Improved Particle Energy Reconstruction with Generalized Mass-Constrained Fitting

A New MarlinKinfit Processor (Sim/Reco/Perf sessions)

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Motivation

- With particle-flow based jet energy reconstruction - we can strive to reconstruct as well as possible each jet
- Previously we have shown that applying mass-constrained fits to the $\pi^0 \to \gamma\gamma$ component of jets can improve the energy resolution of the prompt electro-magnetic component of jets
- We have been working to extend the same techniques to complementary sets of component particles, notably as initial software test-cases $\mathrm{J}/\psi \to \mu^+\mu^-$ and $\eta \to \pi^+\pi^-\gamma$
- Justin has worked on extending the framework that previously existed for specific instances (eg. GammaGammaCandidateFinder, DiTrackCandidateFinder) to not just the more complex mixed case ($\eta \to \pi^+\pi^-\gamma$), but to the general case of an arbitrary number of charged particles and photons including error matrix propagation

Introduction

- Made a MarlinKinfit processor for iLCSoft
- Uses invariant mass information to improve the overall Energy Resolution in single decays
- Looked at the performance of mass constraints with different particle topologies
- Looked at the benefit of multiple constraints on the system
- Code is in Justin's github for now - will port to ilcsoft soon

Basics

Now: Applying mass constrained fits to the energy reconstruction of $P \rightarrow \sum_{i=0}^{k} N_i + \sum_{j=0}^{r} C_j$ decays

- Where N_i and C_j are neutral and charged particles, respectively
- Successive decays are also allowed, each decay allows an additional mass constraint

Particle Parameterization

Charged Track

- Natural track parameterization is the helix, including the signed curvature $\Omega = 1/R$
- $(d_0, \phi_0, \Omega, z_0, \tan\lambda)$
- LeptonFitObject uses a different parameterization with $\kappa = q/p_T$
- (κ, θ, ϕ) assuming track has negligible (d_0, z_0) and vertex at origin.

Photon

- Photon uses a simple parameterization
- (E, θ, ϕ)
- necessary for JetFitObject

Error Models

Charged Track

- Parameter errors are embodied in the track covariance matrix
- Tracks are usually well measured, so expect σ_κ/κ to be $O(10^{-3})(MS)$
- Track angles typically $O(10^{-4})$

Photon

- Photon resolution scales with energy, E (GeV)
- Use a traditional stochastic model

$$\frac{\sigma_E}{E} = \frac{0.18}{\sqrt{E}} \oplus 0.01 \approx O(10^{-2})$$

- Angular resolution should also scale with energy, empirically expect resolution on $O(10^{-3})$ rads

$$\sigma_{\theta,\phi} = \frac{0.001}{\sqrt{E}}$$

Given relative errors. Photon energy error dominates. IF the constraint is correct - the fit helps to reduce the photon energy error.

The MassConstraint Fitter

Processor Overview: Using the MarlinKinfit framework

- developed a processor that can apply multiple mass constraints to a set of particles.
- processor finds a set of particles with a decay topology and invariant mass consistent with the parent hypothesis
- fitted parameters of parent particle are more precise
- Justin used the following test cases in his Master's thesis

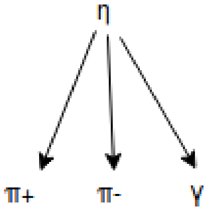
  $\mathrm{J}/\psi \rightarrow \mu^+\mu^-$
  $\pi^0 \rightarrow \gamma\gamma$
  $\eta \rightarrow \pi^+\pi^-\gamma$
  $\eta \rightarrow \pi^+\pi^-\pi^0,\ \pi^0 \rightarrow \gamma\gamma$

  We are now playing with

  $\mathrm{J}/\psi \rightarrow \pi^+\pi^-\mathrm{K}^+\mathrm{K}^-$
  $\mathrm{B}^+ \rightarrow \mathrm{J}/\psi\,\mathrm{K}^+,\ \mathrm{J}/\psi \rightarrow \mu^+\mu^-$ (τ set to 0)
  $\mathrm{H} \rightarrow \mu^+\mu^-$ and $\mathrm{H} \rightarrow \mu^+\mu^-\mu^+\mu^-$
  $\mathrm{K}^+ \rightarrow \pi^+\pi^-\pi^+$ (τ set to 0)

Walkthrough for a 1C fit

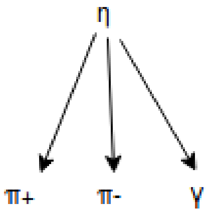
Suppose we have a mixed decay e.g. $\eta \to \pi^+\pi^-\gamma$
We want to apply a mass constraint $M_\eta = 0.547862$ [GeV]



Start with a set of reconstructed particles (often more than needed)
$[C_1, C_2]$ and $[N_1, N_2, N_3]$ 2 tracks and 3 photons
Next impose a mass assumption on the tracks, the $\pi^\pm$ mass
The set becomes: $[\pi^+, \pi^-]$ and $[\gamma_1, \gamma_2, \gamma_3]$

If we have reconstructed 5 particles and need 3, which is the correct combination?

Walkthrough for a 1C fit



- Fit every combination $[\pi^+, \pi^-][\gamma_1]$ and $[\pi^+, \pi^-][\gamma_2]$ and $[\pi^+, \pi^-][\gamma_3]$
- Use the one with the highest χ^2 fit probability as the best guess for the correct combination
- Calculate fitted η 4-vector covariance matrix
- Store all the parameters for evaluation

Walkthrough for a 1C fit

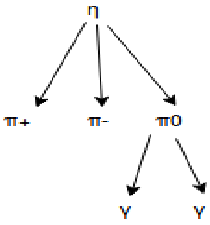
An example of a 20 GeV η well measured event:

Particle	E_{meas}[GeV] $\|\kappa_{meas}$ [GeV^{-1}]	θ_{meas} [rad]	ϕ_{meas} [rad]	Measured ηE [GeV]
γ	9.70 ± 0.6	2.0354 ± 0.0003	-2.3217 ± 0.0003	
π^+	0.1872 ± 0.0002	2.0721 ± 0.0001	-2.3424 ± 0.0002	20.14 ± 0.6
π^-	-0.2583 ± 0.0003	2.0361 ± 0.0002	-2.3187 ± 0.0002	
Particle	E_{fit}[GeV] $\|\kappa_{fit}$ [GeV^{-1}]	θ_{fit} [rad]	ϕ_{fit} [rad]	Fit ηE [GeV]
γ	9.69 ± 0.1	2.0354 ± 0.0003	-2.3217 ± 0.0003	
π^+	0.1872 ± 0.0002	2.0721 ± 0.0001	-2.3424 ± 0.0002	20.13 ± 0.1
π^-	-0.2583 ± 0.0003	2.0361 ± 0.0002	-2.3187 ± 0.0002	

The photon energy error dominates the η resolution. The fit essentially uses the track curvatures and the angular information to adjust the photon energy and increase the overall precision.

Walkthrough for a 2C fit

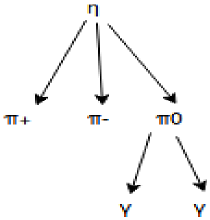
But how do we address more complicated decays?
Now suppose we have a mixed decay e.g. $\eta \to \pi^+\pi^-\pi^0$
We want to apply a mass constraint on the η and π^0 with
$M_\eta = 0.547862$ [GeV] and $M_{\pi^0} = 0.1349766$ [GeV]



Start with the same set of reconstructed particles
$[C_1, C_2]$ and $[N_1, N_2, N_3]$ 2 tracks and 3 photons
Again impose a mass assumption on the tracks, the $\pi^\pm$ mass
The set becomes: $[\pi^+, \pi^-]$ and $[\gamma_1, \gamma_2, \gamma_3]$

If we have reconstructed 5 particles and need 4, which is the correct combination, and which 2 photons compose the π^0?

Walkthrough for a 2C fit



- Again fit every combination, but we have to add another layer of combinations for the π^0
  $[\pi^+, \pi^-][\gamma_1, \gamma_2]$ and $[\pi^+, \pi^-][\gamma_2, \gamma_3]$ and $[\pi^+, \pi^-][\gamma_1, \gamma_3]$
- Use the one with the highest χ^2 fit probability as the best guess for the correct combination
- Calculate fitted η 4-vector covariance matrix
- Store all the parameters for evaluation

For different particle combinations, how is the 4-vector covariance matrix calculated?

Covariance Matrix

With arbitrary amounts of particles the covariance calculation is tricky
For a decay $P \rightarrow \sum_{i=0}^{k} N_i + \sum_{j=0}^{r} C_j$
(with the current parameterization) the fitter spits out a dimension $3(k+r)$ covariance matrix V
The diagonal is the variance of the particle parameters in the order they are added to the fitter
Transforming the matrix V into a 4-vector matrix for the parent requires a Jacobian transformation

$$V_p = J^T V J$$

Details in backup

Simulation and Reconstruction of DataSets

- Generator single particles with specified decay generated in stand-alone Pythia8 with .hepevt output
- Currently using ilcsoft v01-19-04.
- Detector response with $\mathrm{ILD-l4-v02}$ DDSim model
- Particles are reconstructed with v01-19-04 ilcsoft standard reconstruction

The initial reconstruction has some issues. Need to increase the track curvature errors by 20%. Bug related to ECAL cell positions (now fixed), so in studies shown the photon directions are smeared around the true MC photon direction for now. ECAL energy response is non-ideal and still needs to be carefully parametrized. See backup "Calibration slides" for more details.

Results

Simulated and reconstructed 9 data sets to date to test mass constraints and pose as basic test cases for the processor

- 10k 20 GeV $J/\psi \to \mu^+\mu^-$
- 10k 10 GeV $\pi^0 \to \gamma\gamma$
- 10k 20 GeV $\eta \to \pi^+\pi^-\gamma$
- 10k 20 GeV $\eta \to \pi^+\pi^-\pi^0$ 1C & 2C
- 10k 20 GeV $J/\psi \to \pi^+\pi^-K^+K^-$
- 10k 30 GeV $B^+ \to J/\psi K^+$, $J/\psi \to \mu^+\mu^-$ (2C)
- Higgs samples discussed this morning
- 10k 20 GeV $\mathrm{K}^+ \to \pi^+\pi^-\pi^+$

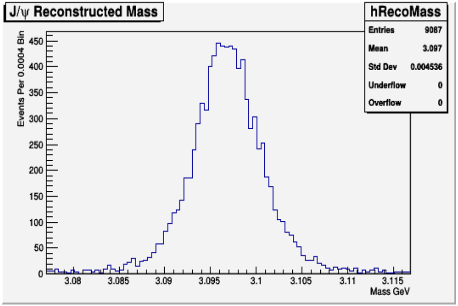
$J/\psi \to \mu^+\mu^-$

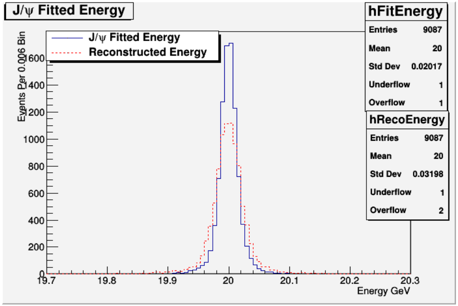
10k 20 GeV J/ψ decaying exclusively

$\Gamma = 0$, phase space only

acceptance requires 2 tracks, a fit probability $> 0.5\%$, and a reconstructed mass within 0.02 GeV

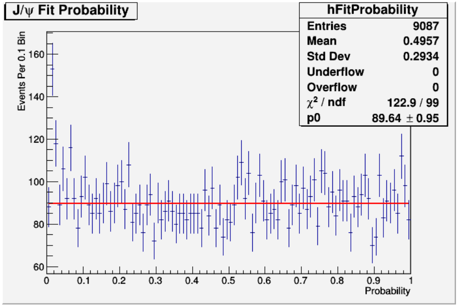
$\sigma_{fit}/\sigma_{meas} = 0.63$ (histogram rms)





$J/\psi \to \mu^+\mu^-$

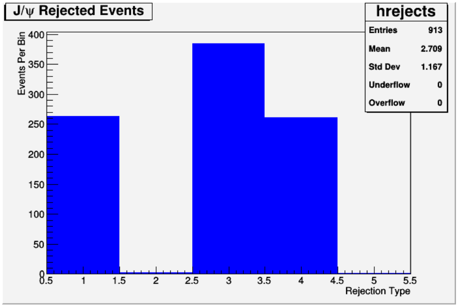
Fit probability distribution expected to be uniform



Reasonable. Some outliers at low fit probability.

$J/\psi \rightarrow \mu^+\mu^-$ Fit Failures

Rejection No.	Description
(1)	Not enough particles to satisfy hypothesis
(2)	No fitted covariance matrix
(3)	Fit probability cut not met
(4)	$\Delta M = \lvert M_{meas} - M_g \rvert$ cut not met
(5)	Fit converged but particles are missing (fitter bugs)

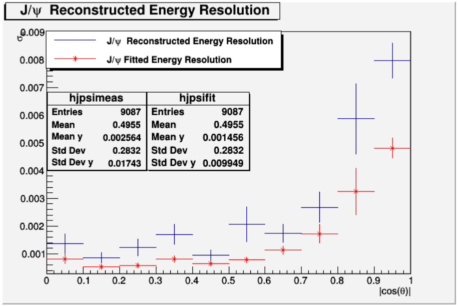


Overall efficiency : 90.87 ± 0.29 %

Same key used in later slides

$J/\psi \rightarrow \mu^+\mu^-$

Energy resolution degrades, before and after mass-constraint, in the endcap/forward polar angle region



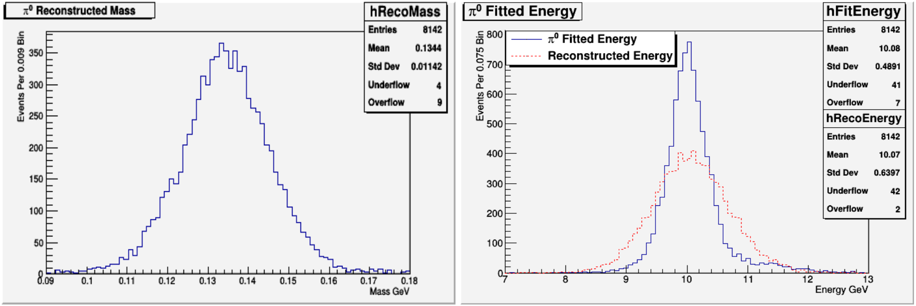
$\pi^0 \to \gamma\gamma$

10k 10 GeV π^0 decaying inclusively

$\Gamma = 0$, phase space decay

acceptance requires 2 photons, a fit probability $> 0.5\%$, and a reconstructed mass within 0.05 GeV

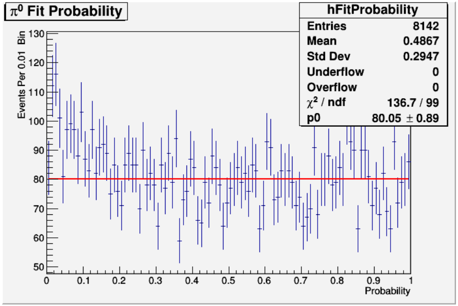
$\sigma_{fit}/\sigma_{meas} = 0.76$

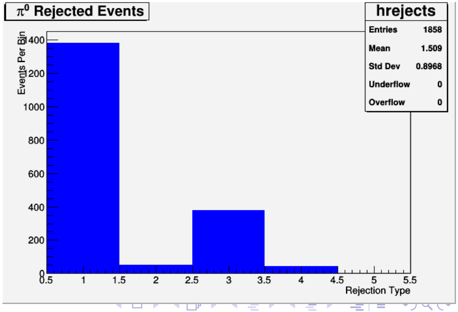


$\pi^0 \to \gamma\gamma$

Remaining energy scale calibration issues may contribute to the low fit probability excess

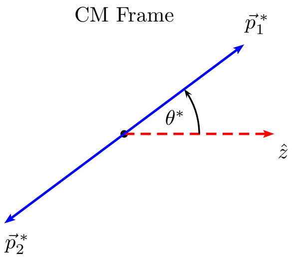
Rejected events largely due to unobserved particles (low energy photons presumably from asymmetric decay)

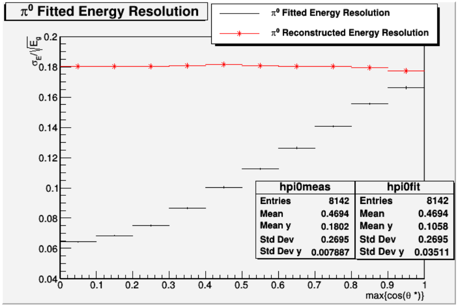




10 GeV $\pi^0 \to \gamma\gamma$

Stochastic resolution reaches 6% ($\sigma_E/E = 6\%/\sqrt{E}$) for symmetric decay (photon energies of 5 GeV, and minimum lab opening angle). θ^* is angle between a CM photon and the boost direction





Overall efficiency : 81.42 ± 0.39 %

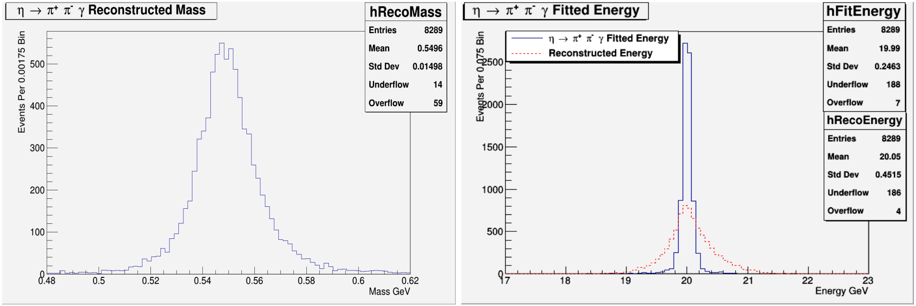
$\eta \to \pi^+\pi^-\gamma$

10,000 20 GeV η decaying exclusively

$\Gamma = 1.31$ keV (negligible). Phase space only.

Acceptance requires 1 photons, 2 tracks, a fit probability $> 0.5\%$, and a reconstructed mass within 0.15 GeV

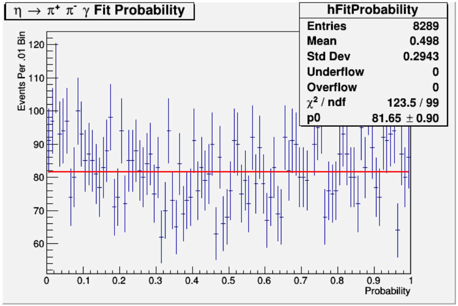
$\sigma_{fit}/\sigma_{meas} = 0.54$

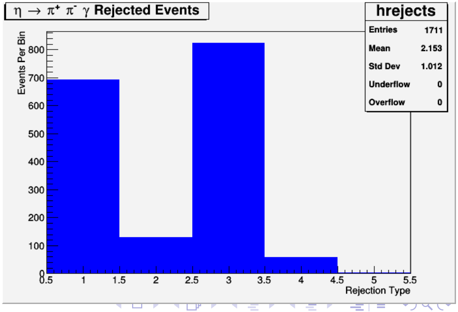


$\eta \to \pi^+\pi^-\gamma$

Photon energy scale problems and wrong combinations likely contribute to low probability excess

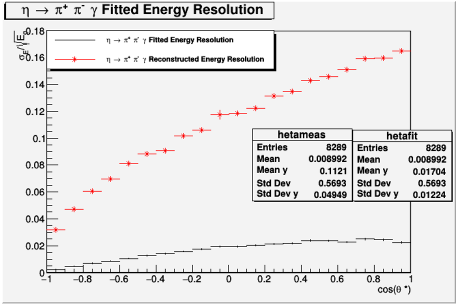
Rejected low fit probabilities possibly contain reconstructed "photons" from pion interactions

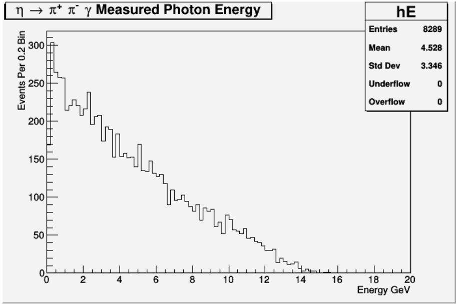




$\eta \to \pi^+\pi^-\gamma$

The constraint works best with low E_γ when the overall system is well measured. θ^* is of the γ. So $\cos\theta^*_\gamma \to -1$ for lowest E_γ





Overall efficiency : 82.89 ± 0.38 %

$\eta \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$ 1C

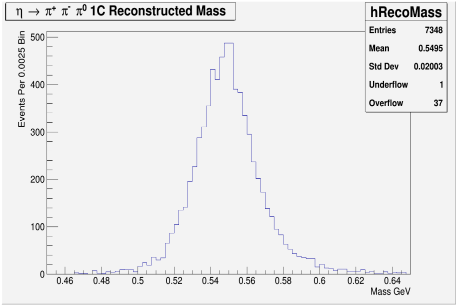
10k 20 GeV η decaying exclusively and π^0 decaying inclusively

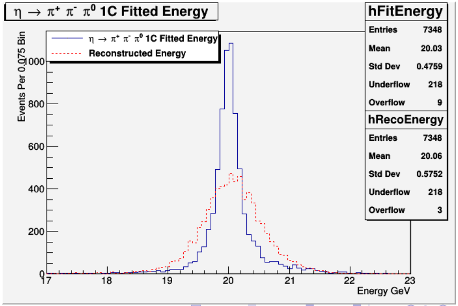
$\Gamma = 1.31$ keV, phase-space only

Acceptance requires 2 photons, 2 tracks, a fit probability $> 0.5\%$, and a reconstructed mass within 0.15 GeV

4 particles are constrained to the η mass

$\sigma_{fit}/\sigma_{meas} = 0.82$



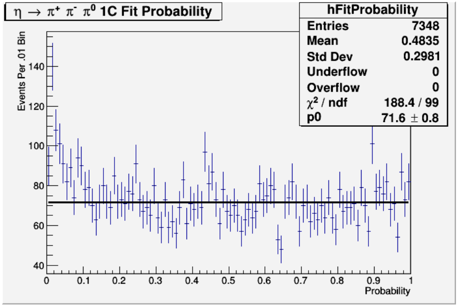


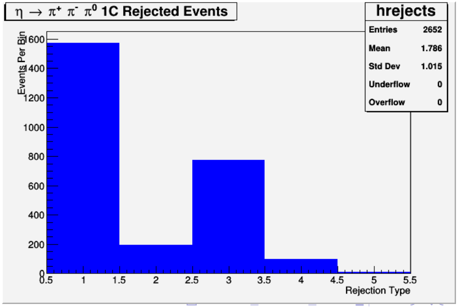
$\eta \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$ 1C

π^0 decay asymmetry makes reconstructing 4 final state particles more tricky

The efficiency is low. Overall efficiency: 73.48 ± 0.44 %

Consistent with estimate of 74% based on J/ψ and π^0 efficiencies

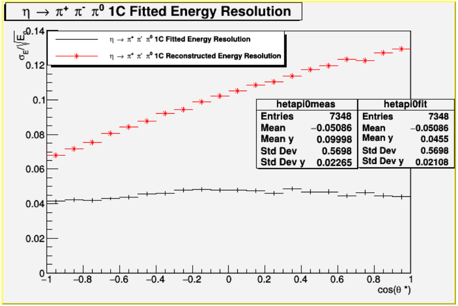




$\eta \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$ 1C

θ^* is angle between CM π^0 and boost axis

Performance is fairly even for all π^0 energies



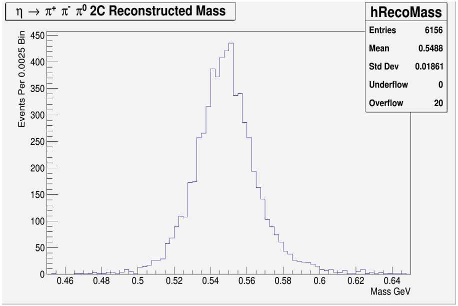
$\eta \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$ 2C

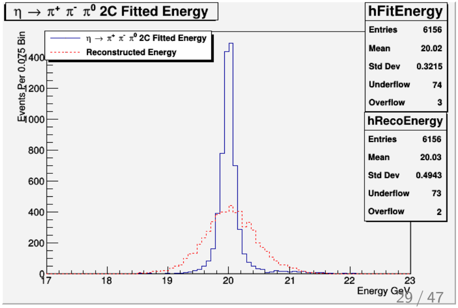
10k 20 GeV η decaying exclusively and π^0 decaying inclusively

Exactly the same events as the 1C fit

4 particles are constrained to the η mass and the 2 photons constrained to the π^0 mass

$\sigma_{fit}/\sigma_{meas} = 0.65$

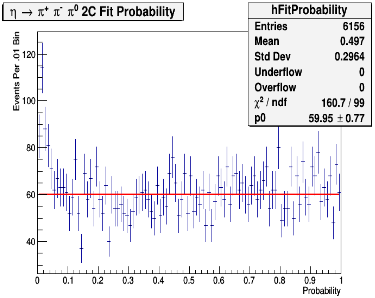


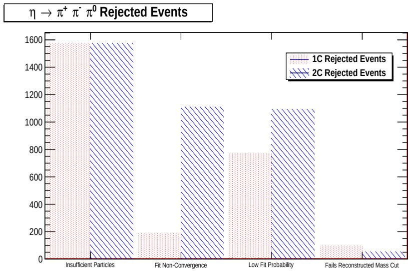


$\eta \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$ 2C

Now 1000 events don't converge in the fitter, these are expected to be wrong combinations, so, additional constraints may be eliminating false positives

The efficiency is lower still. Overall efficiency: 61.56 ± 0.49 %

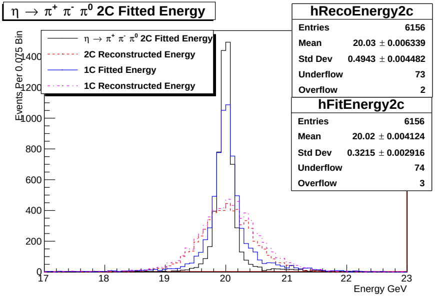


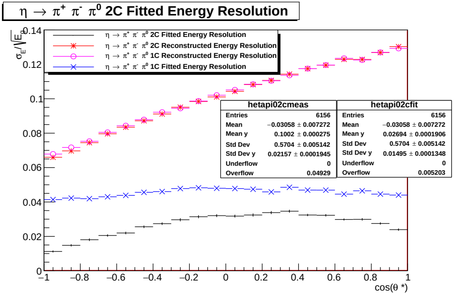


$\eta \to \pi^+\pi^-\pi^0$, $\pi^0 \to \gamma\gamma$ 1C/2C Comparison

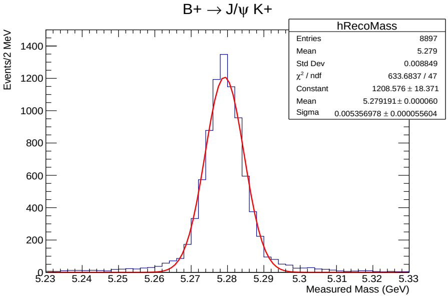
Performance from 1C to 2C is much better by a factor of ~ 1.67

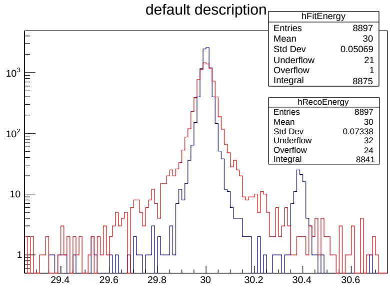
Some evidence resolution may get better at higher π^0 energies?





$B^+ \to J/\psi\, K^+, J/\psi \to \mu^+\mu^-$ 2C





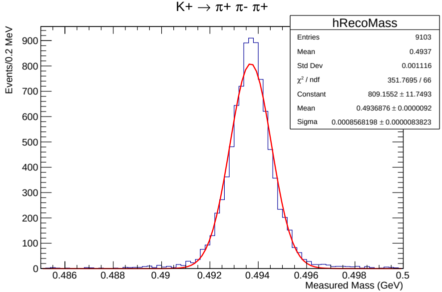
B^+ Energy (GeV)

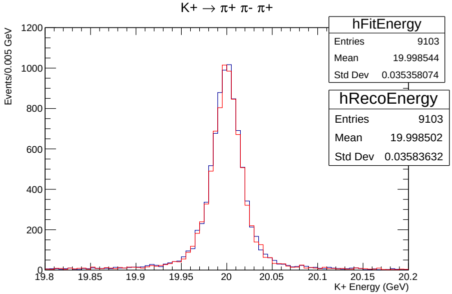
$\sigma_M = 5.4$ MeV

Secondary peak looks to be events fitted as 3μ ....fixme

$K^+ \to \pi^+\pi^-\pi^+$ 1C

20 GeV K^+ artificially decaying at origin. Why? Estimate of potential M_{K^+} measurement in low Q-value mode. PDG error is 32 ppm (scale factor of 2.8 ... disagreements in kaonic atom X-rays). A limiting factor for some potential *p*-scale channels





$\sigma_M = 0.86$ MeV. So 17.4 ppm statistical error with 10,000 events like this

(not much difference - mass not so dependent on *p* given Q-value)

Conclusions/Work-In-Progress/Outlook

- Mass constraints are a powerful tool in improving energy reconstruction and resolution for decays
- General tool has been developed - should be useful for many applications
- Promising results for a variety of test cases
- Working on adding new test cases and improving performance
- Working on extending the implementation to work well for multi-generational constraints using tree-based book-keeping
- Working on more robust rms estimates and polishing plots
- Will redo with ilcsoft v01-19-05
- Eventually will want to integrate this approach with vertex constraints
- Code currently in Justin's github. (Jphsx/constrainedFitter)

Backup Slides

Mass Constraints

How do we improve the 4-vector measurements?

- Suppose a particle decays to a set of particles $P \rightarrow p_i$
- We can describe each particle in the decay with arbitrary measured parameters $p_i(\xi_j, \sigma_j)$
- Build a χ^2 consisting of ξ_j and true parameter estimator $\hat{\xi}_j$
- $\chi^2 = \sum_i \sum_j \frac{(\xi_j - \hat{\xi}_j)^2}{\sigma_j^2}$

  Lagrange Multiplier method allows us to apply constraints

  $C_l \Rightarrow M_l^2 = \left(\sum_\alpha (E_\alpha, \vec{p}_\alpha)\right)^2$
- The χ^2 then becomes
- $\chi^2 = \sum_i \sum_j \frac{(\xi_j - \hat{\xi}_j)^2}{\sigma_j^2} + \sum_l \lambda_l C_l$

- $\chi^2 = \sum_i \sum_j \frac{(\xi_j - \hat{\xi}_j)^2}{\sigma_j^2} + \sum_l \lambda_l C_l$
- Minimization produces new estimates for each parameter along with new estimates for each parameter error
- Add up newly fitted particles to get a better measurement for the parent particle
- Recalculate parent errors based on more precise fitted covariance matrix

How to easily perform constrained fitting?

MarlinKinfit

MarlinKinfit - a kinematic fitting package

- Consists of three pieces
  - a fitting engine to solve our χ^2 equation
  - a constraint to apply to the fit
  - a FitObject to store the parameter information for each particle
- Fitting is easy, just build up FitObjects for each particle in the event, associate particle subsets to the MassConstraint, add the Objects to the fitting engine, and get new fitted values
- However FitObjects rely on a hardcoded parameterization

Covariance Matrix

$$V_p = J^T V J$$

The Jacobian J contains all of the 4-vector parent parameters derivatives w.r.t to each parameter.

Since there are $k + r$ particles we construct a submatrix and concatenate them together to form J

Charge particle submatrix example:

$$\begin{pmatrix} \frac{\partial P_x}{\partial \kappa} & \frac{\partial P_x}{\partial \theta} & \frac{\partial P_x}{\partial \phi} \\ \frac{\partial P_y}{\partial \kappa} & \frac{\partial P_y}{\partial \theta} & \frac{\partial P_y}{\partial \phi} \\ \frac{\partial P_z}{\partial \kappa} & \frac{\partial P_z}{\partial \theta} & \frac{\partial P_z}{\partial \phi} \\ \frac{\partial E}{\partial \kappa} & \frac{\partial E}{\partial \theta} & \frac{\partial E}{\partial \phi} \end{pmatrix} \quad (1)$$

Photon submatrix example:

$$\begin{pmatrix} \frac{\partial P_x}{\partial E_\gamma} & \frac{\partial P_x}{\partial \theta} & \frac{\partial P_x}{\partial \phi} \\ \frac{\partial P_y}{\partial E_\gamma} & \frac{\partial P_y}{\partial \theta} & \frac{\partial P_y}{\partial \phi} \\ \frac{\partial P_z}{\partial E_\gamma} & \frac{\partial P_z}{\partial \theta} & \frac{\partial P_z}{\partial \phi} \\ \frac{\partial E}{\partial E_\gamma} & \frac{\partial E}{\partial \theta} & \frac{\partial E}{\partial \phi} \end{pmatrix} \quad (2)$$

Covariance Matrix

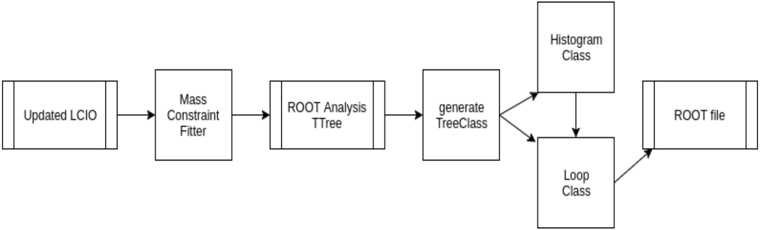
The resulting parent covariance matrix is the following:

$$\begin{pmatrix} \sigma^2_{P_x} & \cdots & \cdots & \\ \vdots & \sigma^2_{P_y} & & \\ \vdots & & \sigma^2_{P_z} & \\ & & & \sigma^2_E \end{pmatrix} \tag{3}$$

This information and all of the other relevant parameters are stored on special data structures for analysis.

Workflow

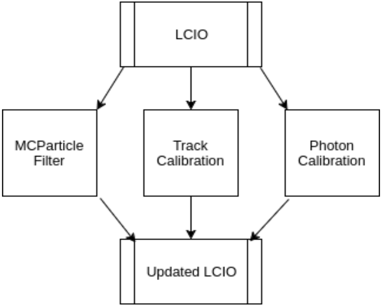
Here is an overview of the data pipeline for the p rocessor



But we are actually missing 3 processors used in this workflow

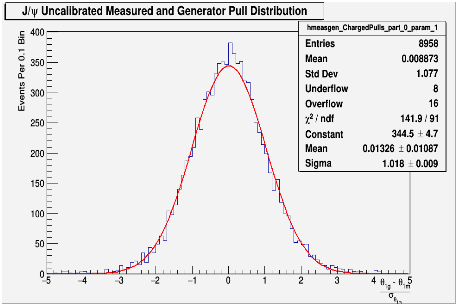
Workflow

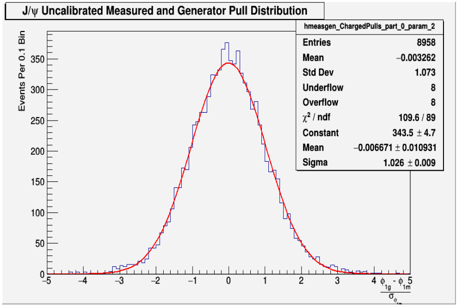
There are two upstream calibration processors that adjust the reconstructed particles, one for tracks and one for photons



Calibration

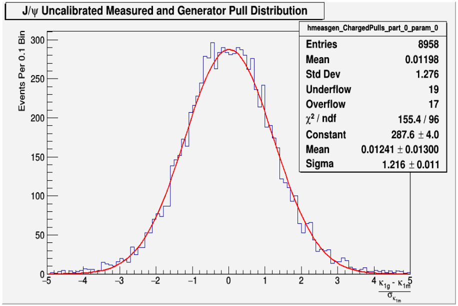
- Calibrations are done mostly from (upcoming) data sets used in the fitter
- To test charged tracks 10,000 20 GeV $J/\psi \to \mu^+\mu^-$ were used
- The efficiency is less than 100% because there are some cuts present
- Pulls shown are for (κ, θ, ϕ) with $\frac{\xi_{meas} - \xi_{gen}}{\sigma_{meas}}$

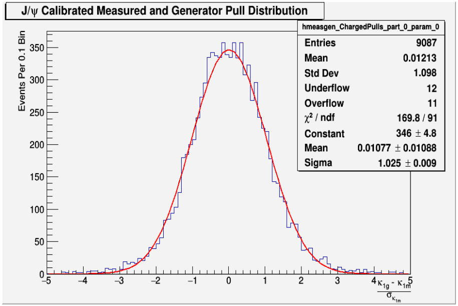




Calibration

The κ distribution is a little wide, increasing the error of $\Omega = 1/R$ by 20% gets the variance closer to unity

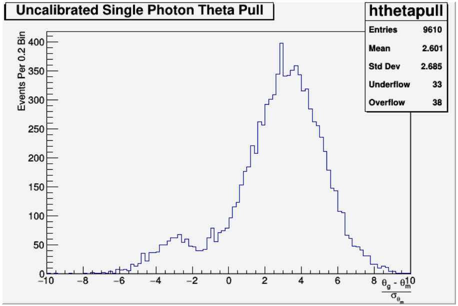


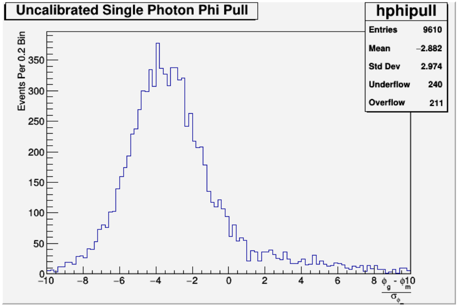


Calibration

The photon calibrations come from 2 datasets (1) 10,000 10 GeV Single photons that where generated uniform in ϕ and $\cos\theta$ and (2) 10,000 10 GeV π^0's decaying inclusively (98% $\gamma\gamma$)

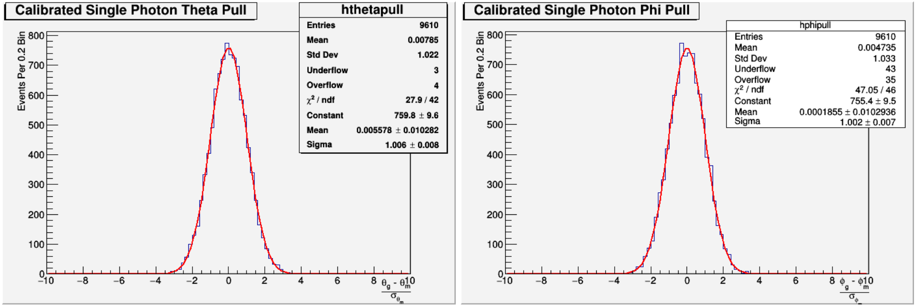
Here are the single photon angular pull distributions





Calibration

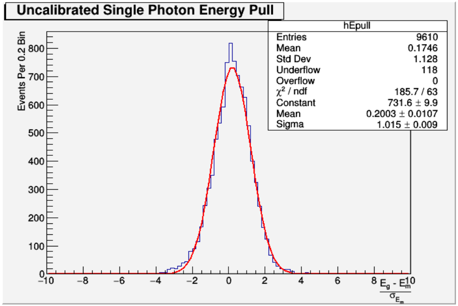
Uncovered an important bug in the ECAL simulation, need to resimulate all of the photon directions based on the used error model

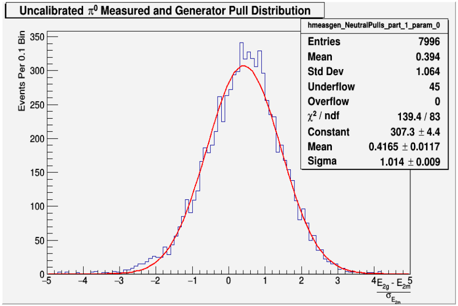


By construction these are now in perfect agreement with the error model, and allow easy study of alternative resolution possibilities.

Calibration

The energy pulls for both single photons and π^0s





Calibration

The single photon energy is reasonable, but the π^0 clearly has a bias, the inconsistency in the energies signifies the need for calbrations sensitive to energy scale. Not enough time to implement this so reduced all photon energy by 5%