SUSY, naturalness, simplicity and falsifiability: why ILC must be built

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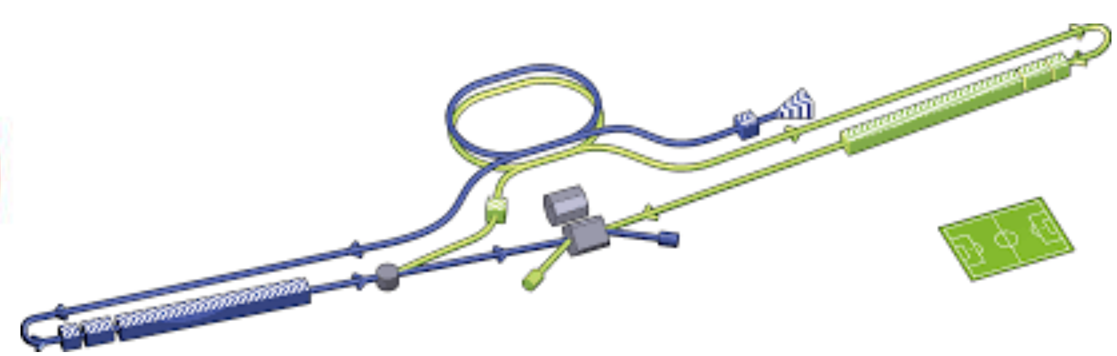
Why

SUSY  

The hypothesis of weak scale SUSY
(that nature is supersymmetric with SUSY breaking at or around the weak scale)
is remarkably simple and solves a host of problems

- SUSY- extension of Poincare group to its most general structure: super-Poincare
- scalar field quadratic divergences cancel thus stabilizing the weak scale: potentially solves SM naturalness problem
- local SUSY: supergravity
- the vague prediction: superpartners around the weak scale

In spirit of Karl Popper, any scientific hypothesis must be falsifiable

SUSY has already met 3 tests:

- measured gauge coupling strengths consistent with SUSY unification
- $m(t)\sim 173$ GeV consistent with SUSY requirement for (radiative) breakdown of EW symmetry
- $m(h)\sim 125$ GeV in accord with narrow MSSM requirement that $m(h)<135$ GeV
- BUT where are the sparticles? And where ought they to be?

The main raison d'etre for SUSY is to address the naturalness question:
works admirably by eliminating quadratic divergences to m(h):
BUT if sparticles too heavy, then re-introduce hierarchy problem in form of
Little Hierarchy: why is m(h)~125 GeV and not m(sparticle)~1-10 TeV?

Working definition of naturalness:

An observable $\mathcal{O}$ is natural if all *independent* contributions to $\mathcal{O} = a_1 + \cdots a_n$ are comparable to or less than $\mathcal{O}$

Or else, if one contribution, say $a_1 \gg \mathcal{O}$, then some other (independent) contribution would have to be *fine-tuned* to a large opposite-sign value to compensate and maintain $\mathcal{O}$ at its measured value

``The appearance of fine-tuning in a scientific theory is like a cry of distress from nature complaining that something needs to be better explained"

EW naturalness: why are m(W,Z,h)~100 GeV
while m(sparticles)~>1 TeV?

$$\text{Let } \mathcal{O} \equiv m_Z^2$$

EW minimization conditions relate m(Z) to SUSY Lagrangian parameters

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2+\Sigma_d^d-(m_{H_u}^2+\Sigma_u^u)\tan^2\beta}{\tan^2\beta-1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u - \mu^2$$

For naturalness:

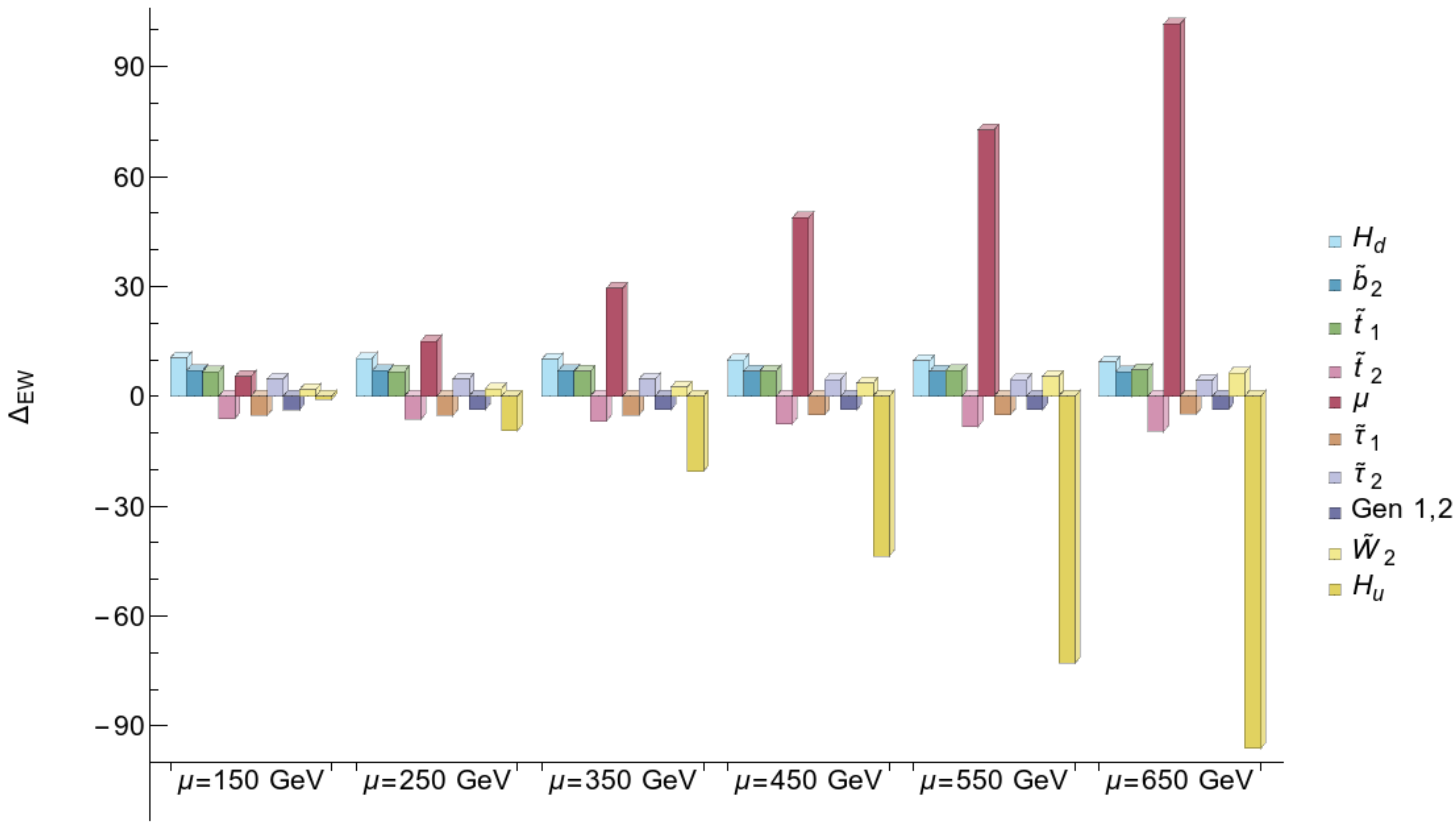
- $m_{H_u}^2$ driven to $\sim -(100-200)^2$ GeV^2 at weak scale
- superpotential Higgs/higgsino mass contribution $\mu \sim 100-200$ GeV
- TeV scale highly mixed top squarks minimize Σ_u^u (and raise $m_h \sim 125$ GeV)

Chan, Chattopadyaya, Nath
HB, Barger, Huang
Perelstein, Shakya
HB, Barger, Huang, Mustafayev, Tata

Low value of $\Delta_{EW} \equiv |max\ each\ term\ on\ RHS|$ is most conservative, unavoidable naturalness condition

Most important inference:
light higgsinos of mass mu~100-200 GeV hard to see at LHC
but easily discovered at ILC with $\sqrt{s} > 2m(higgsino) \sim 200-500$ GeV!
see next talk by Jackie Yan!

How much is too much fine-tuning?



HB, Barger, Savoy

Visually, large fine-tuning has already developed by $\mu \sim 350$ or $\Delta_{EW} \sim 30$

<u>Nature is natural</u> $\Rightarrow \Delta_{EW} < 20 - 30$ (take 30 as conservative)

#3. What about EENZ/BG measure?

$$\Delta_{BG} = max_i \left| \frac{\partial \log m_Z^2}{\partial \log p_i} \right| = max_i \left| \frac{p_i}{m_Z^2} \frac{\partial m_Z^2}{\partial p_i} \right|$$

p_i are the theory parameters

applied to pMSSM, then $\Delta_{BG} \simeq \Delta_{EW}$

apply to high (e.g. GUT) scale parameters

$$\begin{aligned} m_Z^2 \simeq\ & -2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\ & +0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\ & -0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b \\ & -1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\ & +0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\ & +0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\ & +0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2, \end{aligned}$$

applied to most parameters,

Δ_{BG} large, looks fine-tuned for *e.g.* $m_{\tilde{g}} \simeq M_3 > 1.8$ TeV

$$\Delta_{BG}(M_3^2) = 3.84\frac{M_3^2}{m_z^2} \simeq 1500$$

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What if we apply to high (e.g. GUT) scale parameters ?

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For correlated scalar masses $\equiv m_0$,
scalar contribution collapses:
what looks fine-tuned isn't: *focus point SUSY*
multi-TeV scalars are *natural*

Feng, Matchev, Moroi

But wait! in more complete models, soft terms not independent

violates prime directive!

e.g. in SUGRA, for well-specified hidden sector, each soft term calculated as multiple of m(3/2); soft terms must be combined!

e.g. dilaton-dominated SUSY breaking: $m_0^2 = m_{3/2}^2$ with $m_{1/2} = -A_0 = \sqrt{3}m_{3/2}$

$$\begin{aligned} m_{H_u}^2 &= a_{H_u} \cdot m_{3/2}^2, \\ m_{Q_3}^2 &= a_{Q_3} \cdot m_{3/2}^2, \\ A_t &= a_{A_t} \cdot m_{3/2}, \\ M_i &= a_i \cdot m_{3/2}, \\ & \cdots . \end{aligned}$$

since μ hardly runs, then

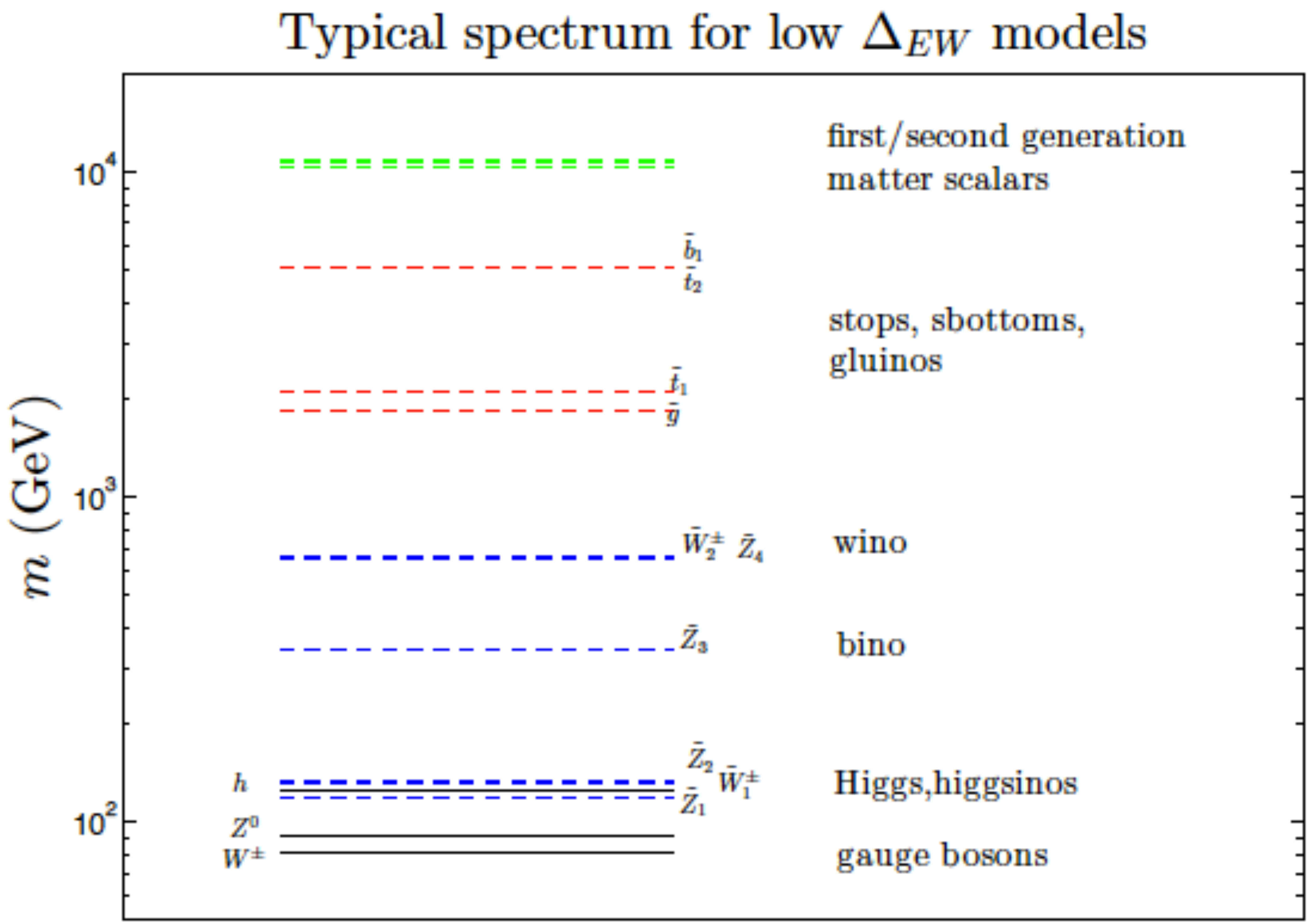
$$\boxed{\begin{aligned} m_Z^2 &\simeq -2\mu^2 + a \cdot m_{3/2}^2 \\ &\simeq -2\mu^2 - 2m_{H_u}^2(weak) \end{aligned}}$$

$$m_{H_u}^2(weak) \sim -(100-200)^2 \text{ GeV}^2 \sim -a \cdot m_{3/2}^2/2$$

using μ^2 and $m_{3/2}^2$ as fundamental,
then $\Delta_{BG} \simeq \Delta_{EW}$ even using high scale parameters!

bounds from naturalness (3%)	BG/DG	Delta_EW
mu	350 GeV	0.35 TeV
gluino	400-600 GeV	5-6 TeV
t1	450 GeV	3 TeV
sq/sl	550-700 GeV	10-30 TeV

h(125) and LHC limits are perfectly compatible with 3-10% naturalness: no crisis for SUSY!

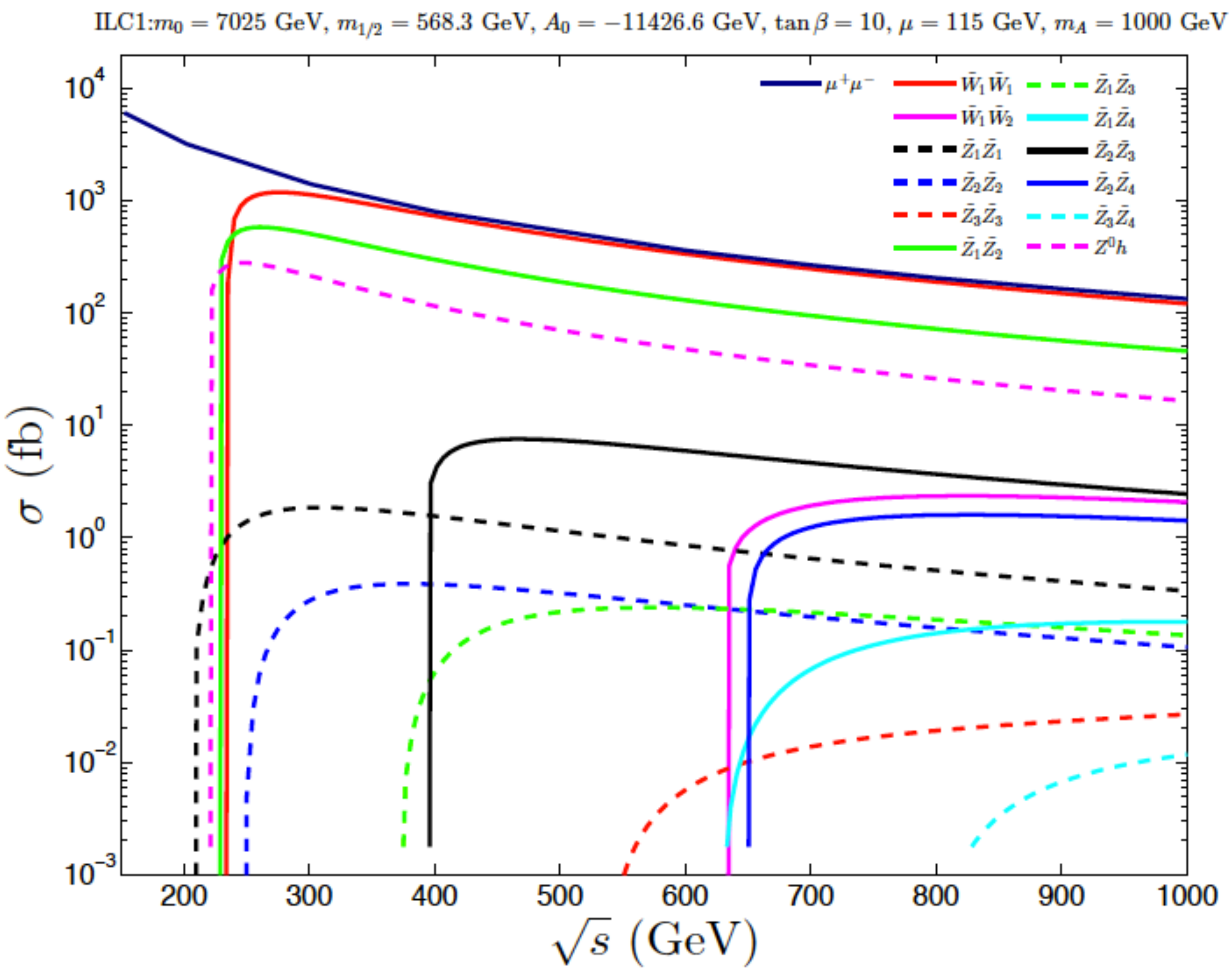


There is a Little Hierarchy, but it is no problem

$$\mu \ll m_{3/2}$$

Smoking gun signature: light higgsinos at ILC:

ILC is Higgs/higgsino factory!



$$\sigma(higgsino) \gg \sigma(Zh)$$

10-15 GeV higgsino mass
gaps no problem
in clean ILC environment

HB, Barger, Mickelson, Mustafayev, Tata
arXiv:1404:7510

ILC either sees light higgsinos or MSSM dead

Why might mu<<m(3/2)?

SUSY mu problem: mu term is SUSY, not SUSY breaking: expect mu~M(Pl) but phenomenology requires mu~m(Z)

- NMSSM: mu~m(3/2); beware singlets!
- Giudice-Masiero: mu forbidden by some symmetry: generate via Higgs coupling to hidden sector
- Kim-Nilles: invoke SUSY version of DFSZ axion solution to strong CP:

KN: PQ symmetry forbids mu term, but then it is generated via PQ breaking

$$\mu \sim \lambda f_a^2/M_P$$

$$m_{3/2} \sim m_{hid}^2/M_P$$

Little Hierarchy due to mismatch between PQ breaking and SUSY breaking scales?

$$f_a \ll m_{hid}$$

Higgs mass tells us where to look for axion!

$$m_a \sim 6.2\mu\text{eV}\left(\frac{10^{12}\ \text{GeV}}{f_a}\right)$$

Little Hierarchy from radiative PQ breaking? exhibited within context of MSY model

Murayama, Suzuki, Yanagida (1992);
Gherghetta, Kane (1995)

Choi, Chun, Kim (1996)

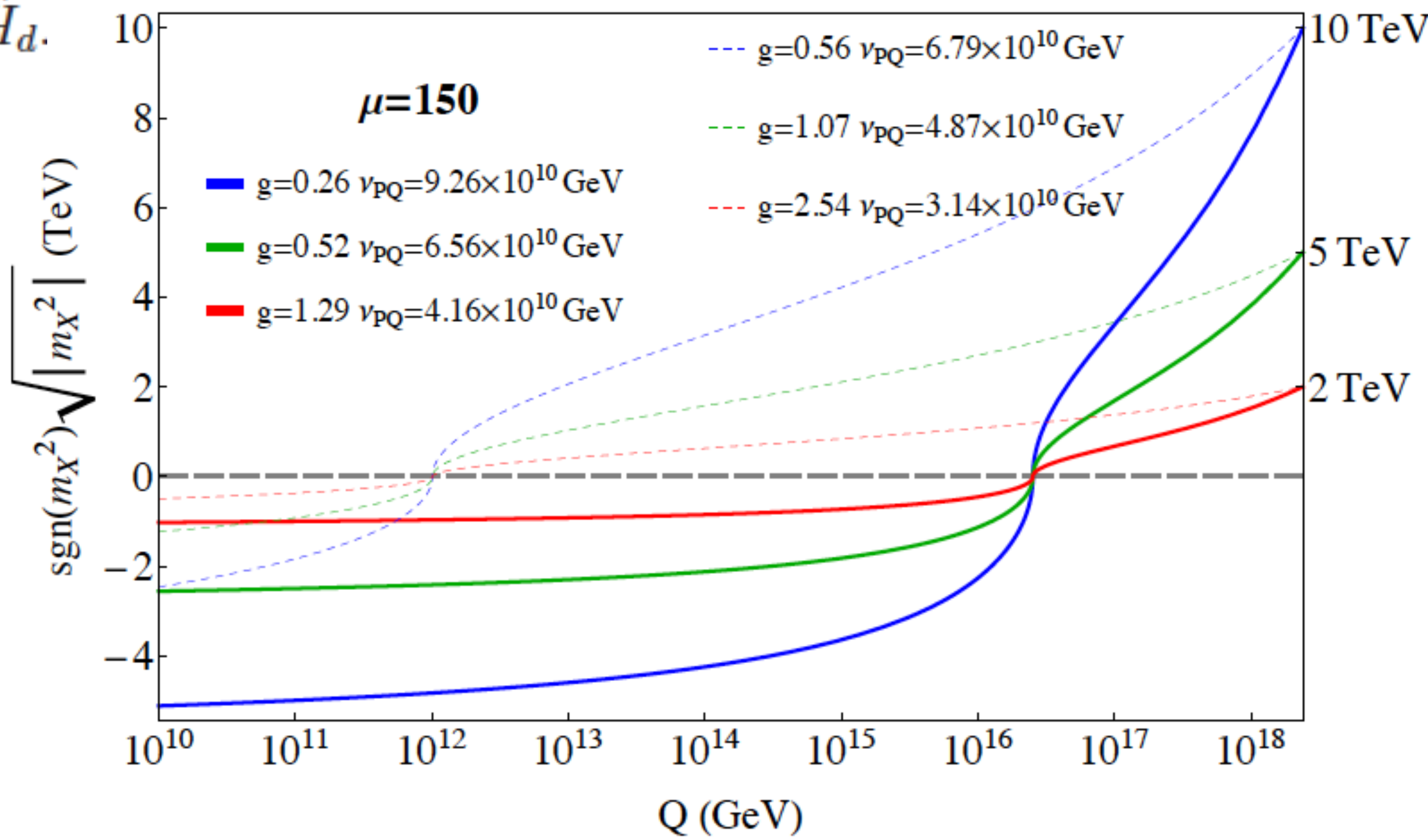
Bae, HB, Serce, PRD91 (2015) 015003

augment MSSM with PQ charges/fields:

$$\hat{f}' = \frac{1}{2} h_{ij} \hat{X} \hat{N}_i^c \hat{N}_j^c + \frac{f}{M_P} \hat{X}^3 \hat{Y} + \frac{g}{M_P} \hat{X} \hat{Y} \hat{H}_u \hat{H}_d.$$

$$M_{N_i^c} = v_X \, h_i|_{Q=v_X}$$

$$\mu = g \frac{v_X v_Y}{M_P} .$$



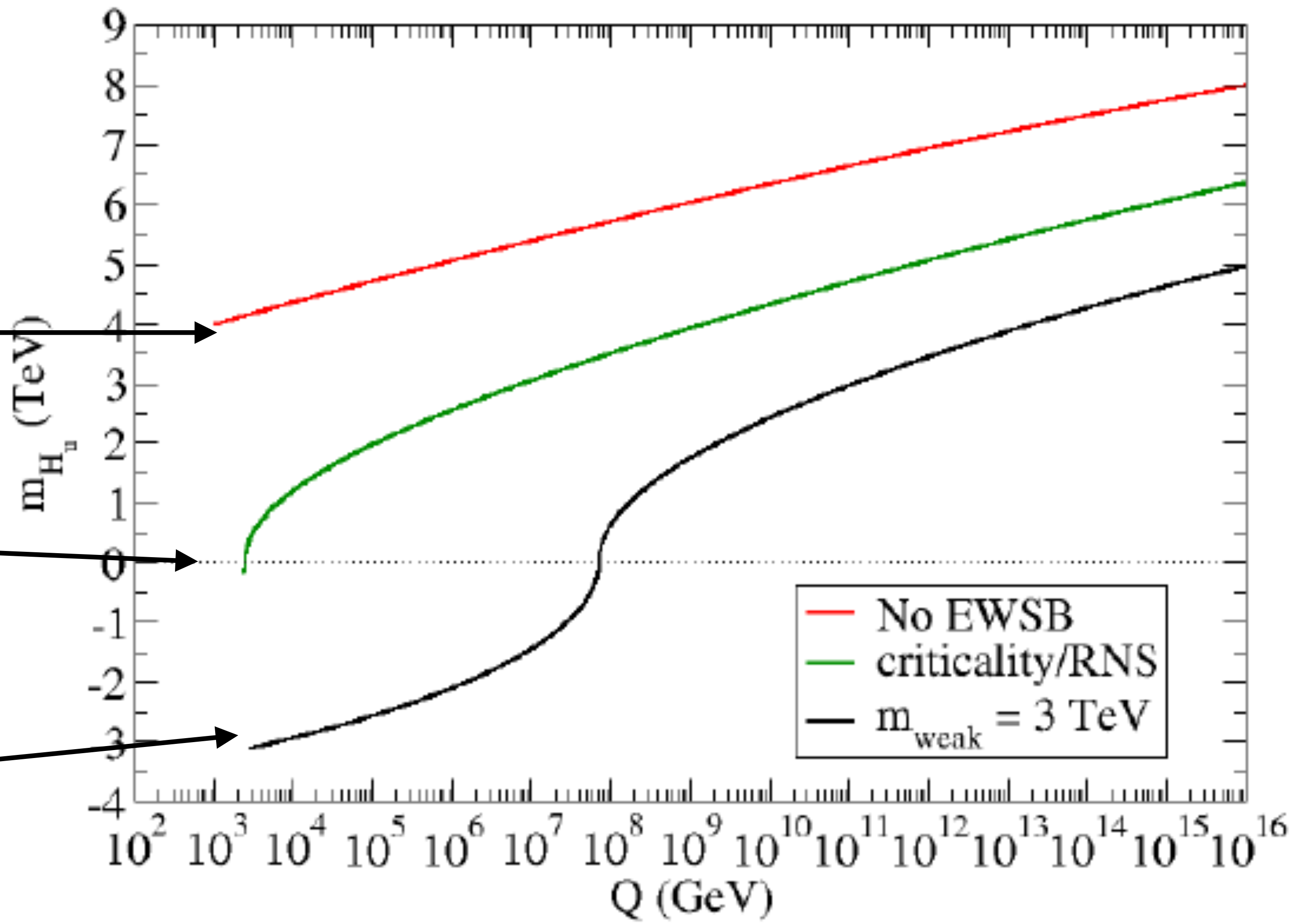
Large $m_{3/2}$ generates small $\mu \sim 100 - 200$ GeV!

radiative corrections drive $m^2_{H_u}$ from unnatural GUT scale values to naturalness at weak scale: radiatively-driven naturalness

EWS not broken

natural: EWS is barely broken

unnatural

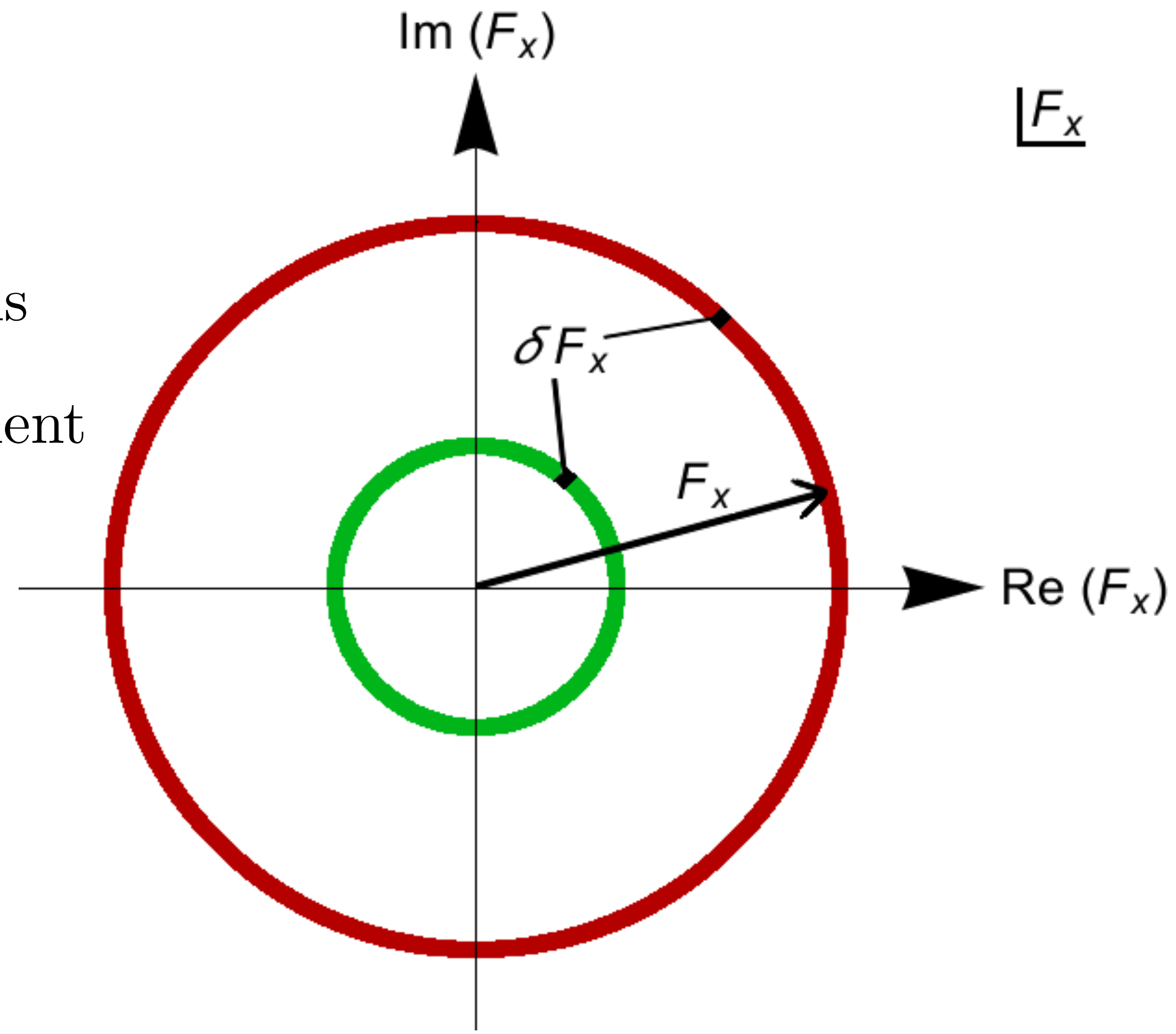


Evolution of the soft SUSY breaking mass squared term $sign(m^2_{H_u})\sqrt{|m^2_{H_u}|}$ vs. Q

Why do soft terms take on values needed for natural (barely-broken) EWSB?

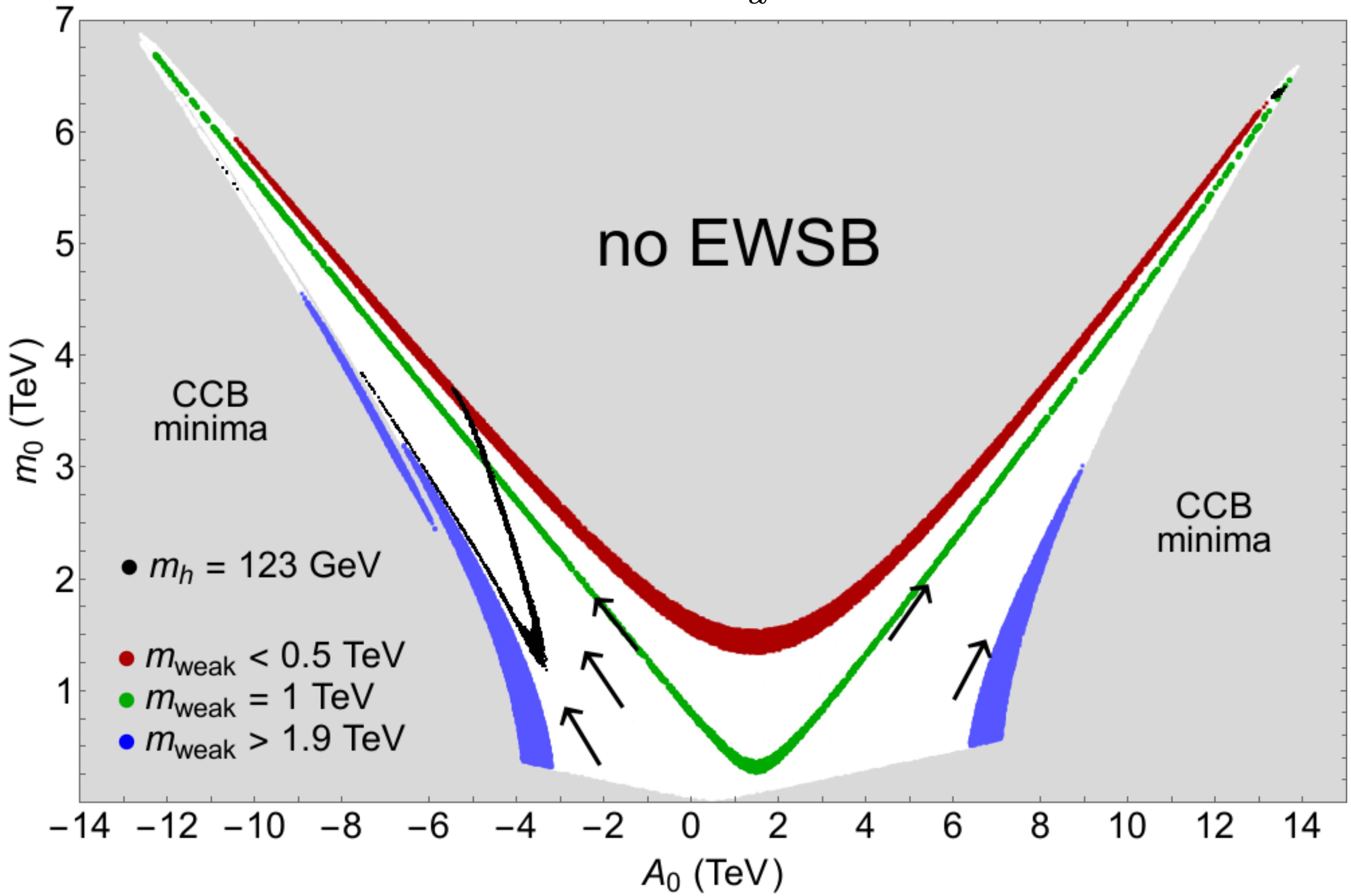
string theory landscape?

- assume model like MSY/CCK where $\mu \sim 100$ GeV
- then $m(weak)^2 \sim |m^2_{H_u}|$
- If all values of SUSY breaking field $\langle F_X \rangle$ equally likely, then mild (linear) statistical draw towards large soft terms
- This is balanced by anthropic requirement of weak scale $m_{weak} \sim 100$ GEV



Anthropic selection of $m_{weak} \sim 100\ GeV$:
If m_W too large, then weak interactions $\sim (1/m_W^4)$ too weak
weak decays, fusion reactions suppressed
elements not as we know them

$$m_{H_u} = 1.3 m_0$$

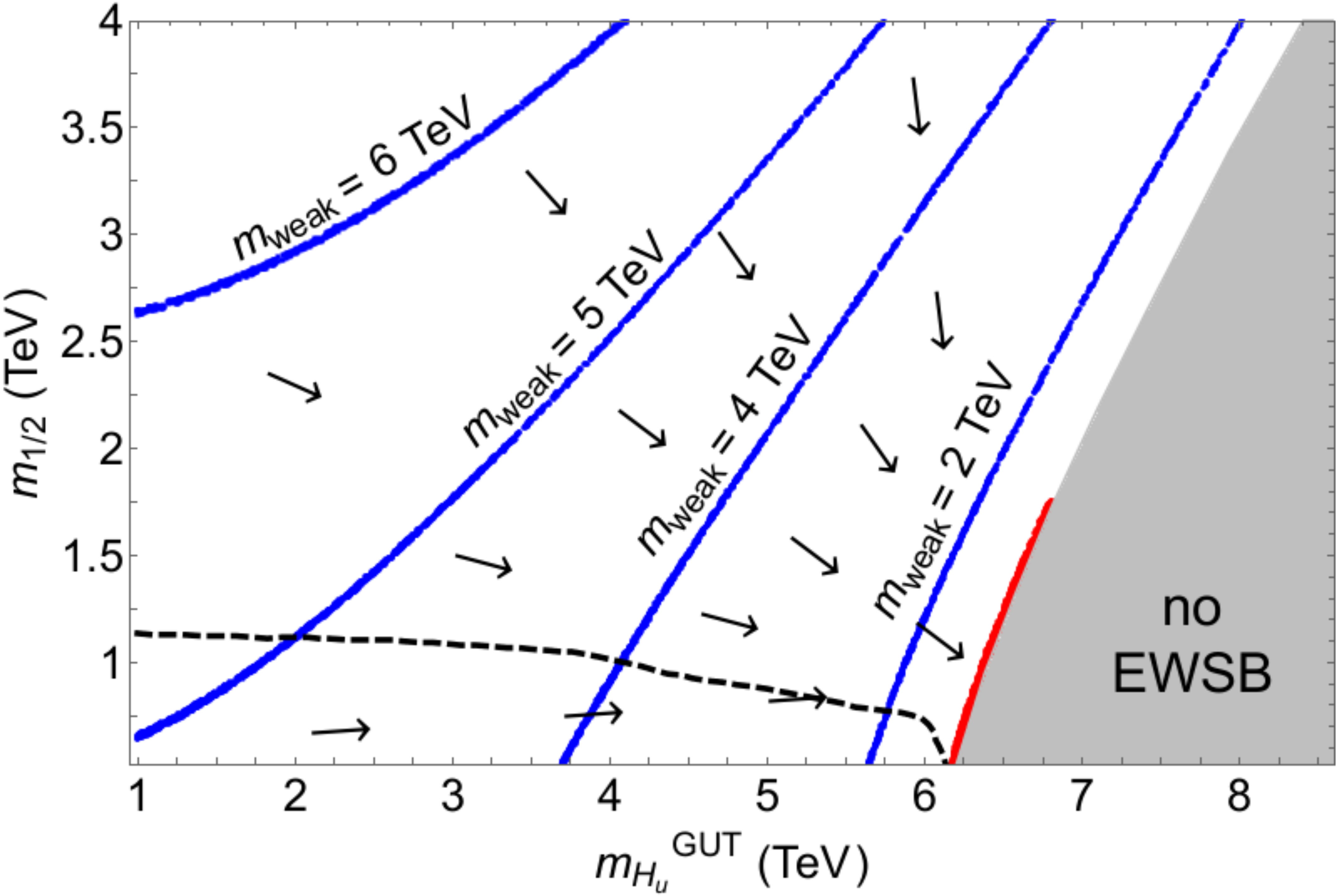


statistical draw to large soft terms balanced by anthropic draw toward red (m(weak)~100 GeV): then m(Higgs)~125 GeV and natural SUSY spectrum!

Giudice, Rattazzi, 2006

HB, Barger, Savoy, Serce, PLB758 (2016) 113

$m_0 = 5\ \mathrm{TeV}$



statistical/anthropic draw toward FP-like region

Mirage mediation:
comparable moduli- & anomaly-mediation

Choi, Falkowski, Nilles, Olechowski, Pokorski

Generalized mirage mediation model:

HB, Barger, Serce, Tata: arXiv:1610.06205

$$M_a = (\alpha + b_a g_a^2)\, m_{3/2}/16\pi^2, \quad (10)$$
$$A_\tau = (-a_3\alpha + \gamma_{L_3} + \gamma_{H_d} + \gamma_{E_3})\, m_{3/2}/16\pi^2, \quad (11)$$
$$A_b = (-a_3\alpha + \gamma_{Q_3} + \gamma_{H_d} + \gamma_{D_3})\, m_{3/2}/16\pi^2, \quad (12)$$
$$A_t = (-a_3\alpha + \gamma_{Q_3} + \gamma_{H_u} + \gamma_{U_3})\, m_{3/2}/16\pi^2, \quad (13)$$
$$m_i^2(1,2) = (c_m\alpha^2 + 4\alpha\xi_i - \dot{\gamma}_i)\, (m_{3/2}/16\pi^2)^2, \quad (14)$$
$$m_j^2(3) = (c_{m3}\alpha^2 + 4\alpha\xi_j - \dot{\gamma}_j)\, (m_{3/2}/16\pi^2)^2, \quad (15)$$
$$m_{H_u}^2 = (c_{H_u}\alpha^2 + 4\alpha\xi_{H_u} - \dot{\gamma}_{H_u})\, (m_{3/2}/16\pi^2)^2, \quad (16)$$
$$m_{H_d}^2 = (c_{H_d}\alpha^2 + 4\alpha\xi_{H_d} - \dot{\gamma}_{H_d})\, (m_{3/2}/16\pi^2)^2, \quad (17)$$

elevate a_3, c_m, c_{m3}, c_{H_u}, c_{H_d} from discrete to continuous:
soft terms depend on location of fields in compactified manifold!

p-space:

$$\alpha,\ m_{3/2},\ c_m,\ c_{m3},\ a_3,\ c_{H_u},\ c_{H_d},\ \tan\beta \quad (GMM)$$
$$\alpha,\ m_{3/2},\ c_m,\ c_{m3},\ a_3,\ \tan\beta, \mu,\ m_A \quad (GMM'). \qquad \Leftarrow$$

allows for natural mirage mediation

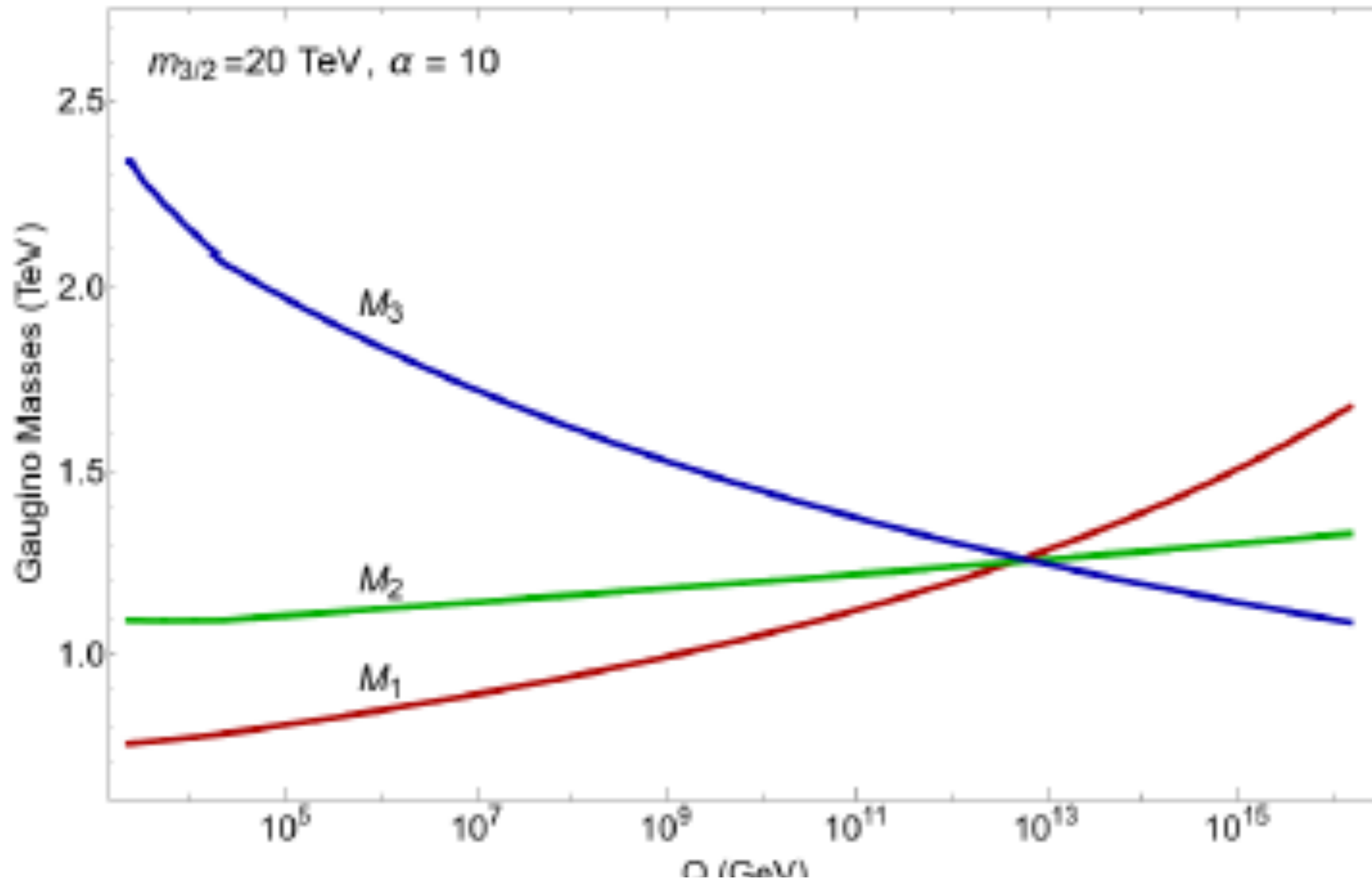
Allows to generate mini-landscape spectra

Lebedev, Nilles, Raby, Ramos-Sanches, Ratz, Vaudrevange

but with radiatively-driven naturalness

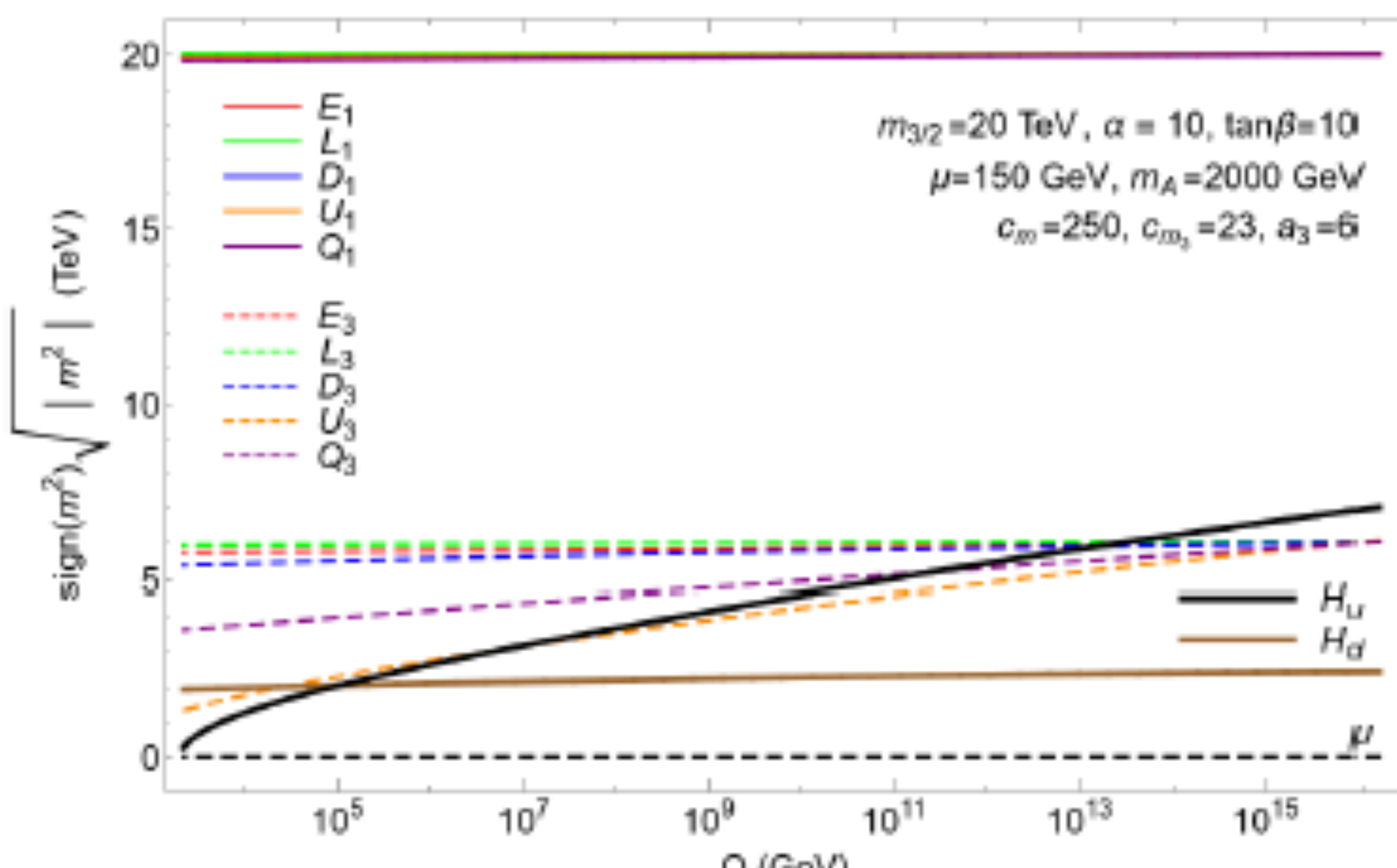
HB, Barger, Savoy, Serce, Tata, arXiv:1705.01578

- Look for fertile patch of landscape giving MSSM
- 1,2 gen lives on orbifold fixed points/tori: in 16 of SO(10)
- 3rd gen, Higgs, gauge live more in bulk: split multiplets
- m(1,2)~m(3/2)~10-30 TeV
- m(3)~m(H)~A's~m(inos)~1-3 TeV
- soft terms that of mirage mediation
- programmed Isajet 7.86



m3/2 =20 TeV, α = 10
M3
M2
M1
Gaugino Masses (TeV)
Q (GeV)

$$\Delta_{EW} = 17.6$$



E1
L1
D1
U1
Q1
E3
L3
D3
U3
Q3
m3/2 =20 TeV, α = 10, tanβ=10
μ=150 GeV, mA =2000 GeV
a3 =6
Hu
Hd
μ
Q (GeV)

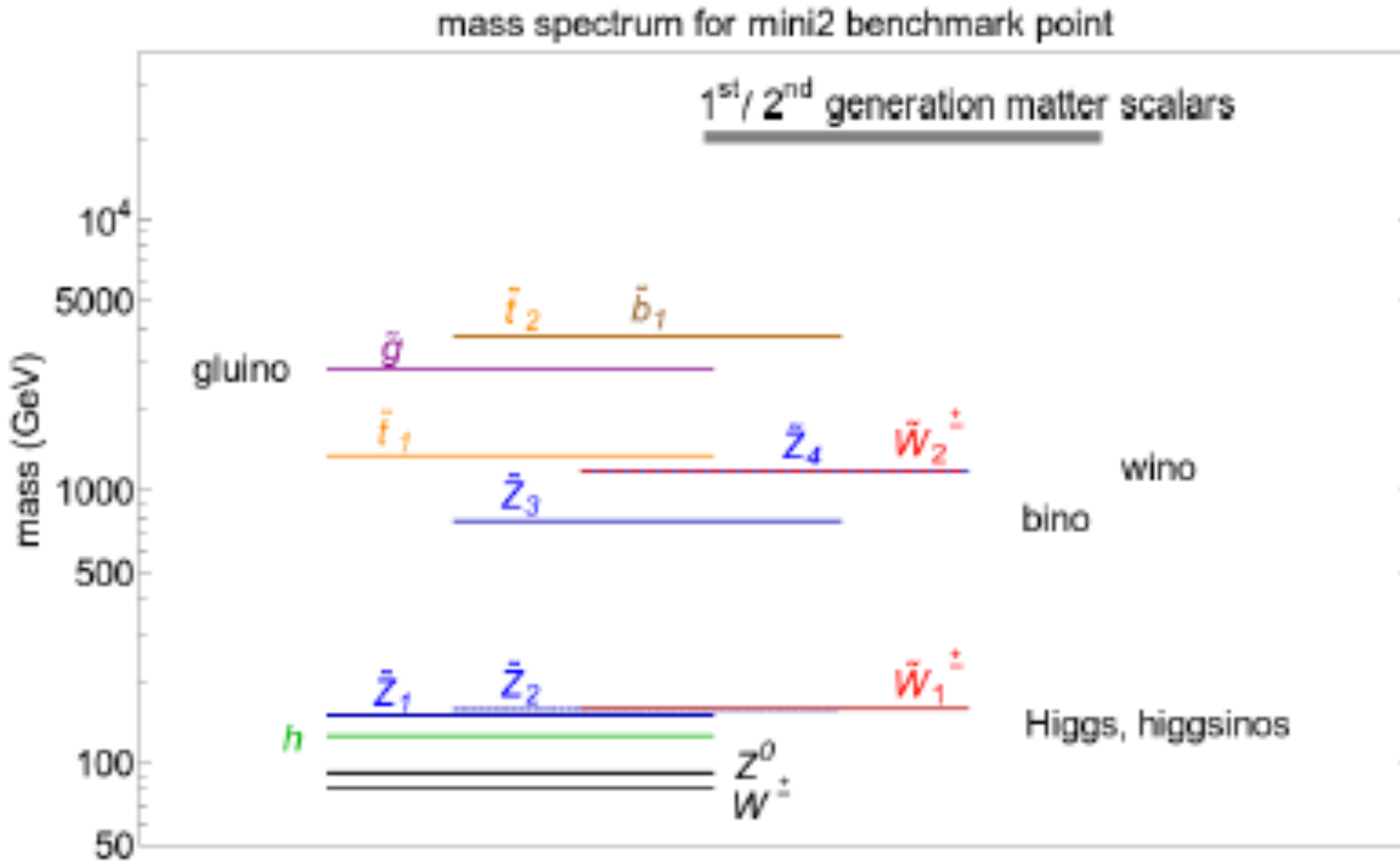


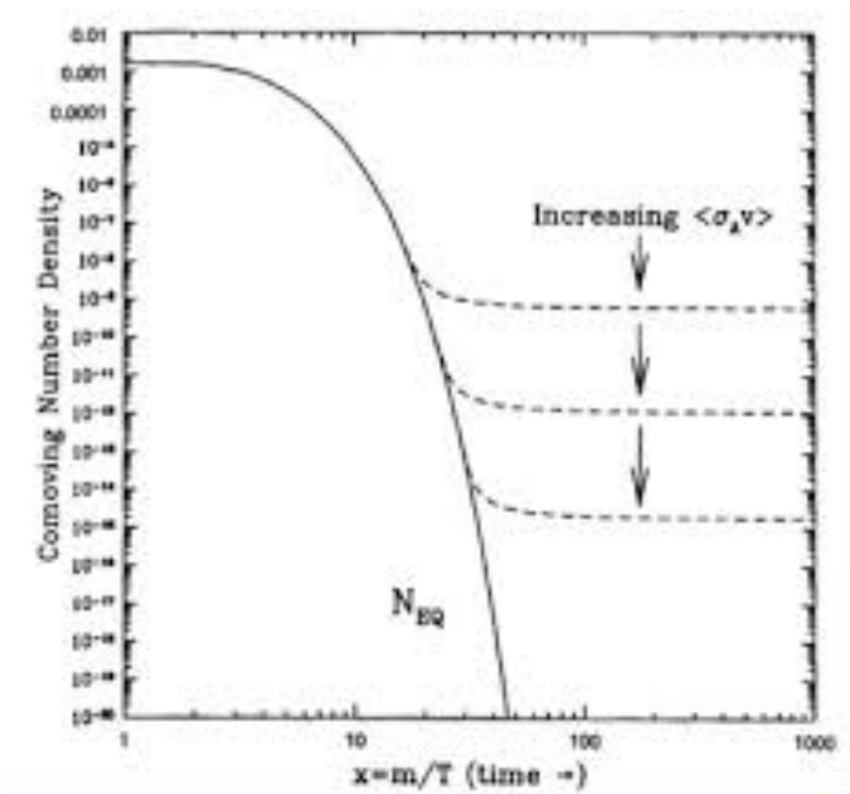
Figure 7: The superparticle mass spectra from the natural mini-landscape point mini2 of Table 1.

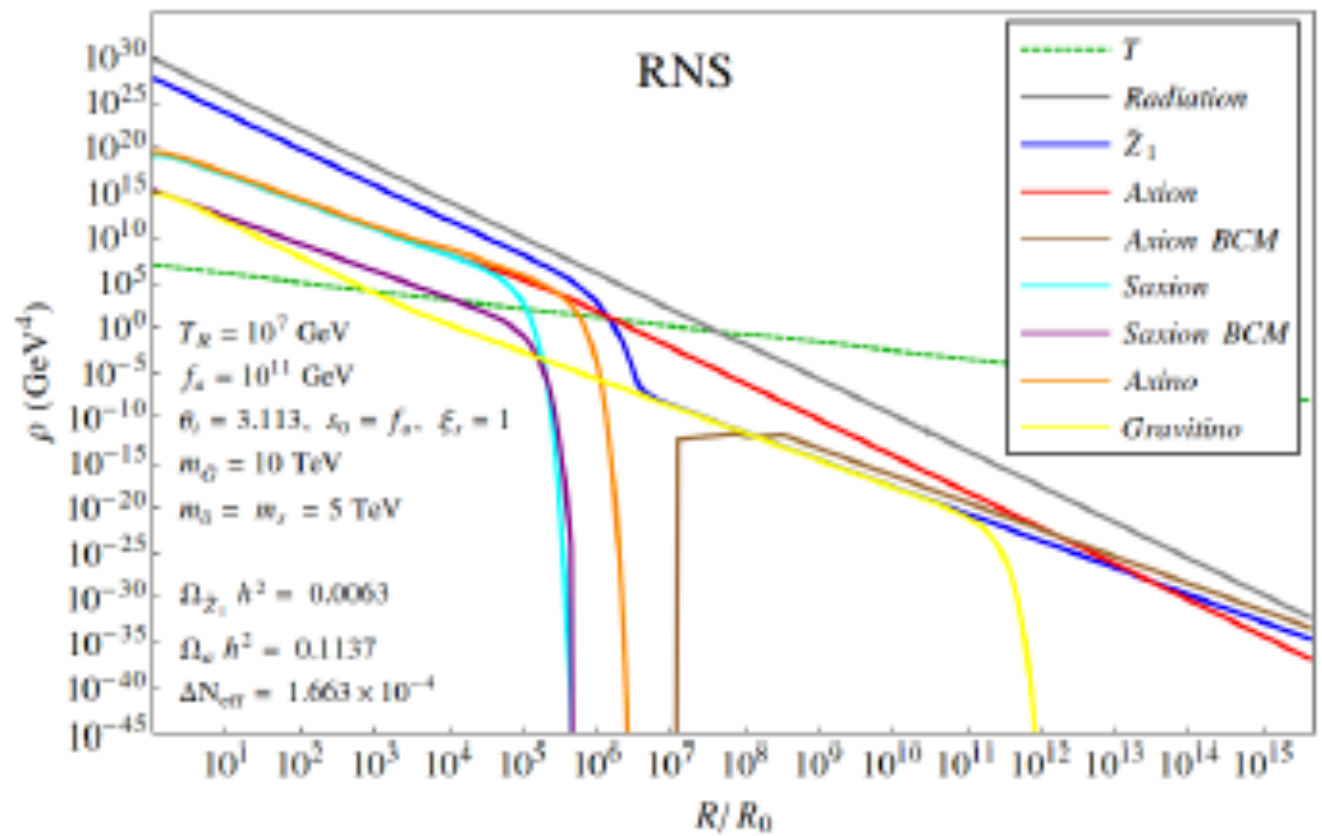
Due to compressed gaugino spectra, minilandscape
can probably hide from HL-LHC while maintaining naturalness

What happens to SUSY WIMP dark matter?

- higgsino-like WIMPs thermally underproduced
- 3 not four light pions => QCD theta vacuum
- EDM(neutron) => axions: no fine-tuning in QCD sector
- SUSY context: axion superfield, axinos and saxions
- DM= axion+higgsino-like WIMP admixture
- DFSZ SUSY axion: solves mu problem with mu<< m_3/2!
- ultimately detect both WIMP and axion?

usual picture => mixed axion/WIMP

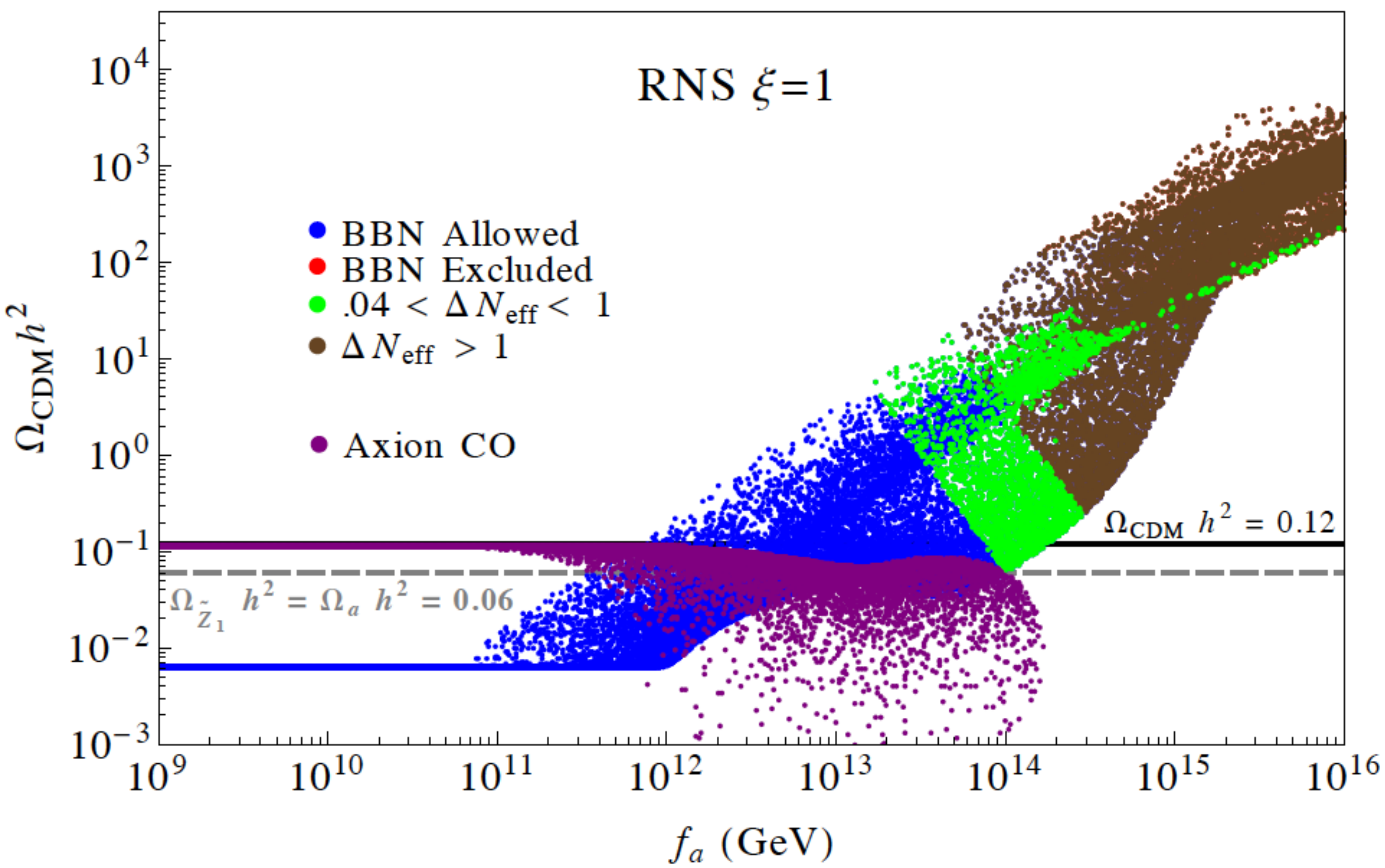




KJ Bae, HB, Lessa, Serce

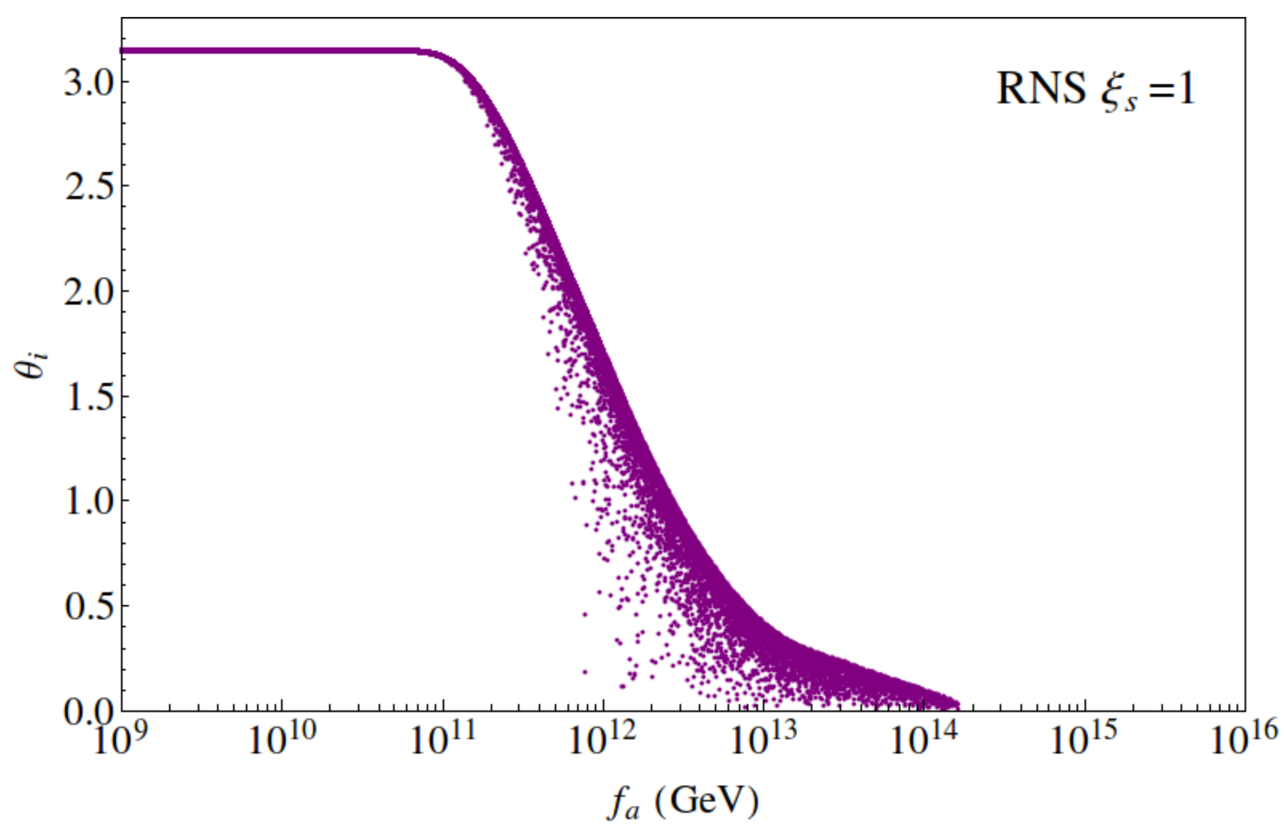
much of parameter space is axion-dominated with 10-15% WIMPs

=>



higgsino abundance

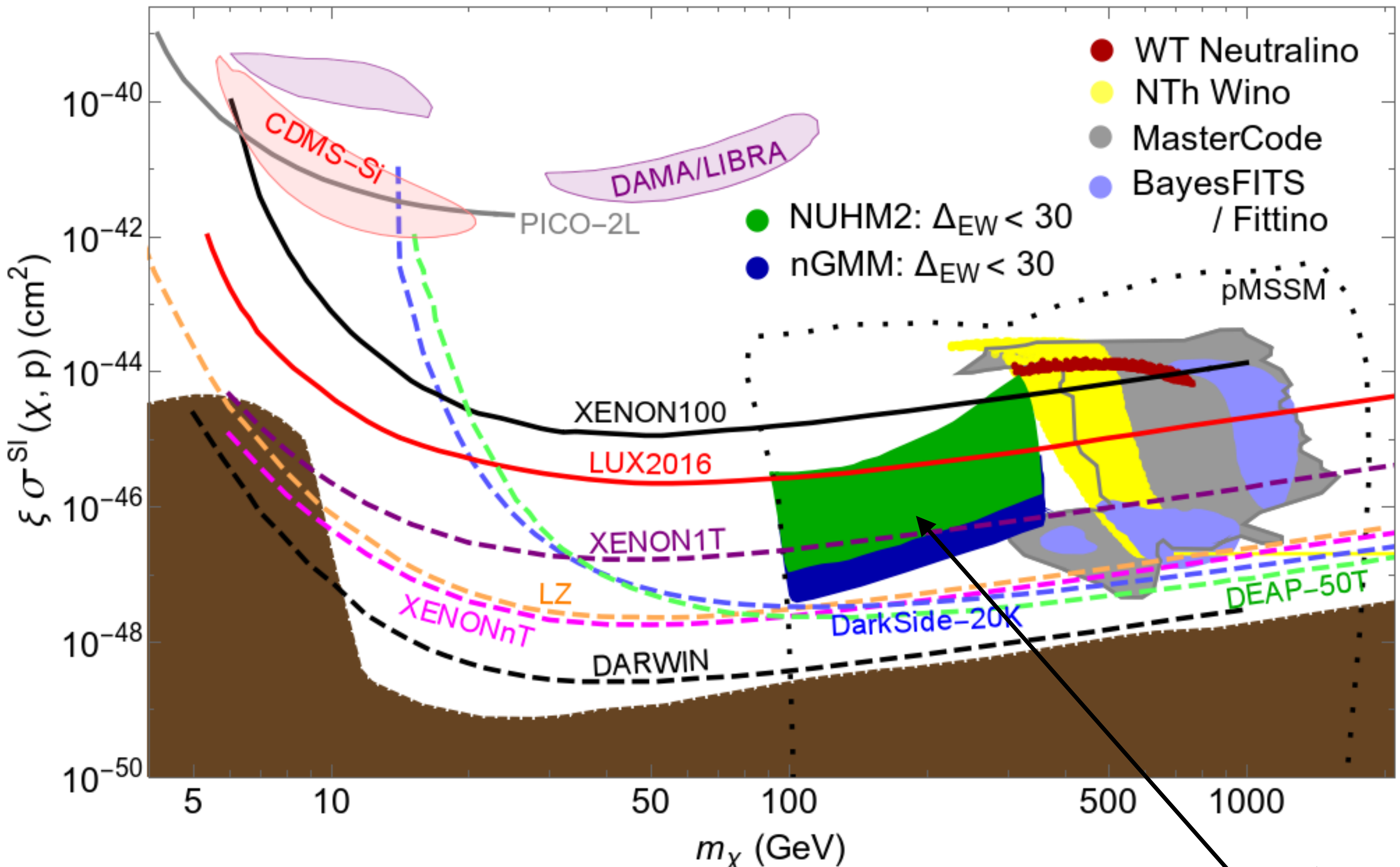
axion abundance



mainly axion CDM for fa<~10^12 GeV; for higher fa, then get increasing wimp abundance

Bae, HB,Lessa,Serce

Direct higgsino detection rescaled for minimal local abundance $\xi \equiv \Omega_\chi^{TP} h^2/0.12$



Bae, HB, Barger, Savoy, Serce

$$\mathcal{L} \ni -X_{11}^h \overline{\widetilde{Z}}_1 \widetilde{Z}_1 h$$

$$X_{11}^h = -\frac{1}{2}\left(v_2^{(1)} \sin\alpha - v_1^{(1)} \cos\alpha\right)\left(g v_3^{(1)} - g' v_4^{(1)}\right)$$

Xe-1-ton now operating!

Can test completely with ton scale detector or equivalent (subject to minor caveats)

Conclusion: SUSY is alive and well!

(in spite of recent NYT article)

- old calculations of naturalness over-estimate fine-tuning
- naturalness: Little Hierarchy mu<< m(SUSY) allowed
- radiatively-driven naturalness: mu~100-200 GeV, m(t1)<3 TeV, m(gluino)<5-6 TeV
- SUSY DFSZ axion: solve strong CP, solve SUSY mu problem; generate mu<< m(SUSY)
- landscape pull on soft terms towards RNS, m(h)~125 GeV
- natural mirage-mediation/mini-landscape
- natural NUHM2: HL-LHC can cover via SSdB+Z1Z2j channels
- natural mirage/mini-landscape may escape detection at HL-LHC; need LHC33!
- expect ILC as higgsino factory
- DM= axion+higgsino-like WIMP admixture: detect both?
- higgsino-like WIMP detection likely; axion more difficult

#2: Higgs mass or large-log fine-tuning Δ_{HS}

It is tempting to pick out one-by-one
quantum fluctuations but must combine log divergences
before taking any limit

$$m_h^2 \simeq \mu^2 + m_{H_u}^2(weak) \simeq \mu^2 + m_{H_u}^2(\Lambda) + \delta m_{H_u}^2$$

$$\frac{dm_{H_u}^2}{dt} = \frac{1}{8\pi^2}\left(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10}g_1^2 S + 3f_t^2 X_t\right) \qquad X_t = m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + A_t^2$$

neglect gauge pieces, S, mHu and running;
then we can integrate from m(SUSY) to Lambda

$$\delta m_{H_u}^2 \sim -\frac{3f_t^2}{8\pi^2}\left(m_{Q_3}^2 + m_{U_3}^2 + A_t^2\right)\ln(\Lambda/m_{SUSY})$$

$$\Delta_{HS} \sim \delta m_h^2/(m_h^2/2) < 10$$

$$m_{\tilde{t}_{1,2},\tilde{b}_1} < 500\ \mathrm{GeV}$$

$$m_{\tilde{g}} < 1.5\ \mathrm{TeV}$$

old natural SUSY

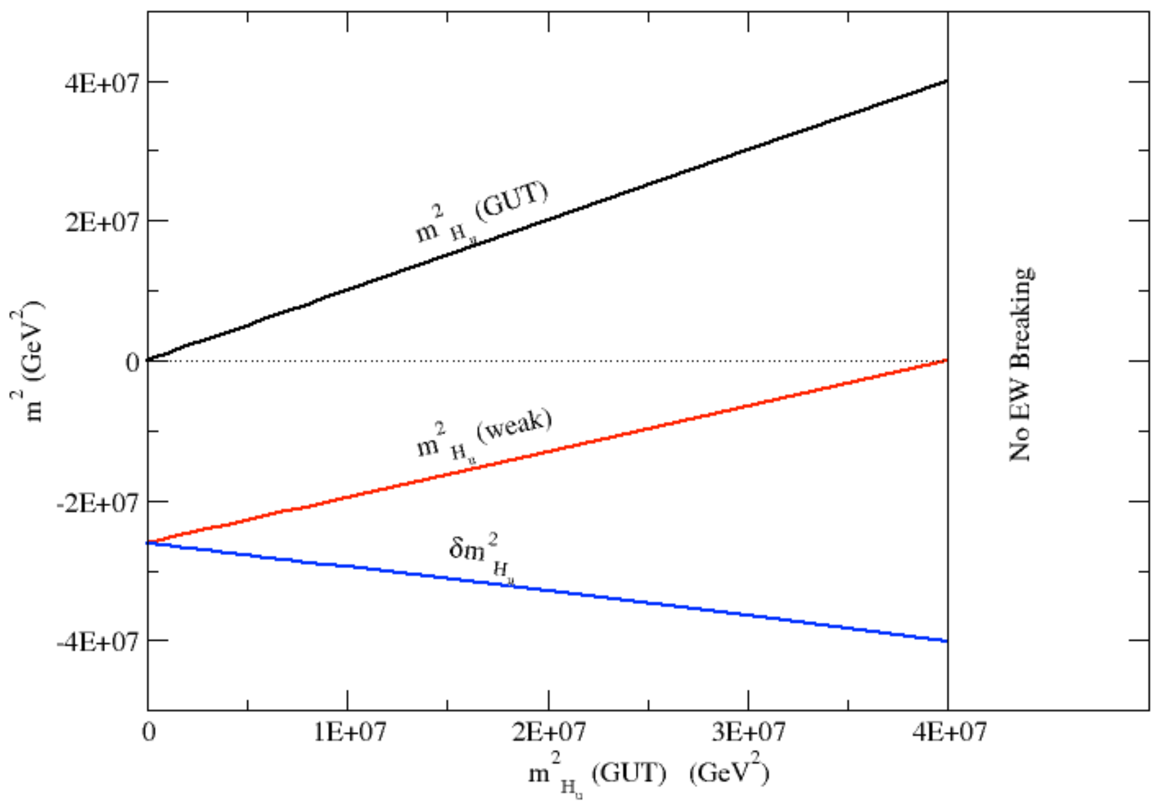
then A_t can't be too big

What's wrong with this argument?

In zeal for simplicity, have made several simplifications: most egregious is that one sets m(Hu)^2=0 at beginning to simplify

$m^2_{H_u}(\Lambda)$ and $\delta m^2_{H_u}$ are *not* independent!

violates prime directive!



The larger $m^2_{H_u}(\Lambda)$ becomes, then the larger becomes the cancelling correction!

HB, Barger, Savoy

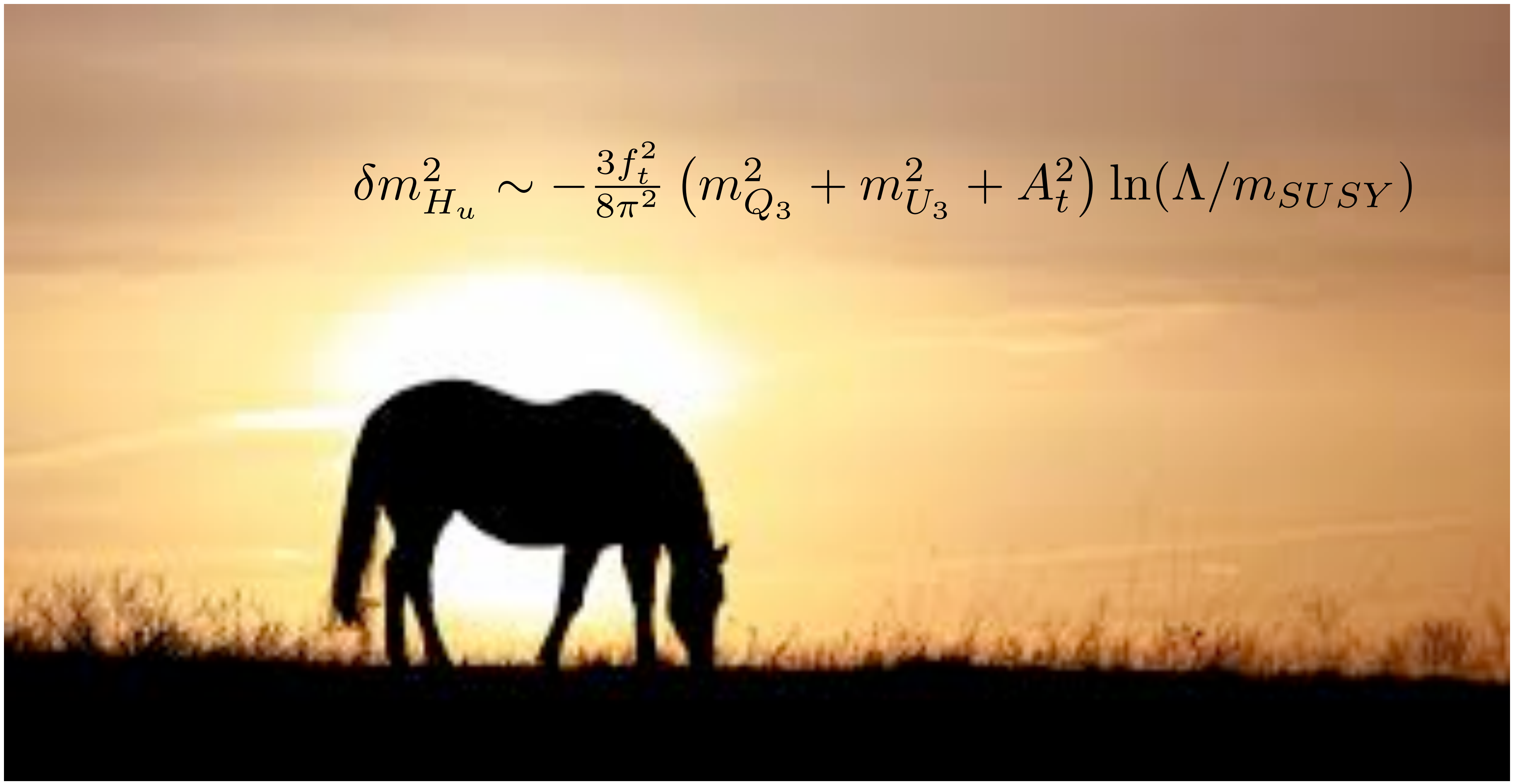
To fix: combine dependent terms:

$m_h^2 \simeq \mu^2 + \left(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2\right)$ where now both μ^2 and $\left(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2\right)$ are $\sim m_Z^2$

After re-grouping: $\Delta_{HS} \simeq \Delta_{EW}$

Instead of: the radiative correction $\delta m_{H_u}^2 \sim m_Z^2$
we now have: the radiatively-corrected $m_{H_u}^2 \sim m_Z^2$

Recommendation: put this horse out to pasture



R.I.P.

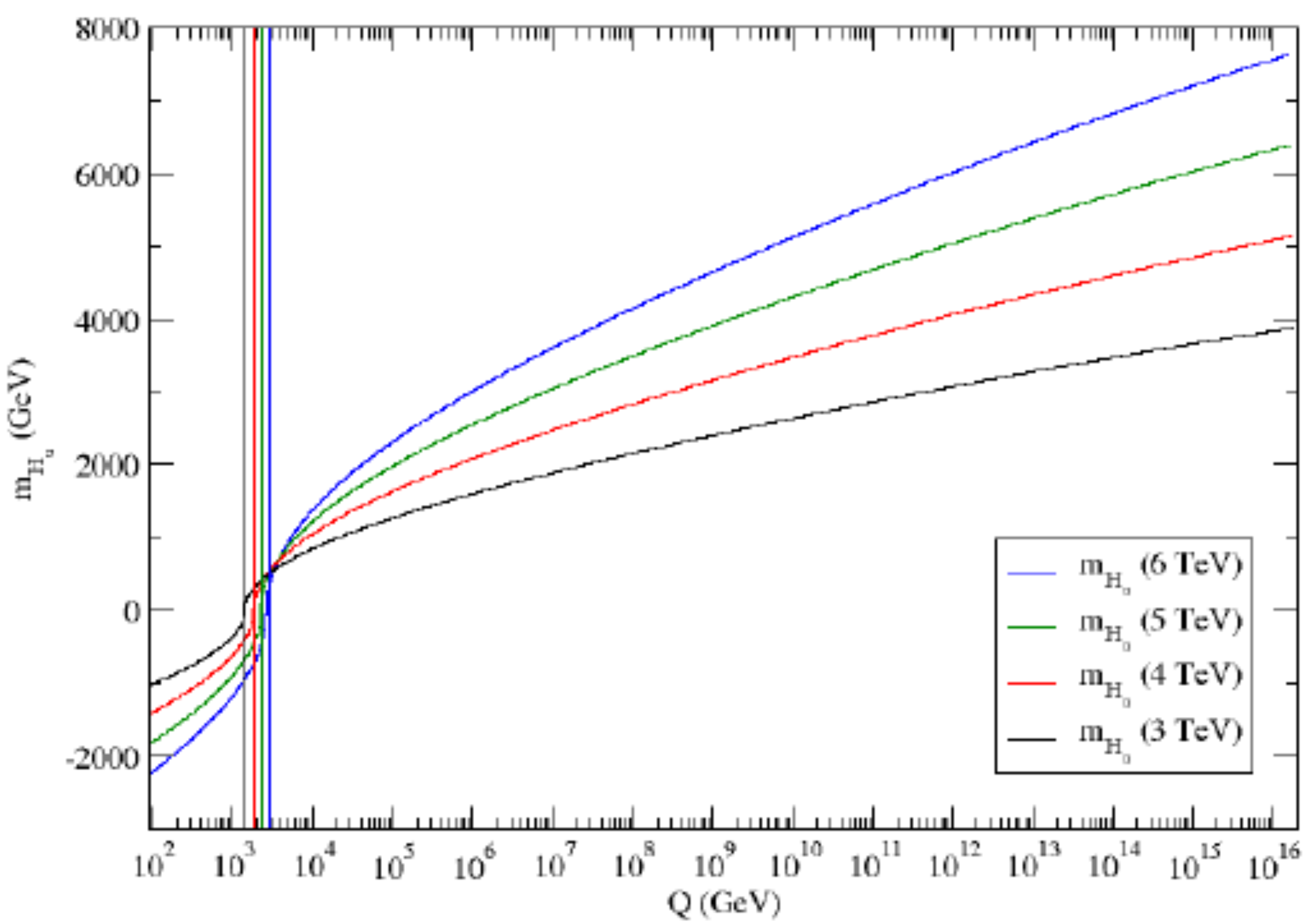
sub-TeV 3rd generation squarks not required for naturalness

If one has the right parameter correlations, can always get generalized focus point behavior for mHu:

$$\begin{aligned} m_0^2 &= m_{3/2}^2 \\ A_0 &= -1.6 m_{3/2} \\ m_{1/2} &= m_{3/2}/5 \\ m_{H_d}^2 &= m_{3/2}^2/2. \end{aligned}$$

$$\mu \simeq 150 \text{ GeV}$$

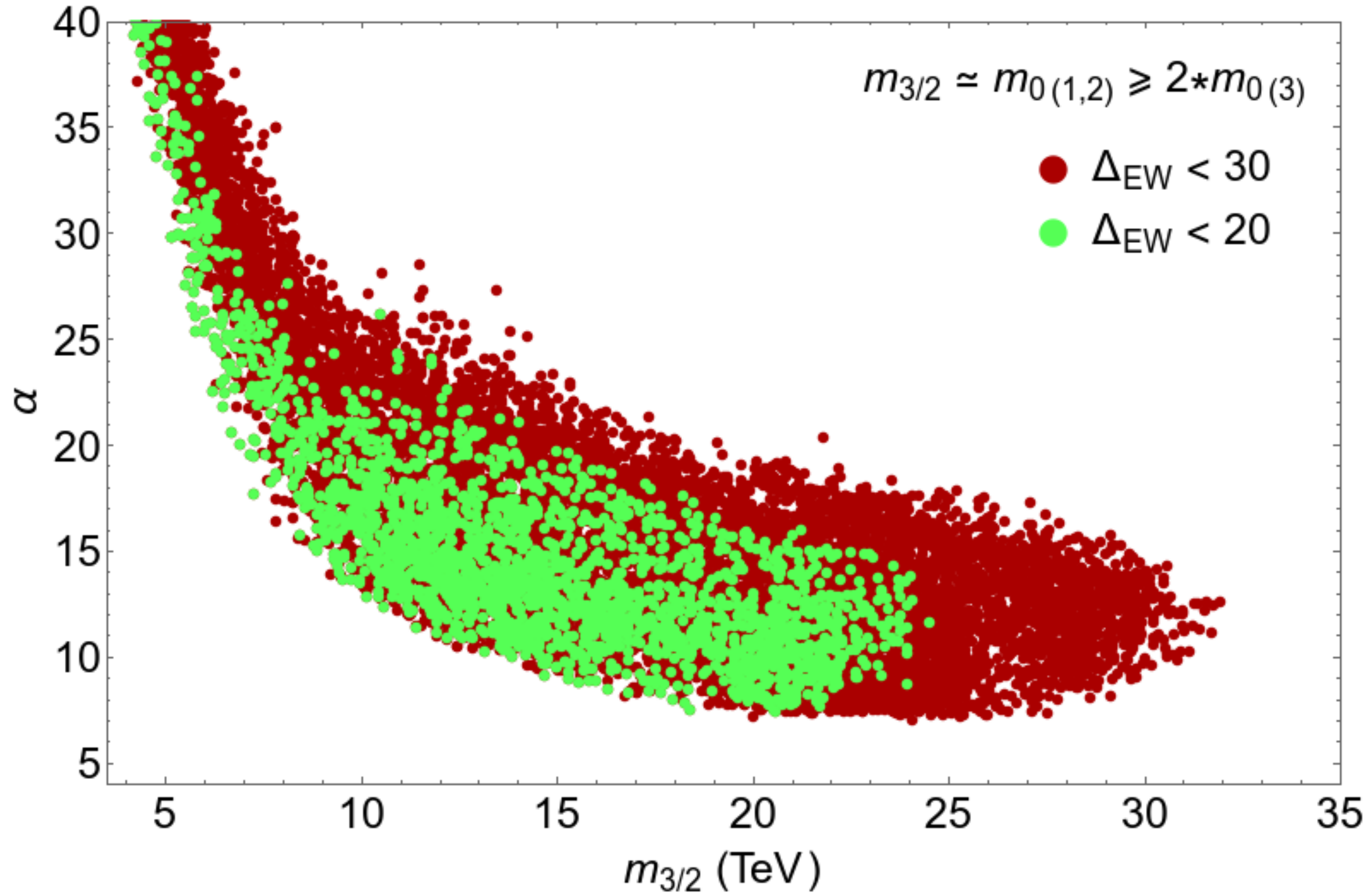
$$m_{H_u}^2(GUT) = 1.8 m_{3/2}^2 - (212.52 \text{ GeV})^2.$$



HB, Barger, Savoy

To generate minilandscape, take:

$$c_m = 16\pi^2/\alpha)^2 \text{ so that } m_0(1,2) \simeq m_{3/2}$$



Then get upper bound $m_{3/2} < 25 - 30$ TeV and $\alpha > 7$ else too large $m_0(1,2)$ drives 3rd generation tachyonic

Martin, Vaughn, 2-loop RGEs

Increased upper bound on m(gluino)<6 TeV

Alpha bound => mirage unif scale >10^11 GeV
(not too much compression of inos)