Top electroweak couplings study using di-muonic state at $\sqrt{s}$ = 500 GeV, ILC

Full ILD detector simulation

ILD Analysis/Software Meeting

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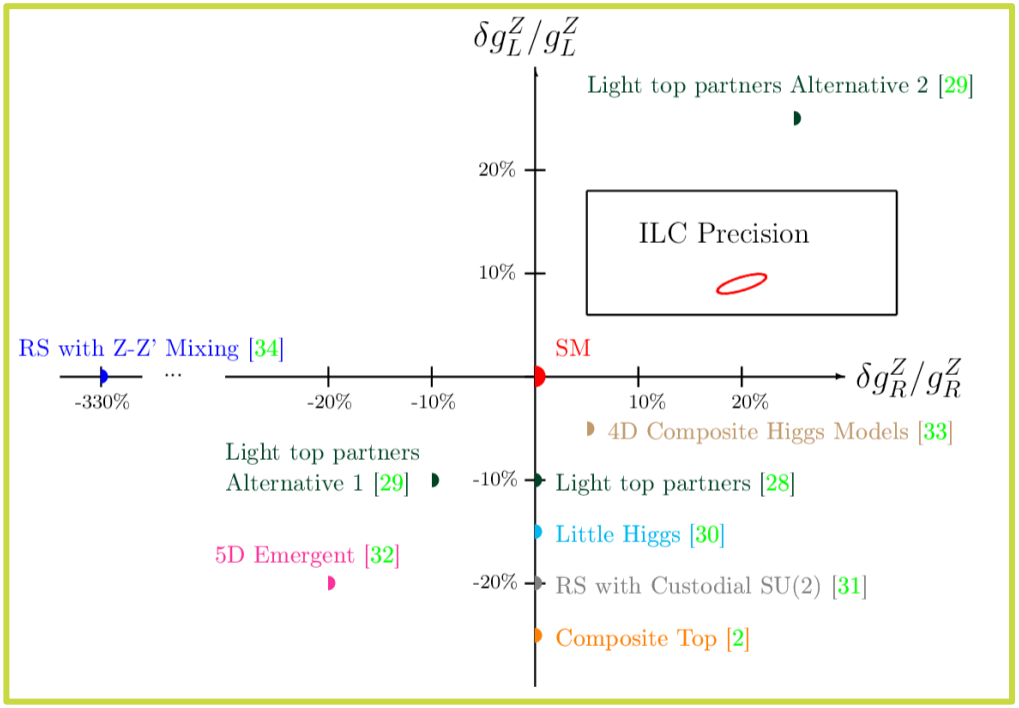
Top electroweak couplings

The top quark mass is comparable with the electroweak symmetry breaking scale. One can speculate that top quark plays a special role for the EWSB, for example such composite models. Therefore top quark electroweak couplings are good probes for New Physics.



Plot shows the predicted deviations from the Standard model of Z^0 couplings to t_L and t_R in composite models

Precision expected at the ILC will allow to distinguish between models.

arXiv:1505.06020 [hep-ph]

Matrix element method

The most efficient method when all the kinematics can be reconstructed.

- Results of previous study show that 10 form factors can be fitted simultaneously at less than a percent precision.

Statistical uncertainties and correlation with the SM LO as normalization

Kheim, E.K. Kurihara, Le Diberder: arXiv:1503:04247

$$\begin{bmatrix}
\mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1V} & \mathcal{R}e\ \delta\tilde{F}^{Z}_{1V} & \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1A} & \mathcal{R}e\ \delta\tilde{F}^{Z}_{1A} & \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2V} & \mathcal{R}e\ \delta\tilde{F}^{Z}_{2V} & \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2A} & \mathcal{R}e\ \delta\tilde{F}^{Z}_{2A} & \mathcal{I}m\ \delta\tilde{F}^{\gamma}_{2A} & \mathcal{I}m\ \delta\tilde{F}^{Z}_{2A} \\
0.0037 & -0.18 & -0.09 & +0.14 & +0.62 & -0.15 & 0 & 0 & 0 & 0 \\
 & 0.0063 & +.14 & -0.06 & -0.13 & +0.61 & 0 & 0 & 0 & 0 \\
 & & 0.0053 & -0.15 & -0.05 & +0.09 & 0 & 0 & 0 & 0 \\
 & & & 0.0083 & +0.06 & -0.04 & 0 & 0 & 0 & 0 \\
 & & & & 0.0105 & -0.19 & 0 & 0 & 0 & 0 \\
 & & & & & 0.0169 & 0 & 0 & 0 & 0 \\
 & & & & & & 0.0068 & -0.15 & 0 & 0 \\
 & & & & & & & 0.0118 & 0 & 0 \\
 & & & & & & & & 0.0069 & -0.17 \\
 & & & & & & & & & 0.0100
\end{bmatrix}$$

500 GeV&500 fb^{-1} Polarization 50/50 between ±80% and ±30%

Emi Kou (LAL-Orsay)
LFC 15, Trento,
7-11 Sep. 2015

This result is at parton level ignoring the detector effect, ISR and so on.

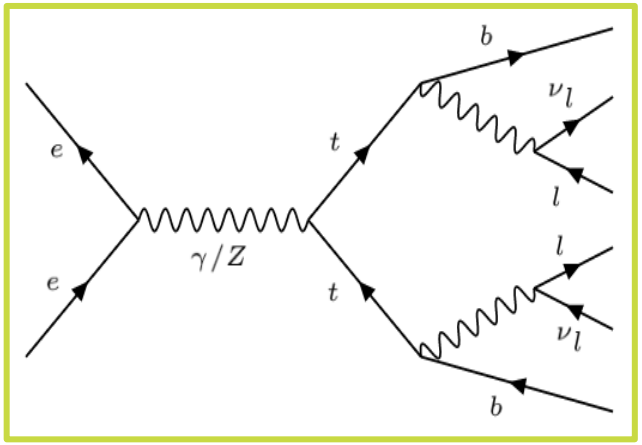
→More realistic study is required !

Setting of this study

Sample

The top pair production di-muonic state;

$t\bar{t} \rightarrow b\bar{b}\mu^{+}\mu^{-}\nu\bar{\nu}$ at $\sqrt{s}$ = 500 GeV



Situation

- ✓ The hadronization of b and $\bar{b}$ quark
- ✓ The detector effects (= full simulation)
- ✓ ISR and beamsstrahlung
- ✓ Gluon emission from top quark
- ✓ γγ → hadrons background
- × Background events

sample A

sample B
=DBD sample

Reconstruction

Process of the reconstruction

- Isolated leptons tagging
- γγ → hadrons background suppression
- B-jets reconstruction using the Thrust axis method
- Kinematical reconstruction

Thrust axis method

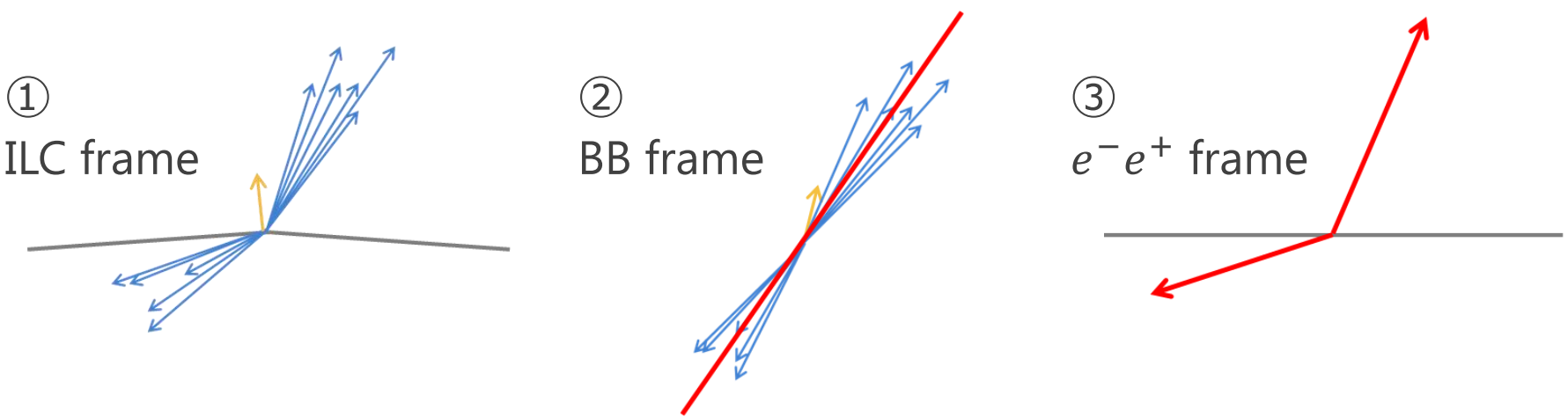
① Collect all hadronized particles in the ILC frame

② Boost them into the BB frame and calculate thrust axis

③ Boost the vectors along the thrust axis into the rest frame of e^-e^+

sample A : use same parameters with ②

sample B : consider ISR/BS effects → introduce the k_{e^-}, k_{e^+}



Considering ISR/BS effects

Collinear approximation:
Photon radiation is assumed to be along the beam lines

$$\vec{e}^{\,-} = \hat{\eta}_{e^-} E_{e^-} \quad (1)$$
$$\vec{e}^{\,+} = \hat{\eta}_{e^+} E_{e^+} \quad (2)$$

with,

$$\hat{\eta}_{e^-} = (\sin\theta_c, 0, \quad \cos\theta_c) \quad (3)$$
$$\hat{\eta}_{e^+} = (\sin\theta_c, 0, -\cos\theta_c) \quad (4)$$
$$E_{e^\pm} = E = 250 \text{ GeV} \quad (5)$$

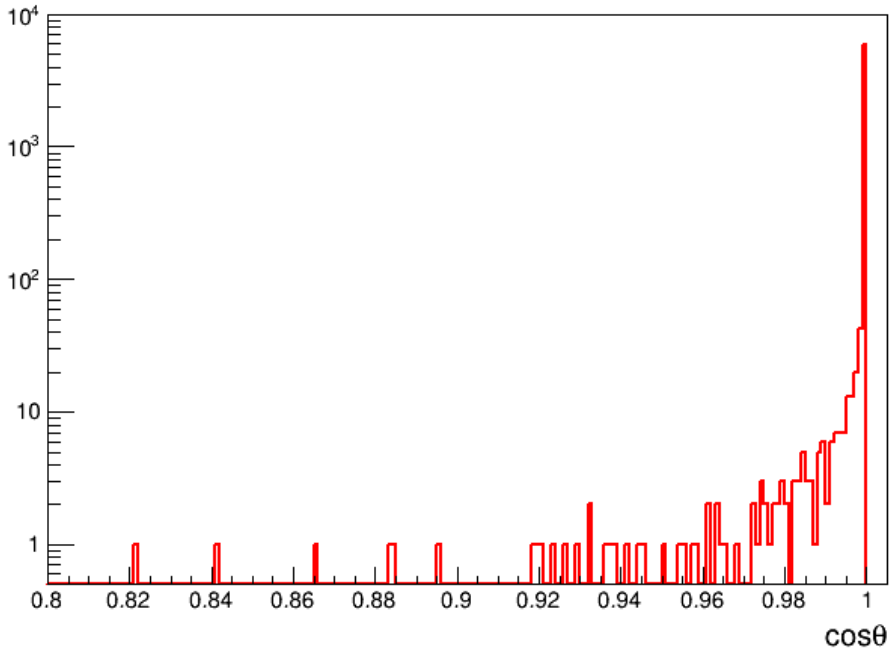
where θ_c is the beam crossing angle, $\theta_c = 7$ mrad.

In this approximation, the directions are not changed but only the energies are changed. Then the electron and positron thre-momenta become:

$$(\vec{e}^{\,-})^* = \hat{\eta}_{e^-} E^*_{e^-} = \hat{\eta}_{e^-} E(1 - k_{e^-}) \quad \text{with} \quad k_{e^-} = \frac{E - E^*_{e^-}}{E} \quad (6)$$
$$(\vec{e}^{\,+})^* = \hat{\eta}_{e^+} E^*_{e^+} = \hat{\eta}_{e^+} E(1 - k_{e^+}) \quad \text{with} \quad k_{e^+} = \frac{E - E^*_{e^+}}{E} \quad (7)$$

where $E^*_{e^\pm}$ is the energy of electron or positron just before collision.

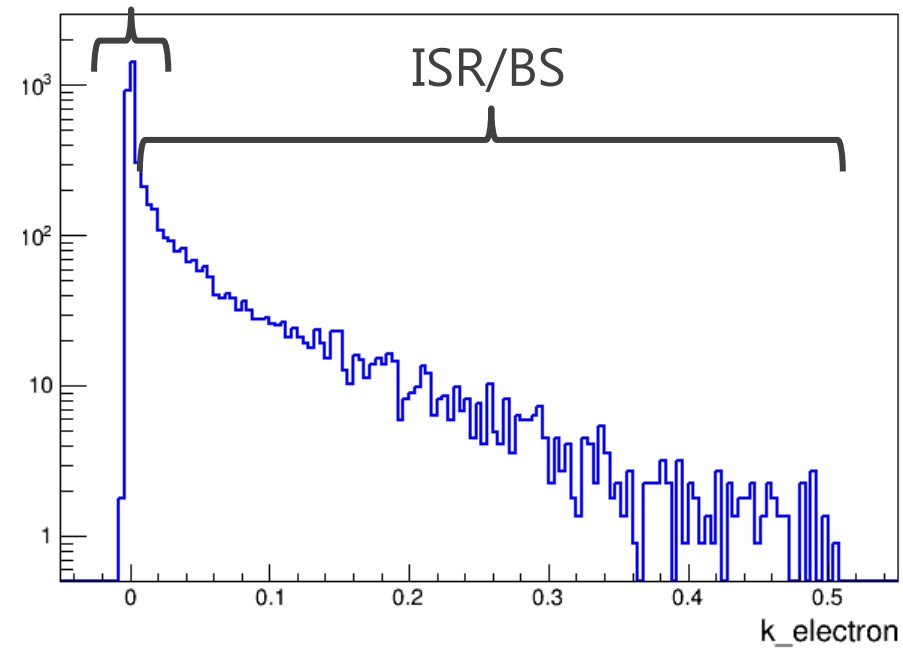
Distribution of cosθ and k_{e^-} (MC truth)



cosθ

(θ : angle between e^{-*} and the beam direction)

Beam spread



$$k_{e^-} = \frac{250 - E^*_{e^-}}{250}$$

Kinematical reconstruction

Strategy of the kinematical reconstruction

- **There are 8 unknown kinematics in this state**

  (= the momenta of two neutrinos and energy of two b-jets)

- **Impose 8 constraints** (= initial state constraints and $m_t, m_{\bar{t}}, m_{W^+}, m_{W^-}$)

  → Solutions are obtained in terms of (θ_t, ϕ_t) → $(\theta_t, \phi_t, m_t, m_{\bar{t}}, m_{W^+}, m_{W^-})$

  But the equation is nonlinear. Furthermore an ambiguity of b-charge remains.

  → **Typically 4 solutions per event.**

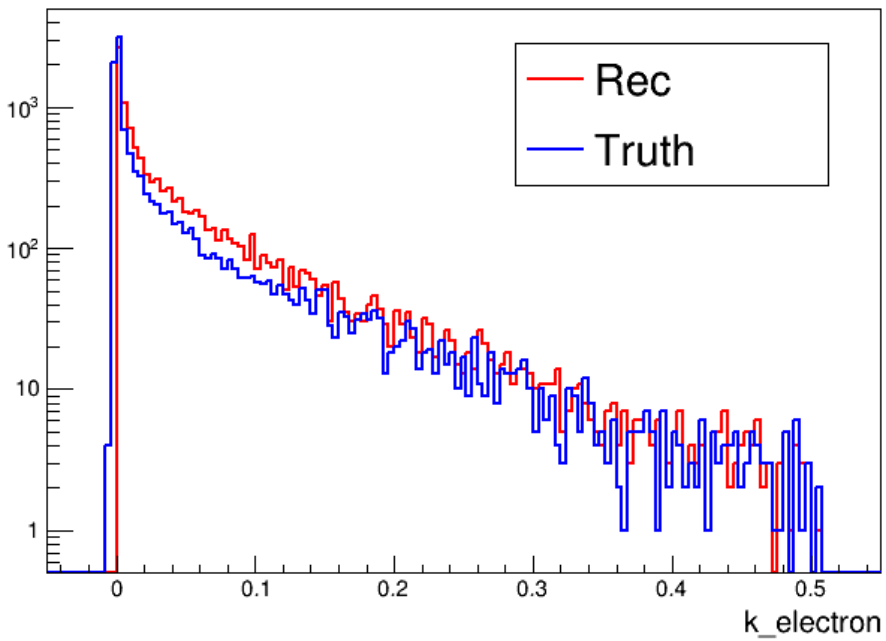
- **Select the optimal solution**

  Compare $E_b^{\text{meas.}}$ (by thrust axis method) and $E_b^{\text{rec.}}$ (by kinematical reconstruction).

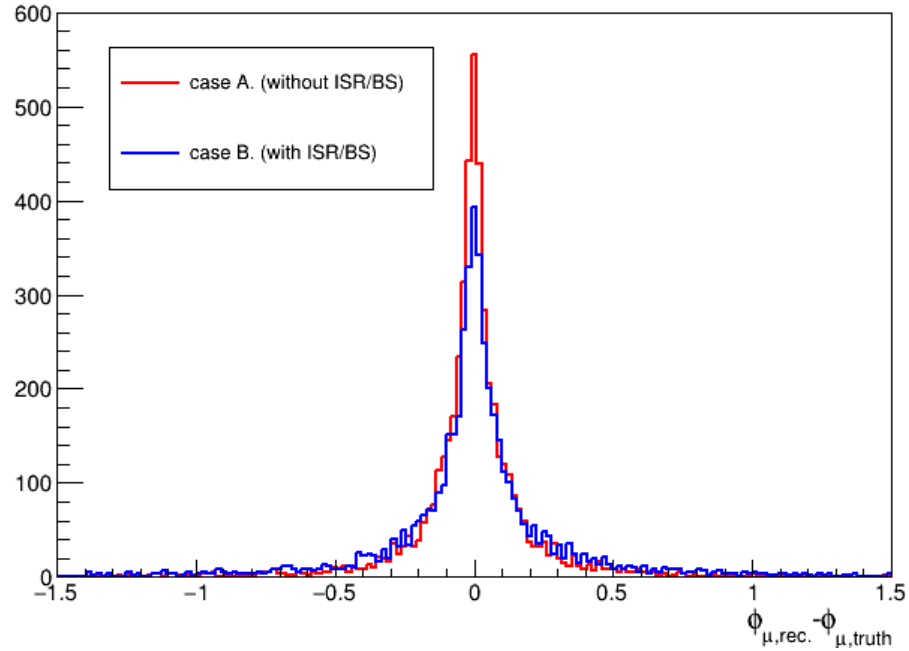
and (k_{e^-}, k_{e^+}) for sample B

Kinematical reconstruction : Results

k_{e^-} (only **sample B**)



$\phi_{\mu^+,rec.} - \phi_{\mu^+,truth}$ (**A** / **B**)

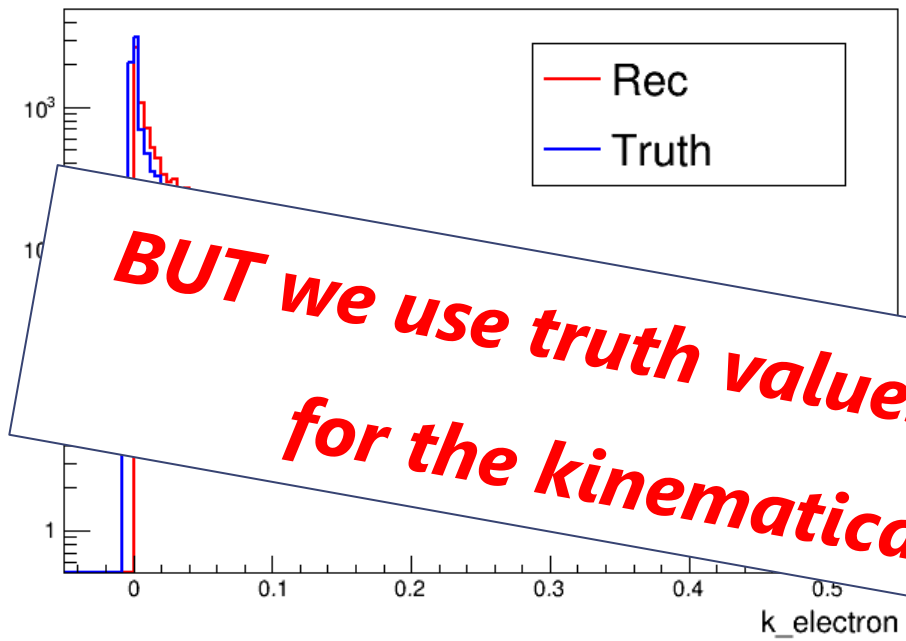


- The k's are free to vary between 0 and 1 without constraint for now
- **Result of sample B** is not that bad comparing with **A**

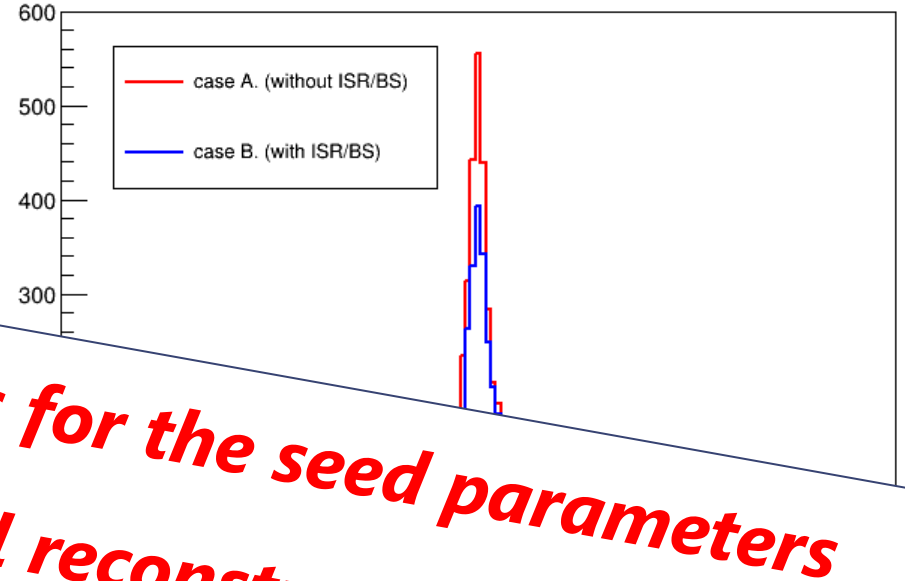
  → Plan to check if it is enough for the analysis using MEM

Kinematical reconstruction : Results

k_{e^-} (only **sample B**)



$\phi_{\mu^+,rec.} - \phi_{\mu^+,truth}$ (**A** / **B**)



BUT we use truth values for the seed parameters for the kinematical reconstruction !!!

- The k's are free to vary between 0 and 1 without constraint
- **Result of sample B** is not that bad comparing with **A**

  → Plan to check if it is enough for the analysis using MEM

Status of the 10 form factors fit (sample A)

Δ_{F1}	-0.0067 $\pm$ 0.0082
Δ_{F2}	0.035 $\pm$ 0.017
Δ_{F3}	-0.056 $\pm$ 0.012
Δ_{F4}	0.035 $\pm$ 0.018
Δ_{F5}	-0.022 $\pm$ 0.026
Δ_{F6}	0.042 $\pm$ 0.045
Δ_{F7}	-0.0081 $\pm$ 0.015
Δ_{F8}	0.010 $\pm$ 0.032
Δ_{F9}	0.013 $\pm$ 0.024
Δ_{F10}	-0.010 $\pm$ 0.022

(Preliminary)

5000 events, after cut on the $\chi^2_{\text{tot.}}$ to keep ~83% of the events without ISR/BS

Some small biases are observed (eg. Δ_{F3}) at few percent level

→ **No show stopper yet !!!**

→ Use **sample B** for the matrix element method

Summary & Plan

- **Started realistic study using sample B (= DBD sample)**
  - Accuracy of kinematical reconstruction is not that bad comparing the **sample A** (without ISR/BS)
  - Seeds issue for the kinematical reconstruction still remains
- **Analysis with Matrix element method**
  - 10 form factors can be fitted at percent precision for **sample A**

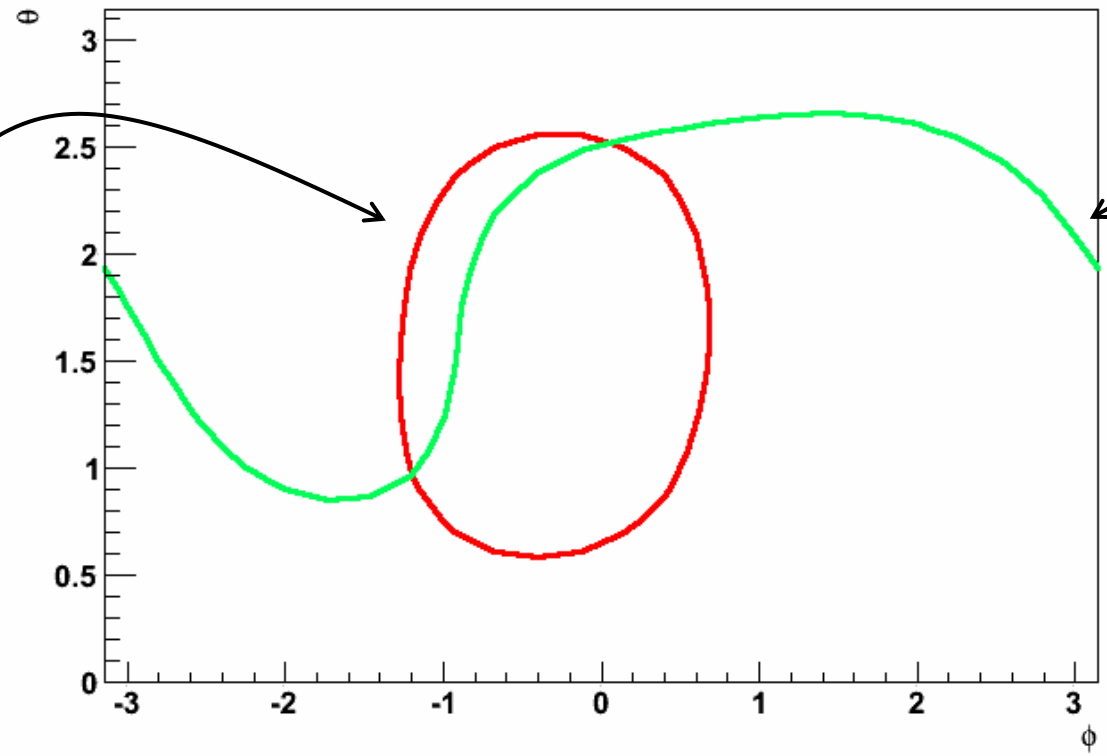
→ Apply the Matrix element method on **sample B**

Back up

Kinematical constraints

In the W rest frame, the energy of isolated lepton is equal to $m_W/2$ (with ignoring ISR and bremsstrahlung)

$$\frac{m_{W^+}}{2} = E_{\mu^+}^{**}$$

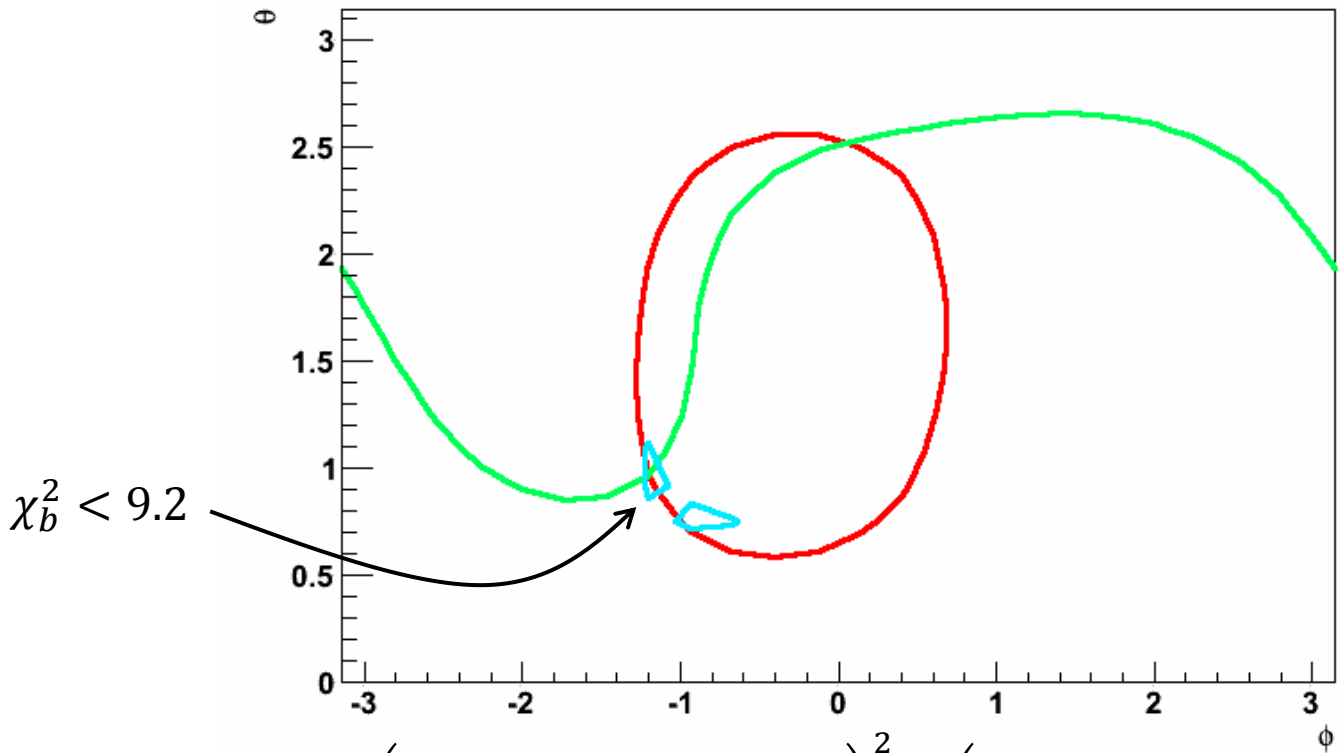


$$\frac{m_{W^-}}{2} = E_{\mu^-}^{**}$$

Measurements of b-quark energies

To select a right solution, we can use the measurement of b-jets energy.

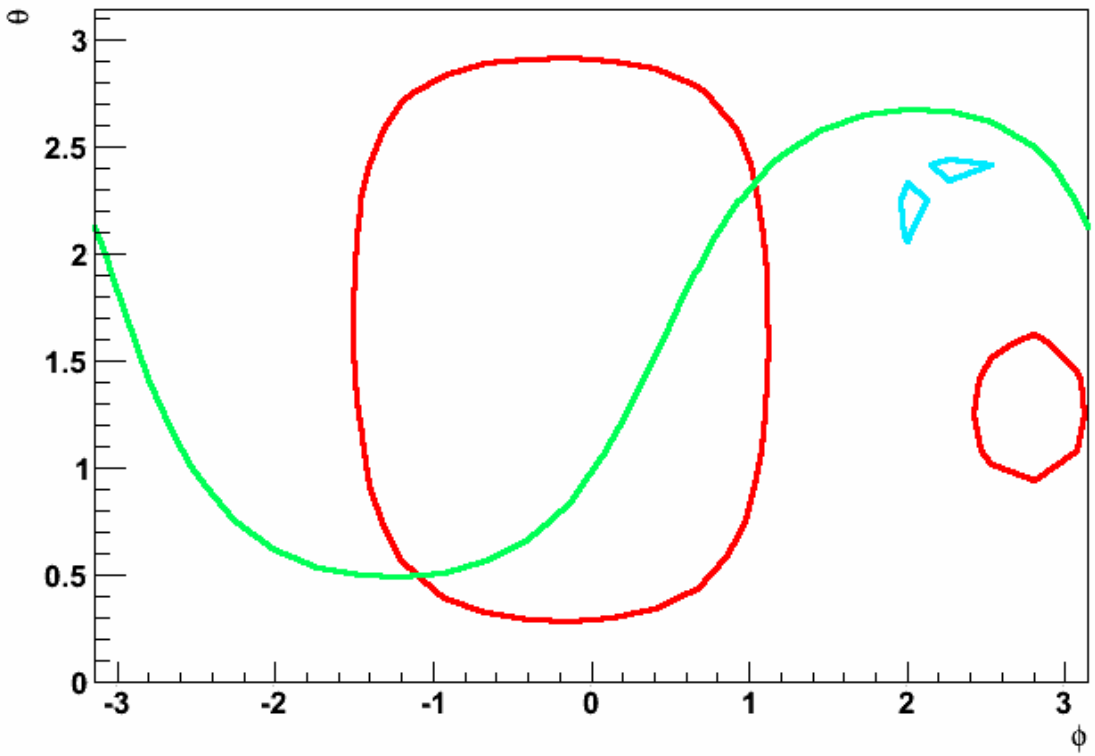
(Because this figure is at parton level, the χ_b^2 doesn't make sense.)



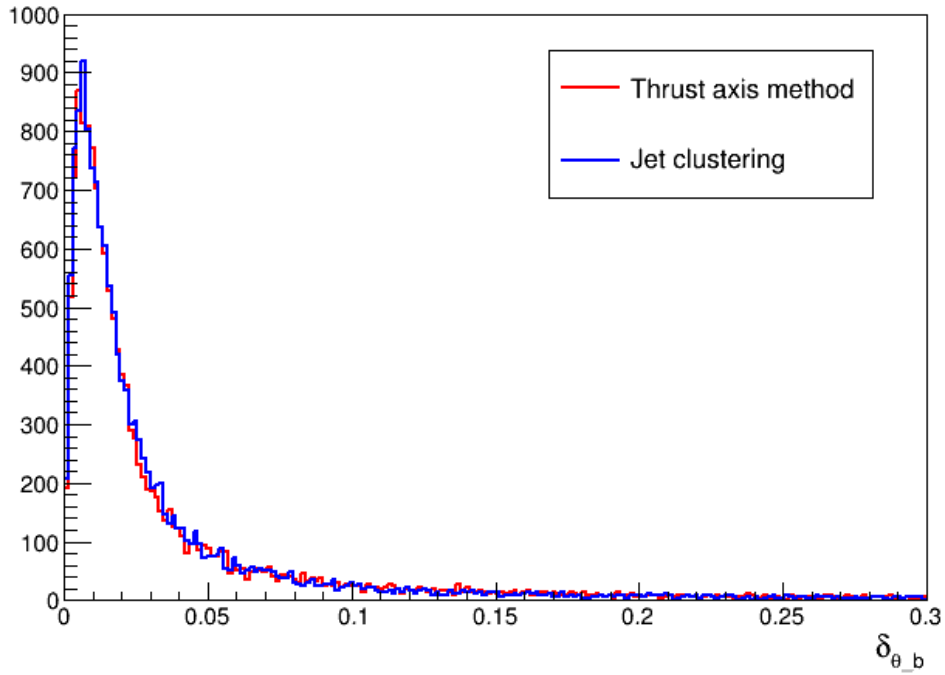
$$\chi_b^2(\phi_t, \theta_t) = \left(\frac{E_{b,mes} - E_{b,reco}(\phi_t, \theta_t)}{\sigma_j \sqrt{E_{b,mes}}}\right)^2 + \left(\frac{E_{\bar{b},mes} - E_{\bar{b},reco}(\phi_t, \theta_t)}{\sigma_j \sqrt{E_{\bar{b},mes}}}\right)^2$$

Miss combination of b-quarks

When we use the anti-b direction for the top reconstruction, the measurements of energy of b-jets excludes this combination.



Comparison Thrust axis method and Jet clustering



The angle between truth direction and reconstructed direction of b quark

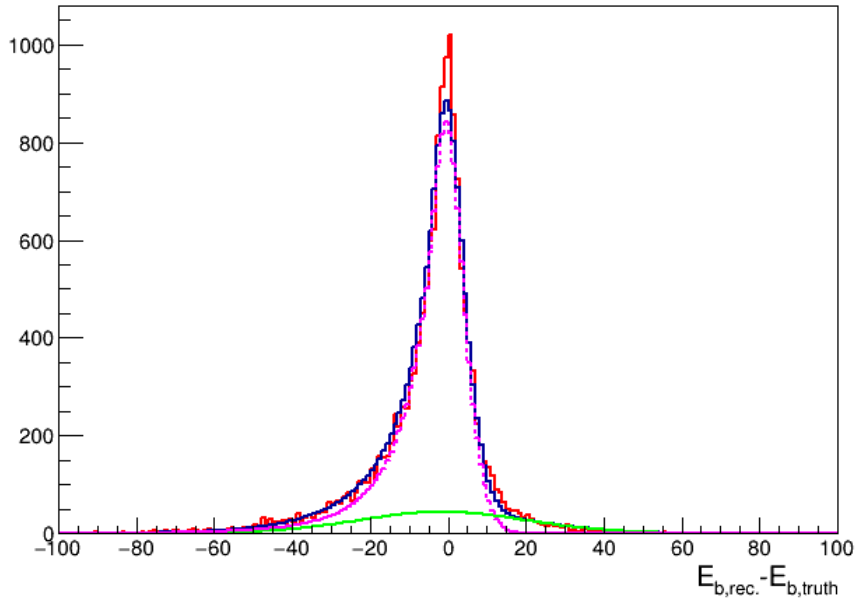
Red : Thrust axis method

Blue : Jet clustering (LCFIPlus)

Two methods produce almost same precision for direction of b-quark.

→We select the thrust axis method for this study so far.

Assessment of energy of b-jet



Deviation of energy of b-jet

(using thrust axis method)

Red : Original

Blue : Fitted

(Blue is a sum of Pink and Green)

To estimate the b-jet energy resolution we use multiple Crystal Ball functions for fitting.

$$CB(x|\alpha, n, \mu, \sigma) = \begin{cases} N \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), & \frac{x-\mu}{\sigma} > -\alpha \\ N \cdot A \cdot \left(B - \frac{x-\mu}{\sigma}\right)^{-n}, & \frac{x-\mu}{\sigma} < -\alpha \end{cases}$$

So far we use $\sigma_{jet} E_b^K$ with two parameters σ_{jet} and K for σ.

χ^2 algorithm : General

1. Define the χ^2_μ ;

$$\chi^2_\mu = \chi^2_{\mu^+} + \chi^2_{\mu^-}, \quad \chi^2_{\mu^\pm} = \left(\frac{E^{**}_{\mu^\pm}(\theta_t, \phi_t, m_t, m_{\bar{t}}, m_{W^+}, m_{W^-}) - m_{W^\pm}/2}{\sigma\left[E^{**}_{\mu^\pm}\right]} \right)^2$$

The energy of $\mu^\pm$ in the $W^\pm$ rest frame, $E^{**}_{\mu^\pm}$, must be equal to $m_{W^\pm}/2$.

2. Define the δ^2_b ;

$$\delta^2_b = -2\log L_b - 2\log L_{\bar{b}}, \quad L_b = \mathrm{CB}\left(E^{\mathrm{meas.}}_b - E^{\mathrm{rec.}}_b(\theta_t, \phi_t, m_t, m_{\bar{t}}, m_{W^+}, m_{W^-})\right)$$

The likelihood function is obtained from the assessment of b-jets energy.

3. Compound $\chi^2_{\mathrm{tot.}}$; $\chi^2_{\mathrm{tot.}} = \chi^2_\mu + \delta^2_b$

One minimizes the $\chi^2_{\mathrm{tot.}}$ to obtain the optimal solution; $(\theta_t, \phi_t, m_t, m_{\bar{t}}, m_{W^+}, m_{W^-})$.

χ^2 algorithm : Optional

The direction of b-jets is obtained by thrust axis method.

→ Add 4 angles $(\theta_b, \phi_b, \theta_{\bar{b}}, \phi_{\bar{b}})$ to the minimization parameters

→ Add constraints of 4 angles $(\theta_b, \phi_b, \theta_{\bar{b}}, \phi_{\bar{b}})$ to $\chi^2_{\text{tot.}}$ as follows;

$$\chi^2_{\text{direction}} = \chi^2_{\theta_b} + \chi^2_{\phi_b} + \chi^2_{\theta_{\bar{b}}} + \chi^2_{\phi_{\bar{b}}}, \qquad \chi^2_{\theta_b} = \left(\frac{\theta_b^{\text{meas.}} - \theta_b}{\sigma[\theta_b^{\text{meas.}}]}\right)^2$$

$(\chi^2_{\phi_b}, \chi^2_{\theta_{\bar{b}}}, \chi^2_{\phi_{\bar{b}}}$ are same as $\chi^2_{\theta_b})$

$$(\chi^2_{\text{tot.}})' = \chi^2_{\text{tot.}} + \chi^2_{\text{direction}}$$

We can use $(\chi^2_{\text{tot.}})'$ instead of $\chi^2_{\text{tot.}}$ to get the optimal solution, which is written in $(\theta_t, \phi_t, m_t, m_{\bar{t}}, m_{W^+}, m_{W^-}, \theta_b, \phi_b, \theta_{\bar{b}}, \phi_{\bar{b}})$

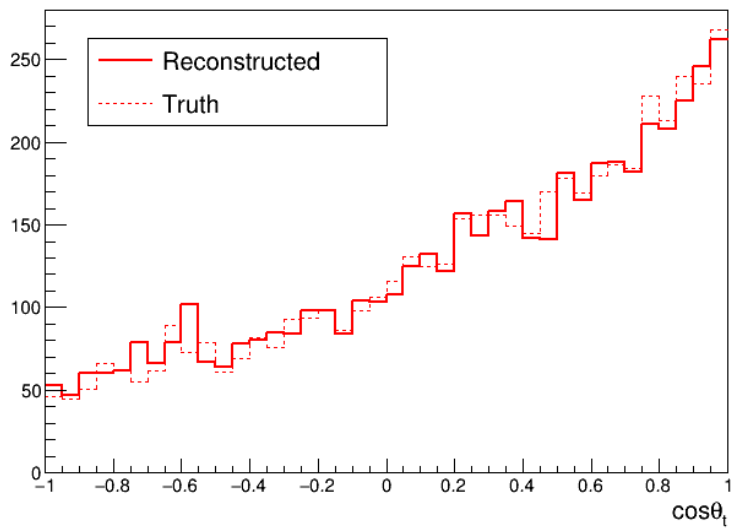
Kinematical reconstruction : Results

Reconstructed particles → 9 helicity angles :

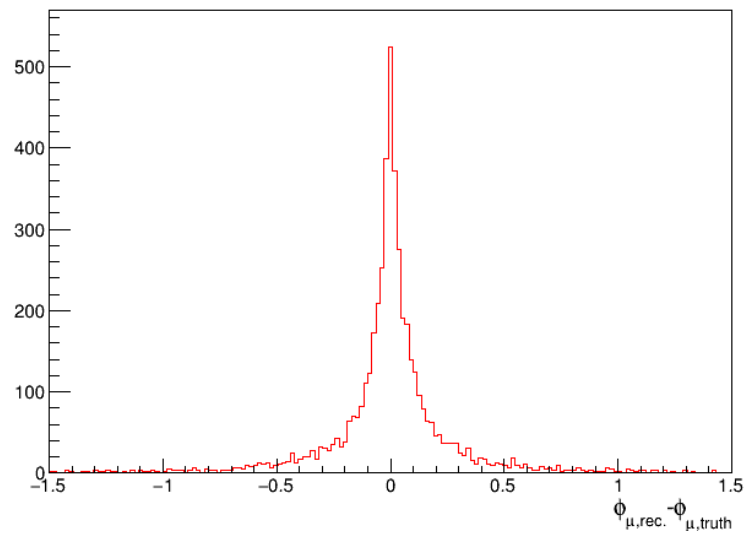
$$\cos\theta_t\,,\cos\theta_{W^+}\,,\phi_{W^+},\cos\theta_{\mu^+}\,,\phi_{\mu^+},\cos\theta_{W^-}\,,\phi_{W^-},\cos\theta_{\mu^-}\,,\phi_{\mu^-}$$

(→ Matrix element squared → Fit Form Factors)

eg.) $\cos\theta_t$



eg.) $\phi_{\mu^+,rec.} - \phi_{\mu^+,truth}$



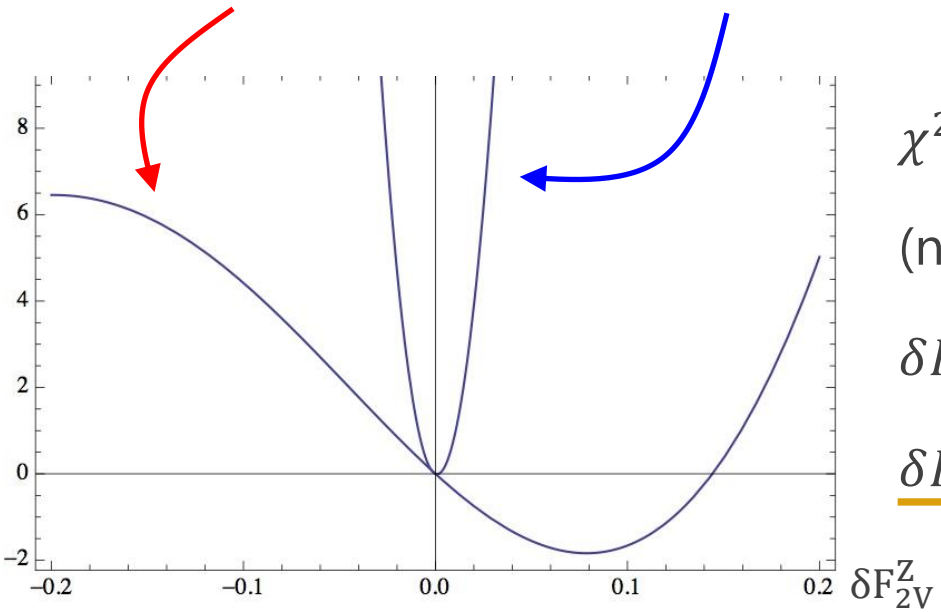
The ratio of wrong assignment of b-quark is only 2.1 % !

(cf. ~16 % for the semi-leptonic analysis, Yo Sato Top@LC 2016)

Analysis with Matrix element method

Illustration of analysis : 5000 unpolarized events

- using only $\cos\theta_t$
- using the complete set of 9 helicity angles



$\chi^2(\delta F_{2V}^Z)$ function

(normalized such that $\chi^2(0) = 0$)

$\delta F_{2V}^Z = 0.080 \pm 0.05$ by $\cos\theta_t$

$\delta F_{2V}^Z = 0.001 \pm 0.01$ by the complete set

Number of events not used on purpose to compare the intrinsic power of these two to determine a single form factor, F_{2V}^Z.