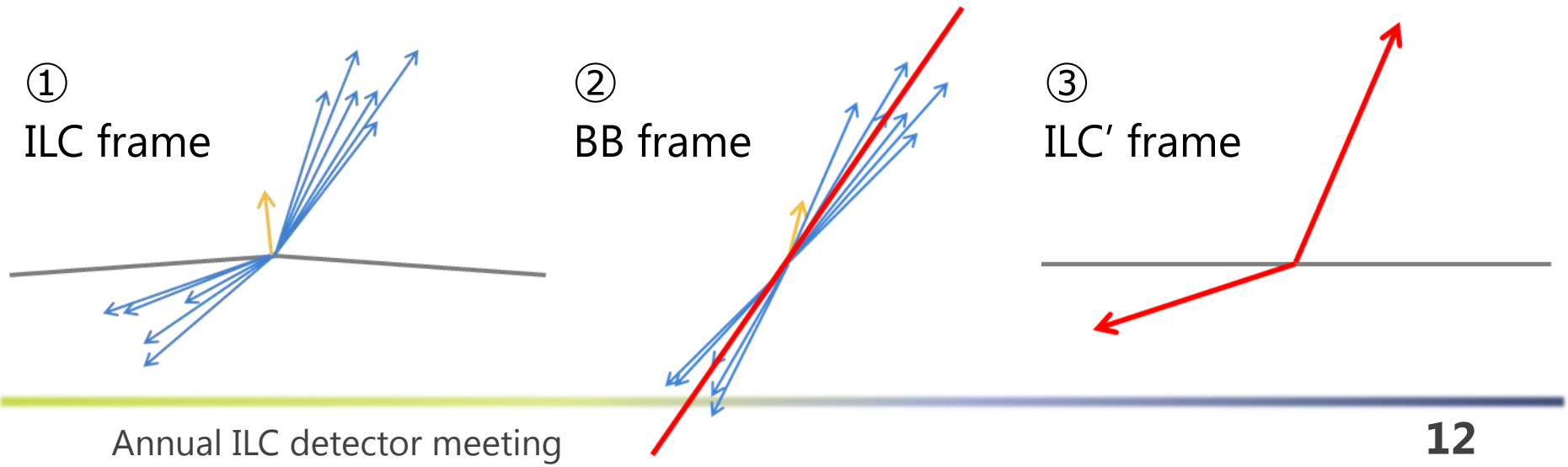
We use the thrust axis method for the measurement of 2 b-jets.

① Collect all hadronized particles and photons from isolated leptons in the ILC frame

② Boost them to their rest frame and calculate thrust axis in this frame (defined as the BB frame in this slide)

③ Boost the vectors along thrust axis to the ILC' frame

(ILC' frame : the frame in which head-on-collision occurs)



Kinematical reconstruction

1. Define the χ^2_μ ;

$$\chi^2_\mu = \chi^2_{\mu^+} + \chi^2_{\mu^-},\quad \chi^2_{\mu^\pm} = \left(\frac{E^{**}_{\mu^\pm}(\theta_t,\phi_t) - m_{W^\pm}/2}{\sigma\left[E^{**}_{\mu^\pm}\right]}\right)^2$$

The energy of $\mu^\pm$ in the $W^\pm$ rest frame, $E^{**}_{\mu^\pm}$, must be equal to $m_{W^\pm}/2$ and it can be written by two parameters (θ_t, ϕ_t).

2. Define the χ^2_b ;

$$\chi^2_b = \left(\frac{E^{meas.}_b - E^{rec.}_b(\theta_t,\phi_{t,})}{\sigma\left[E^{meas.}_b\right]}\right)^2 + \left(\frac{E^{meas.}_{\bar{b}} - E^{rec.}_{\bar{b}}(\theta_t,\phi_t)}{\sigma\left[E^{meas.}_{\bar{b}}\right]}\right)^2$$

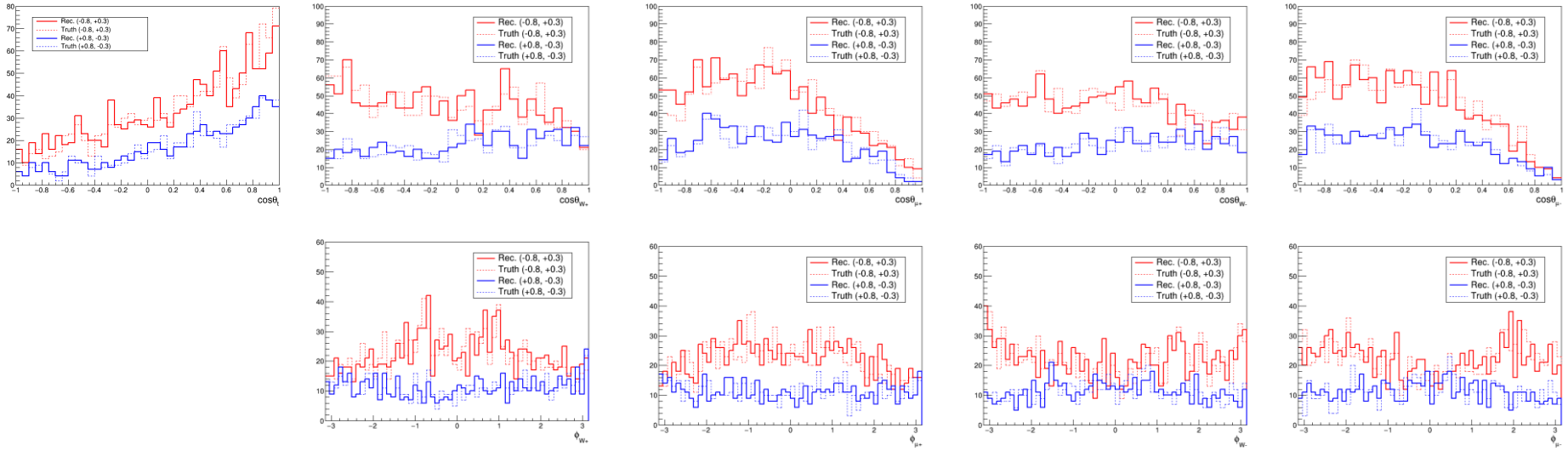
Although the energy of b quarks can be only poorly measured, we can eliminate b-charge ambiguity by comparing the measured energy to the reconstructed energy.

3. Compound χ^2_{tot} ; $\chi^2_{tot} = \chi^2_\mu + \chi^2_b$

One minimizes the χ^2_{tot} to obtain optimal solution of (θ_t, ϕ_t).

Results : Distributions of the 9-angles

$$\cos\theta_t\,,\cos\theta_{W^+}\,,\phi_{W^+},\cos\theta_{\mu^+}\,,\phi_{\mu^+},\cos\theta_{W^-}\,,\phi_{W^-},\cos\theta_{\mu^-}\,,\phi_{\mu^-}$$



- Different between Left and Right polarization

→ These have the information of the polarization of top

Results : Variance matrix of 10 form factors fit

Preliminary (efficiency = ~50 %)

$$
\begin{bmatrix}
\mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1V} & \mathcal{R}e\ \delta\tilde{F}^{Z}_{1V} & \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1A} & \mathcal{R}e\ \delta\tilde{F}^{Z}_{1A} & \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2V} & \mathcal{R}e\ \delta\tilde{F}^{Z}_{2V} & \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2A} & \mathcal{R}e\ \delta\tilde{F}^{Z}_{2A} & \mathcal{I}m\ \delta\tilde{F}^{\gamma}_{2A} & \mathcal{I}m\ \delta\tilde{F}^{Z}_{2A} \\
0.0108 & -0.14 & -0.03 & +0.09 & +0.62 & -0.10 & +0.02 & -0.06 & +0.04 & -0.02 \\
 & 0.0187 & +0.09 & -0.02 & -0.10 & +0.61 & -0.06 & +0.02 & -0.01 & +0.03 \\
 & & 0.0156 & -0.11 & -0.01 & +0.03 & +0.02 & 0 & +0.05 & +0.01 \\
 & & & 0.0246 & -0.01 & 0 & +0.01 & +0.03 & +0.01 & +0.05 \\
 & & & & 0.0317 & -0.17 & +0.03 & -0.12 & 0 & -0.03 \\
 & & & & & 0.0504 & -0.11 & -0.06 & -0.02 & 0 \\
 & & & & & & 0.0211 & -0.17 & -0.04 & +0.01 \\
 & & & & & & & 0.0360 & -0.01 & -0.45 \\
 & & & & & & & & 0.0206 & -0.13 \\
 & & & & & & & & & 0.0297
\end{bmatrix}
$$

Results : No cut & Loose cut & Tight cut

No cut (efficiency = ~92 %)

Left 8.9% Right 6.0%

$$\begin{bmatrix} \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1V} & -0.0054 \pm 0.0081 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{1V} & -0.0228 \pm 0.0142 \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1A} & -0.0421 \pm 0.0117 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{1A} & +0.0587 \pm 0.0185 \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2V} & -0.0681 \pm 0.0233 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{2V} & -0.0129 \pm 0.0387 \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2A} & +0.0179 \pm 0.0153 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{2A} & -0.0207 \pm 0.0274 \\ \mathcal{I}m\ \delta\tilde{F}^{\gamma}_{2A} & -0.0287 \pm 0.0150 \\ \mathcal{I}m\ \delta\tilde{F}^{Z}_{2A} & -0.0011 \pm 0.0218 \end{bmatrix}$$

Loose cut (efficiency = ~80 %)

Left 8.1% Right 5.5%

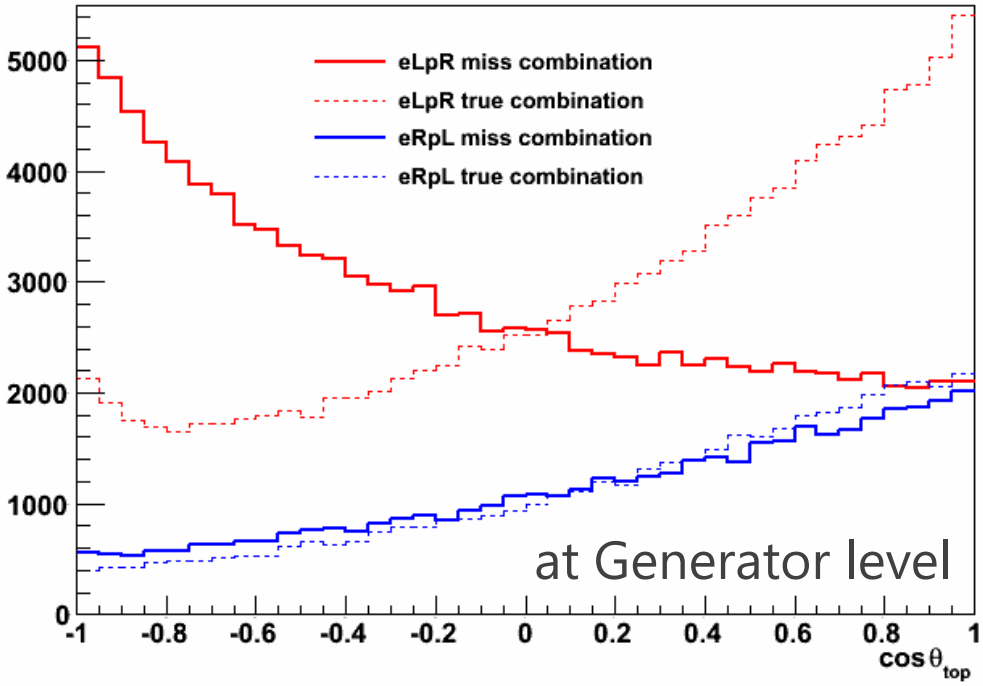
$$\begin{bmatrix} \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1V} & -0.0047 \pm 0.0088 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{1V} & -0.0236 \pm 0.0154 \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1A} & -0.0460 \pm 0.0126 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{1A} & +0.0631 \pm 0.0198 \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2V} & -0.0669 \pm 0.0253 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{2V} & -0.0206 \pm 0.0417 \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2A} & +0.0011 \pm 0.0160 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{2A} & -0.0370 \pm 0.0283 \\ \mathcal{I}m\ \delta\tilde{F}^{\gamma}_{2A} & -0.0143 \pm 0.0163 \\ \mathcal{I}m\ \delta\tilde{F}^{Z}_{2A} & -0.0110 \pm 0.0237 \end{bmatrix}$$

Tight cut (efficiency = ~50 %)

Left 5.5% Right 3.0%

$$\begin{bmatrix} \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1V} & -0.0015 \pm 0.0108 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{1V} & -0.0271 \pm 0.0187 \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1A} & -0.0314 \pm 0.0156 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{1A} & +0.0277 \pm 0.0246 \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2V} & -0.0266 \pm 0.0317 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{2V} & -0.0702 \pm 0.0504 \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2A} & -0.0082 \pm 0.0211 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{2A} & -0.0164 \pm 0.0360 \\ \mathcal{I}m\ \delta\tilde{F}^{\gamma}_{2A} & -0.0427 \pm 0.0206 \\ \mathcal{I}m\ \delta\tilde{F}^{Z}_{2A} & +0.0220 \pm 0.0297 \end{bmatrix}$$

True and Miss pairing of Wb



Right-handed electron case (eRpL), Blue line

Little difference between true and miss associated distributions

Left-handed electron case (eLpR), Red line

Miss association changes angular distribution significantly

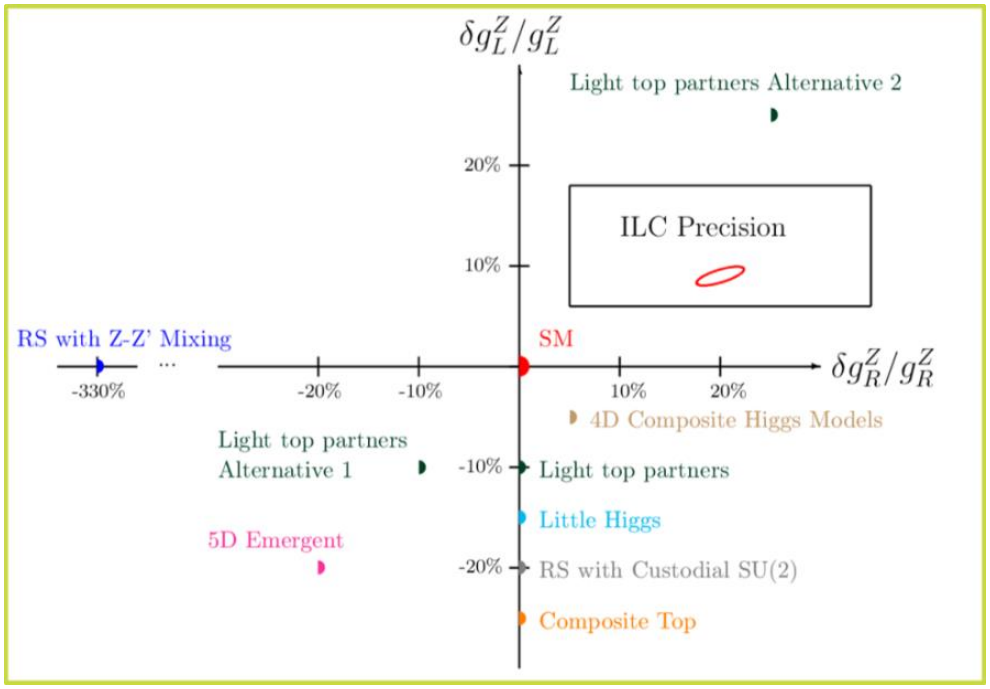
→**The migration effect happens in only the case of eLpR**

The $t\bar{t}Z^0/\gamma$ couplings : CP-conserving

$$\mathcal{L}_{\rm int} = \sum_{v=\gamma,Z} g^v \left[V_l^v \bar{t}\gamma^l (F_{1V}^v + F_{1A}^v \gamma_5) t + \frac{i}{2m_t} \partial_\nu V_l \bar{t}\sigma^{l\nu} (F_{2V}^v + F_{2A}^v \gamma_5) t \right]$$

$$g_L^Z = F_{1V}^Z - F_{1A}^Z, \qquad g_R^Z = F_{1V}^Z + F_{1A}^Z$$

- Deviation from the SM of g_L^Z, g_R^Z will be typically 10% in composite models



arXiv:1505.06020 [hep-ph]

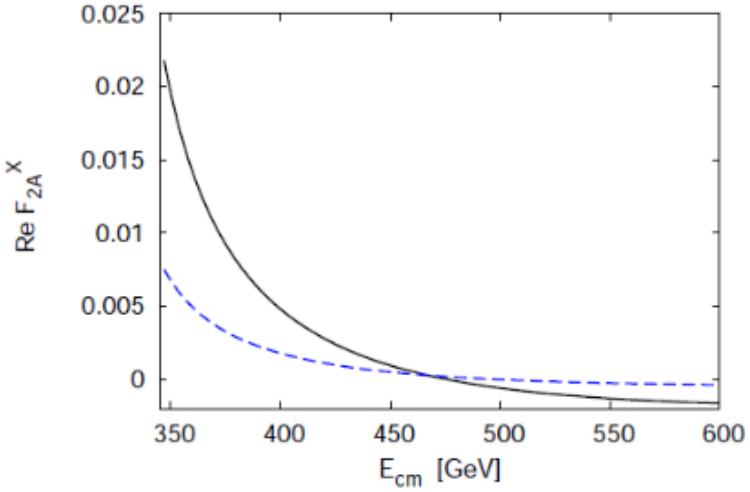
The $t\bar{t}Z^0/\gamma$ couplings : CP-violating

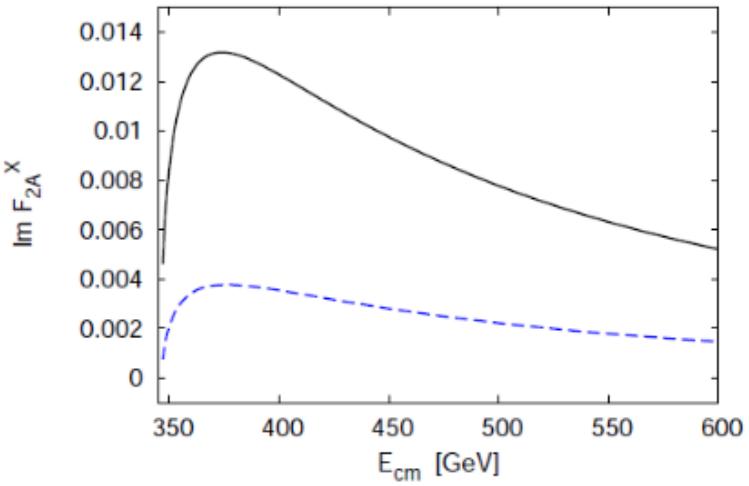
$$\mathcal{L}_{\text{int}} = \sum_{v=\gamma,Z} g^v \left[V_l^v \bar{t}\gamma^l (F_{1V}^v + F_{1A}^v \gamma_5) t + \frac{i}{2m_t} \partial_\nu V_l \bar{t}\sigma^{l\nu} (F_{2V}^v + F_{2A}^v \gamma_5) t \right]$$

- F_{2A}^{γ} is the electric dipole moment which is forbidden in the SM

→ Probes for CP-violating beyond the Kobayashi-Maskawa mechanism

eg) 2HDM can yield ~0.01





M. Vos

@LCWS2016, Iwate

Kinematical Reconstruction : True pairing

- Intersections of green and red lines are solutions in terms of (θ_t, ϕ_t)
- Blue line shows the comparison of the $E_b, E_{\bar{b}}$ between rec. and meas.

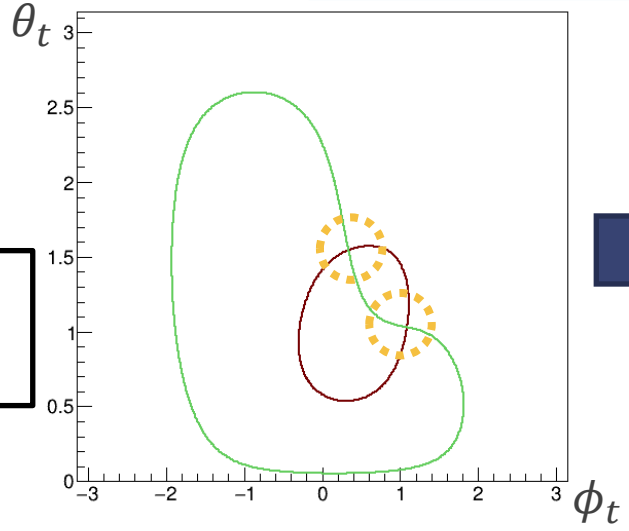
→ The optimal solution is selected obtained

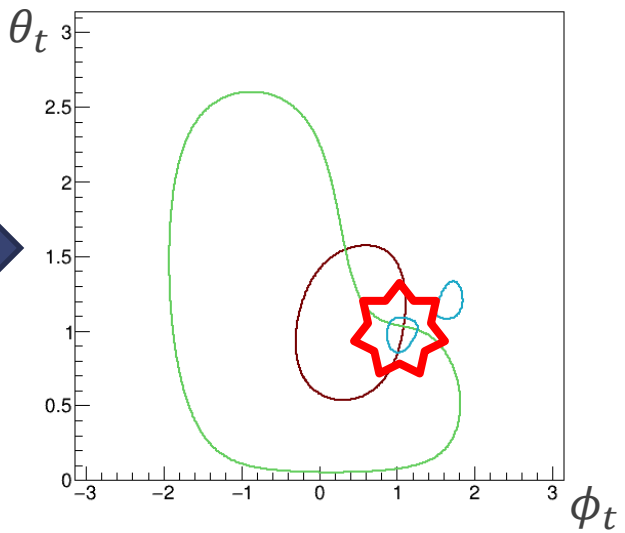
$$E^{**}_{\mu^+}(\theta_t, \phi_t) = \frac{m_{W^+}}{2}$$

$$E^{**}_{\mu^-}(\theta_t, \phi_t) = \frac{m_{W^-}}{2}$$

The energy of $\mu^{\pm}$ in $W^{\pm}$ frame is equal to $m_{W^{\pm}}/2$

$$\chi^2_{\bar{b}} = \left(\frac{E_b^{meas.} - E_b^{rec.}(\theta_t, \phi_t)}{\sigma[E_b^{meas.}]}\right)^2 + \left(\frac{E_{\bar{b}}^{meas.} - E_{\bar{b}}^{rec.}(\theta_t, \phi_t)}{\sigma[E_{\bar{b}}^{meas.}]}\right)^2 = 20$$





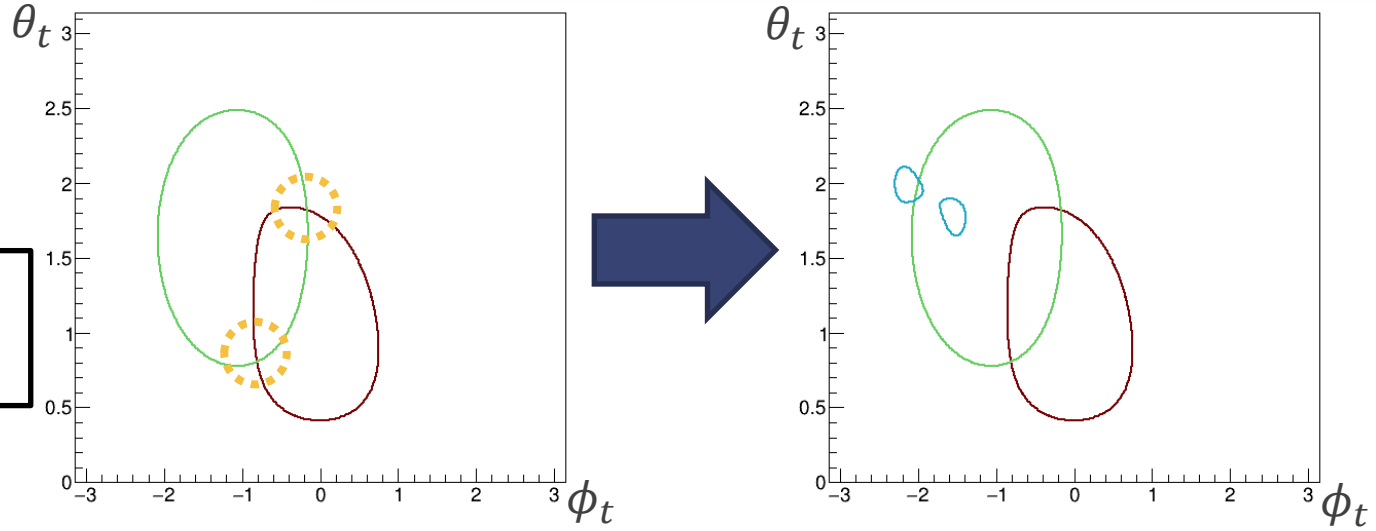
Kinematical Reconstruction : Miss pairing

- Blue line is far from intersections of green and red line

→Miss pairing is excluded by the $E_b, E_{\bar{b}}$ comparision.

$$E^{**}_{\mu^+}(\theta_t, \phi_t) = \frac{m_{W^+}}{2}$$

$$E^{**}_{\mu^-}(\theta_t, \phi_t) = \frac{m_{W^-}}{2}$$

The energy of $\mu^{\pm}$ in $W^{\pm}$ frame is equal to $m_{W^{\pm}}/2$

$$\chi^2_{\bar{b}} = \left(\frac{E_b^{meas.} - E_b^{rec.}(\theta_t, \phi_t)}{\sigma[E_b^{meas.}]}\right)^2 + \left(\frac{E_{\bar{b}}^{meas.} - E_{\bar{b}}^{rec.}(\theta_t, \phi_t)}{\sigma[E_{\bar{b}}^{meas.}]}\right)^2 = 20$$