

Transverse Mode Coupling and its Impact on Emittance Dilution

- Injected emittance: $\epsilon_{x,y} = 8e-6, 2e-8$ m.rads
- Injection beam energy 5 GeV \Rightarrow O/P 500 GeV c.m. energy
- $\gamma_{i,f} \sim 9782, \text{ and } 489,000$
- All simulations are made with the code LIAR.
- 500 bunches in all simulations.
- Initial vertical and horizontal offset $\sim 10 \mu\text{m}$ and $400 \mu\text{m}$.
- c.f. initial beam $\sigma_{y,x} (= [\beta \epsilon_n / \gamma]^{1/2}) \sim 10.1 \mu\text{m}, 270 \mu\text{m}$.
- $\sigma_{x',y'} (= [(1 + \alpha^2) / \beta \epsilon_n / \gamma]^{1/2}) \sim 5.1, 0.26 \mu\text{rads}$
- Initial $\beta_{x,y} \sim 89.3, 50.7$ m



TESLA 9-Cell Cavity



Thanks to *Chris Adolphsen*, Nicoleta Baboi,
Joe Frisch, *Roger Miller*, Tor Raubenheimer, Mark Ross, *Peter Tenenbaum*.

Why?

- A perfectly elliptical TESLA cavity has identical (rotated) x-y fields; their frequencies are degenerate.
- In reality the geometry of the vertical plane will be slightly different from the horizontal plane.
- Thus, the freq. degeneracy is split quite naturally as a consequence of quite relaxed tolerances in making several thousand cavities.
- The plane of the eigenmodes will be shifted.



Overview

Make several injection offset simulations to investigate issue of emittance dilution.

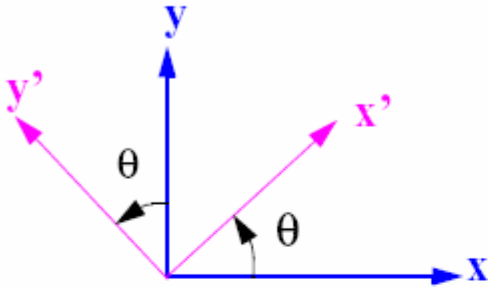
- 1. Force the angle of all couplers relative to the dipole field to be static.**
- 2. Allow complete azimuthal randomization of couplers (0 to 2π).**
- 3. Limit the azimuthal spread of the couplers.**

- **For *detailed* simulations consider a single machine for the above cases.**
- **For general emittance dilution consider 200 separate machines.**

Two cases are investigated:

- 1. ‘Properly’ damped cavities (all modes damped according to the design)**
- 2. ‘Rogue’ mode miss-damped. What is the impact on the emittance dilution?**

Degenerate Dipole Mode Rotation and E.M. Field Coupling



$$V_{x'}(0) = V_x(0) \cos \theta$$

$$V_{y'}(0) = -V_x(0) \sin \theta$$



$$V_y(t) = V_{x'}(t) \sin \theta + V_{y'}(t) \cos \theta$$

$$\Rightarrow V_y(t) = V_x(0) \sin \theta \cos \theta \exp(i \omega_x t) - V_x(0) \sin \theta \cos \theta \exp(i \omega_y t)$$

$$\Rightarrow V_y(t) = i V_x(0) \sin 2\theta \exp(i \langle \omega \rangle t) \sin (\Delta \omega t / 2)$$

where the splitting of the degenerate frequency is given by:

$$\Delta \omega = \omega_{x'} - \omega_{y'}$$

$$\text{and the mean frequency: } \langle \omega \rangle = (\omega_{x'} + \omega_{y'}) / 2$$

$$V_y = iV_x(0) \sin 2\theta \exp(i\langle\omega\rangle t) \sin(\Delta\omega t/2)$$

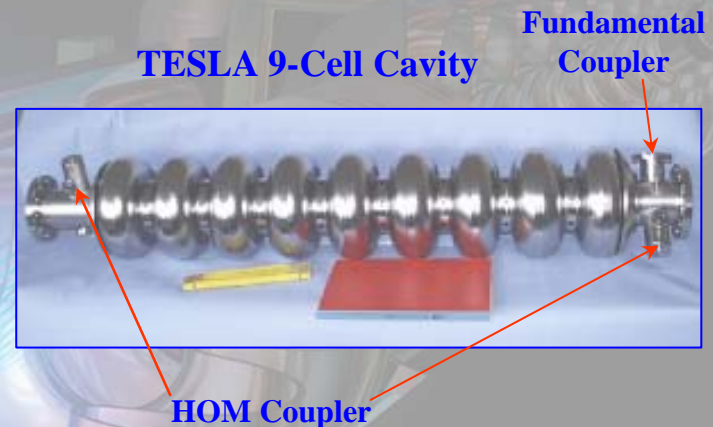
Thus for $\theta = 0$ or $\pi/2$ $V_y(0) = 0$ as expected!

The additional kick to the beam is of the form:

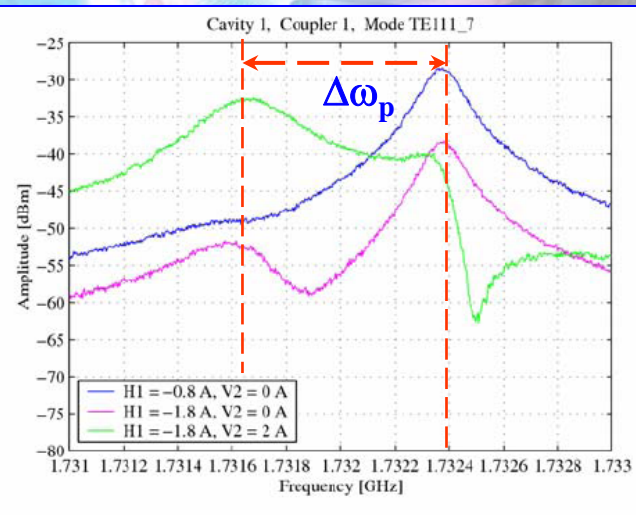
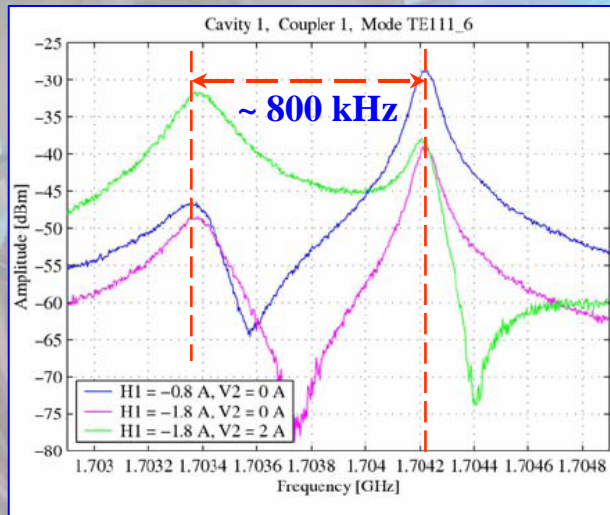
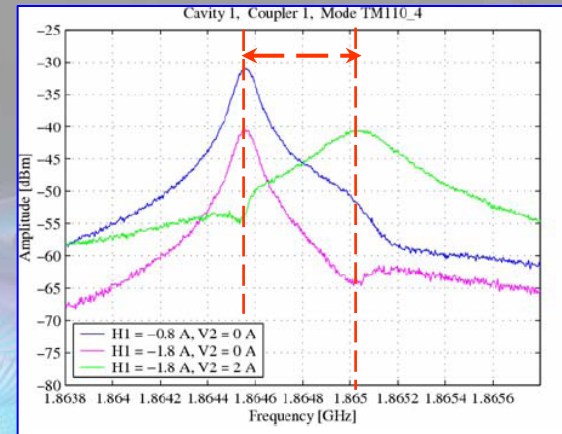
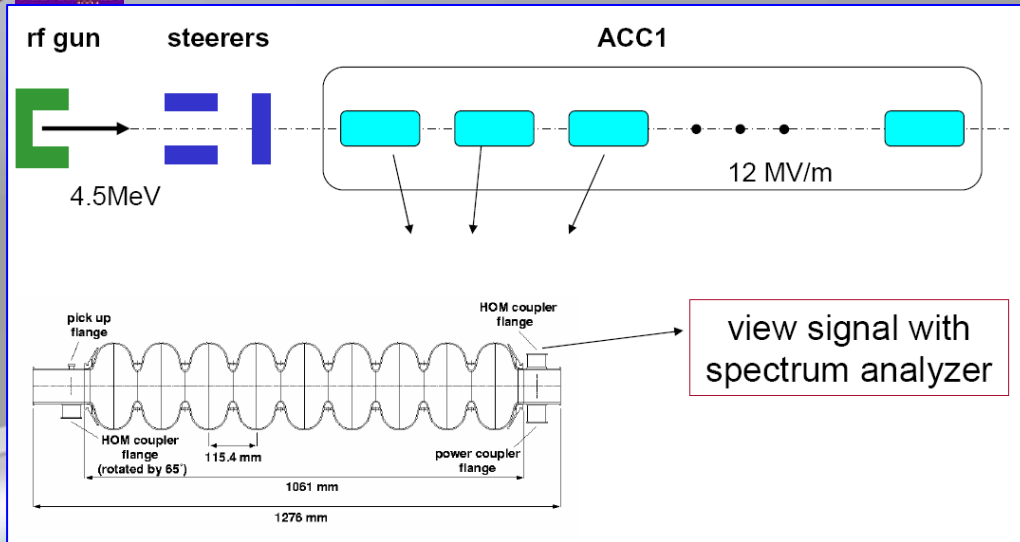
$$V(t) = \sum K_n \sin(\omega_p t) \exp(-\omega_p t/2Q_p) U(t) \hat{y} + \sum K_n \cos(\langle\omega_p\rangle t) \sin(\Delta\omega_p t/2) \exp(-\langle\omega_p\rangle t/2Q_p) U(t) \hat{x}$$

Where the sum is taken over all modes of interest and $U(t)$ is the unit step function

=> Recent TTF experiments (Ross, Napoly & Baboi) suggest that this frequency splitting varies by ~ 400 kHz to 800 kHz. In all simulations presented here we use 600 kHz frequency splitting for all modes.



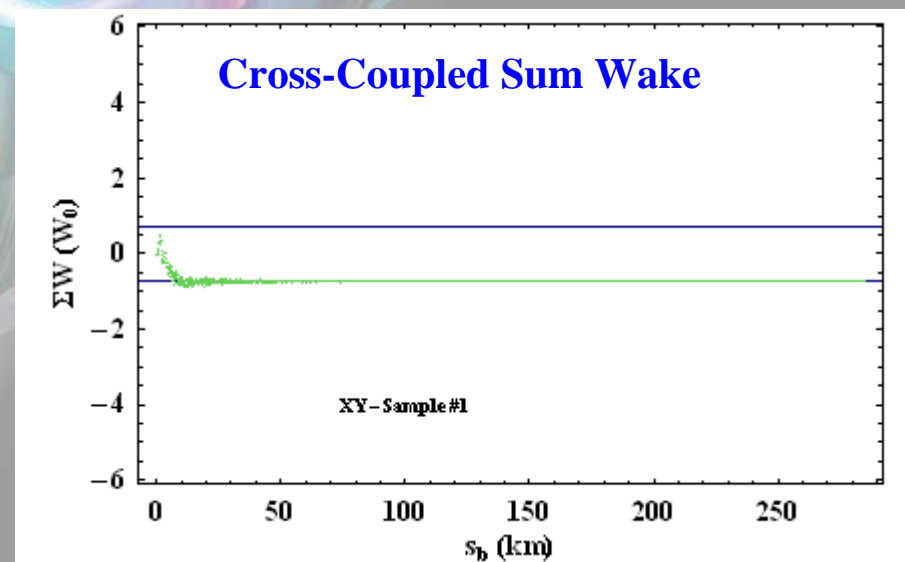
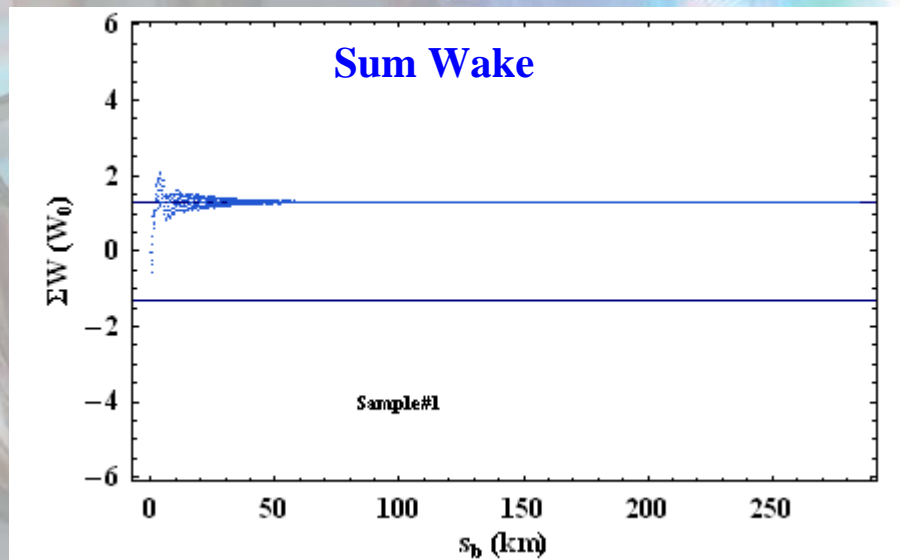
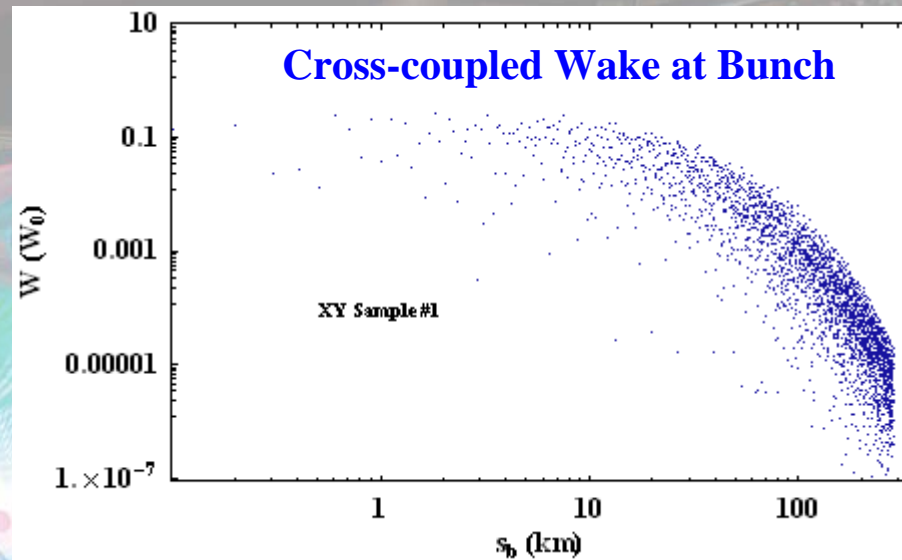
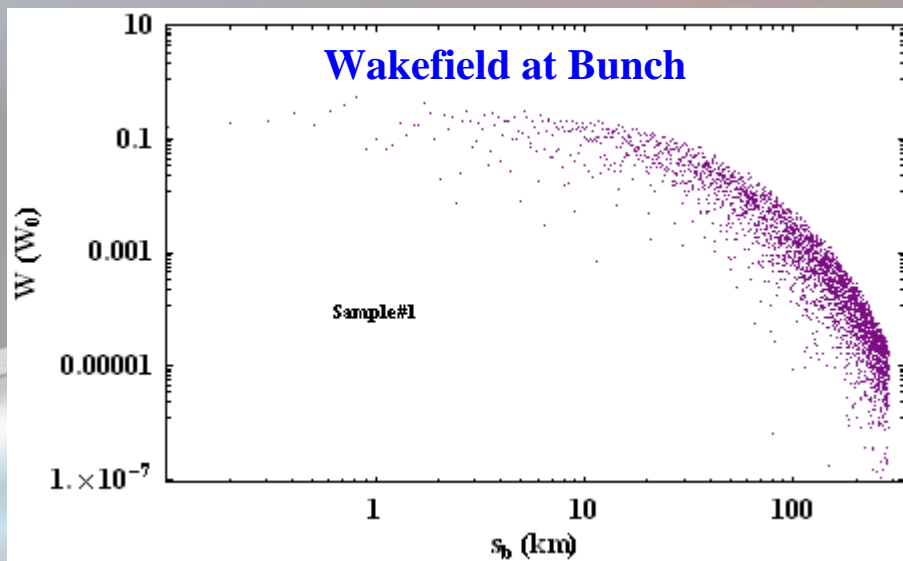
Measurement of HOMs at TTF



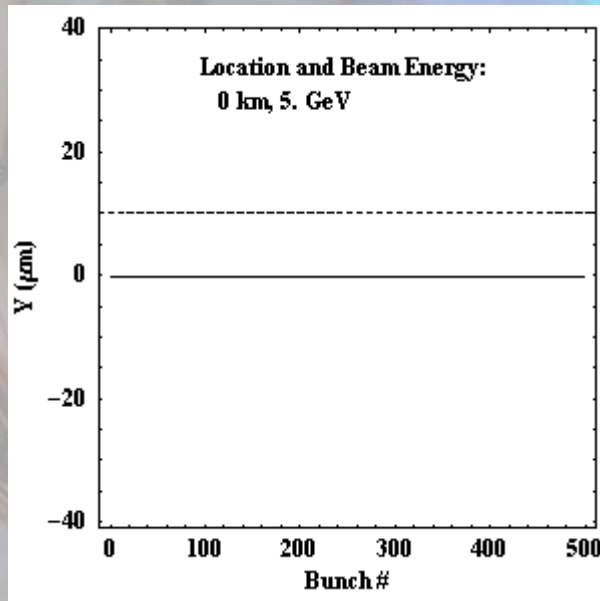
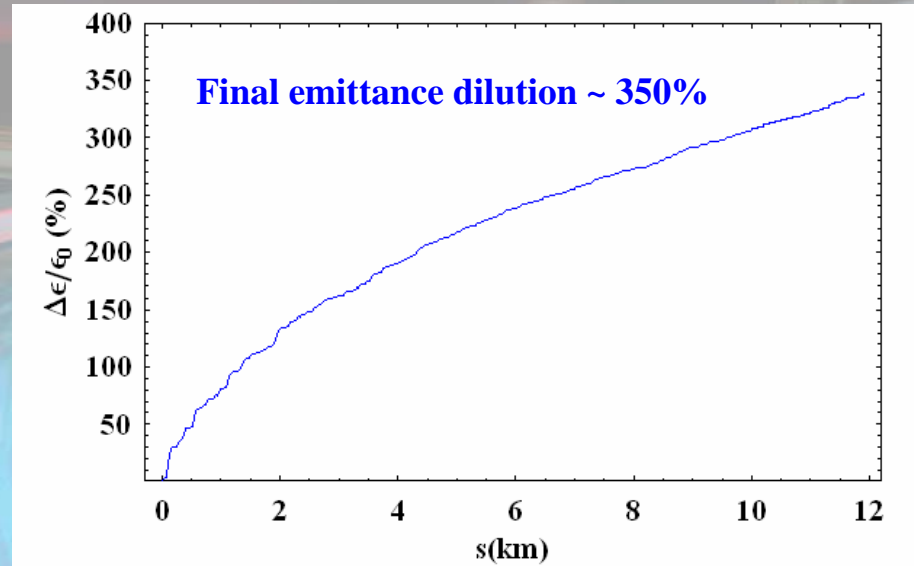
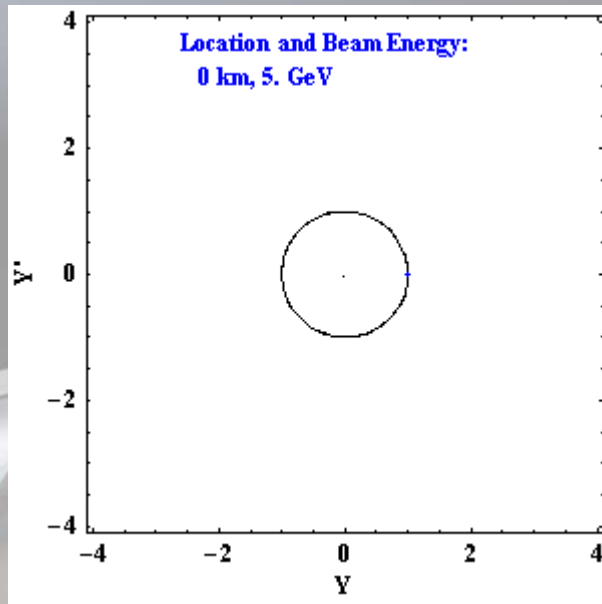
Taken from Ross, Baboi, Napoly et al. See Linac04 paper & SLAC-PUB 11190.

Transverse Wakefields in TESLA-like Cavities.

$$W_y(t) = \sum K_n \sin(\omega_p t) \exp(-\omega_p t / 2Q_p) U(t), \quad W_{xy} = \sum K_n \cos(\langle \omega_p \rangle t) \sin(\Delta \omega_p t / 2) \exp(-\langle \omega_p \rangle t / 2Q_p) U(t).$$

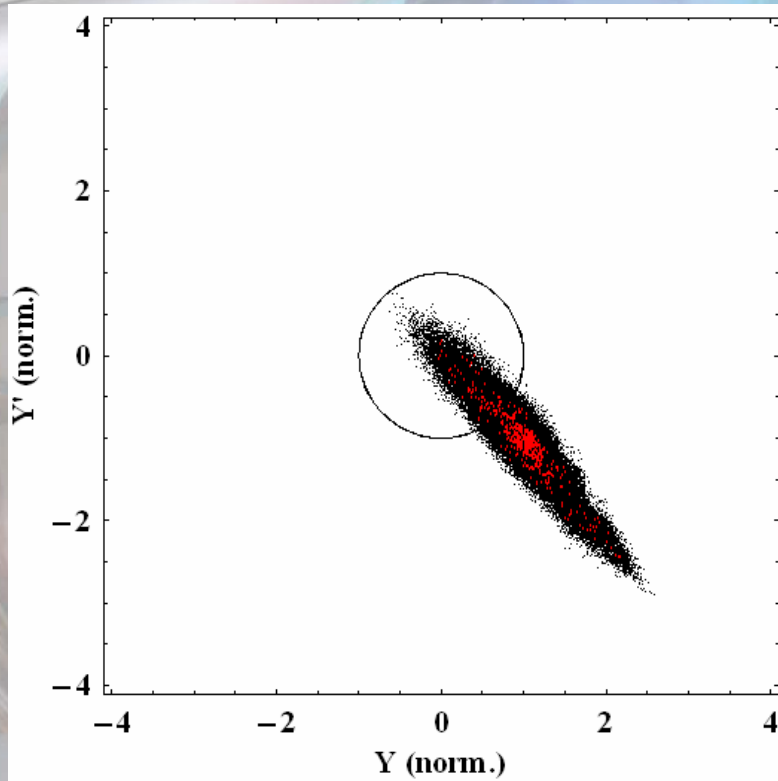
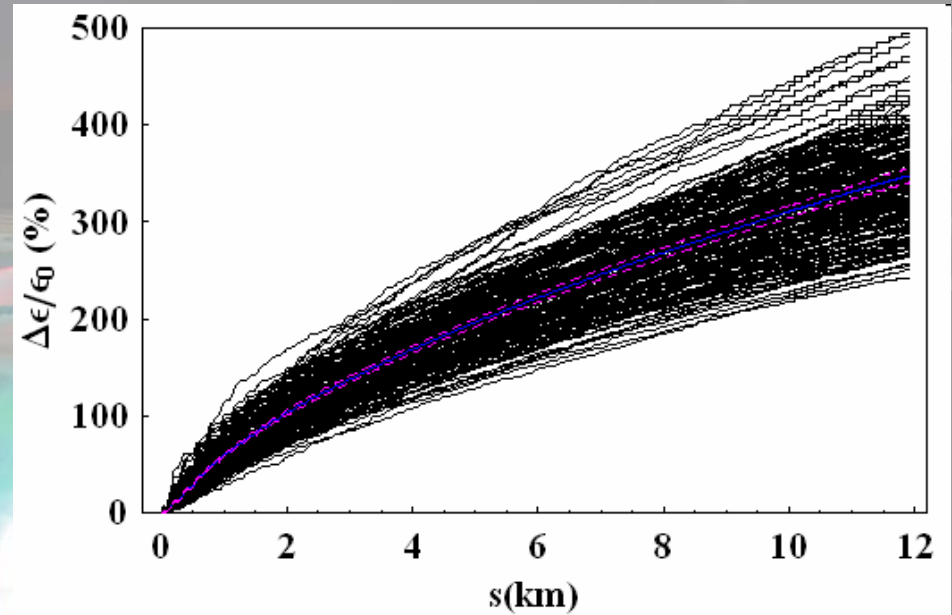


Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{off}=0$).



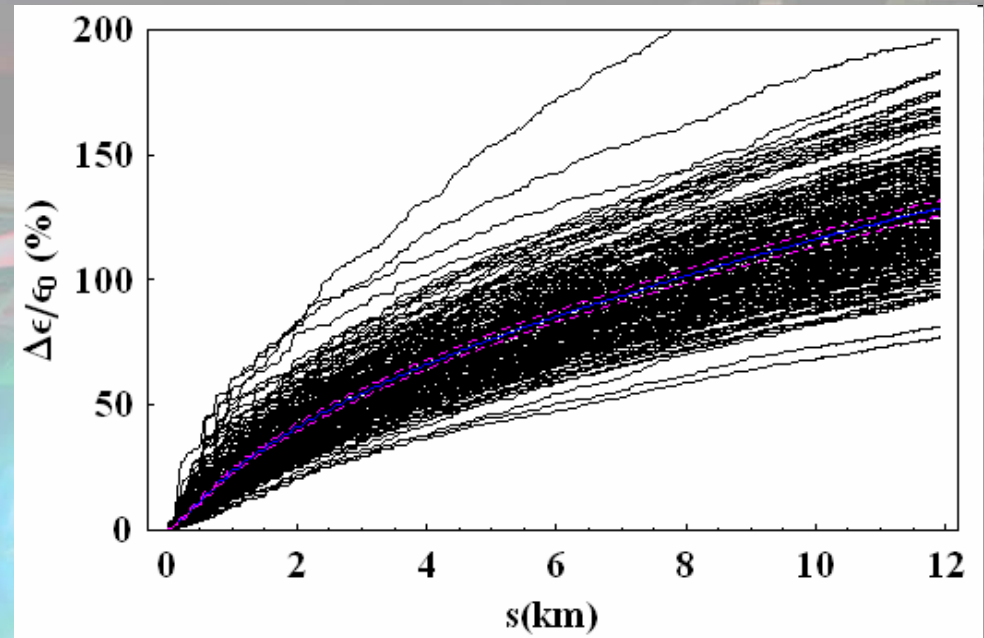
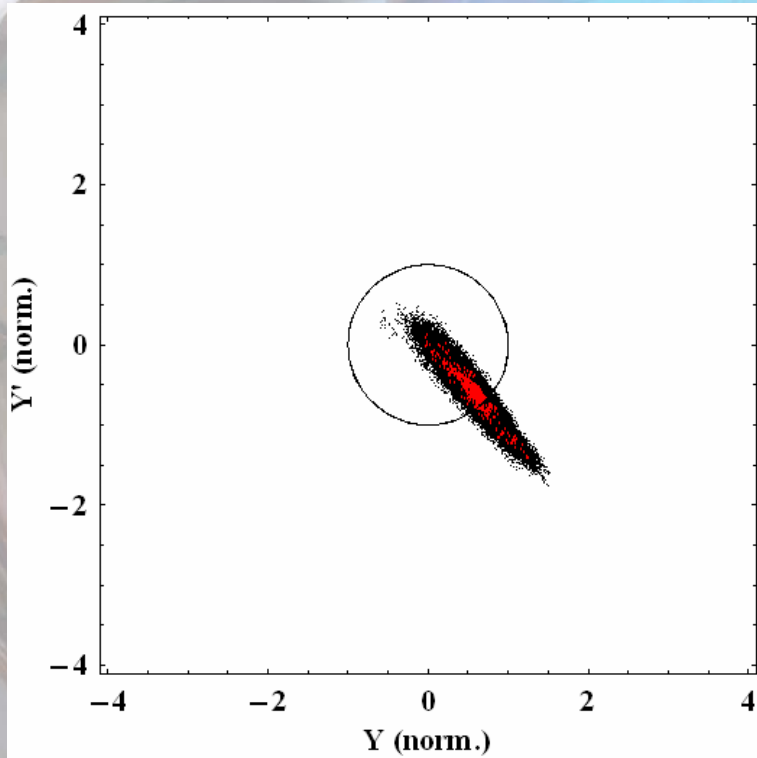
- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- *Fixed azimuthal orientation of couplers.*
- **Long range wakes included in simulations**

Fixed Azimuthal Phase Distribution (Yoff=0).



- 500 particles in all simulations and 200 machines shown
- Initial vertical and horizontal offset $0 \mu\text{m}$ and $270 \mu\text{m}$ ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal orientation of couplers.
- Long range wakes included in simulations

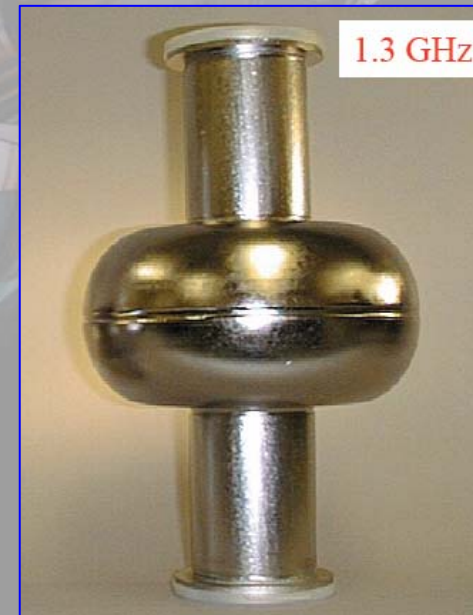
Fixed Azimuthal Phase Distribution ($Y_{\text{off}}=0$).



- 500 particles in all simulations and 200 machines shown
- Initial vertical and horizontal offset $0 \mu\text{m}$ and $270 \mu\text{m}$ ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- *Random azimuthal orientation of couplers (0 to 2π about a mean value of $\pi/4$).*
- **Long range wakes included in simulations**

Amelioration of Large Emittance Dilution?

- Increase the damping of the HOM modes? At present the damping is $\sim 10^4 - 10^5$. It may be difficult to reduce the damping even further?
- The dipole frequency degeneracy's are already split – by ~ 400 kHz to 800 kHz. Increasing the splitting (which can be achieved by making markedly asymmetrical cavities) to say 10 MHz may allow the influence of mode coupling to be minimal over the length of the linac.
- Splitting the tune of the linac. Present design is 60/60 – horizontal/vertical. This may be the most straightforward to implement. \Rightarrow We investigate splitting the tune in this talk.



Re-entrant cavity shape is shown. Taken from Rong-Li Geng SRF 2005

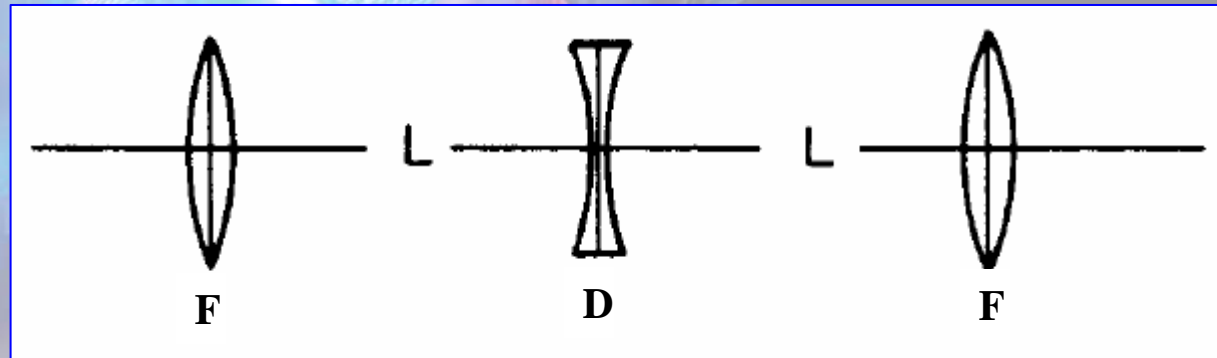
Split Tune of Lattice

- Present lattice has phase advance per cell of 60 degrees per cell.
- Each lattice cell consists of two groups of:
quadupoles + 2 cryo-modules + quadrupole + 2 cryo-modules.
- There are 12 cavities per cryo-module.
- If we split the tune then the coupling should be out of resonance.
- We look at 61/60 through 90/60.

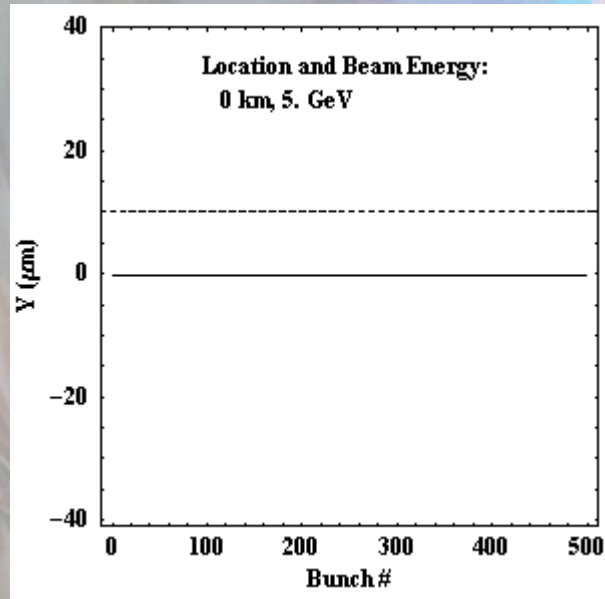
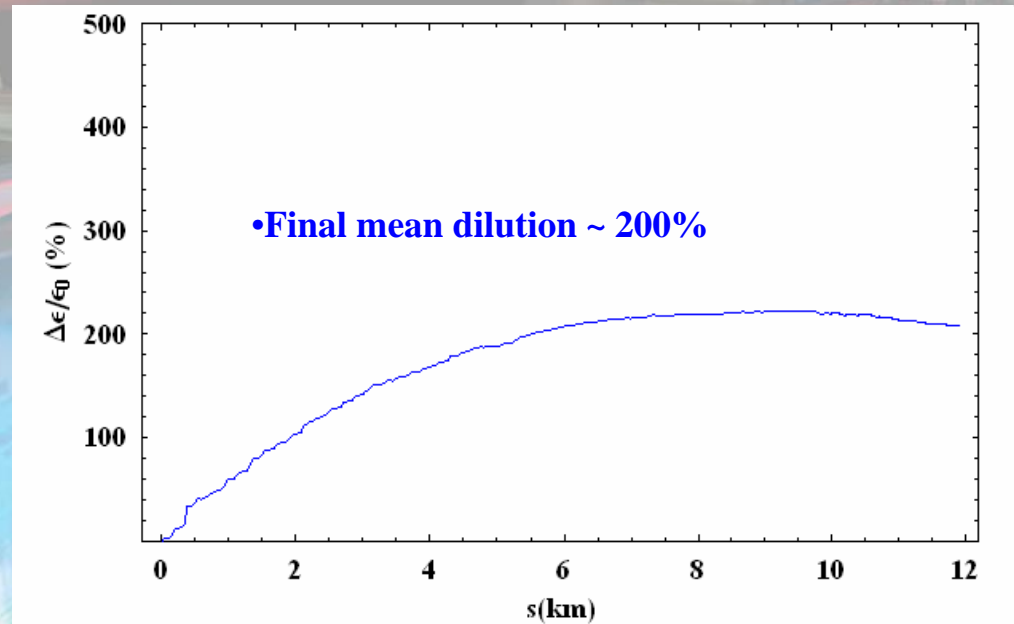
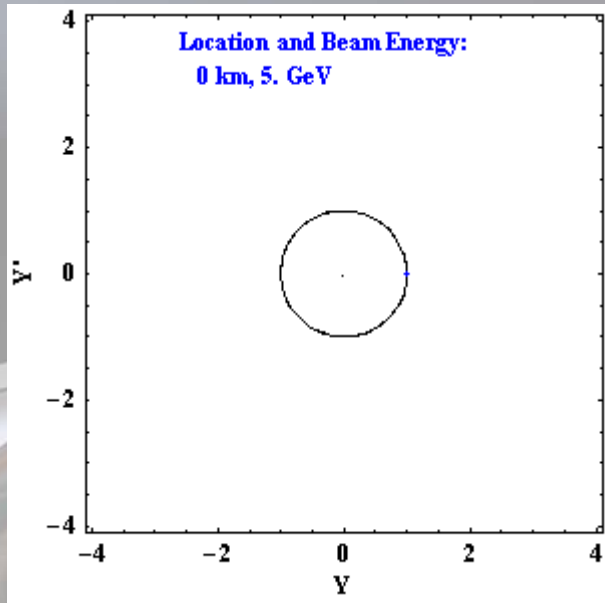
• Single FODO array shown.

• We split the tune by raising the F functions for the focusing quadrupole with respect to the D quadrupole.

• The D quadrupole is then readjusted in order to achieve 60 in the vertical plane.

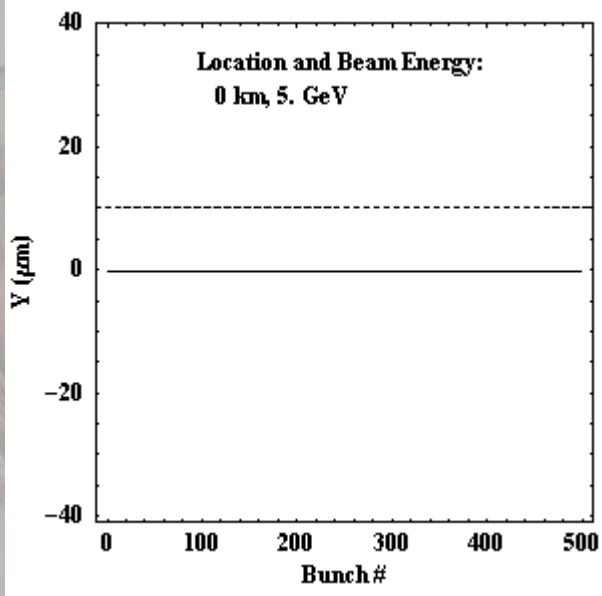
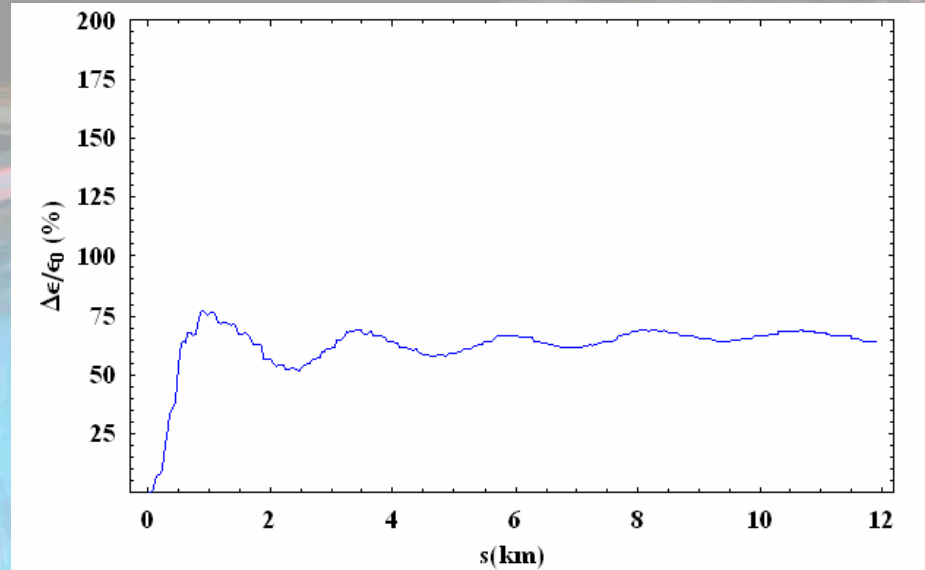
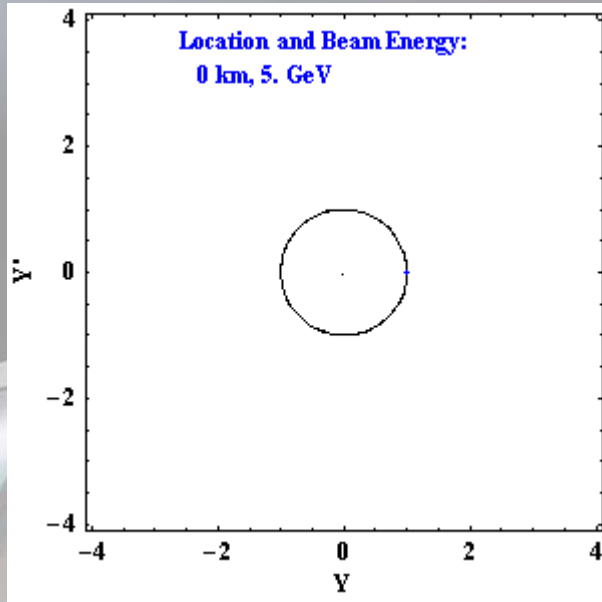


Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{off}=0$). *Split Tune 61/60*



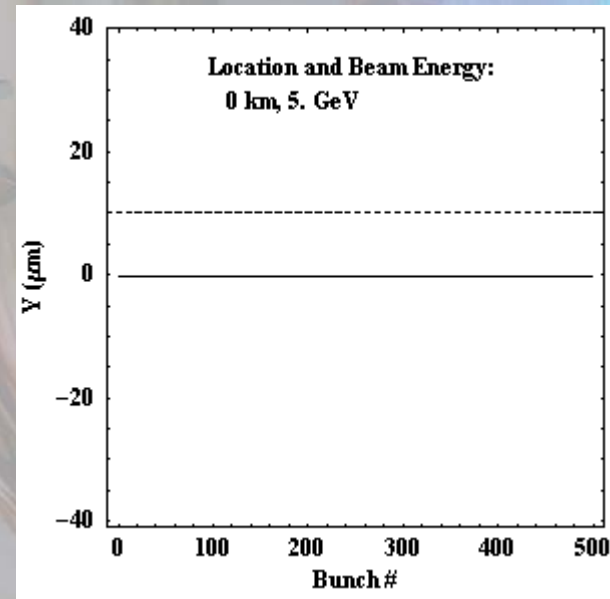
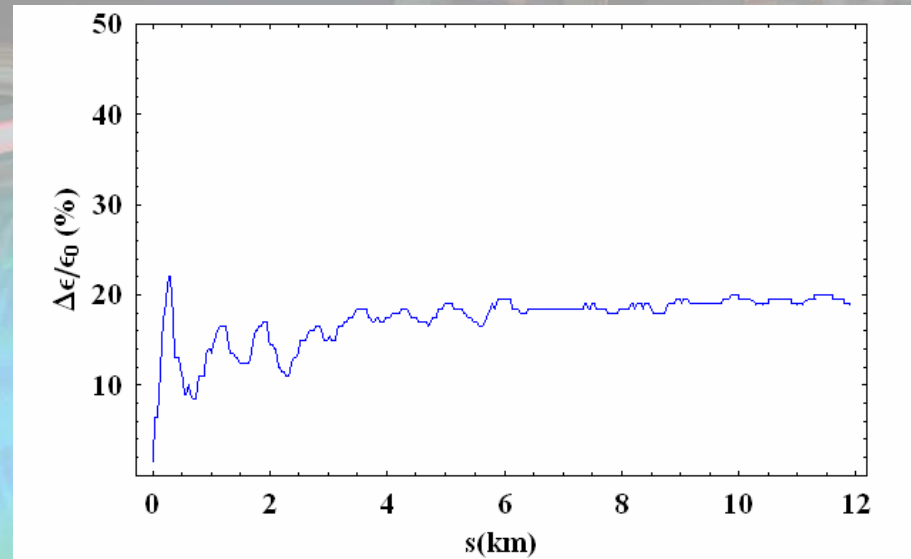
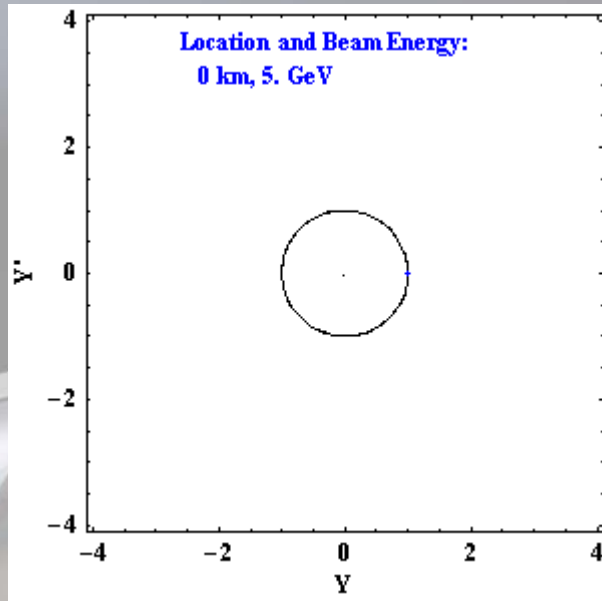
- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal orientation of couplers.
- Long range wakes included in simulations

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{\text{off}}=0$). *Split Tune 70/60*



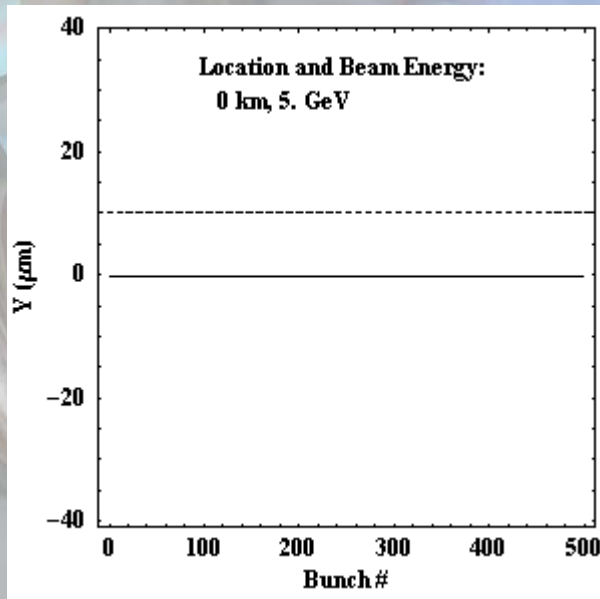
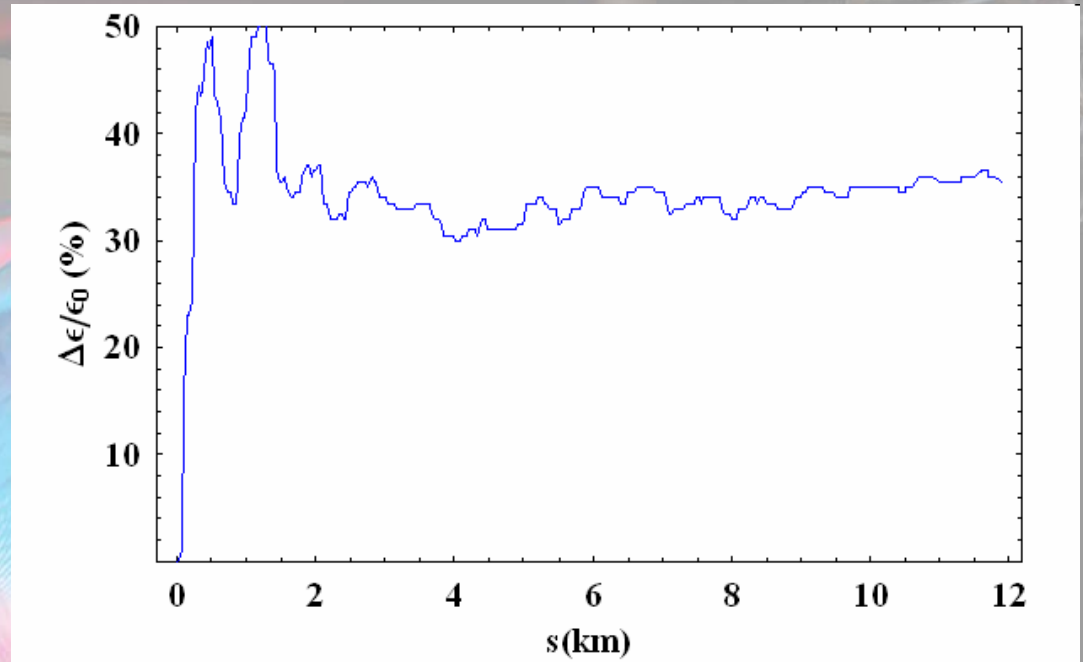
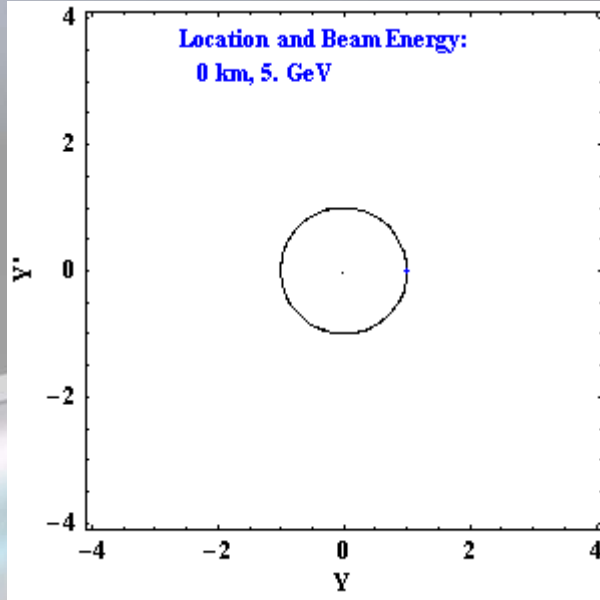
- 500 particles in all simulations
- Initial vertical and horizontal offset $0 \mu\text{m}$ and $270 \mu\text{m}$ ($\sim \sigma_x$),
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- *Random azimuthal orientation of couplers (0 to 2π).*
- **Long + Short range wakes included in simulations**

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{off}=0$). *Split Tune 90/60*



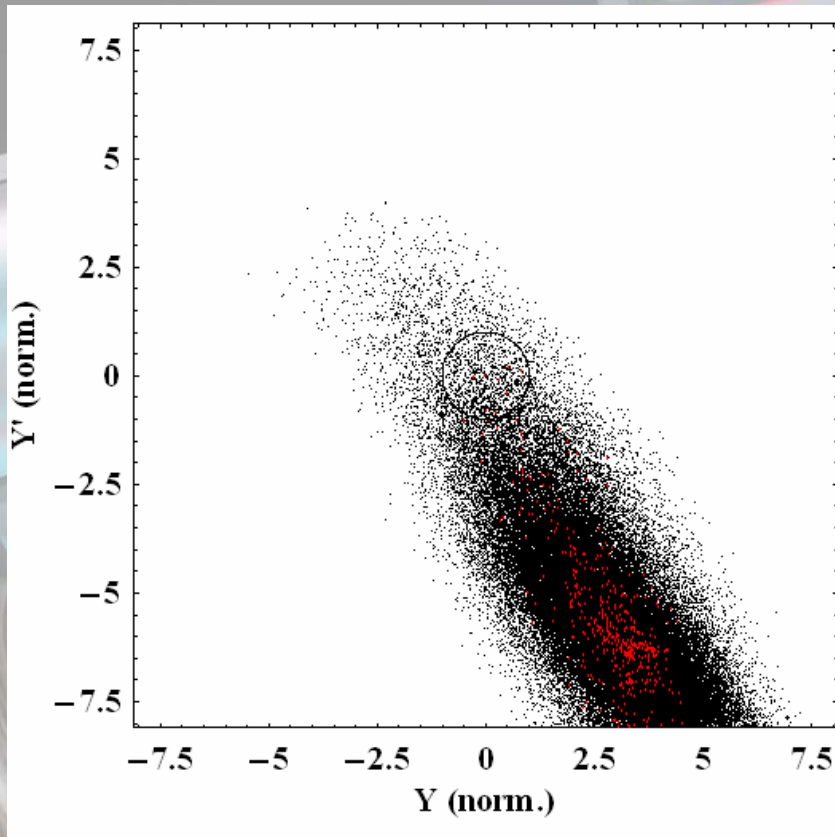
- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}$, 270 μm)
- *Fixed azimuthal orientation of couplers.*
- **Long range wakes included in simulations**

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{off}=0$). *Split Tune 90/60*

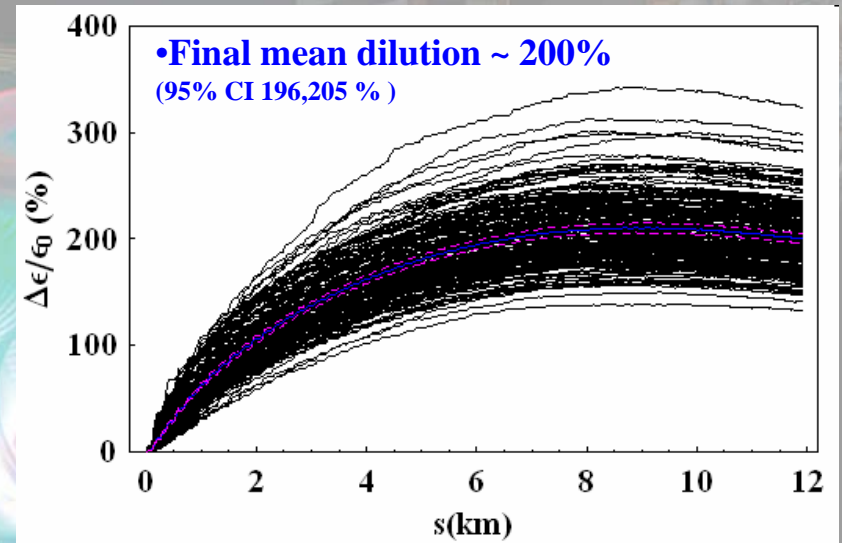


- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- *Limited randomization of azimuthal phase*
– Max 10% of mean.
- **Long range wakes included in simulations**

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{off}=0$). *Split Tune 61/60*

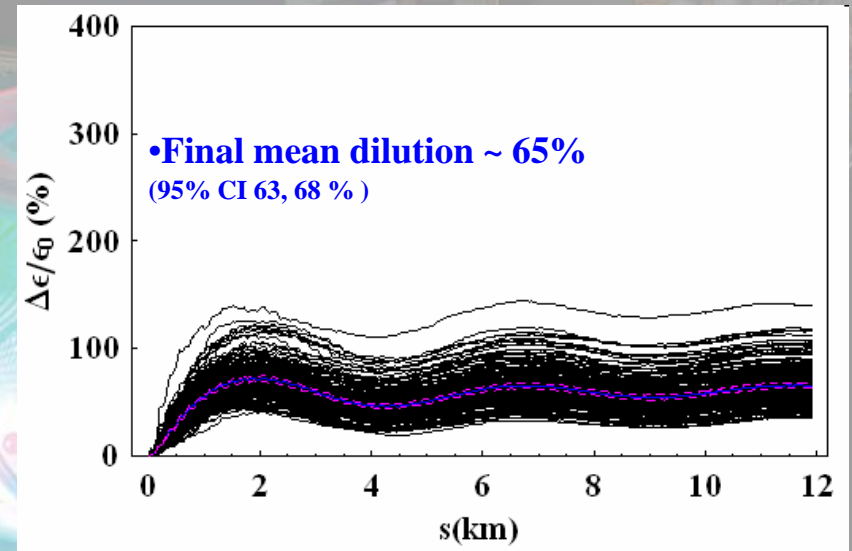
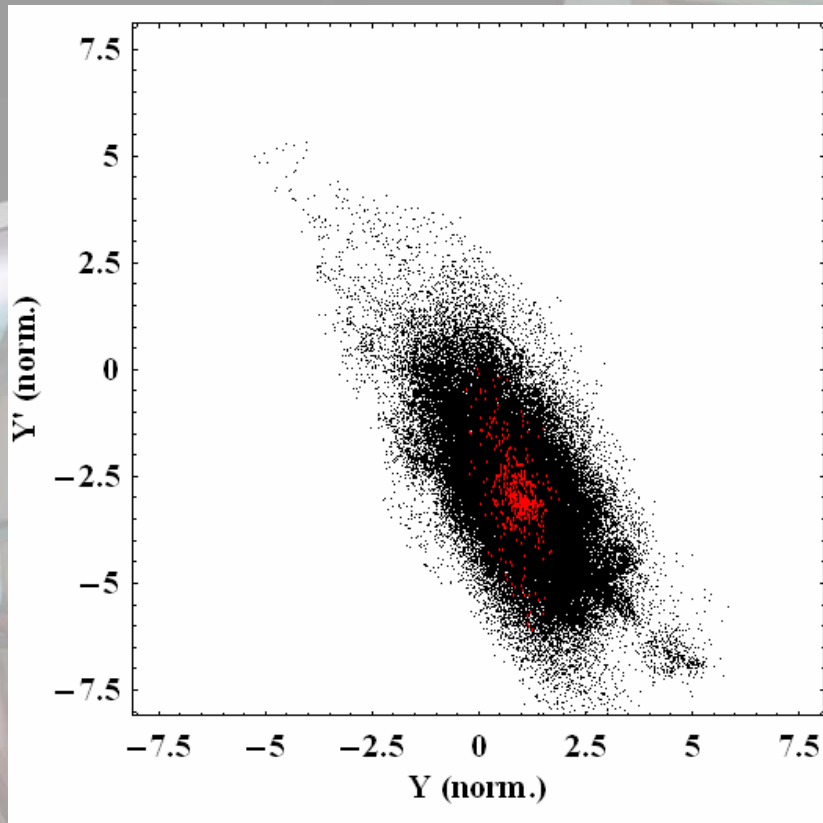


Small circle represents scaling
w.r.t. energy at *end* of linac.



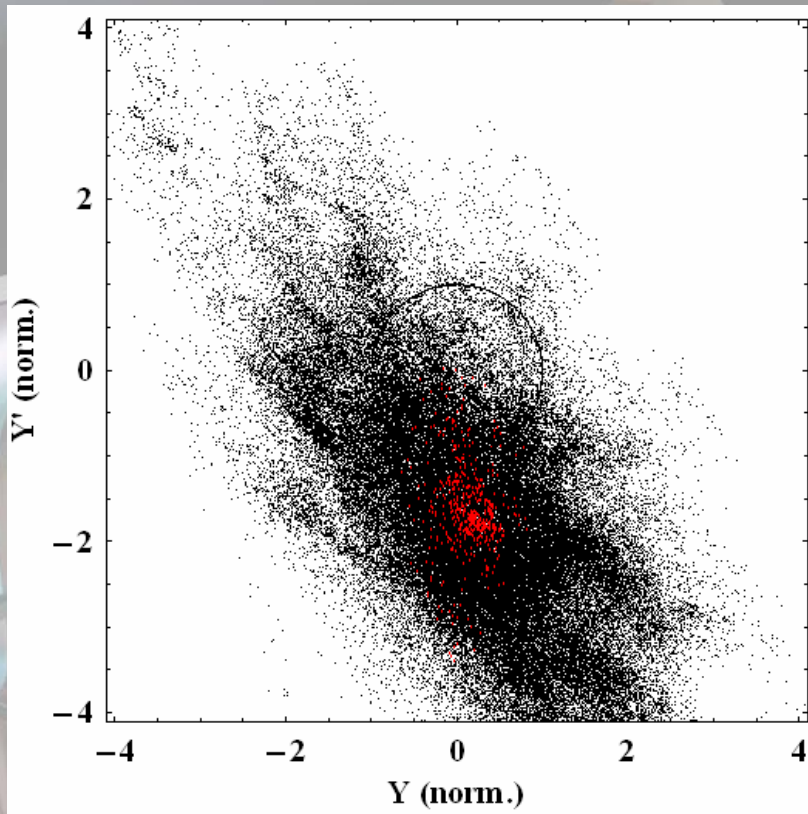
- 500 particles in all simulations and 200 machines are illustrated
- The dashed line marks the injection offset of all bunches
- Initial vertical and horizontal offset $0 \mu\text{m}$ and $270 \mu\text{m}$ ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal phase.
- Long range wakes included in simulations

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{off}=0$). *Split Tune 65/60*



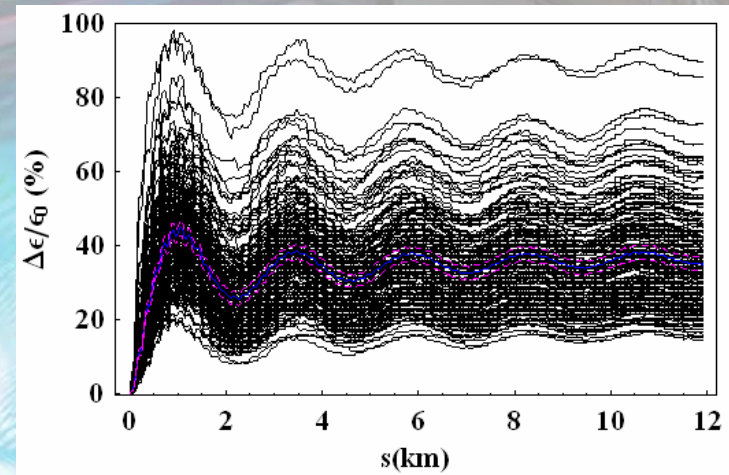
- 500 particles in all simulations and 200 machines are illustrated
- The dashed line marks the injection offset of all bunches
- Initial vertical and horizontal offset $0 \mu\text{m}$ and $270 \mu\text{m}$ ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal phase.
- Long range wakes included in simulations

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{\text{off}}=0$). *Split Tune 70/60*



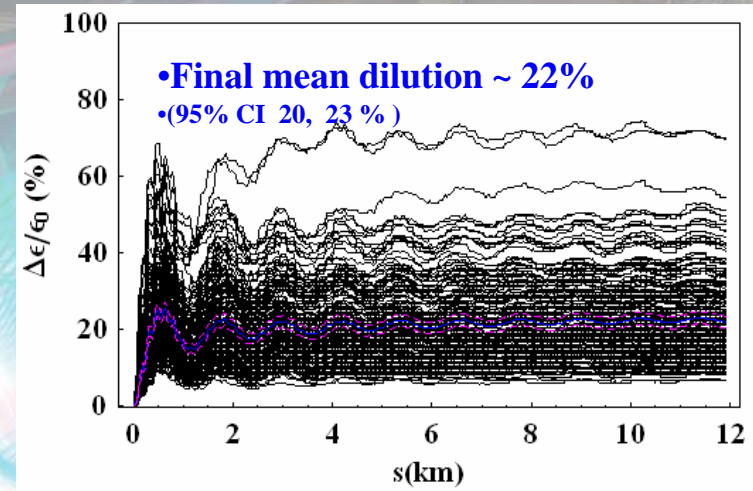
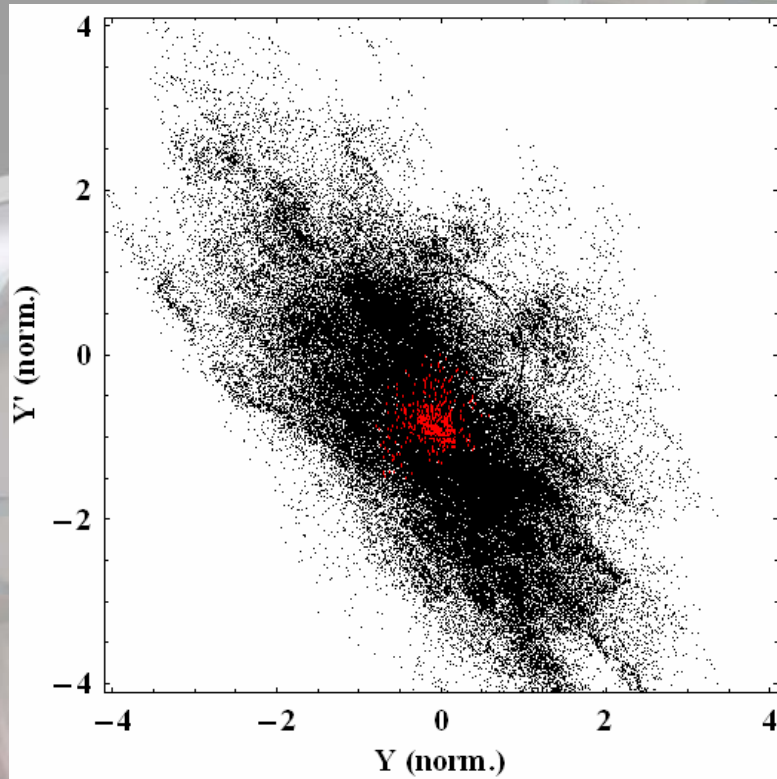
These have been rescaled
relative to the previous ones

•Final mean dilution ~ 35% (95% CI 33, 37 %)



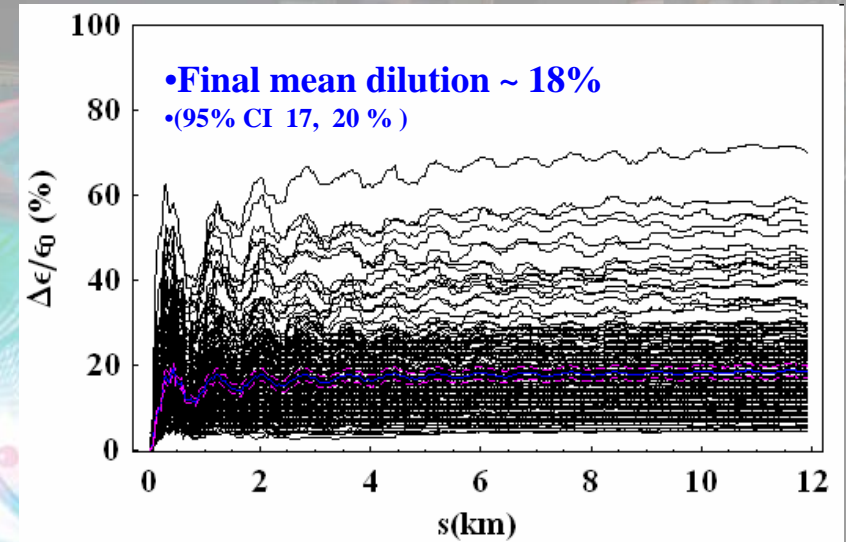
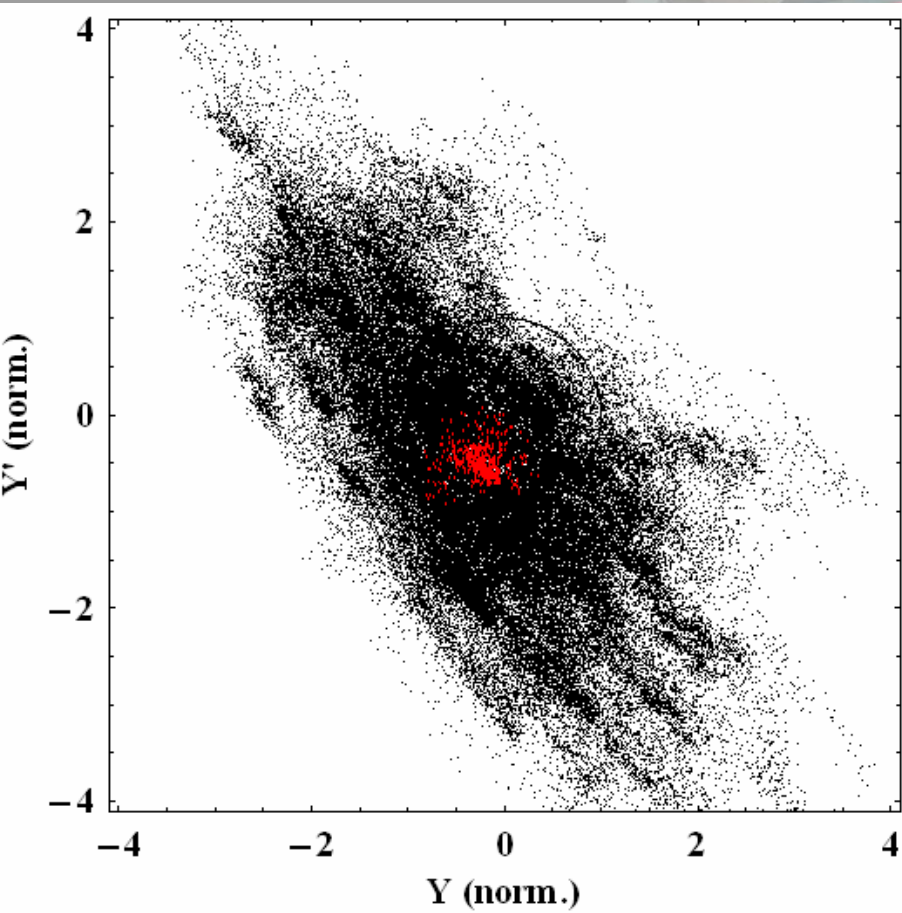
- 500 particles in all simulations and 200 machines are illustrated
- The dashed line marks the injection offset of all bunches
- Initial vertical and horizontal offset $0 \mu\text{m}$ and $270 \mu\text{m}$ ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal phase.
- Long range wakes included in simulations

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{\text{off}}=0$). *Split Tune 80/60*



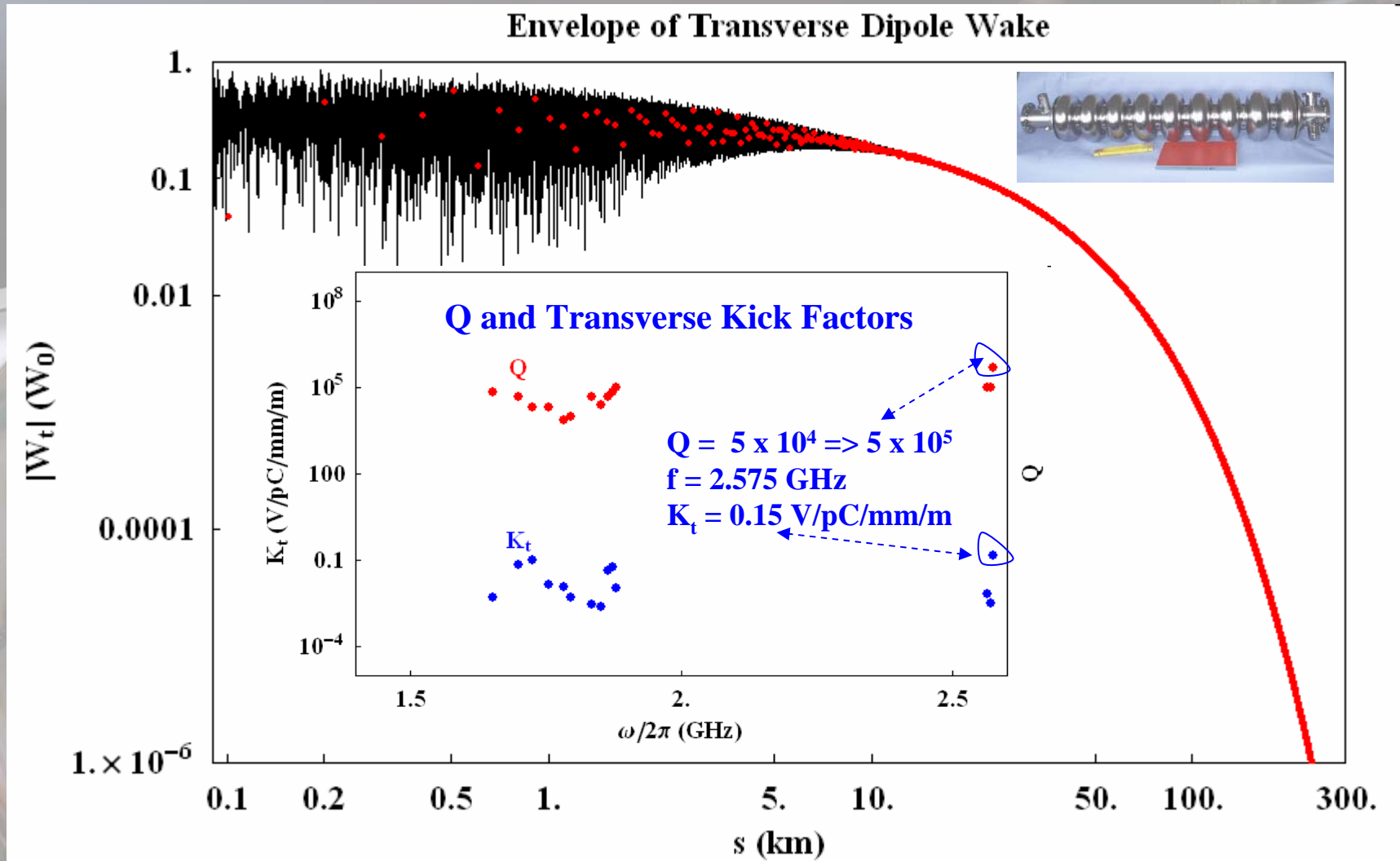
- 500 particles in all simulations and 200 machines are illustrated
- The dashed line marks the injection offset of all bunches
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal phase.
- Long range wakes included in simulations

Cross-Coupled Motion At Fixed Azimuthal Phase Distribution ($Y_{off}=0$). *Split Tune 90/60*

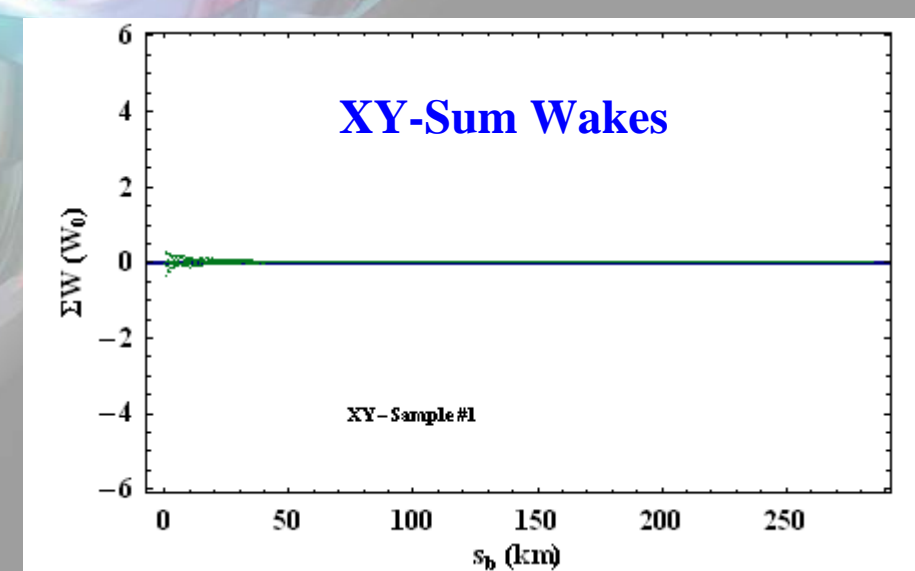
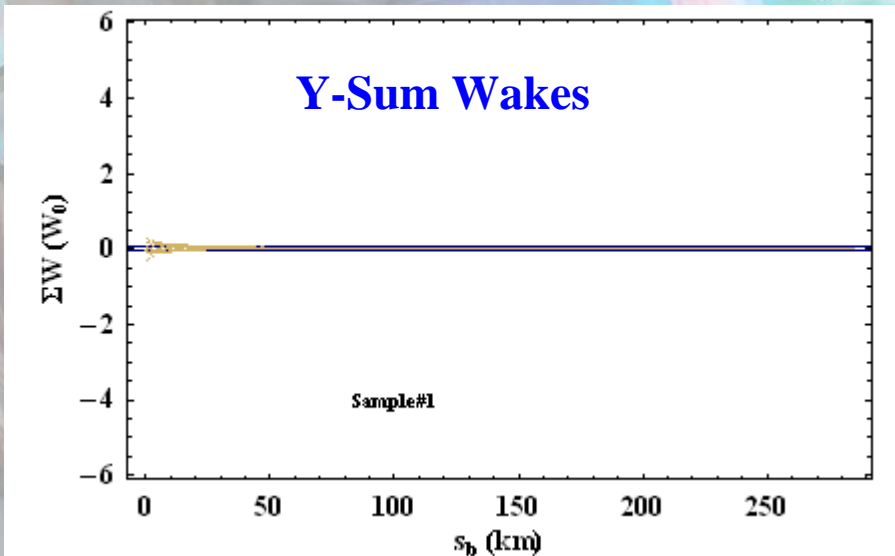
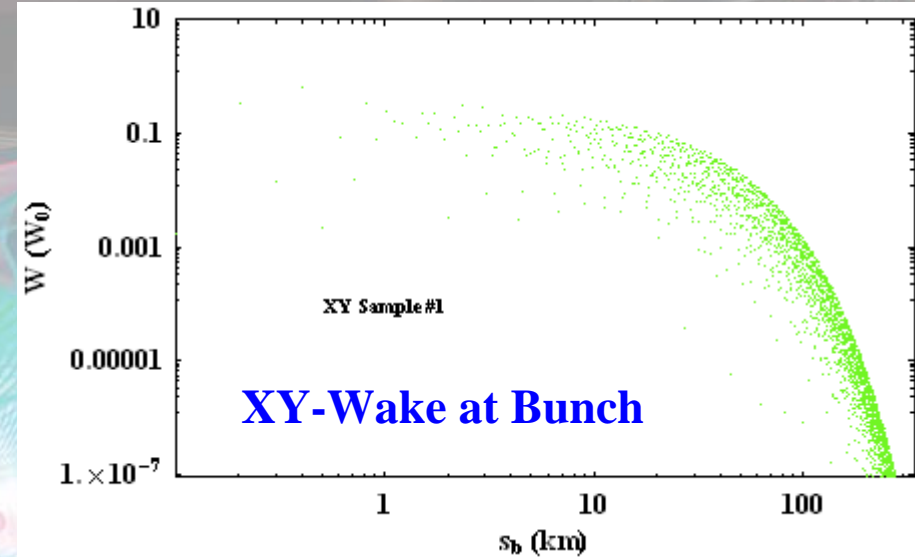
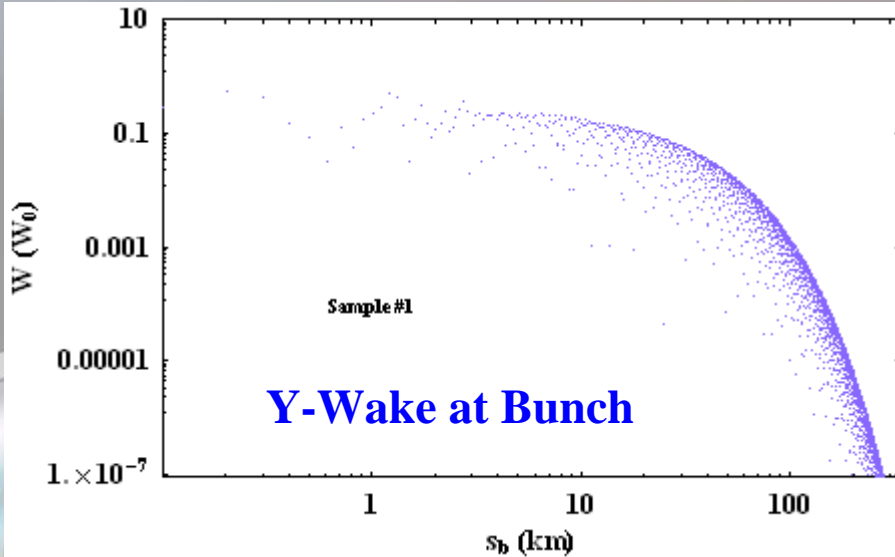


- 500 particles in all simulations and 200 machines are illustrated
- The dashed line marks the injection offset of all bunches
- Initial vertical and horizontal offset $0 \mu\text{m}$ and $270 \mu\text{m}$ ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal phase.
- Long range wakes included in simulations

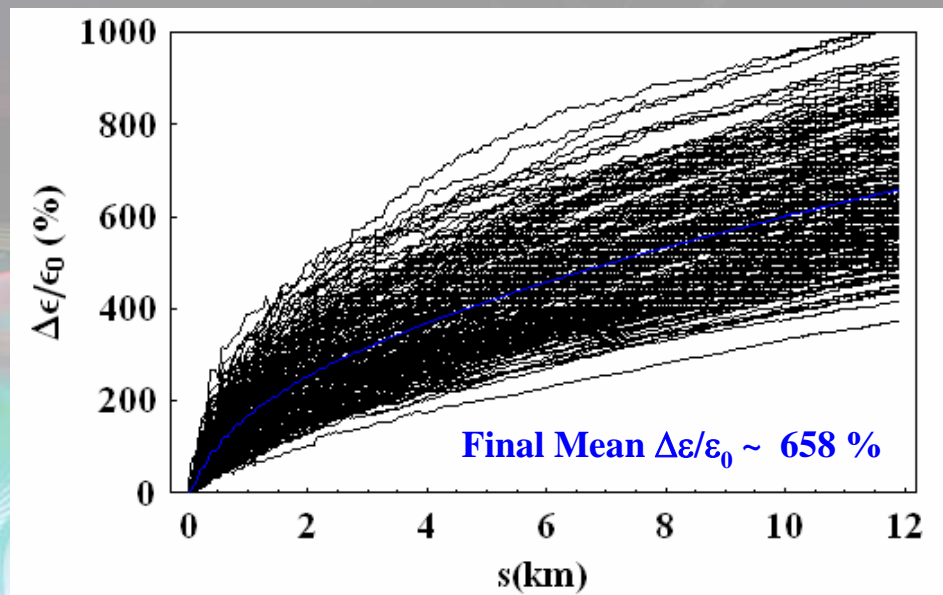
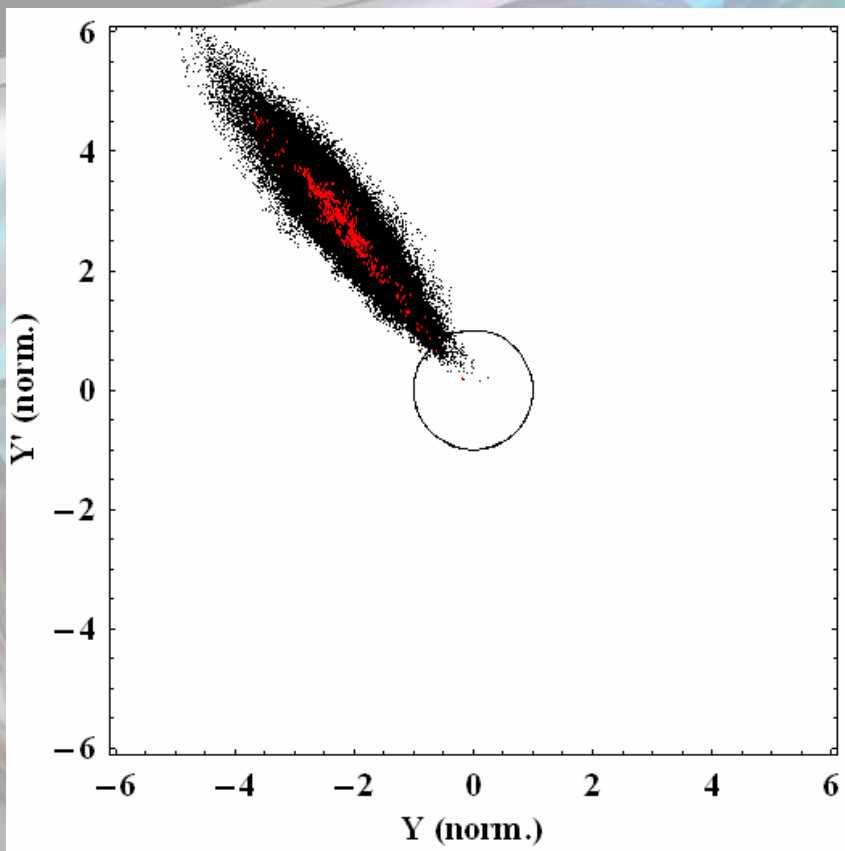
Wakes for Cross-Coupled Motion + Rogue Mode



Wakes Incl. Rogue Mode & Random Errors

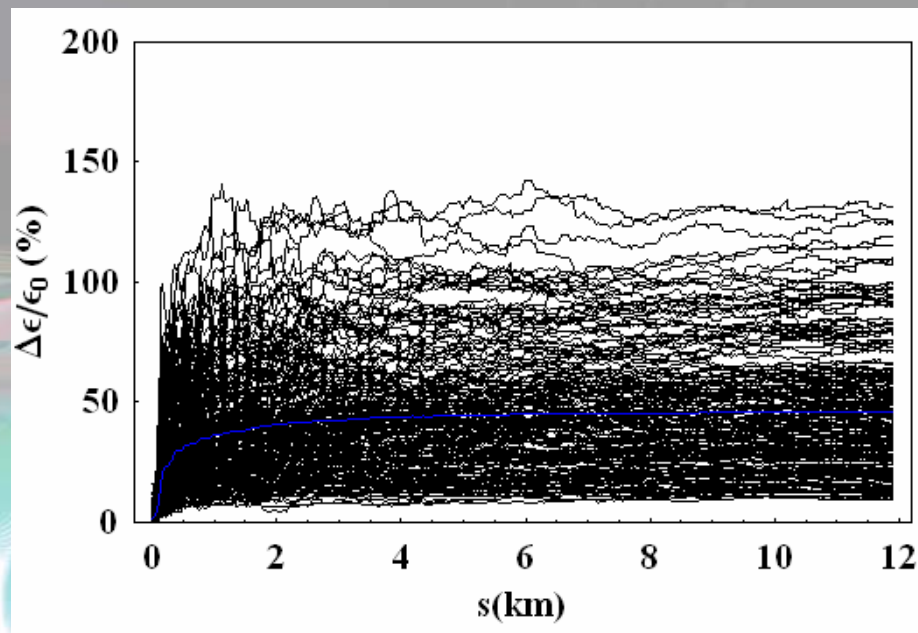
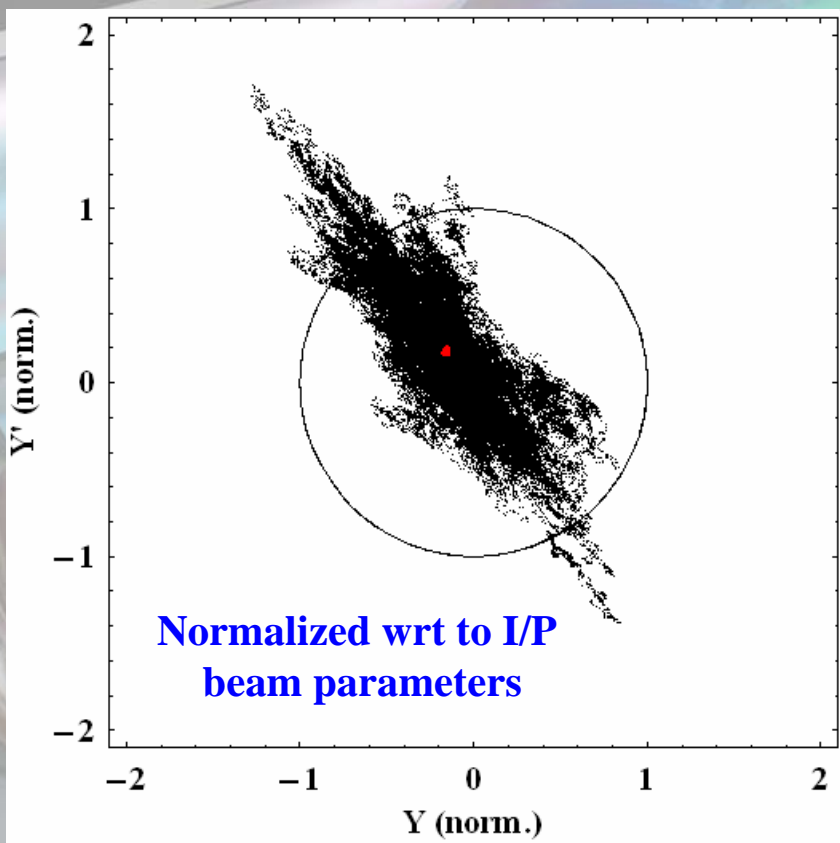


Cross-Coupled Motion At Fixed Azimuthal Phase Distribution + Rogue Mode



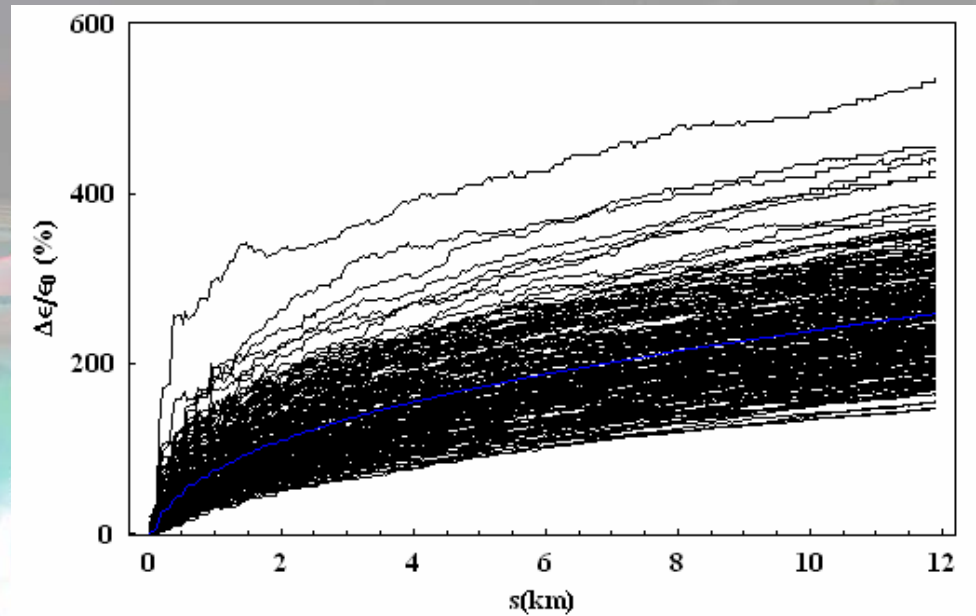
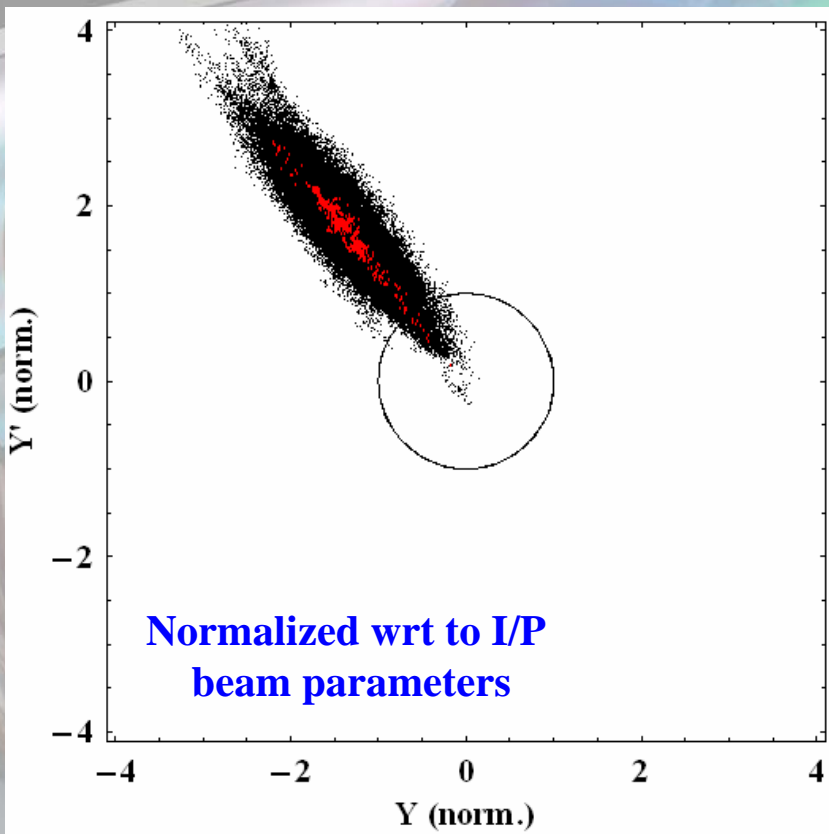
- 500 particles in all simulations.
- The initial position of all bunches in phase space is indicated by the blue dot.
- Initial vertical and horizontal offset $\sim 10 \mu\text{m}$ and $400 \mu\text{m}$.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Azimuthal phase fixed at $\pi/4$
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- Long range wakes included in simulations

Cross-Coupled Motion With Random Azimuthal Phase Distribution + Rogue Mode



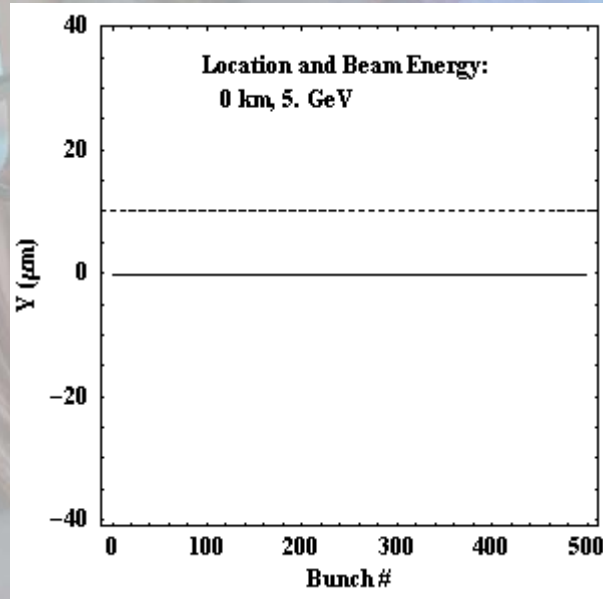
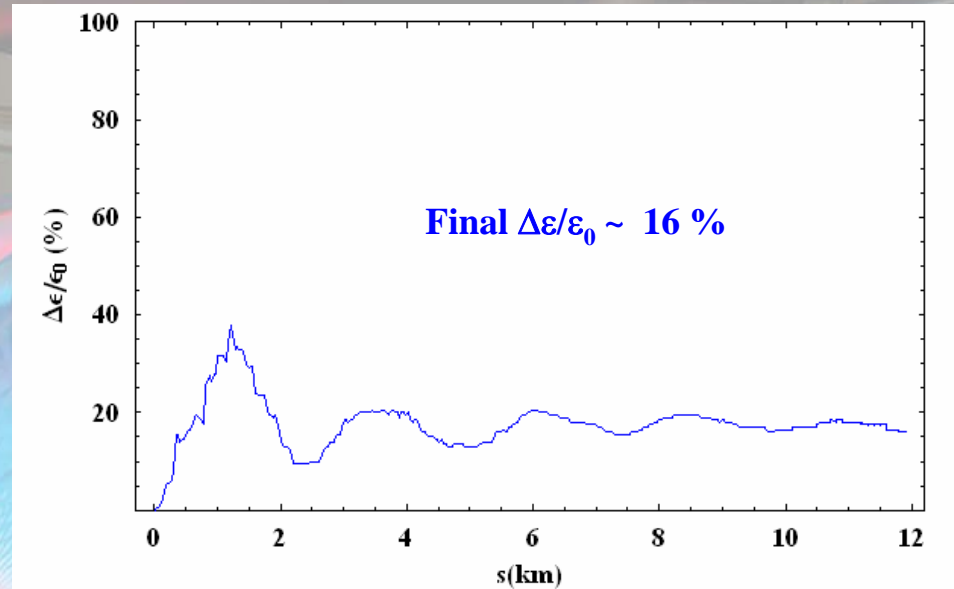
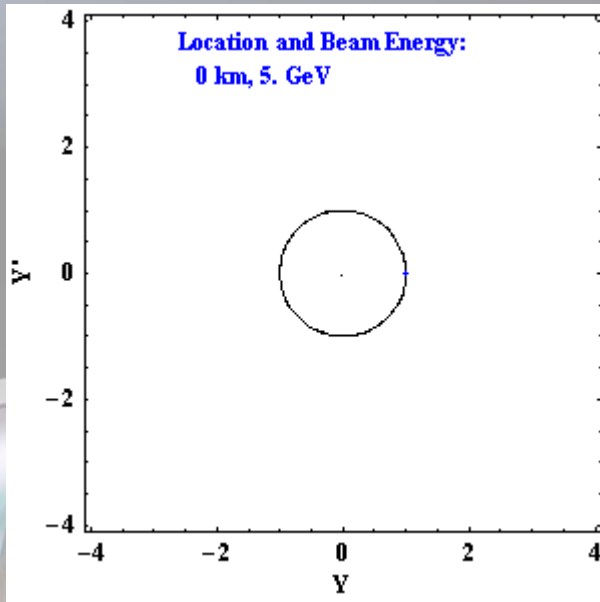
- 500 particles in all simulations.
- The initial position of all bunches in phase space is indicated by the blue dot.
- Initial vertical and horizontal offset $\sim 10 \mu\text{m}$ and $400 \mu\text{m}$.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Random Azimuthal phase (mean $\pi/4$)
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- Long range wakes included in simulations

Cross-Coupled Motion With Limited Random Azimuthal Phase Distribution + Rogue Mode



- 500 particles in all simulations.
- The initial position of all bunches in phase space is indicated by the blue dot.
- Initial vertical and horizontal offset $\sim 10 \mu\text{m}$ and $400 \mu\text{m}$.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Azimuthal phase randomly distributed – Max 10% of mean.
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- Long range wakes included in simulations

Split Tune 70/60.

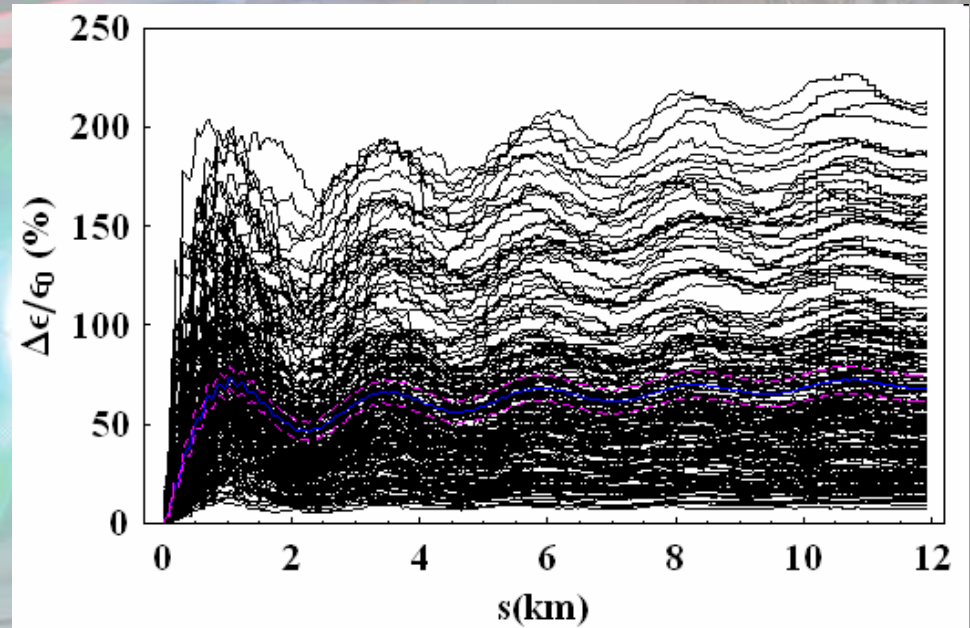
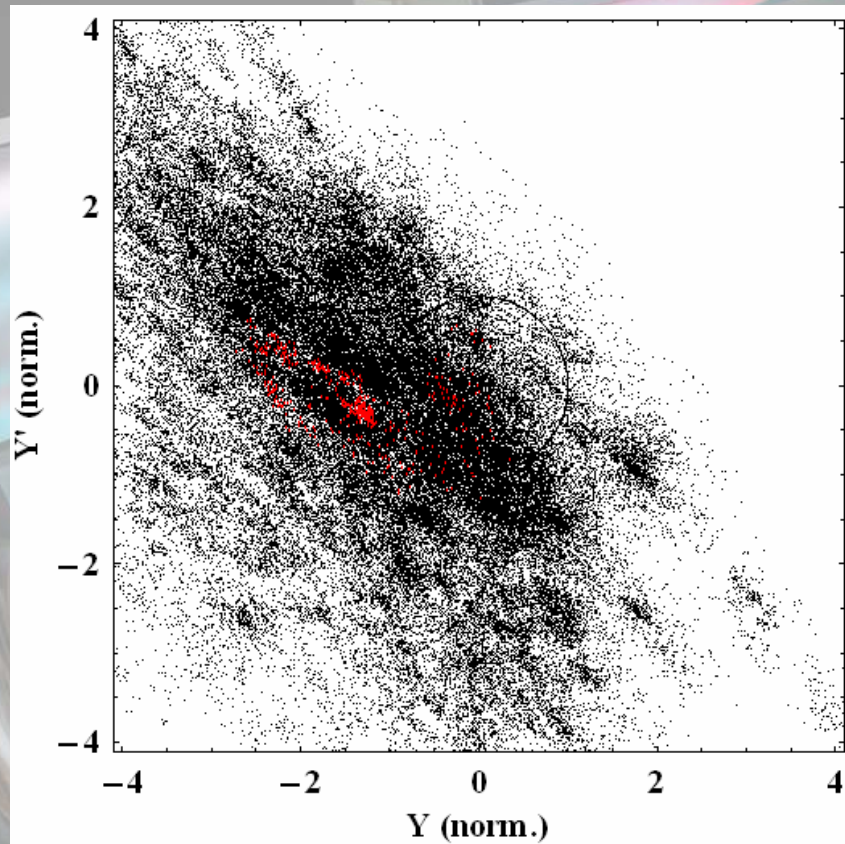


- Only mode-cross coupling gives rise to emittance dilution
- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal orientation of couplers.
- Long range wakes included in simulations

Cross-Coupled Motion

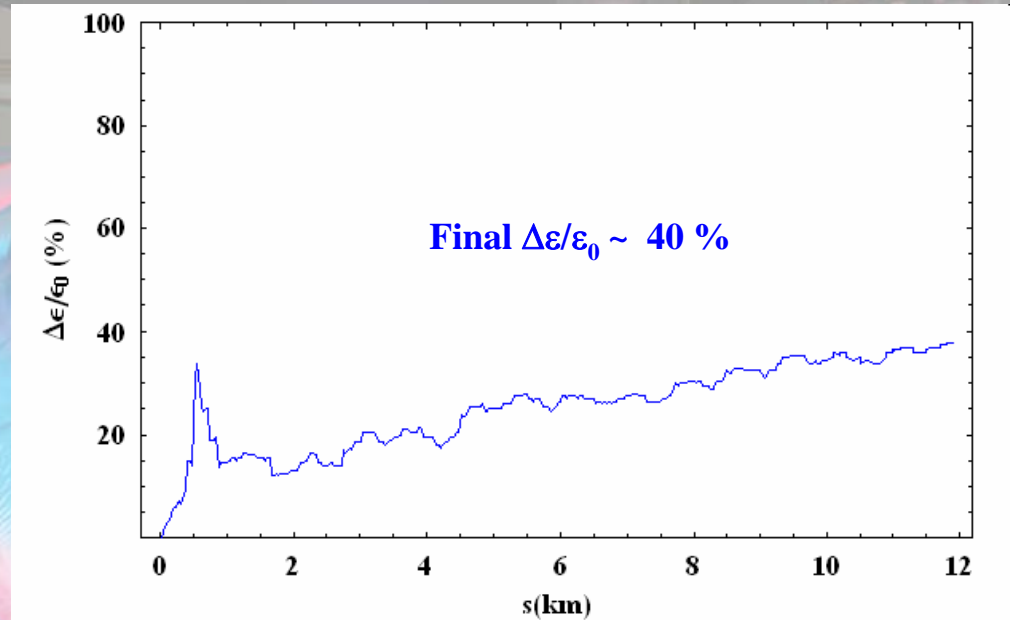
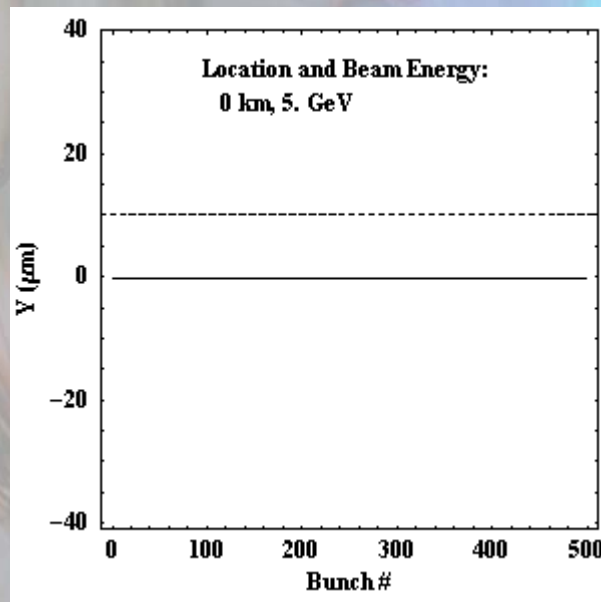
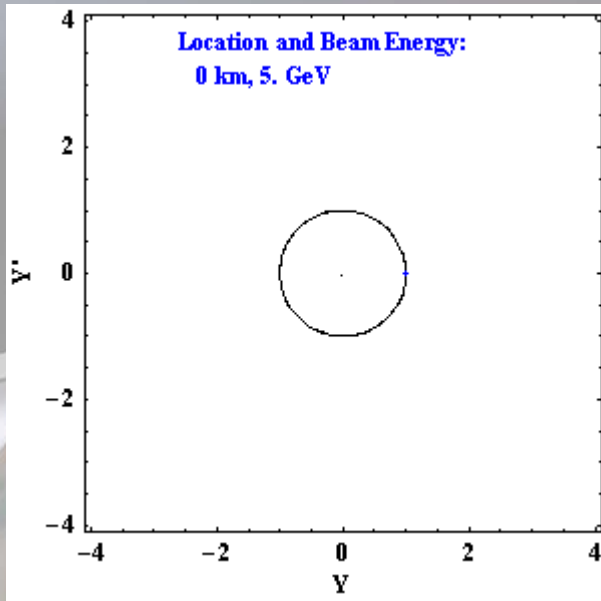
Zero Randomization of Azimuthal Orientation of HOM Couplers + Rogue Mode

$X_{\text{off}} \sim \sigma_x$, $Y_{\text{off}}=0$. *Split Tune 70/60.*



- 500 particles in all simulations.
- Initial vertical and horizontal offset =0 and 270 μm .
- c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$.
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- Long range wakes included

Split Tune 90/60.

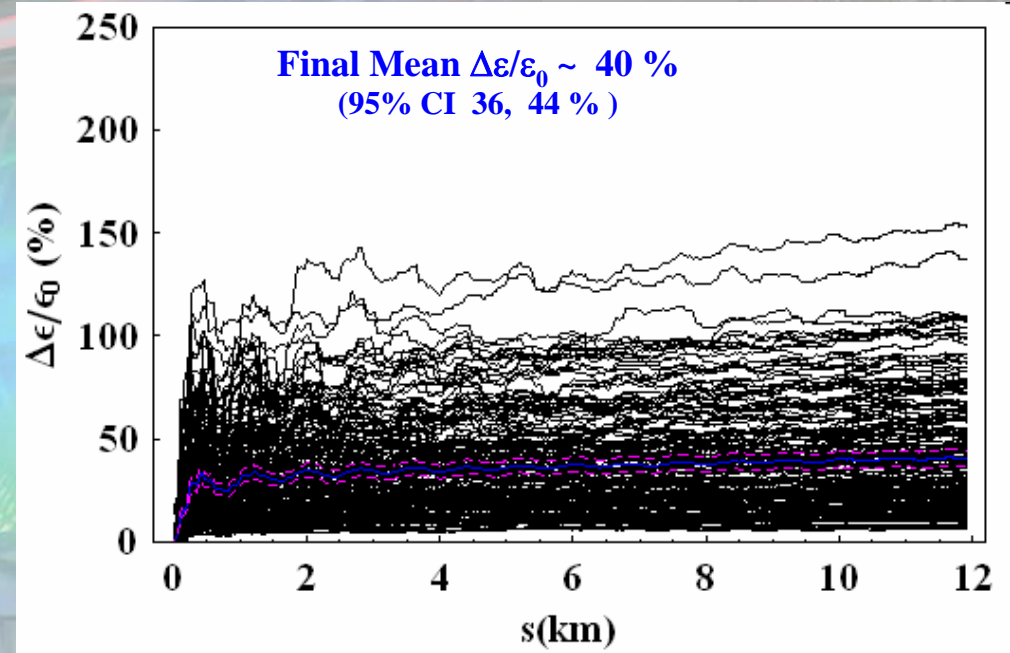
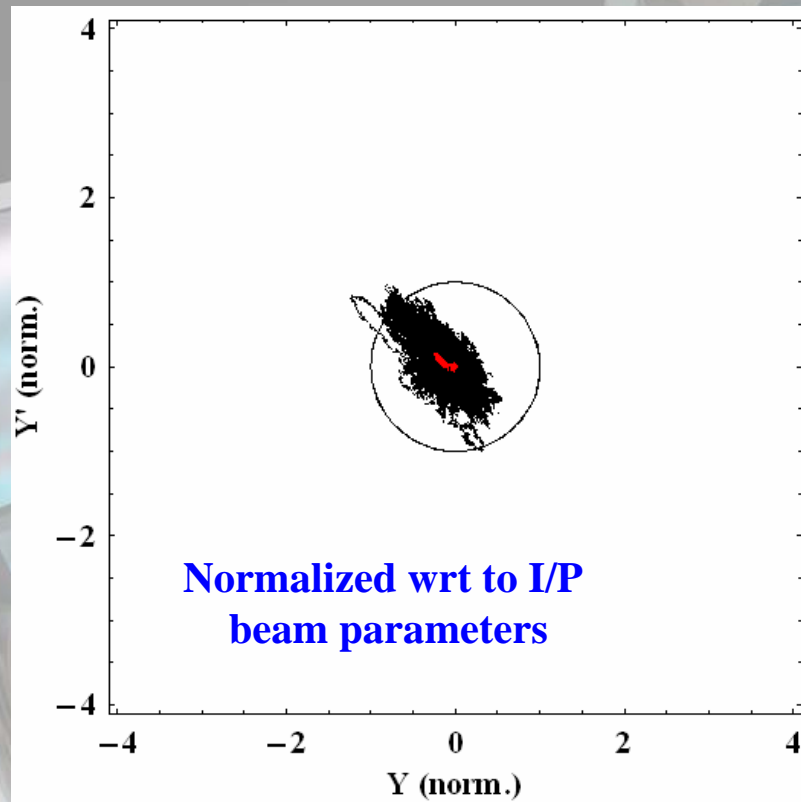


- Only mode-cross coupling gives rise to emittance dilution
- 500 particles in all simulations
- Initial vertical and horizontal offset 0 μm and 270 μm ($\sim \sigma_x$), respectively.
- (c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$)
- Fixed azimuthal orientation of couplers.
- Long range wakes included in simulations

Cross-Coupled Motion

Zero Randomization of Azimuthal Orientation of HOM Couplers + Rogue Mode

$X_{\text{off}} \sim \sigma_x$, $Y_{\text{off}}=0$. *Split Tune 90/60.*



- 500 particles in all simulations.
- Initial vertical and horizontal offset =0 and 270 μm .
- c.f. initial beam $\sigma_{y,x} \sim 10.1 \mu\text{m}, 270 \mu\text{m}$.
- Final mean of the emittance dilution from 200 machines is indicated by the blue line
- Long range wakes included

Mode Coupling Summary

- Mode-coupling is a significant source of emittance dilution.
- Emittance dilution can be ameliorated by splitting the tune of the lattice.
- Further experimental data is needed on the actual splitting of the dipole degeneracies in frequencies that is likely to occur in the fabrication of several thousand ILC cavities (experiments at DESY, TTF already in progress will be helpful in this respect –Ross, Napoly, Baboi et al). In addition, simulations are in progress on the splitting of the tune encountered due to the HOM couplers at SLAC (Z. Li, K. Ko et al).
- Analysis of the influence of higher order bands on emittance dilution in the presence of mode coupling is required.
- The effect of purposely distorting the shapes of the cavities in order to increase the frequency degeneracy splitting requires further study.

Further R&D Needed

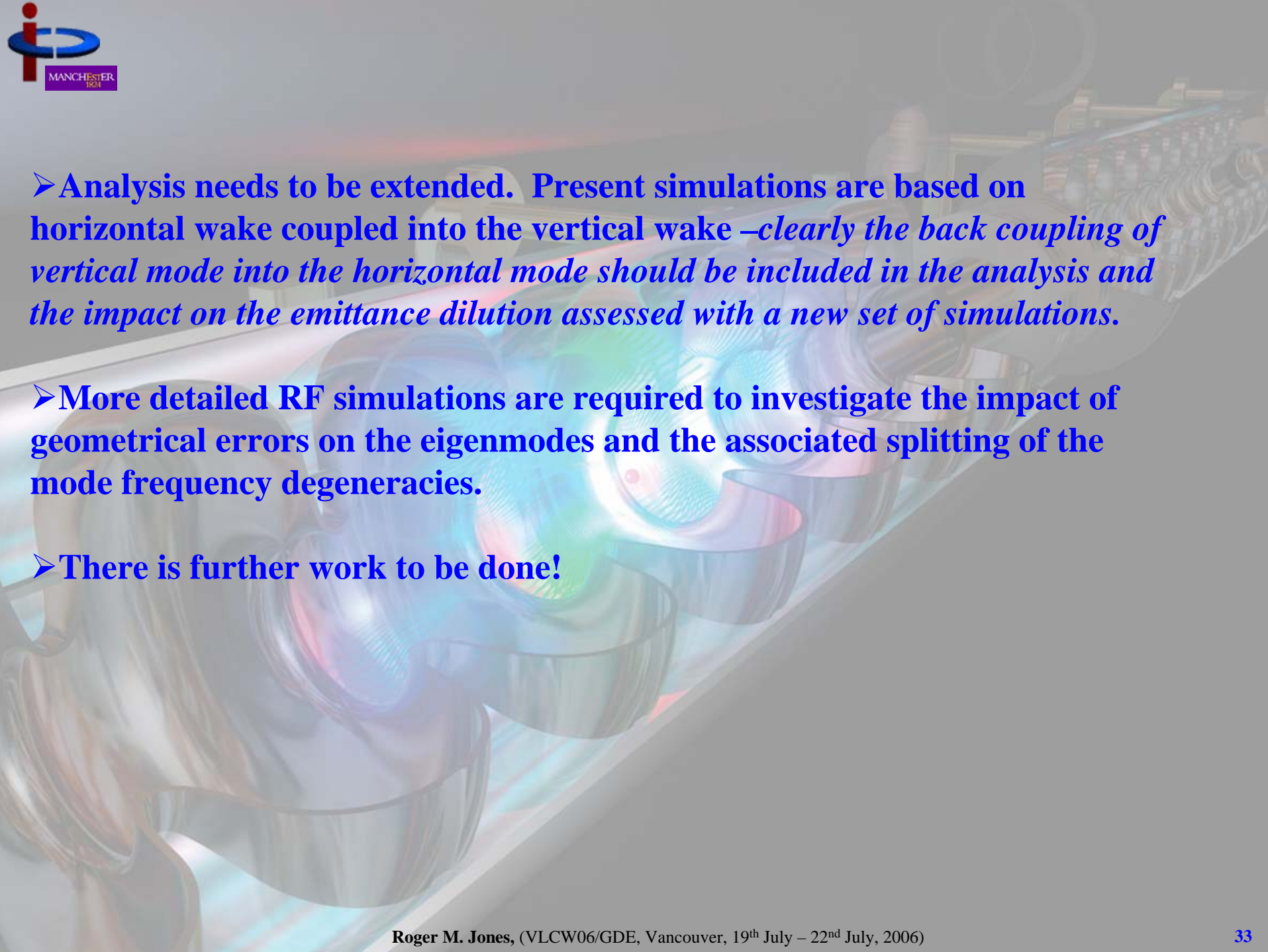
➤ This is one initial set of simulations. Clearly additional simulations are required to verify both the magnitude of this issue and the best means to control it:

a) splitting the tune of the lattice (60/75 lattice implemented in GDE baseline).

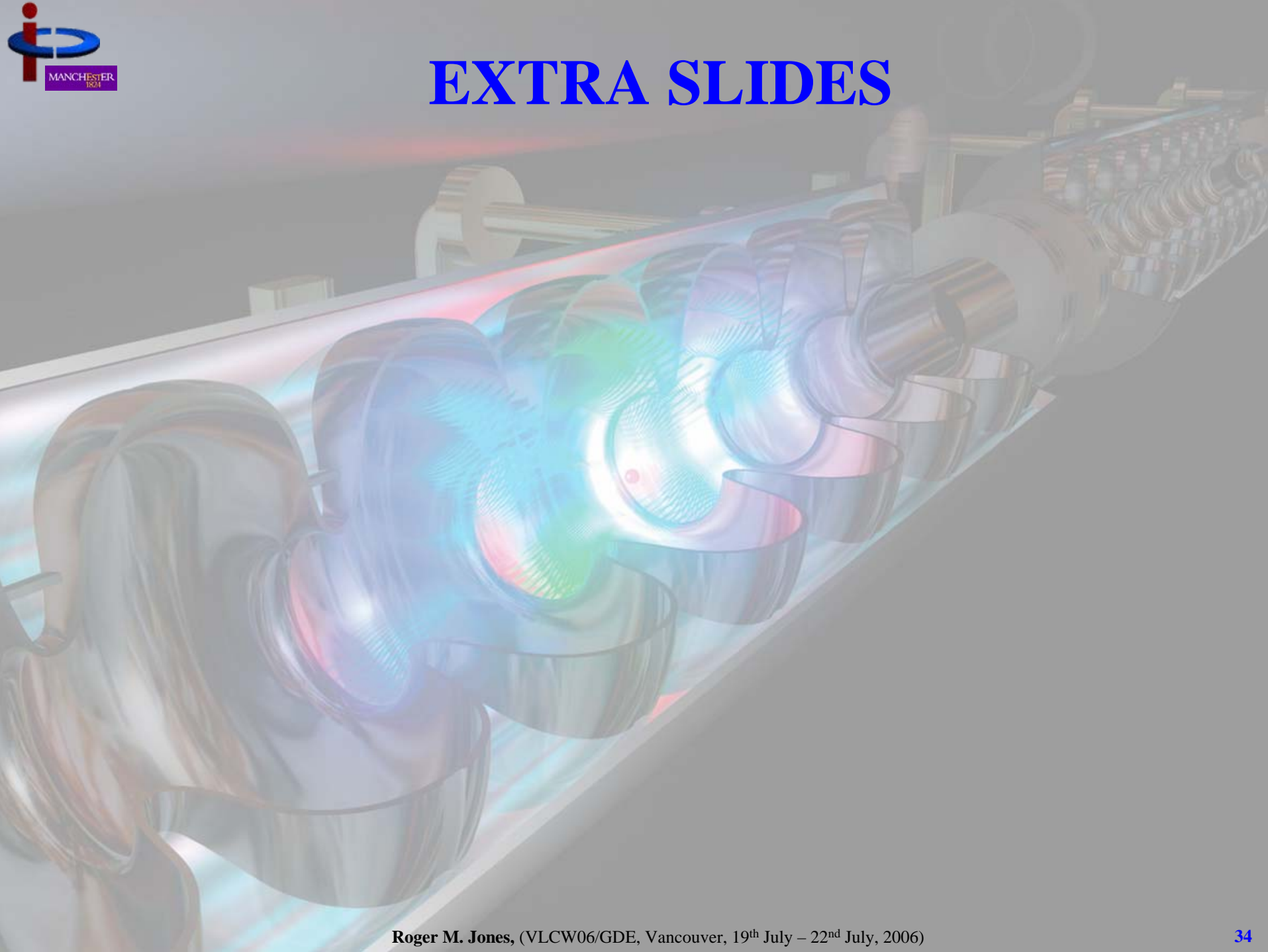
b) by purposely introducing significant fabrication errors.

➤ Several other simulations are needed as part of a careful program (*e.g.* present set includes a fixed freq splitting of 600 kHz. In practice the influence of random frequency splitting through the linac needs to be considered). More data is expected from the July/Aug. FLASH measurements at DESY.

➤ Placet will be modified (small re-coding?) in order to simulate mode rotation. Lucretia can also simulate this mode coupling effect?

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- **Analysis needs to be extended.** Present simulations are based on horizontal wake coupled into the vertical wake –*clearly the back coupling of vertical mode into the horizontal mode should be included in the analysis and the impact on the emittance dilution assessed with a new set of simulations.*
 - **More detailed RF simulations are required to investigate the impact of geometrical errors on the eigenmodes and the associated splitting of the mode frequency degeneracies.**
 - **There is further work to be done!**

EXTRA SLIDES



Cross-Coupled Motion

Zero Randomization of Azimuthal Orientation of HOM Couplers + Rogue Mode

$X_{\text{off}} \sim \sigma_x$, $Y_{\text{off}} = 0$. *Split Tune 90/60.*

