Light Flavon signals in electron-photon collision at the ILC

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(Based on arXiv: 1707.06524)

2017-07-22 Summer Camp on ILC accelrator and physics / detector @ Norikura

Out line of the talk

1. Introduction
2. Flavon in A_4 model
3. Flavon at colliders
4. Summary

1. Introduction

The SM is very successful model

- ✓ Basically consistent with experimental results
- ✓ Discovery of Higgs boson complete the particle list

But we don’t understand origin/principle of structures in SM

- ✓ We don’t know origin of flavor structure of SM fermion
- ✓ In particular neutrino mass issue has lot of mystery
  - ❖ Dirac type mass? Majarana type mass?
  - ❖ How to understand large mixing angle?

Flavor symmetry model is one idea to deal with flavor structure

In this talk we consider A_4 flavor model for lepton sector

□ A model with A_4 flavor symmetry

- ✓ A_4 is Non Abelian discrete symmetry
- ✓ Assigning SM left-handed leptons as triplet of A_4
- ✓ Easily derive large neutrino mixing angle; tri-bimaximal mixing (TBM)

G. Altarelli and F. Feruglio NPB 720 (2005) 64; NPB 741 (2006) 215

Structure of lepton flavor is induced from VEV alignment of A4 breaking scalar (SM singlets)

⇒ **Flavon fields**

- ✓ Flavons couples to SM leptons (not to quarks)
- ✓ Mass can be light as ~O(100) GeV
- ✓ We discuss possibility to produce flavons at colliders
- ✓ Direct test of flavons/flavor symmetry model

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2. Flavon in A_4 model

Symmetry: $G_{SM} \times A_4 \times Z_3 \times U(1)_R$

Field contents including SM leptons

	l	e_R^c	μ_R^c	τ_R^c	ν_R^c	$h_{u,d}$	ϕ_T	ϕ_S	ξ	$\tilde{\xi}$	ξ'	$\tilde{\xi}'$	ϕ_0^T	ϕ_0^S	ξ_0	ξ_0'
$SU(2)$	2	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1
A_4	**3**	**1**	**1''**	**1'**	**3**	**1**	**3**	**3**	**1**	**1**	**1'**	**1'**	**3**	**3**	**1**	**1'**
Z_3	ω	ω^2	ω^2	ω^2	ω^2	1	1	ω^2	ω^2	ω^2	ω^2	ω^2	1	ω^2	ω^2	ω^2
$U(1)_R$	1	1	1	1	1	0	0	0	0	0	0	0	2	2	2	2

Y.Muramatsu, T. Nomura, Y. Shimizu JHEP 1603 (2016) 192

Scalar fields

- ✓ SM lepton doublets are in 3 dimensional rep. of A_4
- ✓ Right handed neutrino also in 3 rep.
- ✓ Lepton mass structure comes from VEV alignment of scalar fields
- ✓ We calculated VEV in the frame work of SUSY with $U(1)_R$ symmetry
- ✓ SUSY particles are assumed to be heavy and does not affect pheno.
- ✓ Φ_T is the flavon field mainly discussed in this talk

$$\phi_T = (\phi_{T_1}, \phi_{T_2}, \phi_{T_3}),\ \phi_S = (\phi_{S_1}, \phi_{S_2}, \phi_{S_3}),$$

The VEV alignment for Flavon fields

VEV alignment s are derived from condition of potential minimum

$$\left\{\begin{array}{l} \langle\phi_T\rangle = v_T(1,0,0), \quad v_T = \dfrac{3M}{2g} \\ \langle\tilde{\xi}\rangle = \langle\tilde{\xi}'\rangle = 0, \quad \langle\xi\rangle = u, \quad \langle\xi'\rangle = u' \\ \langle\phi_S\rangle = v_S(1,1,1), \quad v_S^2 = \dfrac{g_4}{3g_3}u^2 = \dfrac{g'_4}{3g'_3}u'^2 \end{array}\right.$$

M: flavon mass scale
g, g_3, g_4: trilinear couplings

❖ **These alignments induce flavor structure in leptons sector**

The relation in Flavon masses

- Expanding flavon field around VEV

$$\phi_T = (\phi_{T_1}, \phi_{T_2}, \phi_{T_3}) \longrightarrow (v_T + \varphi_{T_1}, \varphi_{T_2}, \varphi_{T_3})$$

- Resulting flavon mass term

$$L_{mass} = 2M^2 \left(\left|\varphi_{T_1}\right|^2 + 4\left|\varphi_{T_2}\right|^2 + 4\left|\varphi_{T_3}\right|^2 \right)$$

- Masses of flavons

$$(m^2_{\varphi_{T1}}, m^2_{\varphi_{T2}}, m^2_{\varphi_{T3}}) = (2M^2, 8M^2, 8M^2)$$

- The mass relation: $2\, m_{\varphi T1} = m_{\varphi T2} = m_{\varphi T3} = 2\sqrt{2}\, M$

➡ We choose parameter g as $2\, m_{\varphi T1} = m_{\varphi T2} = m_{\varphi T3} = v_T$

Lepton masses from flavon VEV

Yukawa interactions (from superpotential)

$$L \supset y_e \phi_T l e_R^c h_d / \Lambda + y_\mu \phi_T l \mu_R^c h_d / \Lambda + y_\tau \phi_T l \tau_R^c h_d / \Lambda$$

$$+ y_D l \nu_R^c h_u$$

(Λ: cut off scale)

$$+ y_{\phi_S} \phi_S \nu_R^c \nu_R^c + (y_\xi \xi + y_{\tilde{\xi}} \tilde{\xi}) \nu_R^c \nu_R^c + (y_{\xi'} \xi' + y_{\tilde{\xi}'} \tilde{\xi}') \nu_R^c \nu_R^c$$

$$\begin{cases} M_\ell = \dfrac{v_d v_T}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \quad M_D = y_D v_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\[2em] M_N = \underbrace{y_{\phi_S} v_S \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} + y_\xi u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}}_{\text{Giving TBM}} + \underbrace{y_{\xi'} u' \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}}_{\text{Deviation from TBM}} \end{cases}$$

Interactions among flavon and leptons

$$\mathcal{L}_\ell = y_e \left(\phi_T \bar{l}\right) e_R h_d/\Lambda + y_\mu \left(\phi_T \bar{l}\right) \mu_R h_d/\Lambda + y_\tau \left(\phi_T \bar{l}\right) \tau_R h_d/\Lambda + h.c.,$$

$$\begin{aligned}\mathcal{L}_\ell^{\rm FY} &= \begin{pmatrix}\bar{e}_L & \bar{\mu}_L & \bar{\tau}_L\end{pmatrix}\begin{pmatrix}\frac{m_e}{v_T} & 0 & 0\\ 0 & \frac{m_\mu}{v_T} & 0\\ 0 & 0 & \frac{m_\tau}{v_T}\end{pmatrix}\begin{pmatrix}e_R\\ \mu_R\\ \tau_R\end{pmatrix}\varphi_{T1}\\ &+ \begin{pmatrix}\bar{e}_L & \bar{\mu}_L & \bar{\tau}_L\end{pmatrix}\begin{pmatrix}0 & \frac{m_\mu}{v_T} & 0\\ 0 & 0 & \frac{m_\tau}{v_T}\\ \frac{m_e}{v_T} & 0 & 0\end{pmatrix}\begin{pmatrix}e_R\\ \mu_R\\ \tau_R\end{pmatrix}\varphi_{T2}\\ &+ \begin{pmatrix}\bar{e}_L & \bar{\mu}_L & \bar{\tau}_L\end{pmatrix}\begin{pmatrix}0 & 0 & \frac{m_\tau}{v_T}\\ \frac{m_e}{v_T} & 0 & 0\\ 0 & \frac{m_\mu}{v_T} & 0\end{pmatrix}\begin{pmatrix}e_R\\ \mu_R\\ \tau_R\end{pmatrix}\varphi_{T3} + h.c..\end{aligned}$$

❖ **Couplings are determined by flavon VEV and lepton masses**

Remnant Z_2 symmetry and mass limit

- **A_4 breaking by flavon VEV induce remnant Z_3 symmetry**

Z_3 transformation for flavons:

$$\varphi_{T1} \to \varphi_{T1}, \quad \varphi_{T2} \to \omega^2 \varphi_{T2}, \quad \varphi_{T3} \to \omega \varphi_{T3}, \quad (\omega^3 = 1)$$

For leptons

$$e \to e, \quad \mu \to \omega\mu, \quad \tau \to \omega^2 \tau,$$

This symmetry forbid LFV process like $\mu \to e\gamma$

LFV constraints is not very strong

- The strongest constraint from $\tau \to \mu\mu e$: $v_T = 2m_{\varphi_{T1}} = m_{\varphi_{T2}} = m_{\varphi_{T3}} > 60\ GeV$

Light flavon can be produced at colliders

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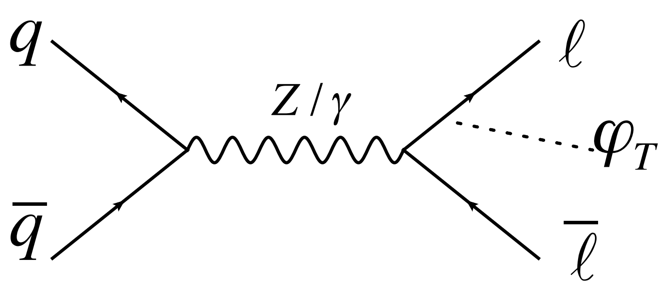
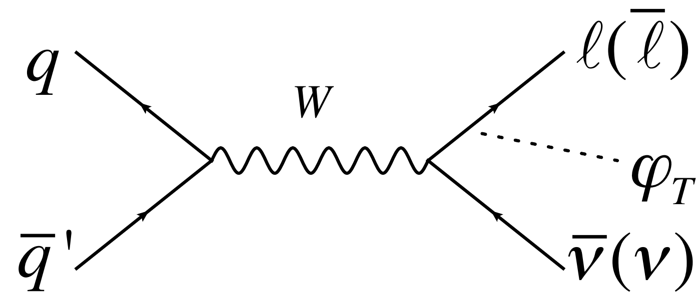
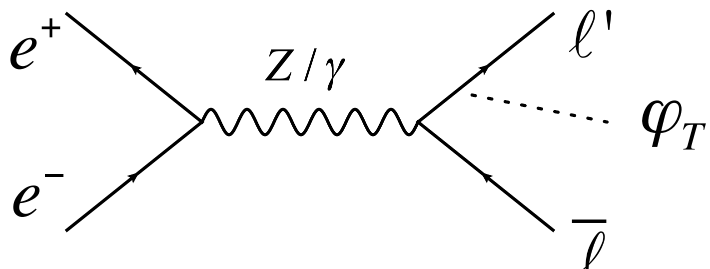
Light flavons can be produced at colliders

① Flavon production process at the LHC

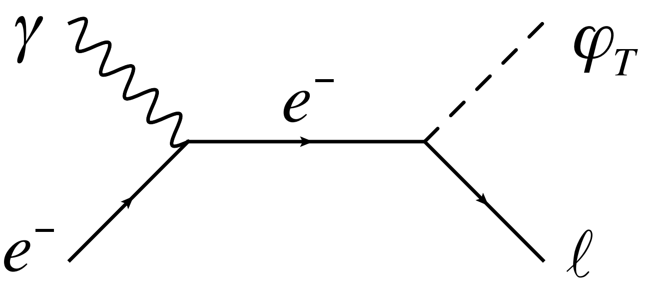




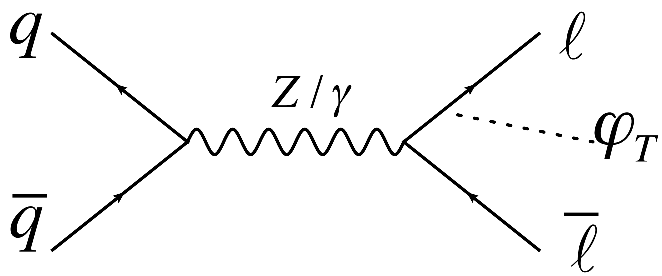
② Flavon production process in e⁺ e⁻ collision at the ILC

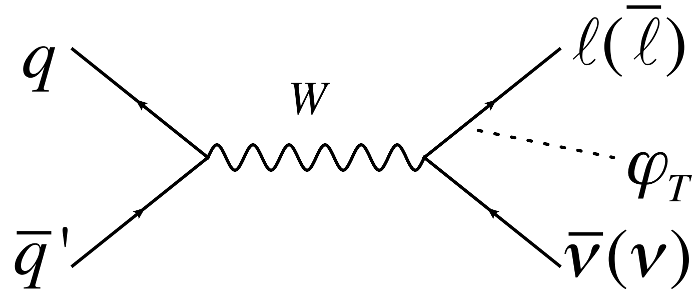


③ Flavon production process in e⁻ γ collision at the ILC



① Flavon production process at the LHC





Cross sections (calculated with MadGraph5)

Final state	$\varphi_{T1}\tau^-\bar{\nu}_\tau$	$\varphi_{T1}\tau^-\tau^+$	$\varphi_{T2}\tau^-\bar{\nu}_\mu$	$\varphi_{T2}\tau^-\mu^+$	$\varphi_{T3}\tau^-\bar{\nu}_e$	$\varphi_{T3}\tau^-e^+$
Cross section [fb]	2.2	1.5×10^{-1}	1.7×10^{-5}	8.4×10^{-6}	1.7×10^{-5}	8.4×10^{-6}

Relatively large cross section for φ_{T1} with tau lepton

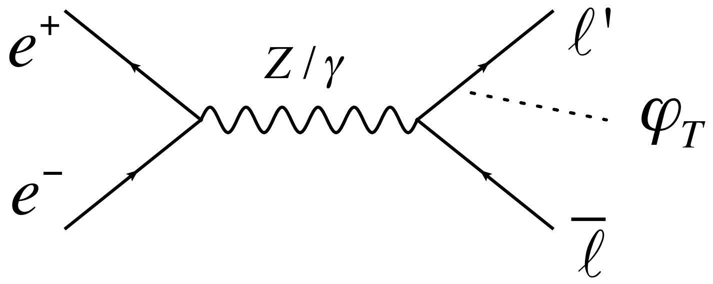
- ✓ Associated coupling is proportional to m_τ
- ✓ φ_{T1} is lighter than the others
- ✓ φ_{T1} decays into tau lepton pair

$$2m_{\varphi_{T1}} = m_{\varphi_{T2}} = m_{\varphi_{T3}}$$

To find the signature tau tagging is important; at least three tau leptons

We find very small number of event even with 3000 fb^{-1} luminosity

② Flavon production process in $e^+ e^-$ collision at the ILC



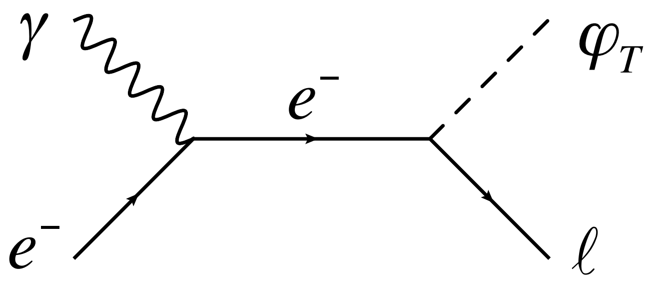
✧ Final states are similar to previous LHC case

- 4 tau lepton final state is dominant
- 4 tau tagging for analyzing the signal

Our simulation indicate tau tagging is not so efficient

- ✓ It is partly due to small p_T tau from flavon decay
- ✓ We can not obtain sufficient discovery significance
- ✓ Improvement of tau tagging may help

③ Flavon production process in $e^- \gamma$ collision at the ILC



This processe has advantages

- ✓ Number of interaction vertex is smaller than previous cases
- ✓ Final states are two body (for on-shell flavon)
- ✓ Number of tau leptons can be smaller

→ **Larger cross section and efficiency are expected**

From here we focus on the process in our simulation study

Beam energy and luminosity in eγ collision

We make rough estimation for our analysis

- Photon beam comes from Compton scattering
  - ✓ Luminosity: $L_\gamma \sim \sqrt{0.036 L_e}$

    V.I. Telnov, Nucl. Instrum. Meth. A 355, 3 (1995)
    K.Fujii et al. arXiv: 1604.1640
  - ✓ Beam energy has peak around 80% of electron beam

- Electron-electron beam is more efficient than electron-positron beam
  - ✓ Factor of 3 luminosity enhancement

We simply use: $L_{e^-\gamma} \sim 3\sqrt{0.036 L_{e^+e^-}} \sim 0.6 L_{e^+e^-}, \quad E_\gamma = 0.8 E_e$

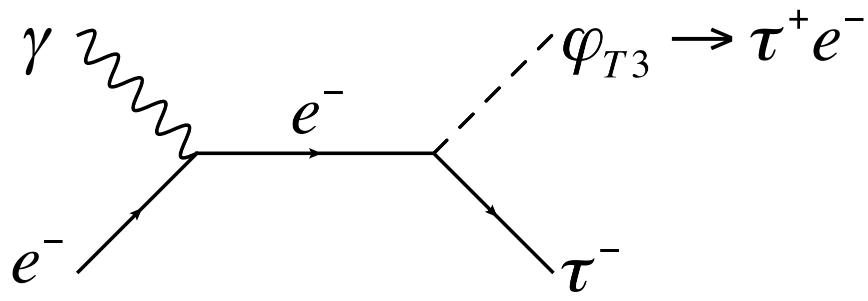
	ILC		upgraded ILC		
Beam energy [GeV^2]	250×200	125×100	500×400	250×200	125×100
Luminosity [fb^{-1}]	300	300	4800	2400	1200

*Luminosity of e+e- collision is taken from T. Barklow et al. arXiv:1506.07830

We perform event simulation with MadGraph, Pythia, Delphes

□ Simulation study for flavor conserving final state

Consider the process:



$\tau^+\tau^-e^-$ final state

❖ **The cross sections for the other processes are small**

The SM background process: $e^-\gamma \to e^- Z/\gamma^* \to e^-\tau^-\tau^+$

Cross sections for signal and BG

$$v_T = 2m_{\varphi_{T1}} = m_{\varphi_{T2}} = m_{\varphi_{T3}} = 65\,GeV$$

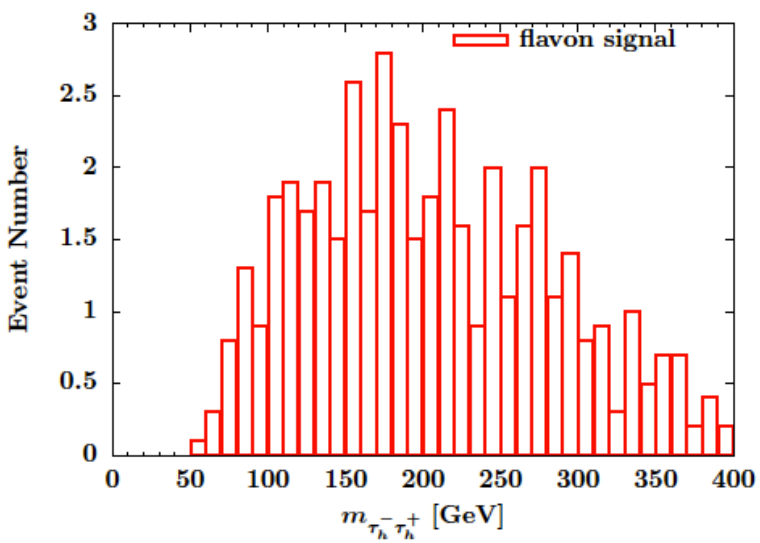
Beam energy [GeV^2]	500×400	250×200	125×100
Signal cross section [fb]	2.2	8.6	3.1×10
Background cross section [fb]	3.0×10	1.1×10^2	3.6×10^2

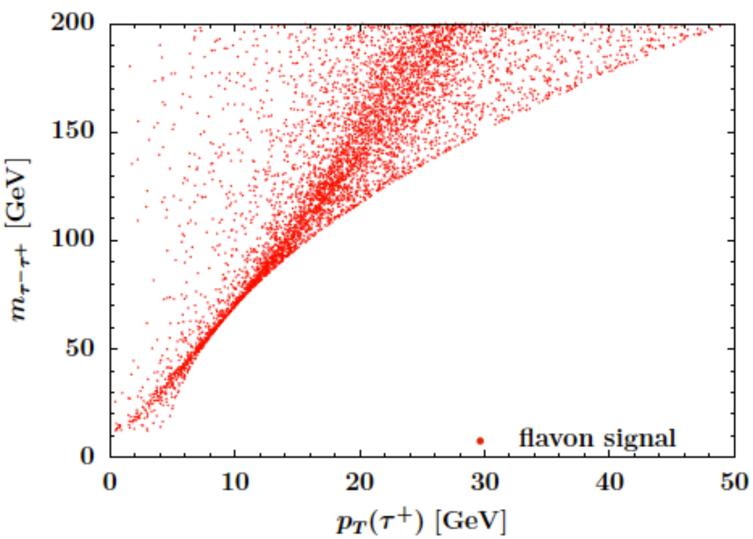
Larger cross section for smaller beam energy: $|M|^2 \propto \dfrac{1}{s^2}$

Event selection rules

1) We chose signal event : $\tau_h^+ \tau_h^- e^-$ (τ_h is hadronic tau)
2) Apply cut for invariant mass of τ pair: $m_{\tau\tau}$
3) Apply cut for invariant mass of $\tau_h^+ e^-$: $m_{e\tau}$

For 2) we check IM distribution





(beam energy 250×200 GeV^2, flavon mass 65 GeV)

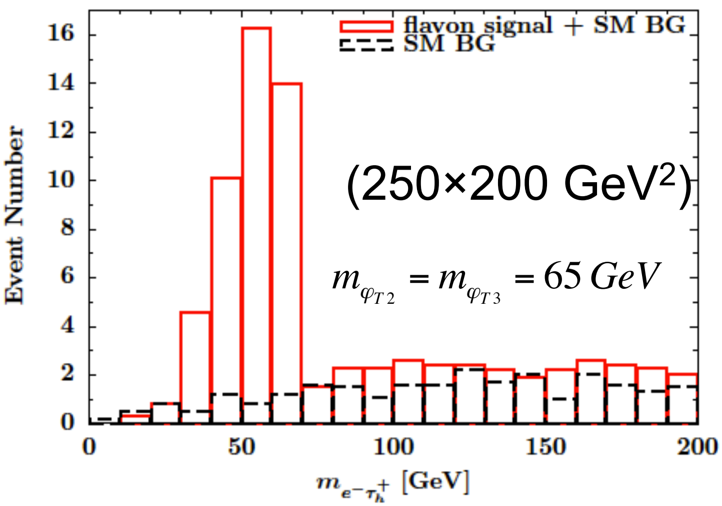
Event selection rules

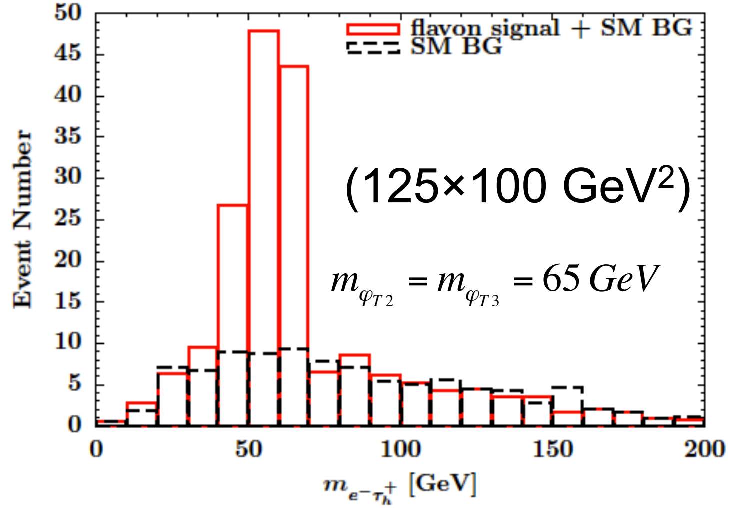
250×200 [GeV^2]	R_S [%]/R_{BG} [%]	N_S/N_{BG}	S_{cL}
contain $\tau_h^+\tau_h^-e^-$	1.7/1.8	45/590	1.8
$m_{\tau_h^-\tau_h^+} > M_Z + 15.0$ [GeV]	1.6/0.15	41/49	5.3
30 [GeV] $< m_{e^-\tau_h^+} <$ 65 [GeV]	1.5/0.00093	38/3.1	12
125×100 [GeV^2]	R_S [%]/R_{BG} [%]	N_S/N_{BG}	S_{cL}
contain $\tau_h^+\tau_h^-e^-$	1.8/1.9	170/2100	3.7
$m_{\tau_h^-\tau_h^+} > M_Z + 5.0$ [GeV]	1.0/0.089	96/96	8.6
25 [GeV] $< m_{e^-\tau_h^+} <$ 70 [GeV]	1.0/0.028	95/31	13
500×400 [GeV^2]	R_S [%]/R_{BG} [%]	N_S/N_{BG}	S_{cL}
contain $\tau_h^+\tau_h^-e^-$	0.91/1.4	97/2100	2.1
$m_{\tau_h^-\tau_h^+} > 290$ [GeV]	0.68/0.12	72/170	5.2
25 [GeV] $< m_{e^-\tau_h^+} <$ 70 [GeV]	0.67/0.0036	72/5.2	17

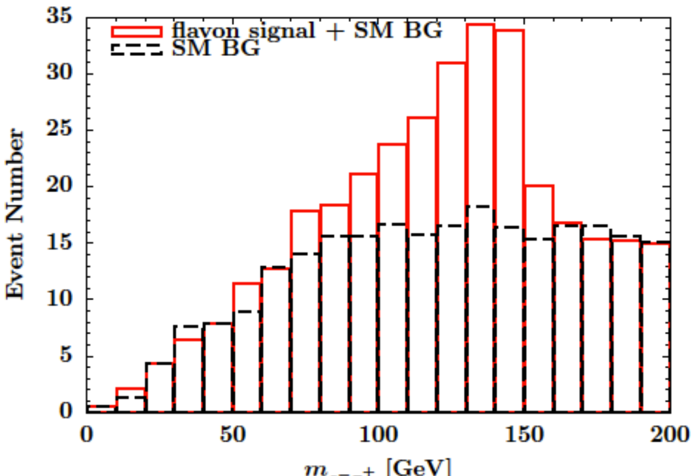
✓ R is ratio between event after cut and before cut

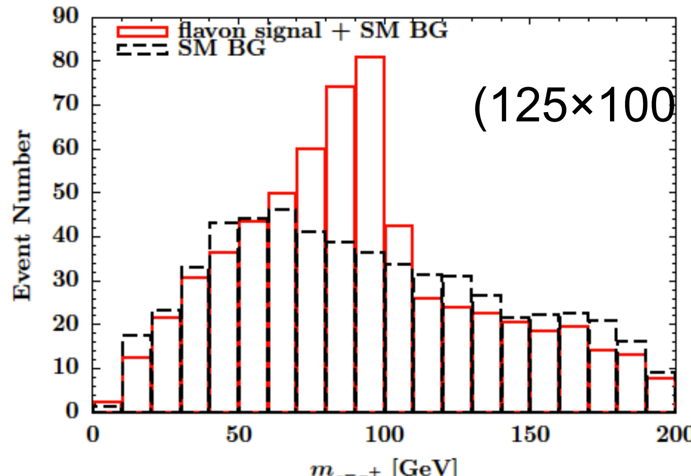
$$S_{cl} = \sqrt{2((N_S + N_{BG})\ln(1 + N_S/N_{BG}) - N_S)},$$

IM distributions for e⁻ τ









(125×100 GeV²)

(250×200 GeV²) $m_{\varphi_{T2}} = m_{\varphi_{T3}} = 150\ GeV$

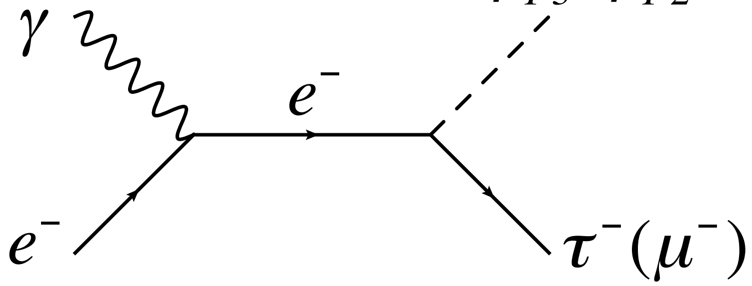
$m_{\varphi_{T2}} = m_{\varphi_{T3}} = 100\ GeV$

We can see clear peak around flavon mass

◻ Simulation study for flavor violating final states

Consider the processes:

$$\varphi_{T3}(\varphi_{T2}) \to (\tau^- \mu^+)\tau^+ \mu^-$$



$\tau^+\mu^-\mu^-$

$\mu^+\tau^-\tau^-$

final states

❖ **The cross sections for the other processes are small**

Flavor violating process → Almost no SM background

Cross sections for $e+\gamma \to \tau^+\mu^-\mu^-$

$$v_T = 2m_{\varphi_{T1}} = m_{\varphi_{T2}} = m_{\varphi_{T3}} = 65\,GeV$$

Beam energy [GeV²]	500×400	250×200	125×100
Cross section [fb]	7.9×10^{-3}	3.1×10^{-2}	1.1×10^{-1}

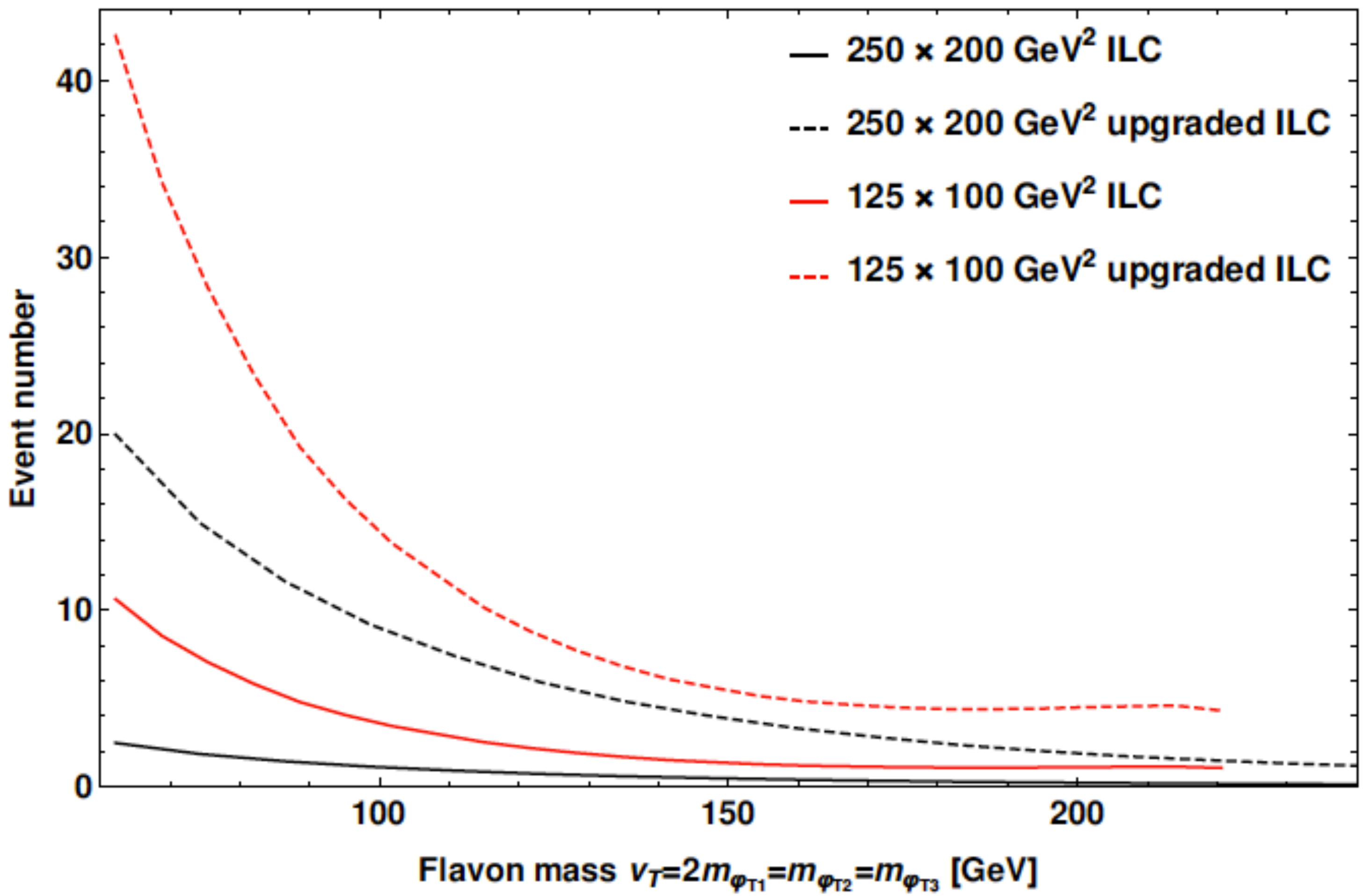
Small cross section but testable due to small BG

Counting number of events

	Cross section [fb]	Final state	Event rate [%]	Event number at 300(300) [fb^{-1}]	Event number at 2400(1200) [fb^{-1}]
$\tau^+\mu^-\mu^-$ mode	3.1×10^{-2} (1.1×10^{-1})	sum	19(23)	1.8(7.4)	14(30)
		$\tau_h^+\mu^-\mu^-$	6.5(8.9)	0.59(2.9)	4.8(12)
		$\mu^+\mu^-\mu^-$	6.5(7.2)	0.59(2.3)	4.7(9.3)
		$e^+\mu^-\mu^-$	6.5(6.8)	0.60(2.2)	4.8(8.8)
$\mu^+\tau^-\tau^-$ mode	3.1×10^{-2} (1.1×10^{-1})	sum	6.5(7.1)	0.59(2.3)	4.8(9.2)
		$\mu^+\tau_h^-\mu^-$	3.2(3.5)	0.29(1.1)	2.3(4.6)
		$\mu^+\tau_h^-\tau_h^-$	2.2(2.3)	0.20(0.76)	1.6(3.0)
		$\mu^+\mu^-\mu^-$	1.1(1.2)	0.10(0.40)	0.84(1.6)

- ✓ Summing events from hadronically and leptonically decaying τ modes
- ✓ S_{cL}=3 can be obtained with 4 events if BG event is 1
- ✓ Sufficient discovery significance can be realized

Counting number of events



Summary and Discussions

- Flavon in A4 model
  - ✓ A4 symmetry for neutrino mass structure
  - ✓ Remnant Z2 symmetry forbid some LFV process
  - ✓ Flavon mass can be as light as ~60 GeV
- Flavon production at colliders
  - ✓ Small cross section at the LHC
  - ✓ Electron-photon collision at the ILC has advantages
  - ✓ Flavor conserving and violating signals
  - ✓ O(100)~O(200) GeV flavon can be tested

Thanks for listening !

Appendix

Full scalar potential in the model

$$
\begin{aligned}
w &\equiv w_Y + w_d + h.c., \\
w_Y &\equiv w_\ell + w_D + w_N, \\
w_\ell &= y_e \phi_T l e^c_R h_d/\Lambda + y_\mu \phi_T l \mu^c_R h_d/\Lambda + y_\tau \phi_T l \tau^c_R h_d/\Lambda, \\
w_D &= y_D l \nu^c_R h_u, \\
w_N &= y_{\phi_S} \phi_S \nu^c_R \nu^c_R + (y_\xi \xi + y_{\tilde{\xi}} \tilde{\xi}) \nu^c_R \nu^c_R + (y_{\xi'} \xi' + y_{\tilde{\xi}'} \tilde{\xi}') \nu^c_R \nu^c_R, \\
w_d &\equiv w_d^T + w_d^S, \\
w_d^T &= -M \phi_0^T \phi_T + g \phi_0^T \phi_T \phi_T, \\
w_d^S &= g_1 \phi_0^S \phi_S \phi_S - g_2 \phi_0^S \phi_S \tilde{\xi} - g_2' \phi_0^S \phi_S \tilde{\xi}' \\
&\quad + g_3 \xi_0 \phi_S \phi_S - g_4 \xi_0 \xi^2 - g_5 \xi_0 \xi \tilde{\xi} - g_6 \xi_0 \tilde{\xi}^2 \\
&\quad + g_3' \xi_0' \phi_S \phi_S - g_4' \xi_0' \xi'^2 - g_5' \xi_0' \xi' \tilde{\xi}' - g_6' \xi_0' \tilde{\xi}'^2,
\end{aligned}
$$

Potential analysis

$$w_d^T = -M\left(\phi_{01}^T\phi_{T1} + \phi_{02}^T\phi_{T3} + \phi_{03}^T\phi_{T2}\right)$$
$$+ \frac{2g}{3}\left[\phi_{01}^T\left(\phi_{T1}^2 - \phi_{T2}\phi_{T3}\right) + \phi_{02}^T\left(\phi_{T2}^2 - \phi_{T1}\phi_{T3}\right) + \phi_{03}^T\left(\phi_{T3}^2 - \phi_{T1}\phi_{T2}\right)\right]$$

$$w_d^S = \frac{2g_1}{3}\left[\phi_{01}^S\left(\phi_{S1}^2 - \phi_{S2}\phi_{S3}\right) + \phi_{02}^S\left(\phi_{S2}^2 - \phi_{S1}\phi_{S3}\right) + \phi_{03}^S\left(\phi_{S3}^2 - \phi_{S1}\phi_{S2}\right)\right]$$
$$- g_2\left(\phi_{01}^S\phi_{S1} + \phi_{02}^S\phi_{S3} + \phi_{03}^S\phi_{S2}\right)\tilde{\xi} - g_2'\left(\phi_{01}^S\phi_{S3} + \phi_{02}^S\phi_{S2} + \phi_{03}^S\phi_{S1}\right)\tilde{\xi}'$$
$$+ g_3\xi_0\left(\phi_{S1}^2 + 2\phi_{S2}\phi_{S3}\right) - g_4\xi_0\xi^2 - g_5\xi_0\xi\tilde{\xi} - g_6\xi_0\tilde{\xi}^2$$
$$+ g_3'\xi_0'\left(\phi_{S2}^2 + 2\phi_{S1}\phi_{S3}\right) - g_4'\xi_0'\xi'^2 - g_5'\xi_0'\xi'\tilde{\xi}' - g_6'\xi_0'\tilde{\xi}'^2.$$

$$\begin{pmatrix} \phi_T = (\phi_{T1}, \phi_{T2}, \phi_{T3}), \quad \phi_S = (\phi_{S1}, \phi_{S2}, \phi_{S3}), \\ \phi_0^T = (\phi_{01}^T, \phi_{02}^T, \phi_{03}^T), \quad \phi_0^S = (\phi_{01}^S, \phi_{02}^S, \phi_{03}^S). \end{pmatrix}$$

Potential analysis

$$V \equiv V_T + V_S \ ,$$

$$V_T = \sum_i \left| \frac{\partial w_d^T}{\partial \phi_{0i}^T} \right|^2 + h.c.$$

$$= 2\left| -M\phi_{T1} + \frac{2g}{3}\left(\phi_{T1}^2 - \phi_{T2}\phi_{T3}\right) \right|^2 + 2\left| -M\phi_{T3} + \frac{2g}{3}\left(\phi_{T2}^2 - \phi_{T1}\phi_{T3}\right) \right|^2$$

$$+ 2\left| -M\phi_{T2} + \frac{2g}{3}\left(\phi_{T3}^2 - \phi_{T1}\phi_{T2}\right) \right|^2 ,$$

$$V_S = \sum_i \left| \frac{\partial w_d^S}{\partial X_i} \right|^2 + h.c.$$

$$= 2\left| \frac{2g_1}{3}\left(\phi_{S1}^2 - \phi_{S2}\phi_{S3}\right) - g_2\phi_{S1}\tilde{\xi} - g_2'\phi_{S3}\tilde{\xi}' \right|^2 + 2\left| \frac{2g_1}{3}\left(\phi_{S2}^2 - \phi_{S1}\phi_{S3}\right) - g_2\phi_{S3}\tilde{\xi} - g_2'\phi_{S2}\tilde{\xi}' \right|^2$$

$$+ 2\left| \frac{2g_1}{3}\left(\phi_{S3}^2 - \phi_{S1}\phi_{S2}\right) - g_2\phi_{S2}\tilde{\xi} - g_2'\phi_{S1}\tilde{\xi}' \right|^2$$

$$+ 2\left| g_3\left(\phi_{S1}^2 + 2\phi_{S2}\phi_{S3}\right) - g_4\xi^2 - g_5\xi\tilde{\xi} - g_6\tilde{\xi}^2 \right|^2$$

$$+ 2\left| g_3'\left(\phi_{S2}^2 + 2\phi_{S1}\phi_{S3}\right) - g_4'\xi'^2 - g_5'\xi'\tilde{\xi}' - g_6'\tilde{\xi}'^2 \right|^2 , \tag{4}$$

VEV alignment is given by conditions $V_T = 0$, $V_S = 0$

BR of LFV three body decays

$$\begin{aligned} \mathrm{BR}\,(\tau \to \mu\mu\bar{e}) &= \tau_\tau \frac{m_\tau^5}{3072\pi^3}\left(\left|\frac{m_\tau m_\mu}{v_T^2 m_{\varphi T2}^2}\right|^2 + \left|\frac{m_\mu m_e}{v_T^2 m_{\varphi T3}^2}\right|^2\right) \\ &\simeq \frac{2.9\times 10^6\ \mathrm{GeV}^8}{v_T^4(2\sqrt{2}M)^4}\,, \end{aligned}$$

$$\begin{aligned} \mathrm{BR}\,(\tau \to ee\bar{\mu}) &= \tau_\tau \frac{m_\tau^5}{3072\pi^3}\left(\left|\frac{m_\mu m_e}{v_T^2 m_{\varphi T2}^2}\right|^2 + \left|\frac{m_\tau m_e}{v_T^2 m_{\varphi T3}^2}\right|^2\right) \\ &\simeq \frac{68\ \mathrm{GeV}^8}{v_T^4(2\sqrt{2}M)^4}\,, \end{aligned}$$

Multiplication rule of A_4 group

$$
\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} = (a_1b_1 + a_2b_3 + a_3b_2)_{\mathbf{1}} \oplus (a_3b_3 + a_1b_2 + a_2b_1)_{\mathbf{1'}}
$$

$$
\oplus (a_2b_2 + a_1b_3 + a_3b_1)_{\mathbf{1''}}
$$

$$
\oplus \frac{1}{3}\begin{pmatrix} 2a_1b_1 - a_2b_3 - a_3b_2 \\ 2a_3b_3 - a_1b_2 - a_2b_1 \\ 2a_2b_2 - a_1b_3 - a_3b_1 \end{pmatrix}_{\mathbf{3}} \oplus \frac{1}{2}\begin{pmatrix} a_2b_3 - a_3b_2 \\ a_1b_2 - a_2b_1 \\ a_1b_3 - a_3b_1 \end{pmatrix}_{\mathbf{3}} .
$$