Top electroweak couplings study using di-muonic state at $\sqrt{s} = 500$ GeV, ILC with the Matrix Element Method

Yo Sato

Current status

Assessment of Matrix element method

Update the kinematical reconstruction

New χ^2 for assessment of goodness of fit

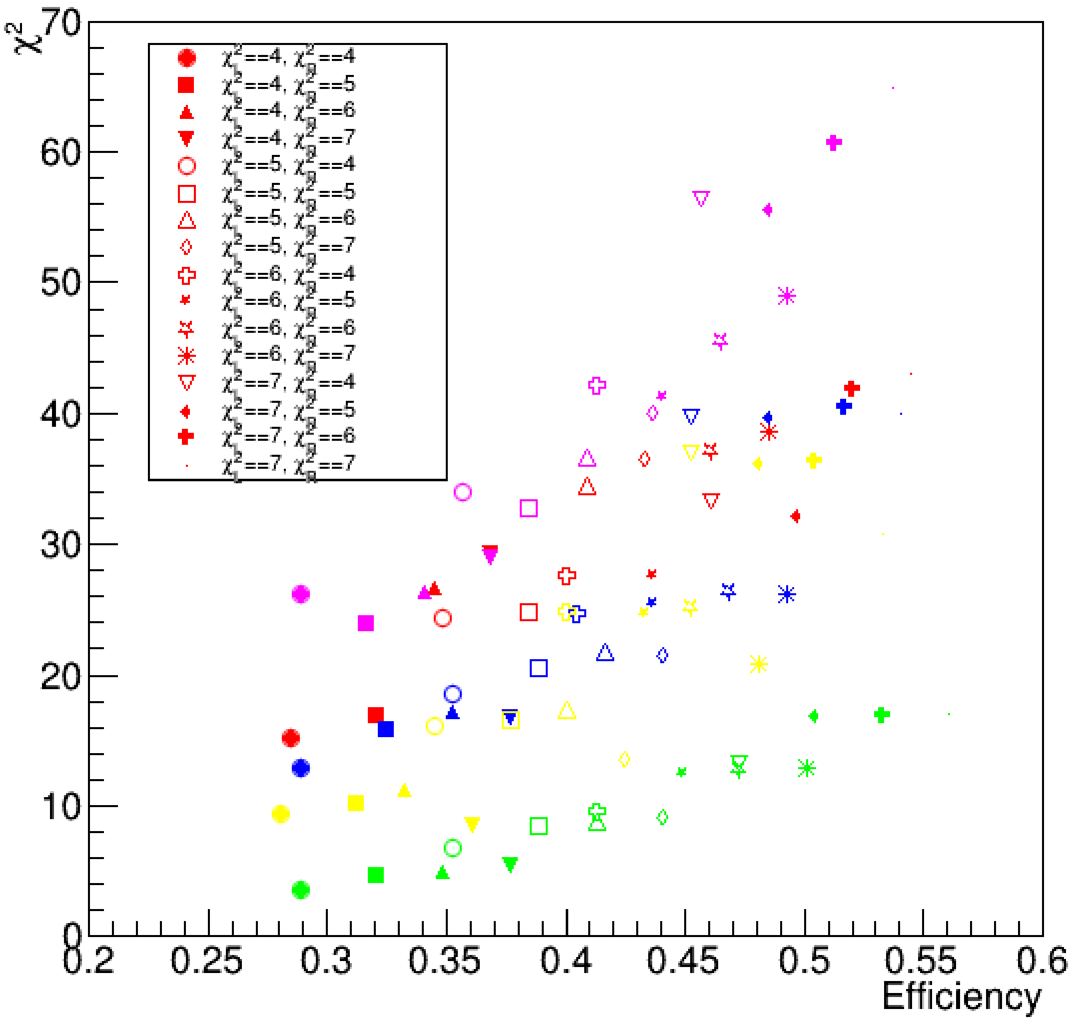
Assessment of MEM

We found that the fluctuation of CL or χ^2_{test} is too large when other samples of events are used. $\chi^2_{\text{test}} = \sum \delta F_i V_{ij}^{-1} \delta F_j$

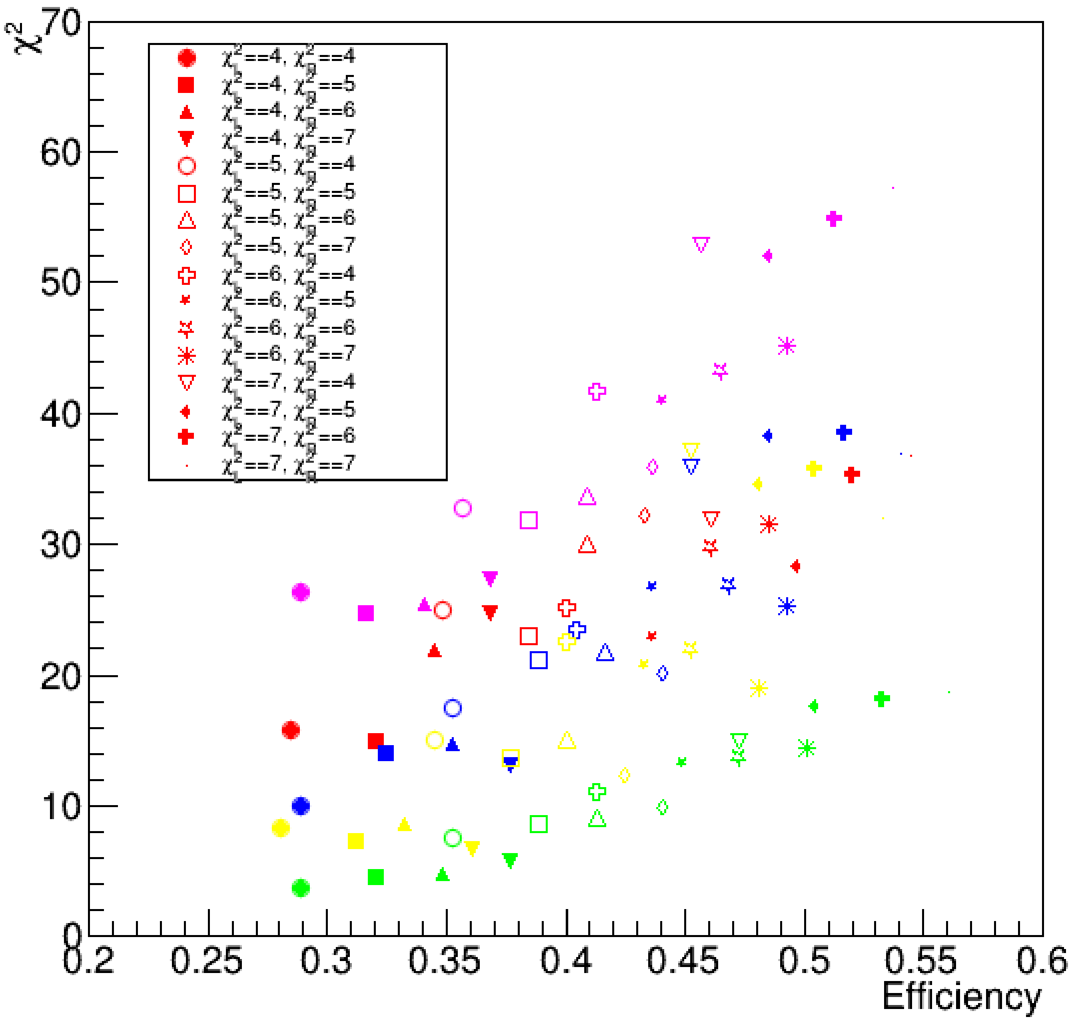
→ **Return back step by step**

- without background
- without background and miss-assignment events
- without background, miss-assignment events and ISR/BS
- compare with MC truth at each step

with bkg (last time)

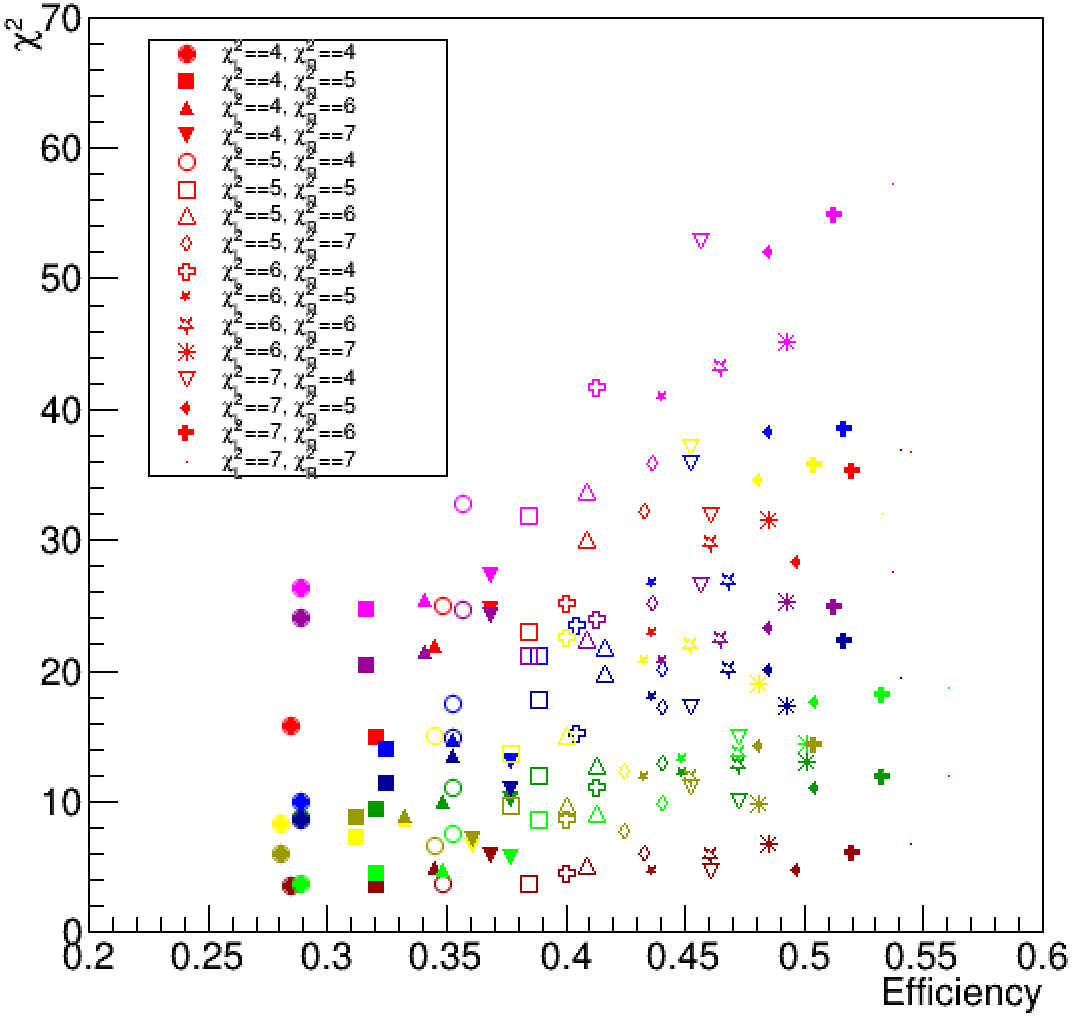


without bkg

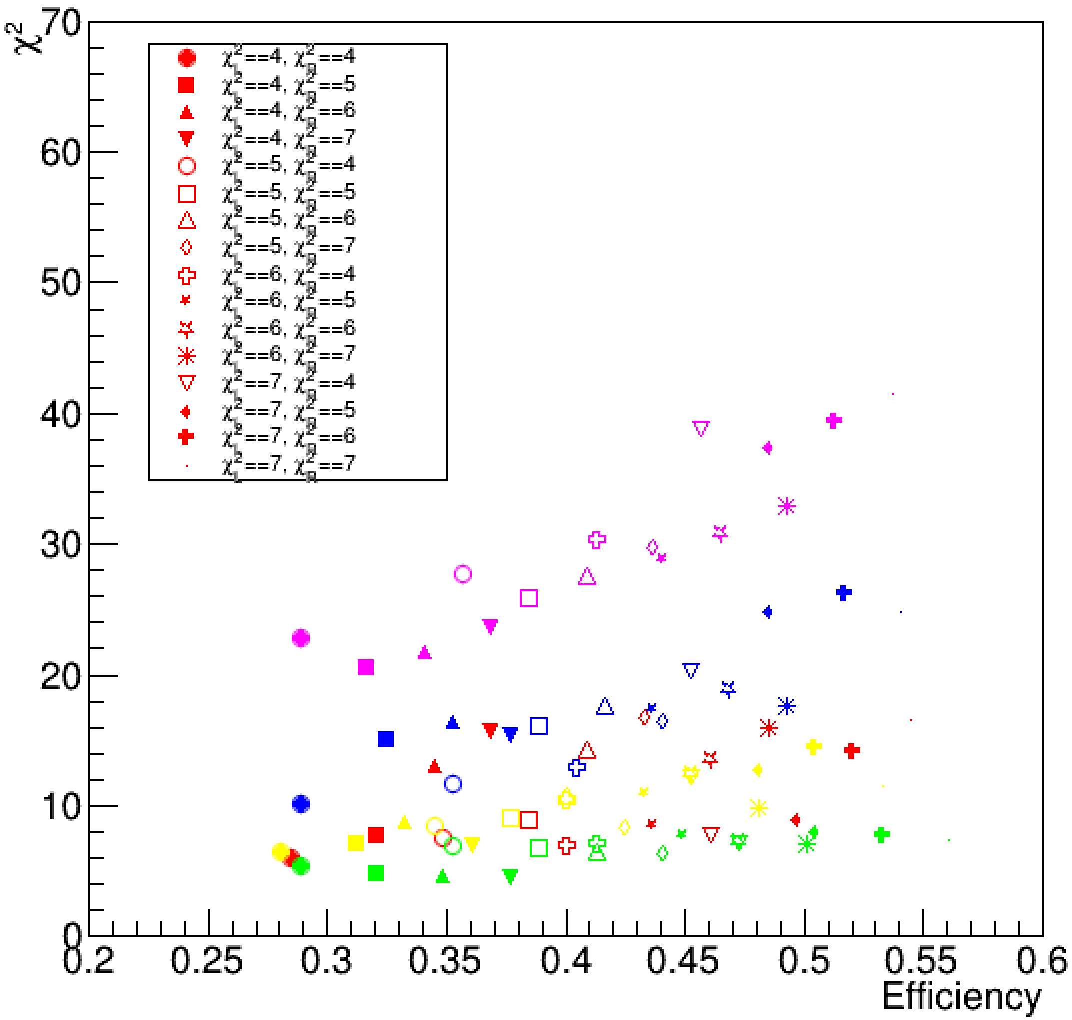


χ^2
Efficiency
0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6
0 10 20 30 40 50 60 70
χ_1^2==4, χ_π^2==4
χ_1^2==4, χ_π^2==5
χ_1^2==4, χ_π^2==6
χ_1^2==4, χ_π^2==7
χ_1^2==5, χ_π^2==4
χ_1^2==5, χ_π^2==5
χ_1^2==5, χ_π^2==6
χ_1^2==5, χ_π^2==7
χ_1^2==6, χ_π^2==4
χ_1^2==6, χ_π^2==5
χ_1^2==6, χ_π^2==6
χ_1^2==6, χ_π^2==7
χ_1^2==7, χ_π^2==4
χ_1^2==7, χ_π^2==5
χ_1^2==7, χ_π^2==6
χ_1^2==7, χ_π^2==7

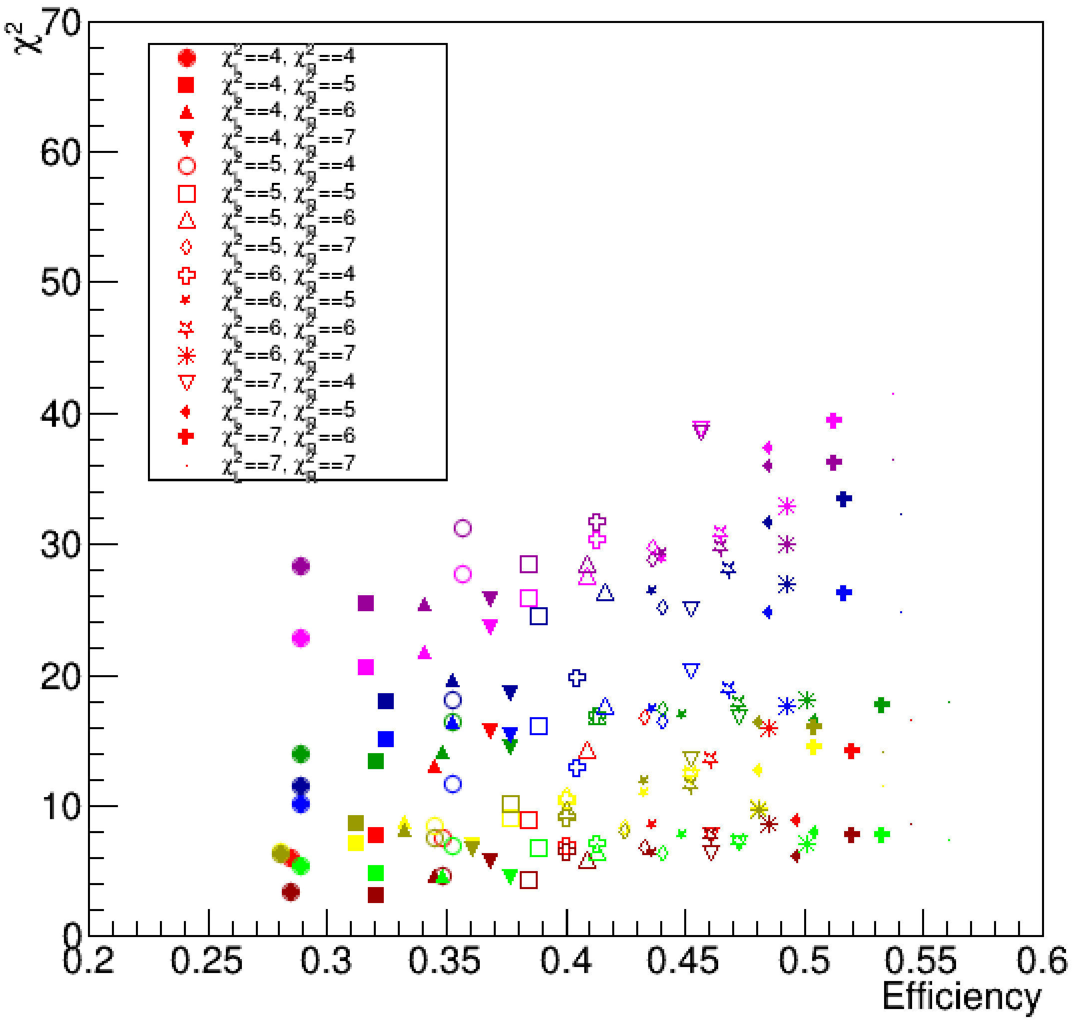
without bkg (with MC, dark color)



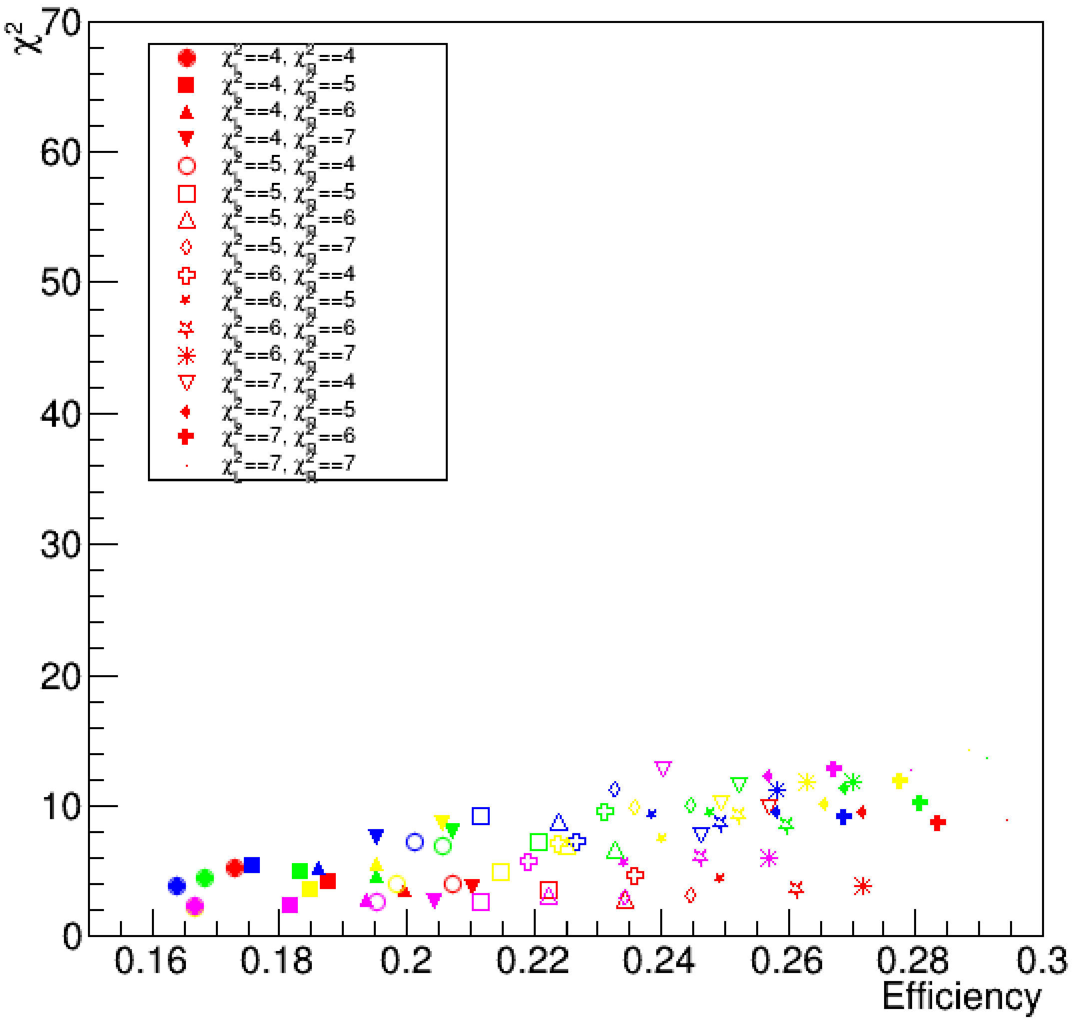
without bkg and miss-assignment



without bkg and miss-assignment (with MC)



without bkg, miss-assignment and ISR

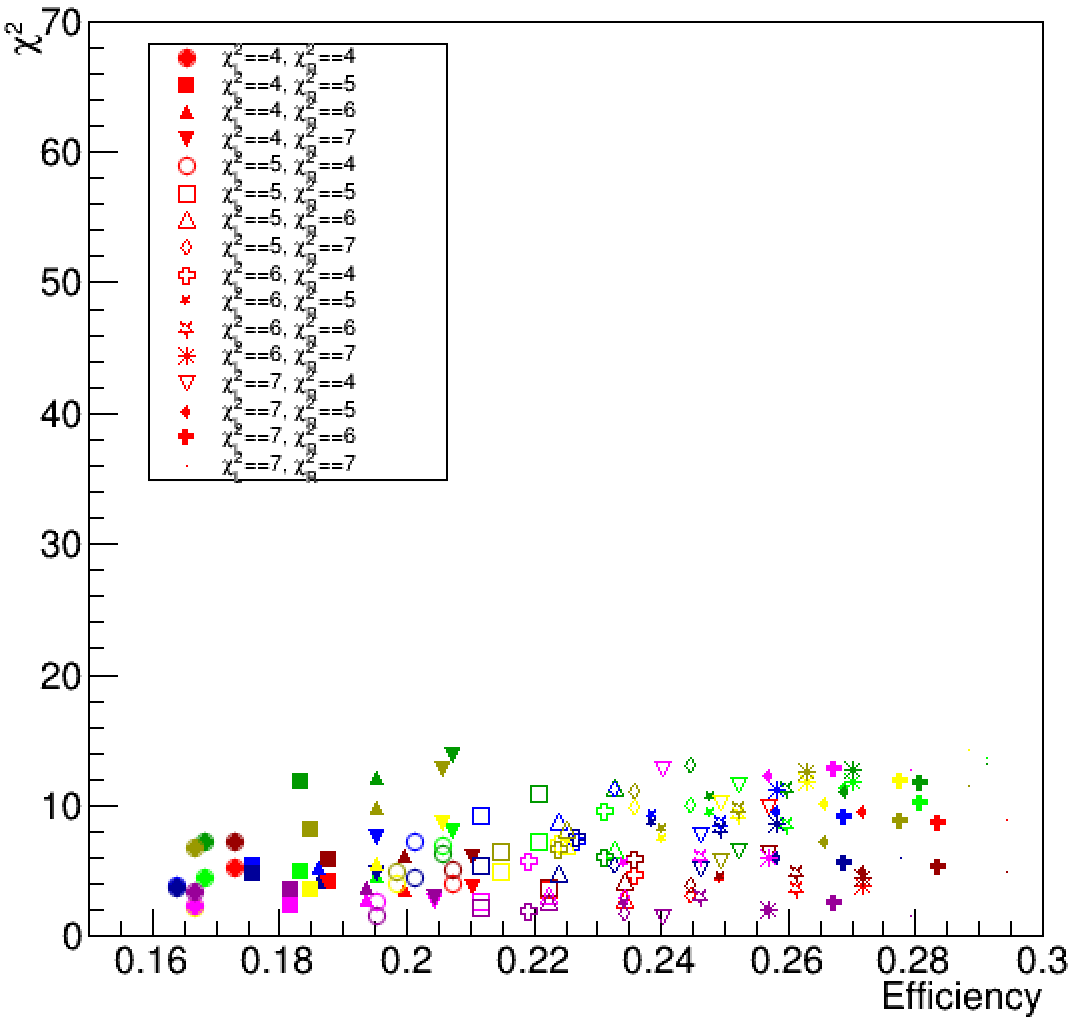


※ Cut on k's is applied as follows

k_electron < 0.03
and
k_positron < 0.03

This criteria is selected to keep about half amount of events.
→ Efficiency is twice smaller than last slides

without bkg, miss-assignment and ISR (with MC)



※ Cut on k's is applied as follows

k_electron < 0.03
and
k_positron < 0.03

This criteria is selected to keep about half amount of events.
→ Efficiency is twice smaller than last slides

Comments

- with bkg. (p.3) <-> without bkg. (p.4)

difference is small → bkg. is not critical effect.

- at without bkg., Rec. (p.4) <-> MC (p.5)

difference is large → miss-assignment has large effect.

- at without bkg. and miss, Rec. (p.6) <-> MC (p.7)

difference is small → detector effect is small

- without bkg. , miss and ISR (p.9)

chi2 is small → ISR has significant effect on the bias.

Kinematic fit : Strategy

Neutrinos and photon of ISR cannot be reconstructed by detectors, because

- Neutrino can't interact by electromagnetism
- Photon of ISR mostly enters the beam pipe.

There are 7 unknowns in di-muonic state of top pair production.

$$P_{x,\nu}, P_{y,\nu}, P_{z,\nu}, P_{x,\overline{\nu}}, P_{y,\overline{\nu}}, P_{z,\overline{\nu}}, P_{z,\gamma_{\mathrm{ISR}}}$$

To recover them, we impose 8 constraints,

- Initial state constraints : $E_{total} = 500\ \mathrm{GeV}, \vec{P}_{\mathrm{total}} = \vec{0}$
- Mass constraints : $m_t = m_{\bar{t}} = 174\ \mathrm{GeV}, m_{W^+} = m_{W^-} = 80.4\ \mathrm{GeV}$

There are enough constraints to determine the missing variables, in principle.

Kinematic fit : Algorithm

Introduce 4 free parameters

$$P_{x,\nu}, P_{y,\nu}, P_{z,\nu}, P_{z,\gamma_{\mathrm{ISR}}}$$

Other missing variables are defined as follows;

$$P_{x,\bar{\nu}} = -\left(P_{x,\mathrm{Visible}} + P_{x,\nu}\right), P_{y,\bar{\nu}} = -\left(P_{y,\mathrm{Visible}} + P_{y,\nu}\right), P_{z,\bar{\nu}} = -\left(P_{z,\mathrm{Visible}} + P_{z,\nu} + P_{z,\gamma_{\mathrm{ISR}}}\right)$$

All physics variables also can be computed using these parameters.

Define the likelihood function;

$$L_0 = BW(m_t, 174)BW(m_{\bar{t}}, 174)BW(m_{W^+}, 80.4)BW(m_{W^-}, 80.4)Gaus(E_{total}, 500)$$

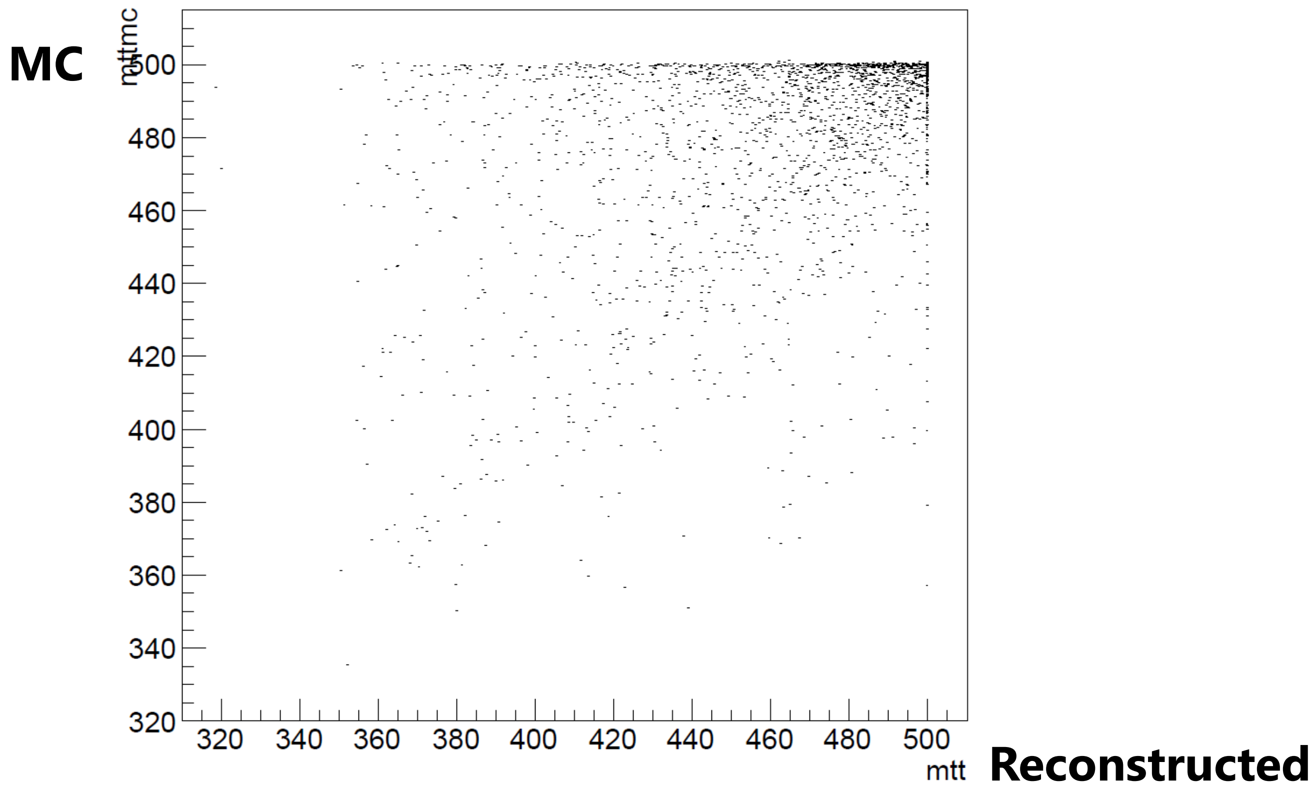
(BW : Breit-Wigner function, $Gaus$: Gaussian function, other parameters are written in backup)

To correct the energy resolution of b-jets reconstruction, we add 2 parameters, $E_b, E_{\bar{b}}$, and resolution functions, R, to the likelihood function.

$$L = L_0 * R\left(E_b, E_b^{\mathrm{reconstructed}}\right)R\left(E_{\bar{b}}, E_{\bar{b}}^{\mathrm{reconstructed}}\right)$$

For simplicity, we define $q^2 = -2\log L$.

Scatter plot of $m_{t\bar{t}}$ (MC vs Rec.)



It seems that there is correlation. This is better than previous algorithm (but not so great). It can be useful to reject events which have hard ISR photons.

New χ^2 for assessment of goodness of fit

Fujii-san suggested that previous definition of χ^2_{test} can't assess the goodness of fit. $\chi^2_{\text{test}} = \sum \delta F_i V_{ij}^{-1} \delta F_j$

We are trying to use another χ^2 (definition is on backup slides). But it's complicated and I don't understand the meaning of that yet...

If you have any suggestions or comments, please let me know !

Plan

- Use new algorithm of the kinematical reconstruction for samples of all events including background.
- Assess the goodness of fit using new χ^2.

Backup

8 Moments and Goodness of fit

The rLO full PDF is:

$$f_{\rm rLO}(\vec{r}) = (1 + \sum_i r_i\omega_i + \sum_{ij} r_i r_j \tilde{\omega}_{ij})\ f_{\rm SM}(\omega,\tilde{\omega}) \tag{19}$$

Provided the above PDF describes correctly the data, the expected values of the ω_i and $\tilde{\omega}_{ij}$ variables, as computed using the events at hand, must satisfy:

$$\langle\omega_k\rangle_{\rm rLO}(r_i) = \Omega_k + \sum_i r_i\Omega_{ik} + \sum_{ij} r_i r_j \tilde{\Omega}_{ij;k} \tag{20}$$

$$\langle\tilde{\omega}_{kl}\rangle_{\rm rLO}(r_i) = \tilde{\Omega}_{kl} + \sum_i r_i\tilde{\Omega}_{kl;i} + \sum_{ij} r_i r_j \tilde{\Omega}_{ijkl} \tag{21}$$

where the Ω's coefficients are computed through Monte Carlo integration using the SM PDF, as:

$$\Omega_k = \int \omega_k\ f_{\rm SM}(\omega,\tilde{\omega}) \tag{22}$$

$$\Omega_{ik} = \int \omega_i\omega_k\ f_{\rm SM}(\omega,\tilde{\omega}) \tag{23}$$

$$\tilde{\Omega}_{ij;k} = \int \tilde{\omega}_{ij}\omega_k\ f_{\rm SM}(\omega,\tilde{\omega}) \tag{24}$$

$$\tilde{\Omega}_{kl} = \int \tilde{\omega}_{kl}\ f_{\rm SM}(\omega,\tilde{\omega}) \tag{25}$$

$$\tilde{\Omega}_{ijkl} = \int \tilde{\omega}_{ij}\tilde{\omega}_{kl}\ f_{\rm SM}(\omega,\tilde{\omega}) \tag{26}$$

It has been checked that if the actual moments computed with Data were to obey these sum-rules (and higher moments sum-rules) then the derivatives of the χ^2 indeed identically vanish, for all r_i. But the converse is not true. If Data are not rLO-Data then the sum-rules are not satisfied. Therefore, these sum-rules provide a means to assess the quality of the fit. A series of tests of goodness of fit can be carried using[23]:

$$\chi^2_k(r_i) = \frac{(\langle\omega_k\rangle - \langle\omega_k\rangle_{\rm rLO}(r_i))^2}{\langle\omega^2_k\rangle - \langle\omega_k\rangle^2} \tag{27}$$

$$\tilde{\chi}^2_{kl}(r_i) = \frac{(\langle\tilde{\omega}_{kl}\rangle - \langle\tilde{\omega}_{kl}\rangle_{\rm rLO}(r_i))^2}{\langle\tilde{\omega}^2_{kl}\rangle - \langle\tilde{\omega}_{kl}\rangle^2} \tag{28}$$