



Status and Future of EWPO at e^+e^- Colliders

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1. Introduction
2. Electroweak Precision Observables
3. Status
4. Future
5. Conclusions

1. Introduction

Experimental situation:

LHC/ILC/CLIC/FCC-ee/CEPC/...

will provide (high!) accuracy measurements!

Theory situation:

- Measurements are performed using theory predictions
- measured observables have to be compared with theoretical predictions (in various models: SM, MSSM, ...)

Full uncertainty is given by the (linear) sum of
experimental and theoretical uncertainties!

Results shown here based on:

Write-up for FCC-ee physics WG2 – Precision EW Calculations

Theoretical uncertainties for electroweak and Higgs-boson precision measurements at the FCC-ee

Conveners: A. Freitas¹, S. Heinemeyer²,

Contributors: M. Beneke³, A. Blondel⁴, A. Hoang⁵, P. Janot⁶, J. Reuter⁷,
C. Schwinn⁸, and S. Weinzierl⁹

⇒ Here: focus on LC precision

⇒ should be taken into account by “exp groups”!

⇒ Here: current status and future of EWPO/Higgs TH calculations
anticipated accuracy of EWPO/Higgs TH calc. in $\mathcal{O}(20)$ years

Where we need theory prediction:

1. Prediction of the measured quantity

Example: M_W

→ at the same level or better as the experimental precision

2. Prediction of the measured process to extract the quantity

Example: $e^+e^- \rightarrow W^+W^-$

→ better than then “pure” experimental precision

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Two types of theory uncertainties:

1. **intrinsic:** missing higher orders

2. **parametric:** uncertainty due to exp. uncertainty in SM input parameters

Example: $m_t, m_b, \alpha_s, \Delta\alpha_{\text{had}}, \dots$

Options for the evaluation of intrinsic uncertainties:

1. Determine all prefactors of a certain diagram class (couplings, group factors, multiplicities, mass ratios) and assume the loop is $\mathcal{O}(1)$
2. Take the known contribution at n -loop and $(n - 1)$ -loop and thus estimate the $n + 1$ -loop contribution:

$$\frac{(n + 1)(\text{estimated})}{n(\text{known})} \approx \frac{n(\text{known})}{(n - 1)(\text{known})}$$

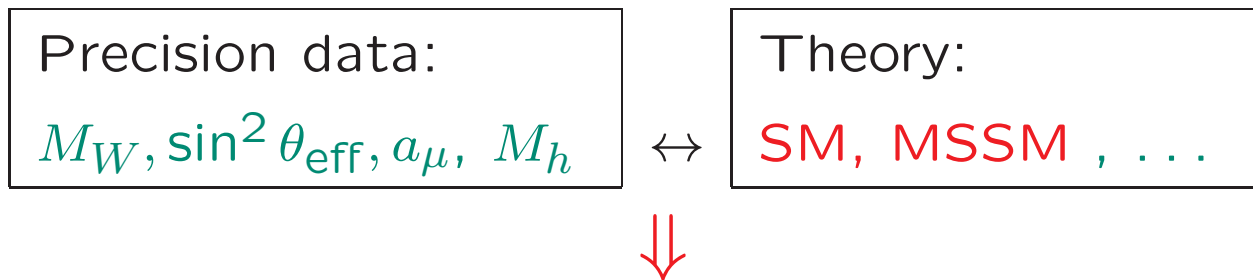
\Rightarrow simplified example! Has to be done
“coupling constant by coupling constant”

3. Variation of $\mu^{\overline{\text{MS}}}$ (QCD!, EW?)
4. Compare different renormalizations

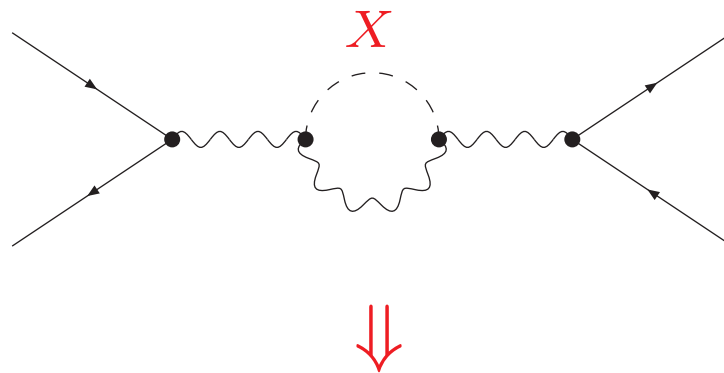
\Rightarrow Mostly used here: 1 & 2

2. Electroweak Precision Observables

Comparison of observables with theory:



Test of theory at quantum level: Sensitivity to loop corrections, e.g. X



SM: limits on M_H , BSM: limits on M_X

Very high accuracy of measurements and theoretical predictions needed
 \Rightarrow only models “ready” so far: SM, MSSM

Precision observables in the SM and the MSSM

M_W , $\sin^2 \theta_{\text{eff}}$, M_h , $(g-2)_\mu$, b physics, ...

A) Theoretical prediction for M_W in terms

of M_Z , α , G_μ , Δr :

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r)$$



loop corrections

Evaluate Δr from μ decay $\Rightarrow M_W$

One-loop result for M_W in the SM:

[A. Sirlin '80], [W. Marciano, A. Sirlin '80]

$$\begin{aligned} \Delta r_{1\text{-loop}} &= \Delta\alpha & - & \frac{c_W^2}{s_W^2} \Delta\rho & + & \Delta r_{\text{rem}}(M_H) \\ &\sim \log \frac{M_Z}{m_f} & & \sim m_t^2 & & \log(M_H/M_W) \\ &\sim 6\% & & \sim 3.3\% & & \sim 1\% \end{aligned}$$

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loop corrections

B) Effective mixing angle:

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4 |Q_f|} \left(1 - \frac{\text{Re } g_V^f}{\text{Re } g_A^f} \right)$$

Higher order contributions:

$$g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f$$

Corrections to $M_W, \sin^2 \theta_{\text{eff}}$ \rightarrow approximation via the ρ -parameter:

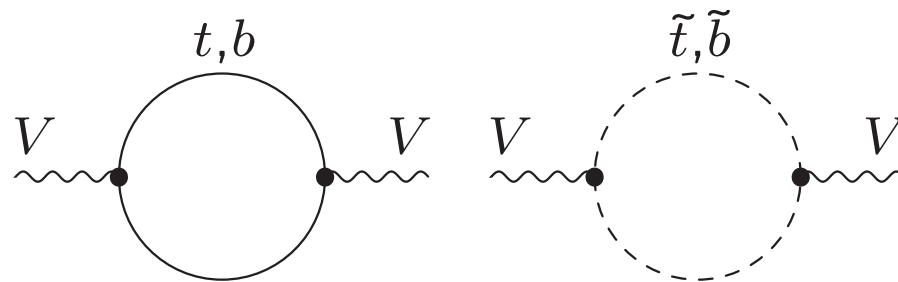
ρ measures the relative strength between
neutral current interaction and charged current interaction

$$\rho = \frac{1}{1 - \Delta\rho} \quad \Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$$

(leading, process independent terms)

$\Delta\rho$ gives the main contribution to EW observables:

$$\Delta M_W \approx \frac{M_W}{2} \frac{c_W^2}{c_W^2 - s_W^2} \Delta\rho, \quad \Delta \sin^2 \theta_W^{\text{eff}} \approx -\frac{c_W^2 s_W^2}{c_W^2 - s_W^2} \Delta\rho$$



$$\Delta\rho^{\text{SUSY}} \text{ from } \tilde{t}/\tilde{b} \text{ loops} > 0 \quad \Rightarrow \quad M_W^{\text{SUSY}} \gtrsim M_W^{\text{SM}}$$

All the EWPO:

M_W (best from threshold scan)

$$\sigma_{\text{had}}^0 = \sum_q \sigma_q(M_Z^2),$$

$$\Gamma_Z = \sum_f \Gamma[Z \rightarrow f\bar{f}], \quad (\text{from a fit to } \sigma_f(s) \text{ at various values of } s)$$

$$R_\ell = \left[\sum_q \sigma_q(M_Z^2) \right] / \sigma_\ell(M_Z^2), \quad (\ell = e, \mu, \tau)$$

$$R_q = \sigma_q(M_Z^2) / \left[\sum_q \sigma_q(M_Z^2) \right], \quad (q = b, c)$$

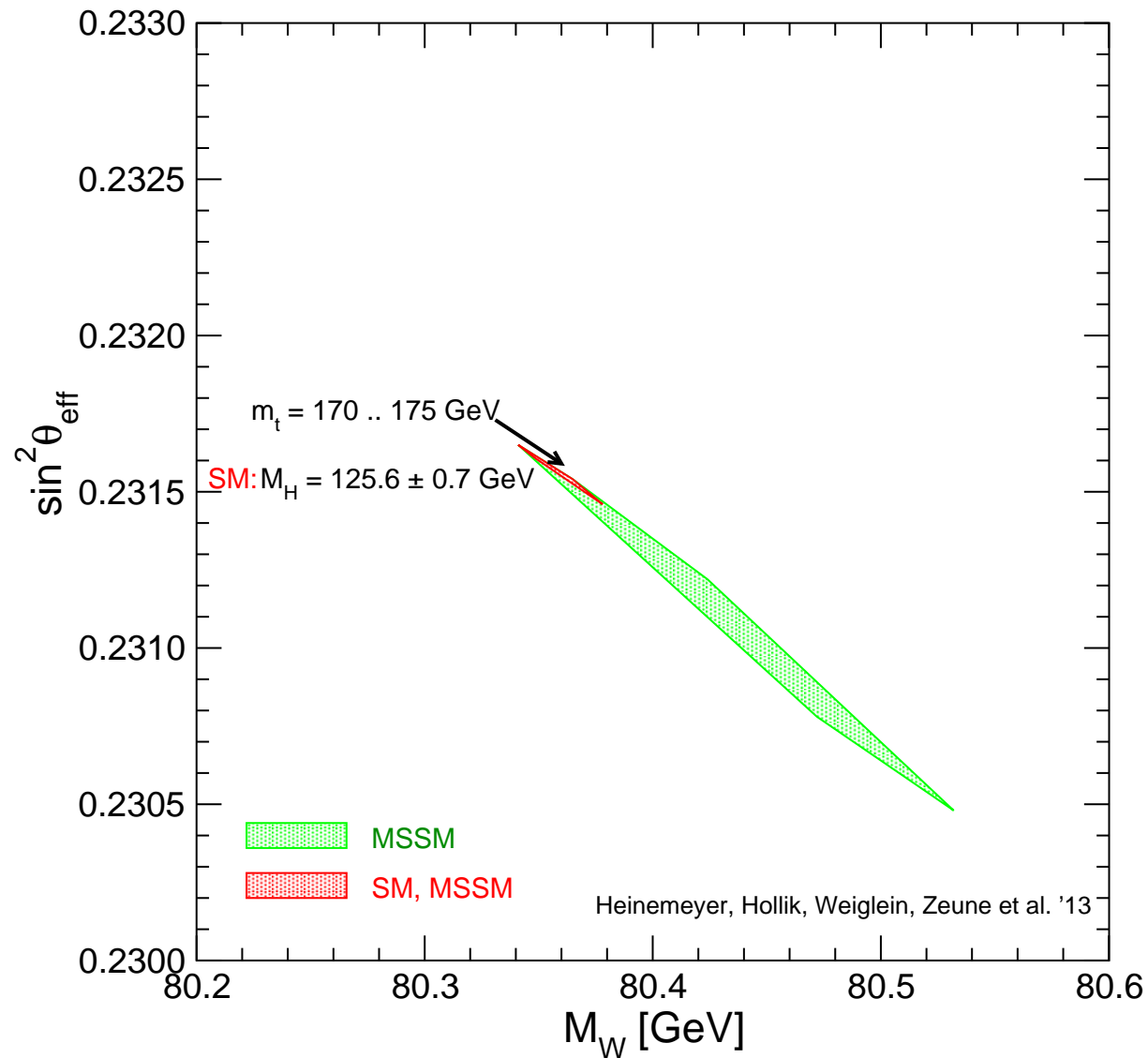
$$A_{\text{FB}}^f = \frac{\sigma_f(\theta < \frac{\pi}{2}) - \sigma_f(\theta > \frac{\pi}{2})}{\sigma_f(\theta < \frac{\pi}{2}) + \sigma_f(\theta > \frac{\pi}{2})} \equiv \frac{3}{4} \mathcal{A}_e \mathcal{A}_f,$$

$$A_{\text{LR}}^f = \frac{\sigma_f(P_e < 0) - \sigma_f(P_e > 0)}{\sigma_f(P_e < 0) + \sigma_f(P_e > 0)} \equiv \mathcal{A}_e |P_e|$$

$$\mathcal{A}_f = 2 \frac{g_{V_f}/g_{A_f}}{1 + (g_{V_f}/g_{A_f})^2} = \frac{1 - 4|Q_f| \sin^2 \theta_{\text{eff}}^f}{1 - 4|Q_f| \sin^2 \theta_{\text{eff}}^f + 8(|Q_f| \sin^2 \theta_{\text{eff}}^f)^2} \quad (f = \ell, b, \dots)$$

What M_W and $\sin^2 \theta_{\text{eff}}$ precision do we want?

[S.H., W. Hollik, G. Weiglein, L. Zeune et al. '13]



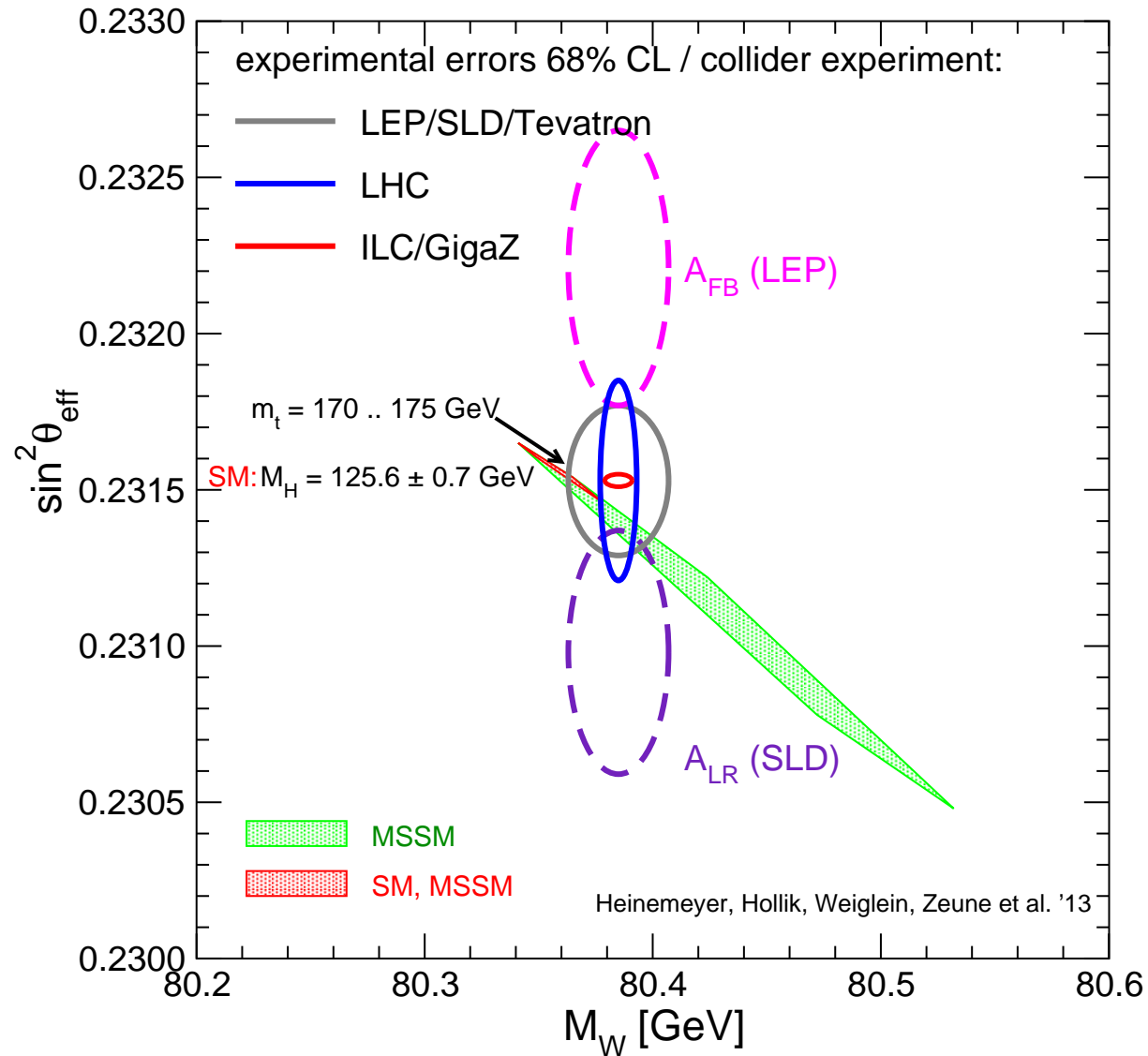
MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of m_t

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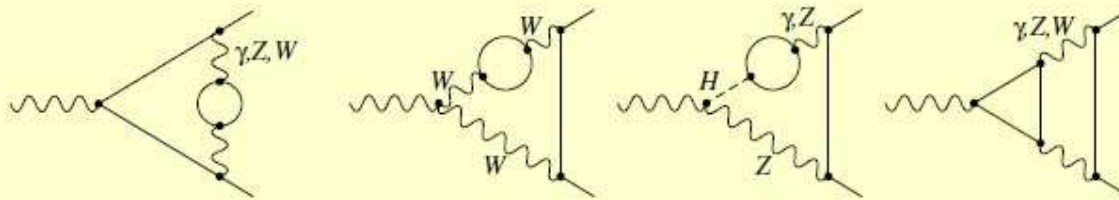
SM band:
variation of m_t

3. EWPO Status

Existing higher-order corrections to the EWPO

[taken from A. Freitas '16]

Known corrections to Δr , $\sin^2 \theta_{\text{eff}}^f$, g_{Vf} , g_{Af} :



- Complete NNLO corrections (Δr , $\sin^2 \theta_{\text{eff}}^l$) Freitas, Hollik, Walter, Weiglein '00
Awramik, Czakon '02; Onishchenko, Veretin '02
Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06
Hollik, Meier, Uccirati '05,07; Degrandi, Gambino, Giardino '14
- “Fermionic” NNLO corrections (g_{Vf} , g_{Af}) Czarnecki, Kühn '96
Harlander, Seidensticker, Steinhauser '98
Freitas '13,14
- Partial 3/4-loop corrections to ρ/T -parameter
 $\mathcal{O}(\alpha_t \alpha_s^2)$, $\mathcal{O}(\alpha_t^2 \alpha_s)$, $\mathcal{O}(\alpha_t \alpha_s^3)$
Chetyrkin, Kühn, Steinhauser '95
Faisst, Kühn, Seidensticker, Veretin '03
Boghezal, Tausk, v. d. Bij '05
Schröder, Steinhauser '05; Chetyrkin et al. '06
Boghezal, Czakon '06

$$(\alpha_t \equiv \frac{y_t^2}{4\pi})$$

Intrinsic uncertainties:

Quantity	current experimental unc.	current intrinsic unc.
M_W [MeV]	15	4 ($\alpha^3, \alpha^2\alpha_s$)
$\sin^2 \theta_{\text{eff}}^\ell$ [10^{-5}]	16	4.5 ($\alpha^3, \alpha^2\alpha_s$)
Γ_Z [MeV]	2.3	0.5 ($\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2$)
R_b [10^{-5}]	66	15 ($\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$)
R_l [10^{-3}]	25	5 ($\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$)

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Parametric uncertainties:

Quantity	$\delta m_t = 0.9$ GeV	$\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$	$\delta M_Z = 2.1$ MeV
δM_W^{para} [MeV]	5.5	2	2.5
$\delta \sin^2 \theta_{\text{eff}}^{\ell, \text{para}}$ [10^{-5}]	3.0	3.6	1.4

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⇒ Current intrinsic/parametric uncertainties are substantially smaller than current experimental uncertainties :-)

Additional uncertainty for M_W from threshold scan:

Not only $e^+e^- \rightarrow W^{(*)}W^{(*)}$, but $e^+e^- \rightarrow WW \rightarrow 4f$ needed

Current status:

full one-loop for $2 \rightarrow 4$ process

[A. Denner, S. Dittmaier, M. Roth, D. Wackerath '99-'02]

\Rightarrow extraction of M_W at the level of ~ 6 MeV

Most recent improvement:

leading 2L corrections from EFT

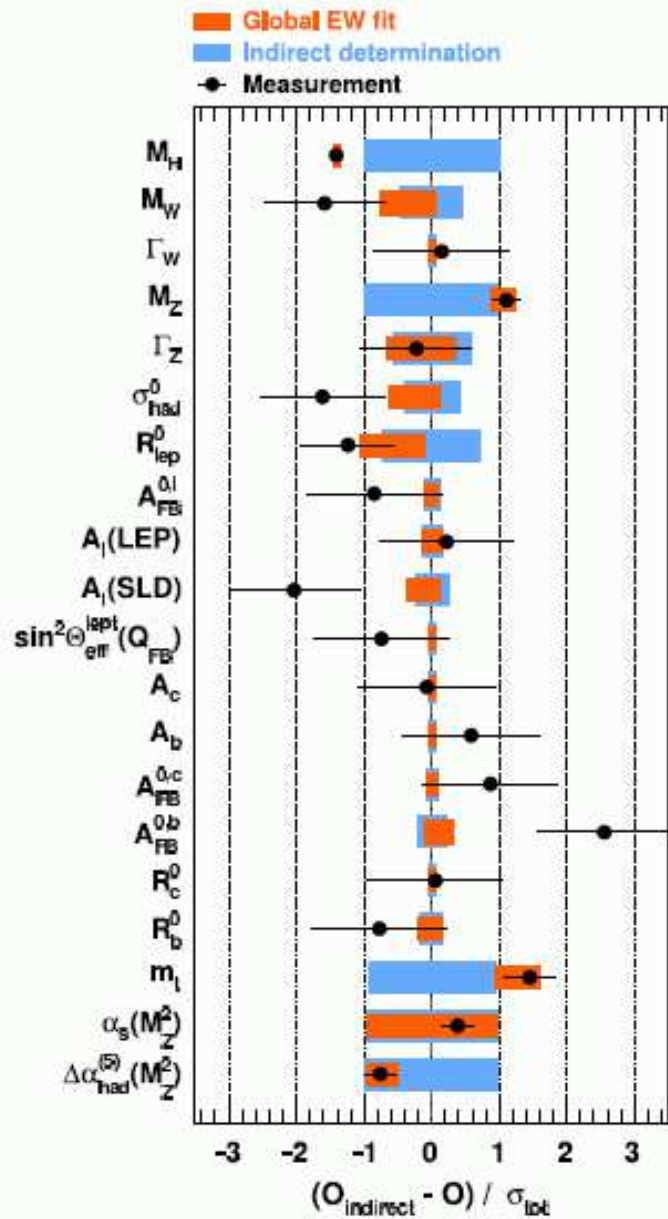
[Actis, Beneke, Falgari, Schwinn '08]

\Rightarrow impact on M_W at the level of ~ 3 MeV

\Rightarrow well under control for LEP data

Overview about all EWPO:

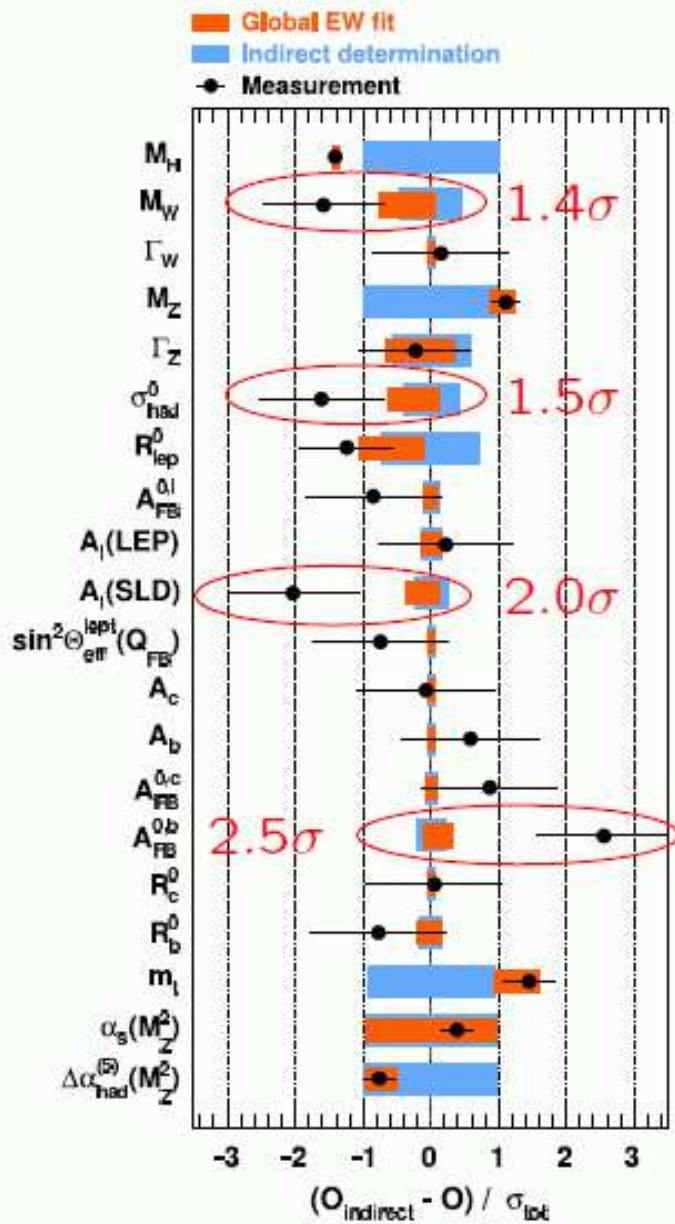
[taken from A. Freitas '16]



Surprisingly good agreement:
 $\chi^2/\text{d.o.f.} = 18.1/14$ ($p = 20\%$)

Most quantities measured with
 1%–0.1% precision

GFitter coll. '14



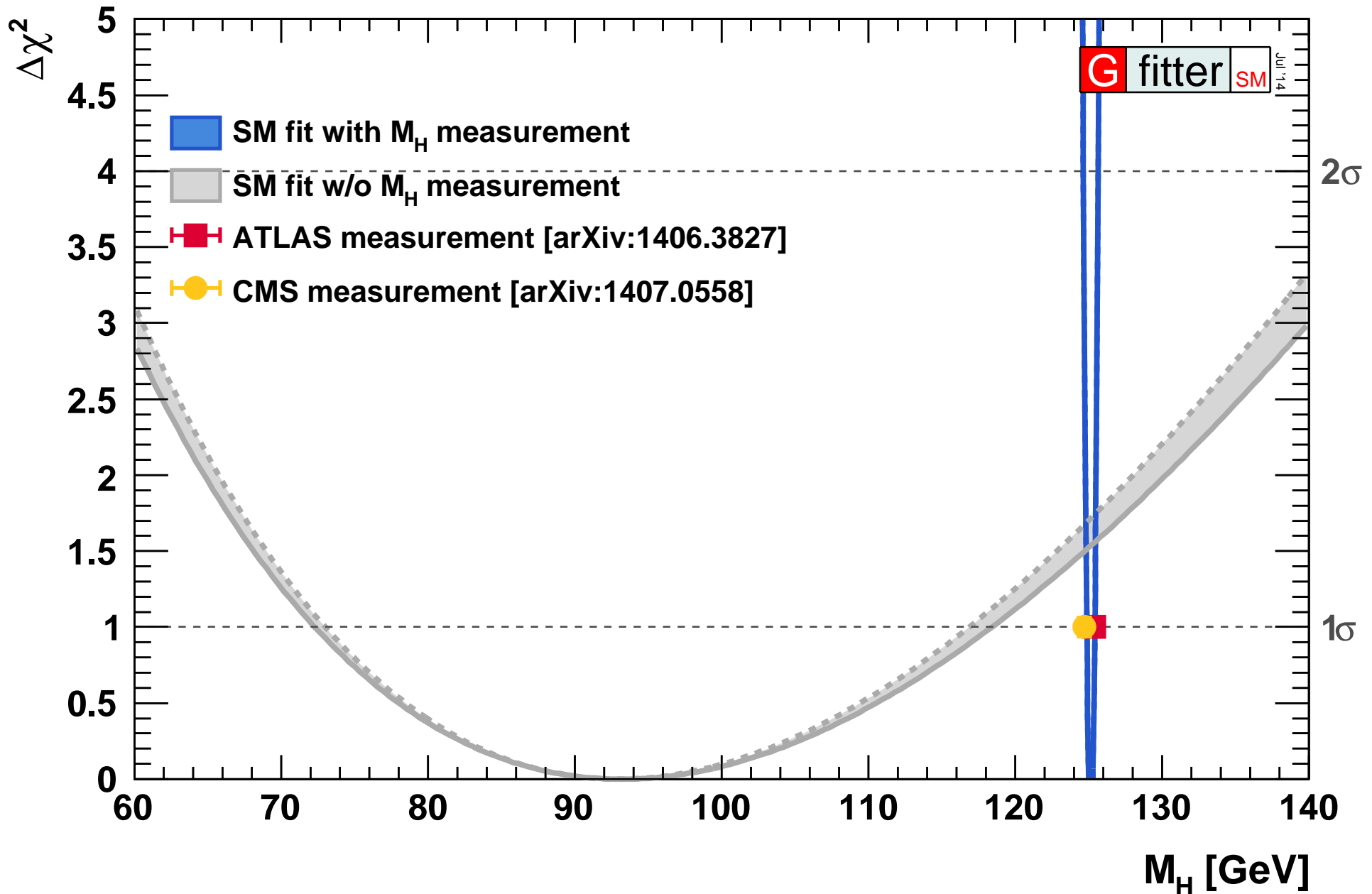
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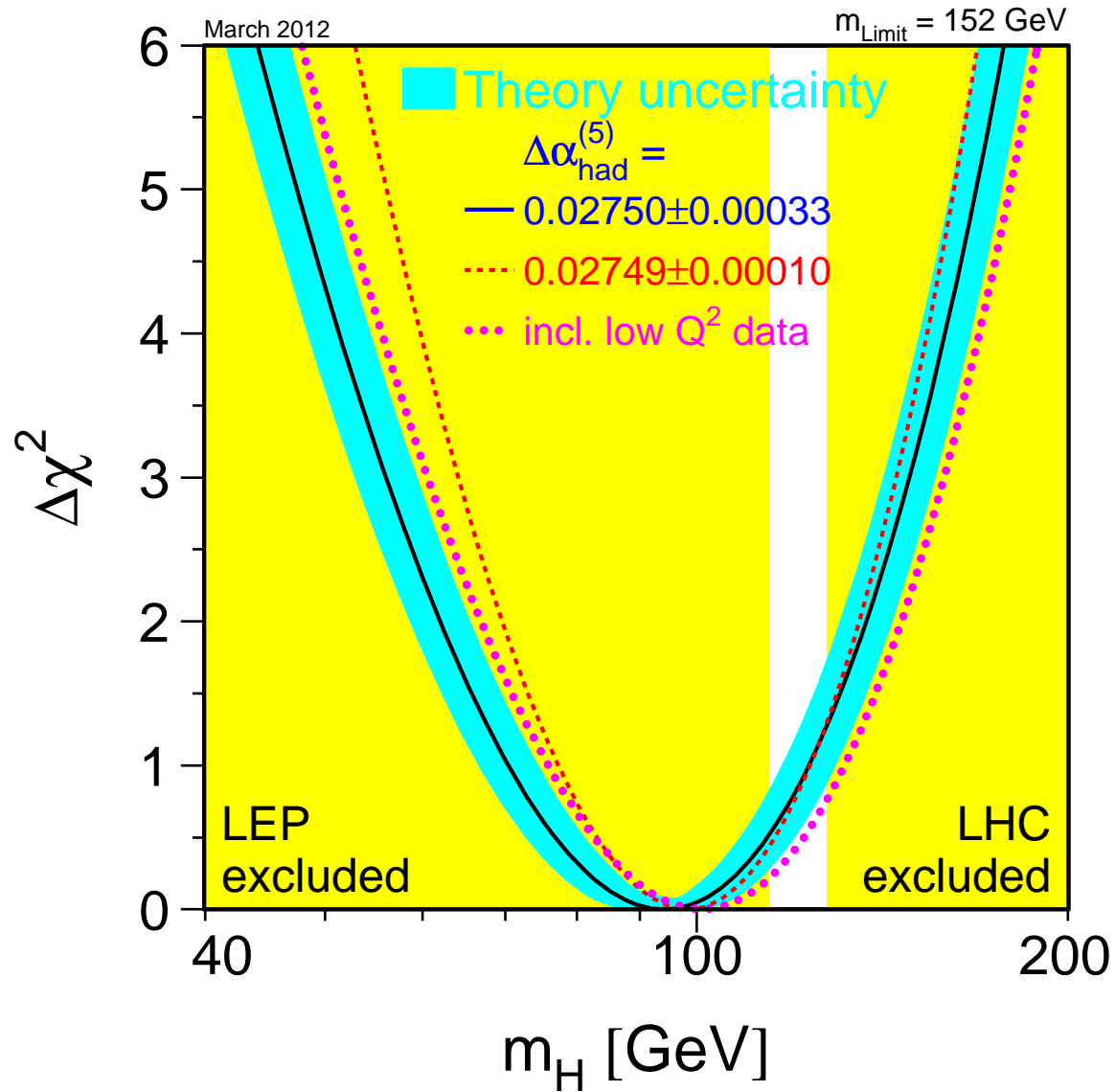
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A few interesting deviations:

- M_W ($\sim 1.4\sigma$)
- σ_{had}^0 ($\sim 1.5\sigma$)
- $A_\ell(\text{SLD})$ ($\sim 2\sigma$)
- A_{FB}^b ($\sim 2.5\sigma$)
- $(g_\mu - 2)$ ($\sim 3\sigma$)

GFitter coll. '14





4. EWPO Future

Our future estimates:

- assume to go **substantially** beyond what is known now
- assume that **many theorists** will put **many² hours** of work into it (motivation?)
- do not assume that magically new calculational methods are invented
- are overall optimistic

⇒ they should be taken seriously!

⇒ An honest evaluation of theory uncertainties will increase the robustness of the LC physics case!

What is needed to match the LC precision?

Compare:

1. LC (pure) **experimental** (anticipated) precision
2. **Intrinsic** uncertainties
3. **Parametric** uncertainties
→ taking into account the improved precision of SM parameters at the LC

Combined uncertainty:

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

Intrinsic uncertainties: \Rightarrow can be the limiting factor!

Quantity	ILC	Current intrinsic unc.	Projected unc.
M_W [MeV]	3	4 ($\alpha^3, \alpha^2\alpha_s$)	1
$\sin^2 \theta_{\text{eff}}^\ell$ [10^{-5}]	1.3	4.5 ($\alpha^3, \alpha^2\alpha_s$)	1.5
Γ_Z [MeV]	1	0.5 ($\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2$)	0.2
R_b [10^{-5}]	15	15 ($\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$)	7
R_l [10^{-3}]	10??	5 ($\alpha_{\text{bos}}^2, \alpha^3, \alpha^2\alpha_s$)	1.5

These calculations are required for the projection:

- complete $\mathcal{O}(\alpha\alpha_s^2)$ corrections
- fermionic $\mathcal{O}(\alpha^2\alpha_s)$ corrections
- double-fermionic $\mathcal{O}(\alpha^3)$ corrections
- leading four-loop corrections enhanced by the top Yukawa coupling
- the $\mathcal{O}(\alpha_{\text{bos}}^2)$ corrections are not the leading uncertainties now

For these calculations, qualitatively new developments of existing loop integration techniques will be required, but no conceptual paradigm shift.

Parametric uncertainties:

1. M_H : better than 50 MeV \Rightarrow negligible
2. M_Z : ~ 0.1 MeV with negligible theory uncertainties \Rightarrow negligible
3. $\alpha_s(M_Z)$: from (mainly) R_ℓ
 $\delta\alpha_s^{\text{exp}} \sim 10^{-4}$, $\delta\alpha_s^{\text{theo}} \sim 1.5 \times 10^{-4}$
4. m_t : from threshold scan
 $\delta m_t^{\text{exp}} \sim \mathcal{O}(10 \text{ MeV})$
 $\delta m_t^{\text{theo}} \sim 50 \text{ MeV}$ (NNNLO/NNLL \oplus 1S \rightarrow $\overline{\text{MS}}$ \oplus $\delta\alpha_s$)
5. m_b : from lattice calculations \Rightarrow negligible for EWPO
 $\delta m_b \sim 10 \text{ MeV}$ (still under discussion, too optimistic?)
6. $\Delta\alpha_{\text{had}}$: BES III and Belle II: $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$
better from measurements “around the Z pole?”

Uncertainty budget for m_t :

[talk by A. Hoang '15]

$\delta\alpha_s(M_z) = 0.001$

Msbar mass error budget (from threshold scan)

$(\delta M_t^{\text{SD-low}})^{\text{exp}}$	$(\delta M_t^{\text{SD-low}})^{\text{theo}}$	$(\delta \bar{m}_t(\bar{m}_t))^{\text{conversion}}$	$(\delta \bar{m}_t(\bar{m}_t))^{\alpha_s}$
40 MeV	50 MeV	7 – 23 MeV	70 MeV

⇒ improvement in α_s crucial

e^+e^- collider: precision measurement:

$$R_l := \frac{\Gamma(Z \rightarrow \text{hadrons})}{\Gamma(Z \rightarrow l^+l^-)}$$

Improvement down to $\delta^{\text{exp}}\alpha_s \sim 0.001 - 0.0001$ possible?!

Note: TH uncertainty (assuming fermionic 3-loop corrections):

$$\delta R_l^{\text{theo}} \sim 0.0015 \Rightarrow \delta\alpha_s^{\text{theo}} \sim 0.00015$$

⇒ hard to beat ...

M_W parametric:

parametric today: $\delta m_t = 0.9$ GeV, $\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$, $\delta M_Z = 2.1$ MeV

$$\delta M_W^{\text{para},m_t} = 5.5 \text{ MeV}, \quad \delta M_W^{\text{para},\Delta\alpha_{\text{had}}} = 2 \text{ MeV}, \quad \delta M_W^{\text{para},M_Z} = 2.5 \text{ MeV}$$

parametric future: $\delta m_t^{\text{fut}} = 0.05$ GeV, $\delta(\Delta\alpha_{\text{had}})^{\text{fut}} = 5 \times 10^{-5}$, $\delta M_Z^{\text{ILC}} = 1$ MeV

$$\Delta M_W^{\text{para,fut},m_t} = 0.5 \text{ MeV}, \quad \Delta M_W^{\text{para,fut},\Delta\alpha_{\text{had}}} = 1 \text{ MeV}, \quad \Delta M_W^{\text{para,fut},M_Z} = 0.2 \text{ MeV}$$

$\sin^2 \theta_{\text{eff}}$ parametric: [10^{-5}]

parametric today: $\delta m_t = 0.9$ GeV, $\delta(\Delta\alpha_{\text{had}}) = 10^{-4}$, $\delta M_Z = 2.1$ MeV

$$\delta \sin^2 \theta_{\text{eff}}^{\text{para},m_t} = 3.0, \quad \delta \sin^2 \theta_{\text{eff}}^{\text{para},\Delta\alpha_{\text{had}}} = 3.6, \quad \delta \sin^2 \theta_{\text{eff}}^{\text{para},M_Z} = 1.4$$

parametric future: $\delta m_t^{\text{fut}} = 0.05$ GeV, $\delta(\Delta\alpha_{\text{had}})^{\text{fut}} = 5 \times 10^{-5}$, $\delta M_Z^{\text{ILC}} = 1$ MeV

$$\Delta \sin^2 \theta_{\text{eff}}^{\text{para,fut},m_t} = 0.2, \quad \Delta \sin^2 \theta_{\text{eff}}^{\text{para,fut},\Delta\alpha_{\text{had}}} = 1.8, \quad \Delta \sin^2 \theta_{\text{eff}}^{\text{para,fut},M_Z} = 0.65$$

Additional uncertainty for M_W from threshold scan:

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Current status:

full one-loop for $2 \rightarrow 4$ process

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Most recent improvement:

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\Rightarrow impact on M_W at the level of ~ 3 MeV

\Rightarrow full 2L for $2 \rightarrow 4$ process not foreseeable

Potentially possible:

2L resummed higher-order terms for $e^+e^- \rightarrow WW$ and $W \rightarrow ff'$

\Rightarrow extraction of M_W at ~ 1 MeV?? \oplus pure exp. uncertainty of ~ 3 MeV

Summary of future parametric uncertainties:

Quantity	ILC	future parametric unc.	Main source
M_W [MeV]	3	1	$\delta(\Delta\alpha_{\text{had}})$
$\sin^2 \theta_{\text{eff}}^l$ [10^{-5}]	1.3	2	$\delta(\Delta\alpha_{\text{had}})$
Γ_Z [MeV]	1	0.5	
R_b [10^{-5}]	15	< 1	$\delta\alpha_s$

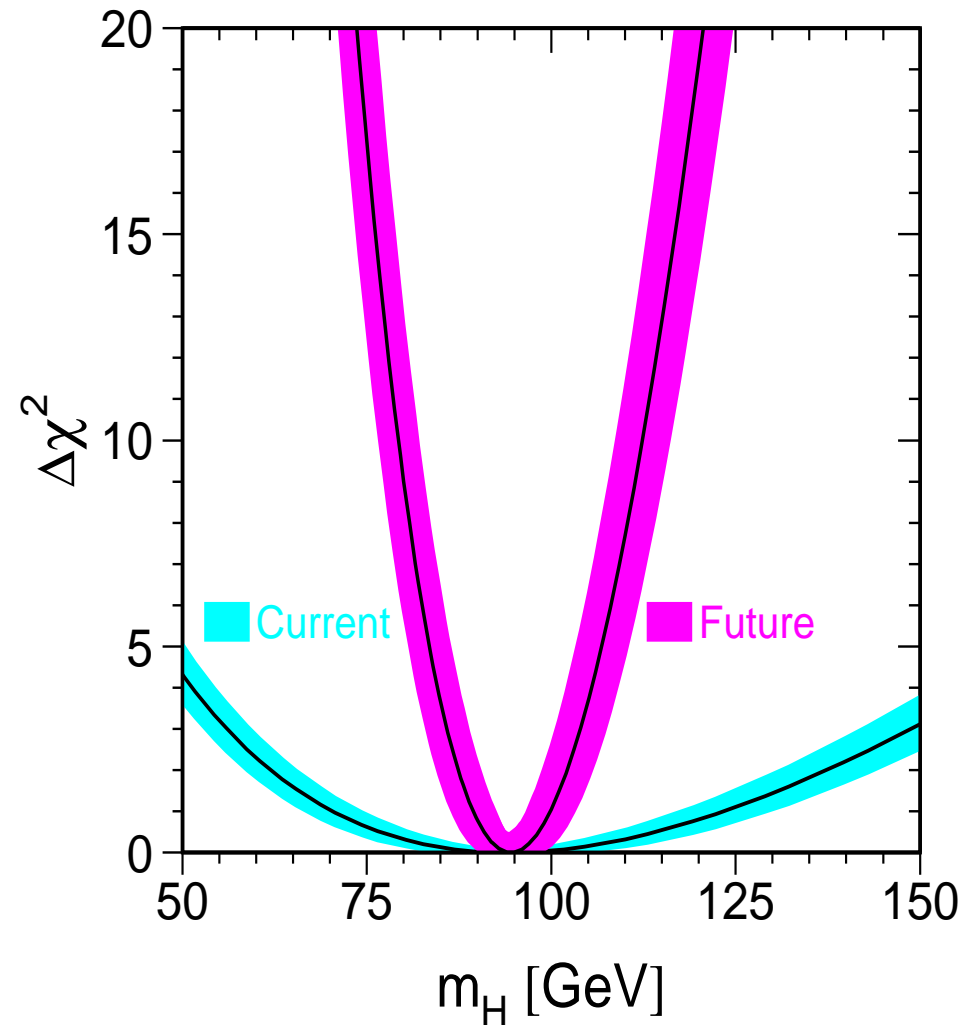
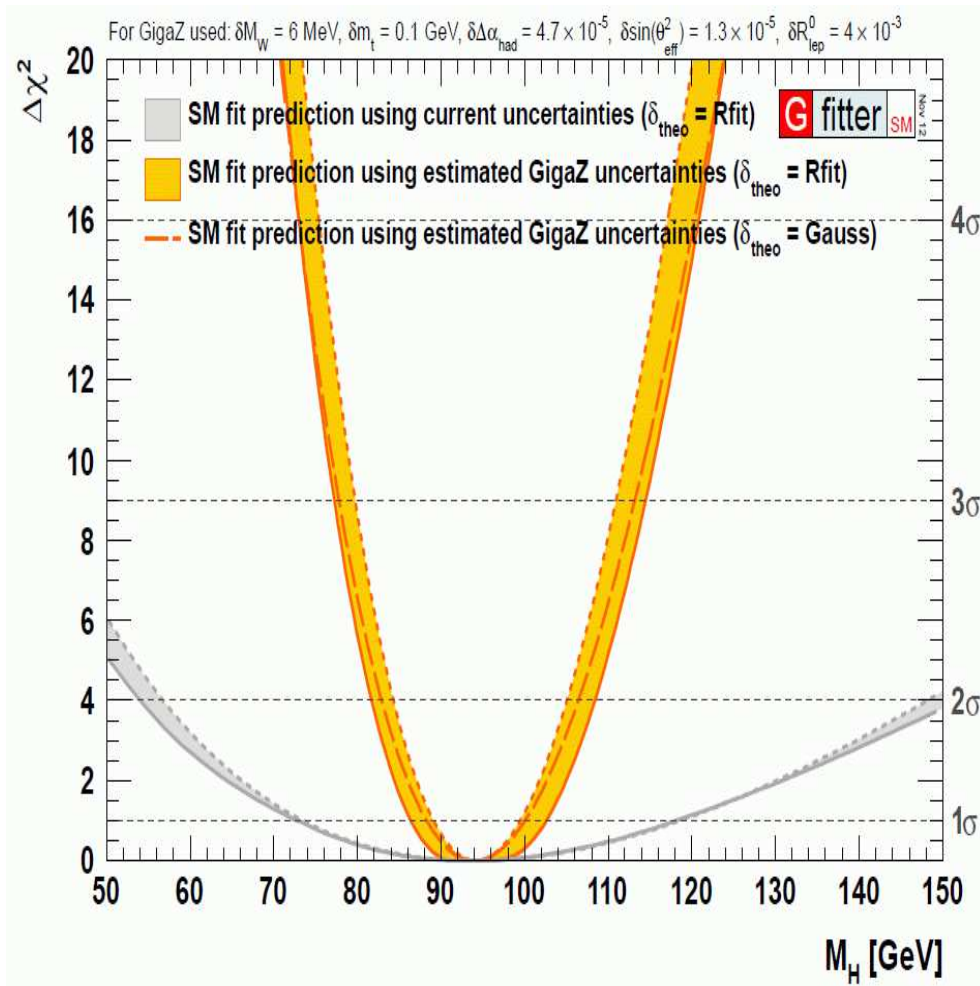
⇒ add quadratic to experimental uncertainties!

⇒ add linearly to intrinsic uncertainties!

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

Precise M_H test with the ILC precision:

[GFitter '13] [LEPEWWG '13]



$\Rightarrow \delta M_H^{\text{ind}} \lesssim 6 \text{ GeV}$

\Rightarrow extremely sensitive test of SM (and BSM) possible

\Leftarrow to be redone incl. all TH unc.

One more word of caution:

The above numbers have all been obtained assuming the SM as calculational framework.

The SM constitutes the model in which highest theoretical precision for the predictions of EWPO can be obtained.

We know that BSM physics must exist! (DM, gravity, ...)

As soon as BSM physics will be discovered, an evaluation of the EWPO in any preferred BSM model will be necessary.

The corresponding theory uncertainties, both intrinsic and parametric, can then be larger (as known for the MSSM).

A dedicated theory effort (beyond the SM) would be needed in this case.

5. Conclusions

- High anticipated experimental precision for EWPO at the LC
- Crucial: theory uncertainties: intrinsic and parametric

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

- We give (realistic/optimistic) estimates for future intrinsic and parametric uncertainties
- intrinsic unc. larger than anticipated experimental unc.
parametric unc. can be larger than experimental uncertainties
⇒ particularly true for M_W and $\sin^2 \theta_{\text{eff}}$
- Uncertainties should be taken into account by experimental analyses!

Further Questions?

