Track fitting in the ILD non-uniform magnetic field

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Outline

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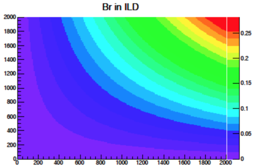
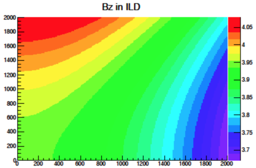
Introduction

- The performance of tracking is essential to the physics program on the future linear collider experiment:
  - **Tracking detectors** should have excellent spatial resolution and minimized track distortion
  - **Tracking algorithm** need to has the ability to extract momentum with high accuracy in the ILD non-uniform magnetic field $\Rightarrow$ **KalTest with segment-wise helical track model**





  - The non-uniformity is $> 10\%$ (F. Gaede, LCWS 2016)
  - Maybe we need to re-consider both **track finding** and **track fitting**.

Outline

1. Introduction
2. Track fitting methods for non-uniform magnetic field
   - Segment-wise helical track model
   - Runge-Kutta based track model
3. Discussion

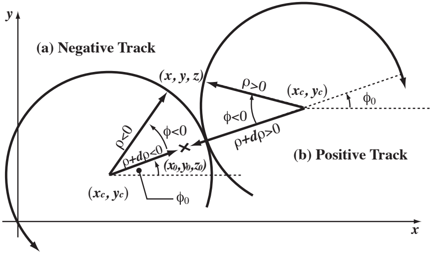
Equation of motion for a charged particle

- The equation of motion of a charged particle in a magnetic field is

$$m\frac{d^2\boldsymbol{x}}{dt^2} = Q\frac{d\boldsymbol{x}}{dt} \times \boldsymbol{B}(\boldsymbol{x}), \tag{1}$$

where m is the relativistic mass, and Q is the charge of particle.

- If the magnetic is uniform, we have a analytical solution



- The **state vector** of a track is defined as

$$\boldsymbol{a}_k = \left(\ d_\rho, \phi_0, \kappa, d_z, \tan\lambda \ \right)^{\mathrm{T}}. \tag{2}$$

Kalman Filter

- For each site, Kalman filter algorithm has two steps:
  - **Prediction**:
$$\boldsymbol{a}_k^{k-1} = \boldsymbol{f}_{k-1}(\boldsymbol{a}_{k-1}), \tag{3}$$
    in which, $\boldsymbol{f}_k$ is **propagation function**. And the corresponding **propagation matrix** is defined by
$$\boldsymbol{F}_{k-1} = \frac{\partial \boldsymbol{f}_{k-1}}{\partial \boldsymbol{a}_{k-1}}. \tag{4}$$
  - **Filtering**:
$$\boldsymbol{a}_k = \boldsymbol{a}_k^{k-1} + \boldsymbol{K}_k \left(\boldsymbol{m}_k - \boldsymbol{h}_k(\boldsymbol{a}_k^{k-1}) \right), \tag{5}$$
    where $\boldsymbol{K}_k$ is the gain matrix which is related to the propagation matrix, $\boldsymbol{h}_k$ is the measurement function.
- Kalman filter is implemented in KalTest[1]by K. Fujii, together with track models and basic detector geometries.

[1]KalTest manual is at http://www-jlc.kek.jp/jlc/en/subg/soft/tracking/kaltest.

Basic idea of segment-wise helical track model

To use the helical track model of KalTest in the non-uniform magnetic field, we can

- assume the magnetic field between two nearby layers is uniform;
- transform the frame to make the z axis point to the direction of magnetic field.

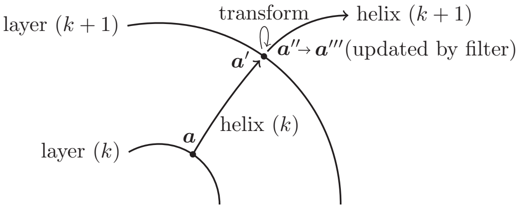


Figure: The updated track propagation procedure.

Therefore we now have a **segment-wise helical track model**, and we should recalculate the propagation function and propagation matrix.

How to transform the frame

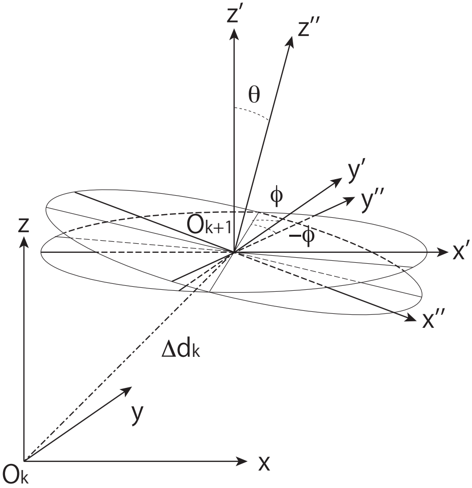


Figure: Transformation: translation and rotation

Modified propagator

- The propagation procedure can be represented by four equations:

$$\begin{cases} \boldsymbol{a}' & = & \boldsymbol{f}_k(\boldsymbol{a}_k) \\ \boldsymbol{p} & = & \boldsymbol{c}(\boldsymbol{a}') \\ \boldsymbol{p}' & = & \boldsymbol{t}(\boldsymbol{p}) \\ \boldsymbol{a}'' & = & \boldsymbol{c}^{-1}(\boldsymbol{p}') \end{cases} . \tag{6}$$

  $\boldsymbol{f}_k$: the original propagation function; $\boldsymbol{c}$: the function converting state vector to momentum, with $\boldsymbol{c}^{-1}$ as its inverse function; $\boldsymbol{t}$: actually the **rotation matrix.**

- Therefore, the propagation matrix should be modified accordingly by the chain rule:

$$\boldsymbol{F}_{k-1}^{m} = \frac{\partial \boldsymbol{a}''}{\partial \boldsymbol{p}'} \frac{\partial \boldsymbol{p}'}{\partial \boldsymbol{p}} \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{a}'} \frac{\partial \boldsymbol{a}'}{\partial \boldsymbol{a}} = \boldsymbol{F}_{k-1}^{r} \boldsymbol{F}_{k-1}. \tag{7}$$

The test conditions of algorithm

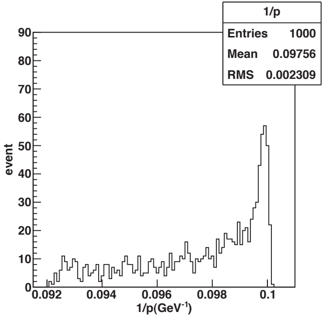
- Suppose the non-uniform magnetic field has a form of

$$\begin{cases} B_x &= B_0 kxz \\ B_y &= B_0 kyz \\ B_z &= B_0(1-kz^2) \end{cases}, \tag{8}$$

  in which, $k = \frac{k_0}{z_m r_m}$, $B_0 = 3$ T, $z_m = r_m = 3000$ mm;
- Runge-Kutta track generator: `TEveTrackPropagator` in ROOT and bisection method are used;
- Track parameters: dip angle $\lambda \in [0, 0.5]$, azimuth angle $\phi \in [0, 2\pi]$;
- Detector:
  - 251 layers;
  - distance between two nearby layers is 6 mm;
  - $R_{\text{in}} = 300$ mm;
  - Point resolution $\sigma_{r\phi} = 100$ µm.
- To see the effect of the non-uniform magnetic field, the track with the same initial parameters are also simulated in uniform magnetic field.

Momentum resolution

- $k_0 = 1$, $p = 10$ GeV;
- Tracks are reconstructed in **uniform** magnetic field.



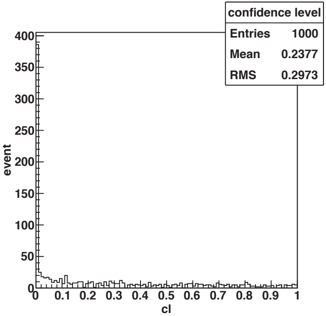


Figure: Momentum and confidence level by the original algorithm.

Momentum resolution

- $k_0 = 1$, $p = 10$ GeV;
- Tracks are reconstructed in **non-uniform** magnetic field.

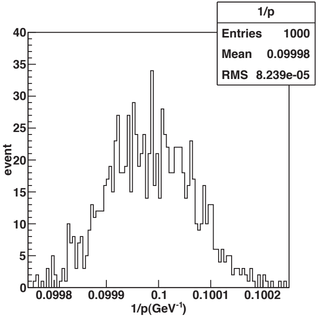


Figure: Momentum and confidence level by the updated algorithm.

Results with different non-uniformity and step size

Table: Mean and RMS of $\frac{1}{p}$ (in units of $10^{-1} \cdot (\mathrm{GeV}/c)^{-1}$ and $10^{-5} \cdot (\mathrm{GeV}/c)^{-1}$ respectively) at $10\,\mathrm{GeV}/c$.

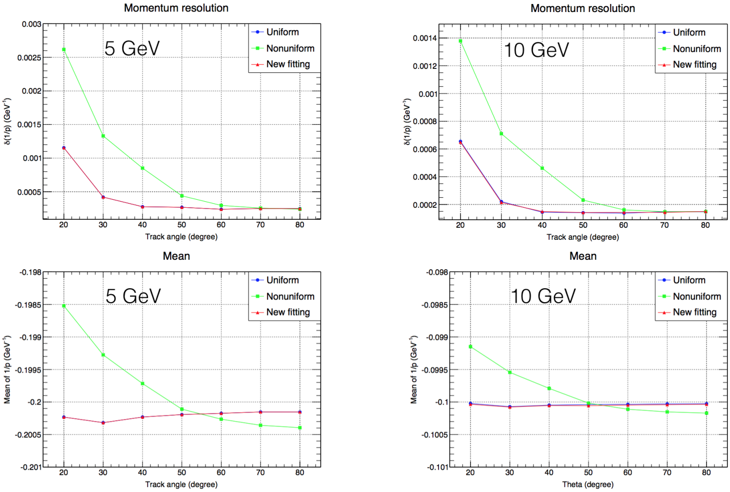
(a) Step size 6 mm

k_0	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$
1	1.0000/8.03	0.9998/7.89	0.9995/7.65
2	1.0000/8.05	0.9997/8.09	0.9990/8.36
3	0.9999/8.07	0.9995/8.31	0.9984/9.20

(b) Step size 1 mm

k_0	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$
1	1.0000/8.03	1.0000/7.89	0.9999/7.65
2	1.0000/8.05	0.9999/8.10	0.9998/8.36
3	1.0000/8.07	0.9999/8.32	0.9997/9.21

Track fitting results in ILD B field



- By taking field non-uniformity into account, the new KalTest gets almost the same results with the original version for uniform field.

The motion of equation in the non-uniform field

- Using the track length as a variable of track, i.e. $\boldsymbol{x}(l)$, and since

$$\frac{d\boldsymbol{x}}{dt} = \frac{d\boldsymbol{x}}{dl}\frac{dl}{dt} = v\frac{d\boldsymbol{x}}{dl},$$

from Eq.(1) we can get

$$\frac{d^2\boldsymbol{x}}{dl^2} = \frac{Q}{p}\frac{d\boldsymbol{x}}{dl} \times \boldsymbol{B(x)}, \tag{9}$$

where $p = mv$ is the momentum of charged particle.

- Defining $\frac{d\boldsymbol{x}}{dl} \equiv \boldsymbol{\alpha}$, then Eq.(9) can be written by

$$\frac{d\boldsymbol{\alpha}}{dl} = \lambda\boldsymbol{\alpha} \times \boldsymbol{B}(\boldsymbol{x}) \equiv \boldsymbol{f}(\boldsymbol{\alpha}, \boldsymbol{x}), \tag{10}$$

where the coefficient $\lambda \equiv \frac{Q(e)}{p(\mathrm{GeV}/c)}\lambda_c$ with $\lambda_c = 2.99792458 \times 10^{-4}$ and l in unit of millimeter.

Runge-Kutta method

- Runge-Kutta method (the fourth order Runge-Kutta) can give numerical solution to Eq.(9) and (10):

$$\begin{cases} \boldsymbol{x} = \boldsymbol{x}_0 + h\boldsymbol{\alpha}_0 + \frac{h^2}{6}(\boldsymbol{K}_0 + \boldsymbol{K}_1 + \boldsymbol{K}_2) \\ \boldsymbol{\alpha} = \boldsymbol{\alpha}_0 + \frac{h}{6}(\boldsymbol{K}_0 + 2\boldsymbol{K}_1 + 2\boldsymbol{K}_2 + \boldsymbol{K}_3) \end{cases}, \tag{11}$$

in which h is step length and

$$\begin{cases} \boldsymbol{K}_0 = \boldsymbol{f}(\boldsymbol{\alpha}_0, \boldsymbol{x}_0) \\ \boldsymbol{K}_1 = \boldsymbol{f}(\boldsymbol{\alpha}_0 + \frac{h}{2}\boldsymbol{K}_0, \boldsymbol{x}_0 + \frac{h}{2}\boldsymbol{\alpha}_0 + \frac{h^2}{8}\boldsymbol{K}_0) \\ \boldsymbol{K}_2 = \boldsymbol{f}(\boldsymbol{\alpha}_0 + \frac{h}{2}\boldsymbol{K}_1, \boldsymbol{x}_0 + \frac{h}{2}\boldsymbol{\alpha}_0 + \frac{h^2}{8}\boldsymbol{K}_1) \\ \boldsymbol{K}_3 = \boldsymbol{f}(\boldsymbol{\alpha}_0 + h\boldsymbol{K}_2, \boldsymbol{x}_0 + h\boldsymbol{\alpha}_0 + \frac{h^2}{2}\boldsymbol{K}_2) \end{cases}$$

or $\boldsymbol{K}_i = \boldsymbol{f}(\boldsymbol{A}_i, \boldsymbol{X}_i) = \lambda\boldsymbol{A_i} \times \boldsymbol{B}(\boldsymbol{X}_i)$.

The Runge-Kutta Track

- Using state vector $\boldsymbol{a}$ and step length h, the Runge-Kutta-based track can be represented as

$$\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{a}, h), \tag{12}$$

instead of $\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{a}, \phi)$.

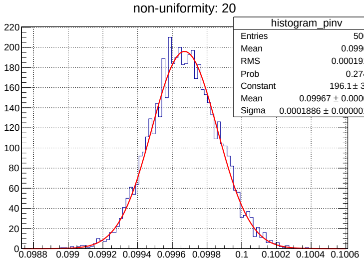
- Similar with the segment-wise helical track model, the propagator can be represented as

$$\begin{aligned} \boldsymbol{F}_{k-1} \equiv \frac{\partial \boldsymbol{a}_k^{k-1}}{\partial \boldsymbol{a}_{k-1}} &= \frac{\partial \boldsymbol{a}''}{\partial \boldsymbol{a}} = \frac{\partial \boldsymbol{a}''}{\partial \boldsymbol{a}'} \frac{\partial \boldsymbol{a}'}{\partial (\boldsymbol{p}', \boldsymbol{x}')} \frac{\partial (\boldsymbol{p}', \boldsymbol{x}')}{\partial (\boldsymbol{p}, \boldsymbol{x})} \frac{\partial (\boldsymbol{p}, \boldsymbol{x})}{\partial \boldsymbol{a}} \\ &= \boldsymbol{F}_4 \boldsymbol{F}_3 \boldsymbol{F}_2 \boldsymbol{F}_1 \end{aligned} \tag{13}$$

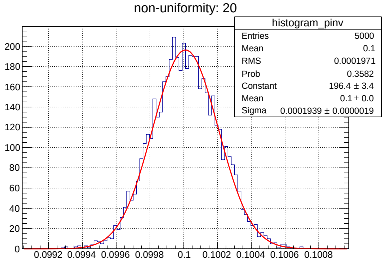
The detailed calculation of each matrix, $\boldsymbol{F}_i$, is omitted.

Test results

- The non-uniformity in Eq.(8): $k_0 = 20$, momentum: 10 GeV.
- At large non-uniformity of magnetic field, the Runge-Kutta method has a better performance.



(a) segmented helix



(b) Runge-Kutta

Test results (cont'd)

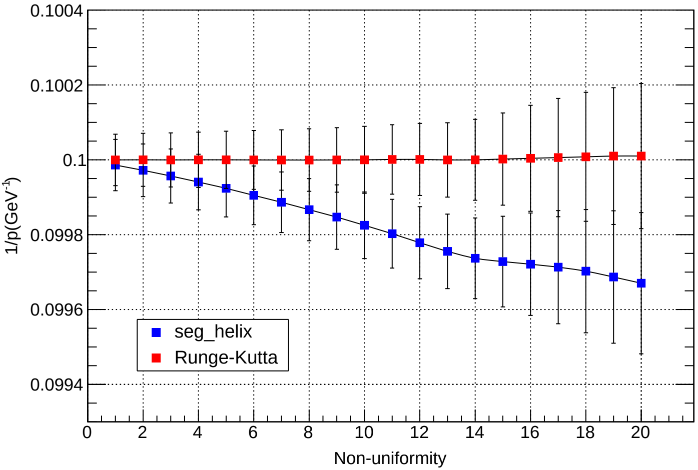


Figure: The results under different non-uniformity by two methods

Outline

Conclusion and discussion

- To deal with the non-uniform magnetic field, two kinds of track models has been implemented in KalTest
  - Segment-wise helical track model
  - Runge-Kutta method based track model
- The ILD magnetic field
  - The current status of study on ILD magnetic field ?
  - The necessity of using the updated version of KalTest for non-uniform magnetic field
    - Implement the field map in new simulation tools: DD4hep + lcgeo ?
    - Physics oriented study: $e^+e^- \to ZH \to l^+l^-X$?