Top electroweak couplings study using di-leptonic state at $\sqrt{s}$ = 500 GeV, ILC with the Matrix Element Method

ILD meeting

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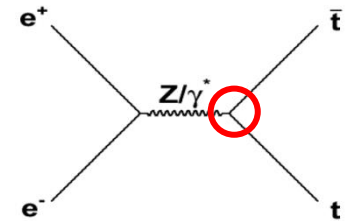
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Introduction

- ☐ Top quark is the heaviest particle in the SM. Its large mass implies that it is strongly coupled to the mechanism of electroweak symmetry breaking (EWSB)

→ **Top EW couplings are good probes for New physics behind EWSB**

$$\mathcal{L}_{\text{int}} = \sum_{v=\gamma,Z} g^v \left[V_l^v \bar{t}\gamma^l (F_{1V}^v + F_{1A}^v \gamma_5)t + \frac{i}{2m_t}\partial_\nu V_l \bar{t}\sigma^{l\nu}(F_{2V}^v + F_{2A}^v \gamma_5)t \right]$$

- ☐ Di-leptonic state of top-pair production have rich observables, so one can get higher intrinsic sensitivity and do multi-parameters fit.

→ **Out target is di-leptonic state**

- ☐ Use the Matrix Element method to handle many observables and many parameters simultaneously.
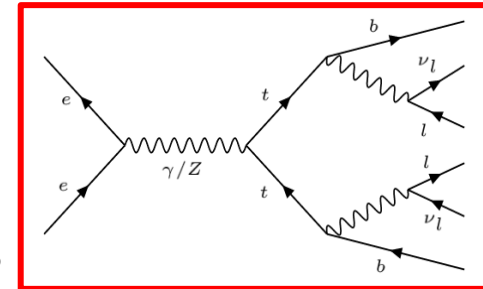
Set Up

|	
$\sqrt{s}$	500 GeV
Polarization (P_{e^-}, P_{e^+})	(-0.8, +0.3) ”Left” / (+0.8, -0.3) ”Right”
Integrated luminosity	500 fb^{-1} (250 fb^{-1} for each polarization)
Generator	Whizard (including ISR/BS, $\gamma\gamma \to$ hadrons)
Detector model	ILD_01_v05 (TDR version)

Sample of events

- Signal : $ee \rightarrow bb\mu\mu\nu\nu$

  - We focus on only the **di-muonic state** which is the most accurate to be reconstructed in the di-leptonic state.

  - This includes **top pair production**, single top production and so on.

- Main background

  - $ee \rightarrow bbll\nu\nu$ (except for $bb\mu\mu\nu\nu$)

  - $ee \rightarrow qqll$ (mainly ZZ)

  - $ee \rightarrow bbl\nu qq$ (mainly top pair production)
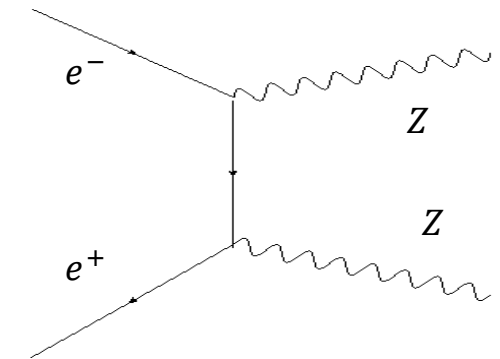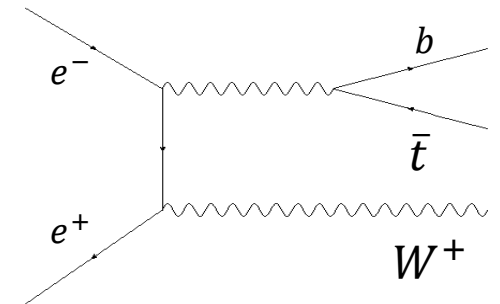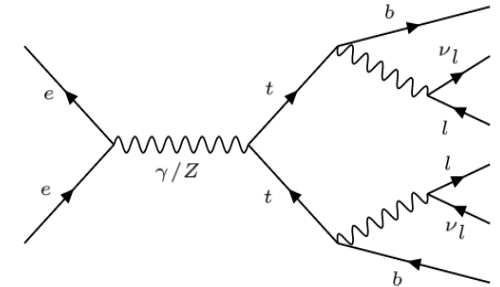
Process of study

Event Reconstruction

- ➢ Isolated muon reconstruction

- ➢ $\gamma\gamma \to$ hadrons suppression

- ➢ b-jet reconstruction

- ➢ Kinematical reconstruction

Analysis and Discussion

- ➢ Helicity angles computation

- ➢ Optimal variables computation

- ➢ Fit form factors with MEM

- ➢ Assessment of goodness of fit

Kinematical Reconstruction : Strategy

Neutrinos and photon of ISR cannot be reconstructed by detectors, because

- Neutrino can't interact by electromagnetism

- Photon of ISR mostly enters the beam pipe.

There are **7 unknowns** in di-muonic state of top pair production.

$$P_{x,\nu}, P_{y,\nu}, P_{z,\nu}, P_{x,\overline{\nu}}, P_{y,\overline{\nu}}, P_{z,\overline{\nu}}, P_{z,\gamma_{\mathrm{ISR}}}$$

To recover them, we impose **8 constraints**,

- Initial state constraints : $E_{total} = 500 \text{ GeV}, \vec{P}_{\mathrm{total}} = \vec{0}$

- Mass constraints : $m_t = m_{\overline{t}} = 174 \text{ GeV}, m_{W^+} = m_{W^-} = 80.4 \text{ GeV}$

There are enough constraints to determine the missing variables.

Kinematical Reconstruction : Algorithm

Introduce **4 free parameters** : $P_{x,\nu}, P_{y,\nu}, P_{z,\nu}, P_{z,\gamma_{ISR}}$

Other missing variables are defined as follows;

$$P_{x,\bar{\nu}} = -\left(P_{x,\text{Visible}} + P_{x,\nu}\right), P_{y,\bar{\nu}} = -\left(P_{y,\text{Visible}} + P_{y,\nu}\right), P_{z,\bar{\nu}} = -\left(P_{z,\text{Visible}} + P_{z,\nu} + P_{z,\gamma_{ISR}}\right)$$

(All physics variables also can be computed using these parameters.)

Define the likelihood function;

$$L_0 = BW(m_t, 174)BW(m_{\bar{t}}, 174)BW(m_{W^+}, 80.4)BW(m_{W^-}, 80.4)Gaus(E_{total}, 500)$$

(BW : Breit-Wigner function, $Gaus$: Gaussian function, other parameters are written in backup)

To correct the energy resolution of b-jets reconstruction, we add **2 parameters,**

$E_b, E_{\bar{b}}$, and resolution functions, R, to the likelihood function.

$$L = L_0 * R\left(E_b, E_b^{reconstructed}\right)R\left(E_{\bar{b}}, E_{\bar{b}}^{reconstructed}\right)$$

Kinematical Reconstruction

For simplicity, we define $q = -2 \log L + C$ (scaled as the minimum value becomes 0)

There are two possibilities for combination of b-jet and muon.

→ Define ***the best candidate*** as a candidate having **smaller** q and q_{min} as q of the best candidate. One can check that it is true or miss combination by generator information.
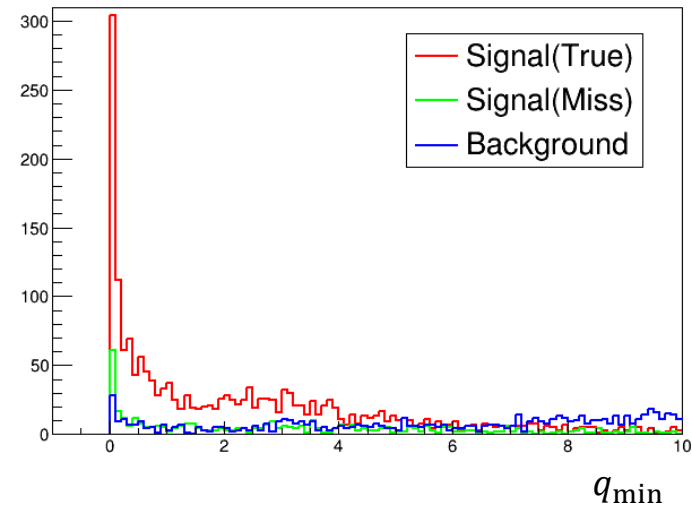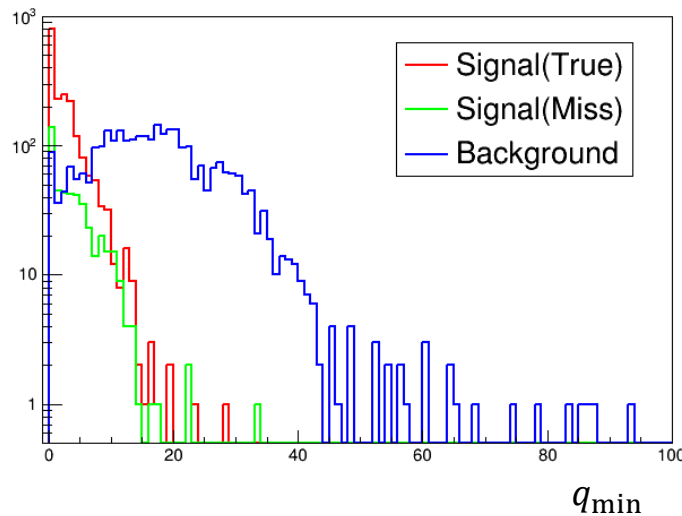
q_{min} distribution of Left polarization events (left : whole distribution, right : zoomed one)

→ *Cut on q_{min} is useful to reduce the background and miss combination events.*

ISR photon
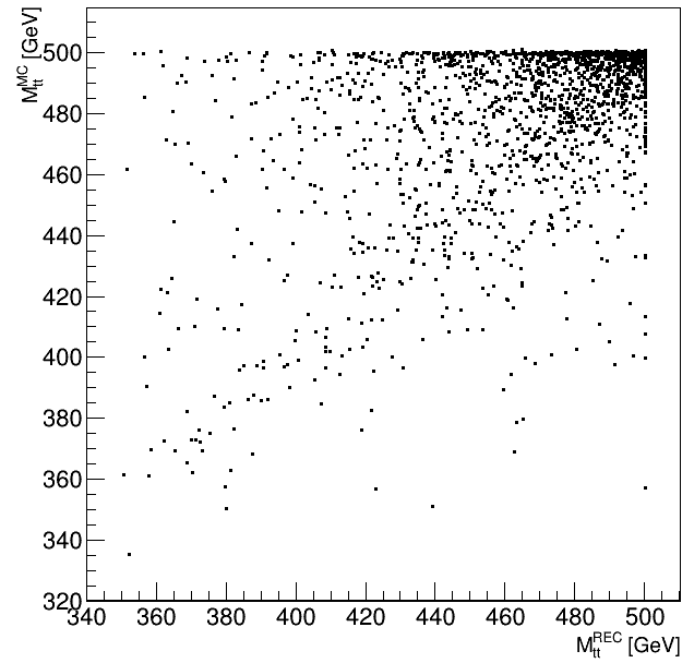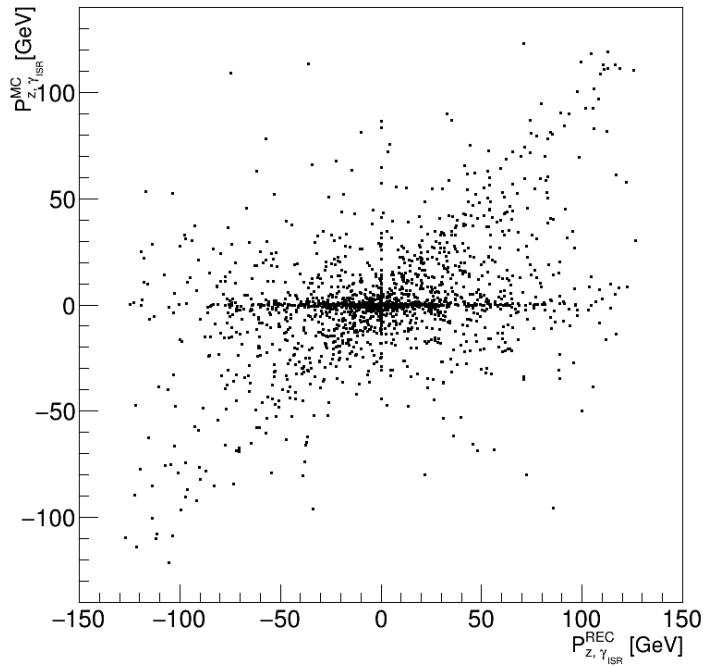
Scatter plots between MC and reconstructed of Left polarization and signal events

(left : $P_{z,\gamma_{ISR}}$, right : mass of top pair)

Small correlations between MC and reconstructed are observed.

→ *Cut on $P_{z,\gamma_{ISR}}$ or M_{tt} is useful to reduce hard ISR events.*

ISR photon
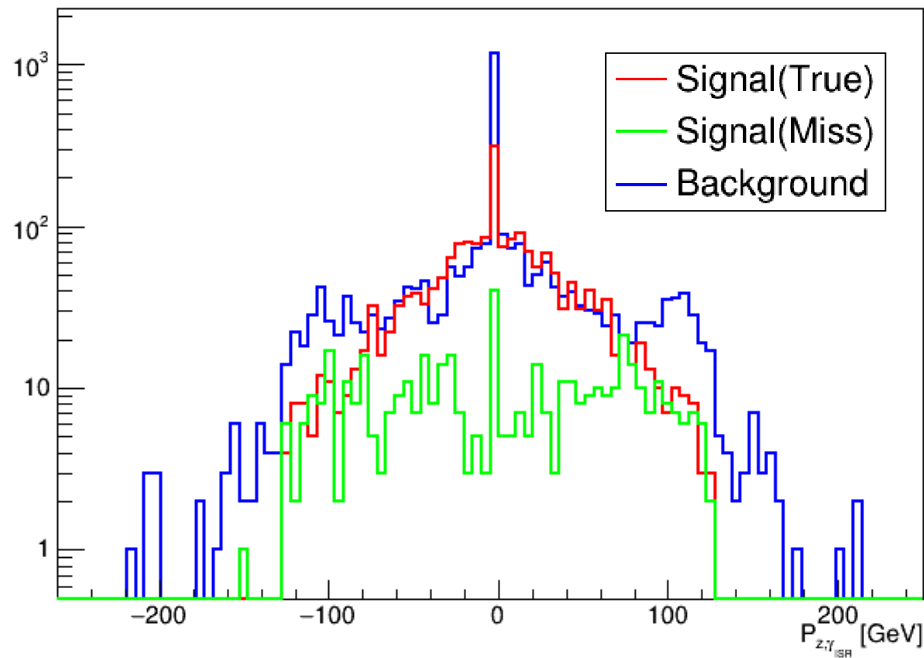
$P_{z,\gamma_{\rm ISR}}$ distribution of Left polarization events

$P_{z,\gamma_{\rm ISR}}$ distribution of miss combination events are wider than true combination

→ *Cut on $P_{z,\gamma_{ISR}}$ is also useful to reduce miss combination events*

Cut table

250 fb⁻¹ (-0.8,+0.3) Left	initial	$\mu^+\mu^-$	b-tag1>0.8 or b-tag2>0.8	$q_{min} < 3$ & $\lvert P_{z,\gamma}\rvert < 50$ GeV
Signal $bb\mu\mu\nu\nu$ **(True)**	2961	2725 (e = 92.0 %)	1921 (80.9%) — (e = 80.2%)	945 (90.7%) — (e = 35.2%)
Signal $bb\mu\mu\nu\nu$ **(Miss)**			453 (19.1%)	97 (9.3%)
$bbll\nu\nu$ **(except** $bb\mu\mu\nu\nu$**)**	23609	387	335	71
$bbl\nu qq$	104114	40	31	3
$qqll$ **(ZZ)**	91478	13800	2519	21
ll **(weight = 4)**	212274 (→ 849096)	74961 (→ 299844)	90 (→ 360)	0
$l\nu l\nu$ **(WW)** **(weight = 4)**	377058 (→ 1508232)	1884 (→ 7536)	3 (→ 12)	0
$lll\nu l\nu$ **(llWW)**	3021	947	19	0

Matrix Element Method

Matrix element method is based on the maximum likelihood method and a likelihood function is written by $|M|^2$ and N;

$$-2\log L(\delta F) = \chi^2(\delta F) =$$

$$-2\left(\sum_{e=1}^{N_{\text{event}}} \log\left(1 + \sum_i \omega_i(\Phi_e)\delta F_i + \sum_{ij} \widetilde{\omega}_{ij}(\Phi_e)\delta F_i \delta F_j\right)\right.$$

$$\left. - N_{\text{event}}\log\left(1 + \sum_i \Omega_i \delta F_i + \sum_{ij} \widetilde{\Omega}_{ij}\delta F_i \delta F_j\right)\right)$$

where Φ_e is helicity angles which have sensitivity for the form factors. $\chi^2(\delta F)$ is scaled to 0 at $\delta F = 0$.

If we use the information of yields with Poisson distribution, the second term can be replaced as $N_{\text{event}}\left(\sum_i \Omega_i \delta F_i^{\text{SM}} + \sum_{ij} \widetilde{\Omega}_{ij}\delta F_i^{\text{SM}}\delta F_j^{\text{SM}}\right)$

Matrix Element Method

What we must do to fit the form factors correctly is to reconstruct ω_i correctly.

Indeed the results of fit are related with ω_i and Ω_i which are called **optimal variables**

- $\delta F_i^{\text{Fit}} \simeq \frac{<\omega_i - \Omega_i>}{<\omega_i{}^2>}$

- Covariance matrix, V_{ij} : $V_{ij}^{-1} \simeq N_{\text{event}} < \omega_i \omega_j >$
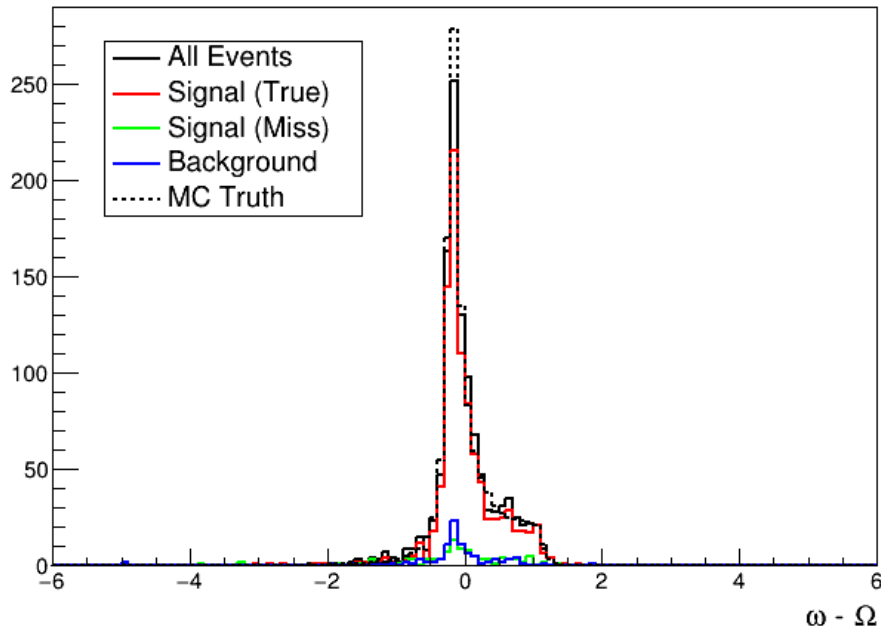
Reconstructed (All Events) are similar with MC Truth

$\omega - \Omega$ distribution for δF_{1V}^{Z} of Left polarization events

Outliers

A few events are distributed far from other events. It can be caused by detector effects and ISR effects, i.e. they are badly reconstructed events.
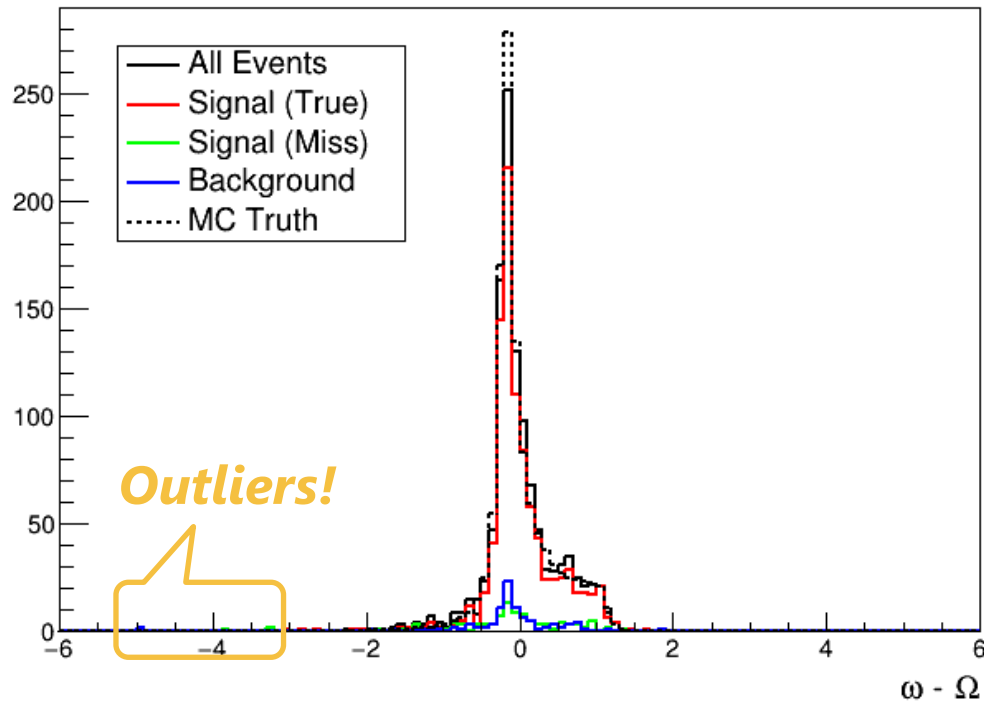
$\omega - \Omega$ distribution for δF_{1V}^{Z} of Left polarization events (same as last slide)

These events easily induce biases on results of fit. → **Outliers**

We fit $\omega - \Omega$ distribution within a region not including *outliers*. Efficiency cost is only 1.6%(0.8%) for Left polarization events.

Preliminary Results without Outliers

Results of 10 parameters multi-fit

$$
\begin{bmatrix}
\mathcal{Re}\ \delta\tilde{F}_{1V}^{\gamma} & -0.0148 \pm 0.0129 \\
\mathcal{Re}\ \delta\tilde{F}_{1V}^{Z} & +0.0232 \pm 0.0226 \\
\mathcal{Re}\ \delta\tilde{F}_{1A}^{\gamma} & +0.0140 \pm 0.0184 \\
\mathcal{Re}\ \delta\tilde{F}_{1A}^{Z} & +0.0309 \pm 0.0286 \\
\mathcal{Re}\ \delta\tilde{F}_{2V}^{\gamma} & -0.0736 \pm 0.0371 \\
\mathcal{Re}\ \delta\tilde{F}_{2V}^{Z} & +0.0564 \pm 0.0601 \\
\mathcal{Re}\ \delta\tilde{F}_{2A}^{\gamma} & -0.0059 \pm 0.0226 \\
\mathcal{Re}\ \delta\tilde{F}_{2A}^{Z} & -0.0377 \pm 0.0389 \\
\mathcal{Im}\ \delta\tilde{F}_{2A}^{\gamma} & +0.0403 \pm 0.0238 \\
\mathcal{Im}\ \delta\tilde{F}_{2A}^{Z} & +0.0007 \pm 0.0343
\end{bmatrix}
$$

Efficiency : 35 %

Confidence level : 35 % (It is discussed in backup slides)

Significance($N_S/\sqrt{N_S + N_B}$) : 37

This precision is comparable with semi-leptonic state analysis considering difference of statistics. ***One can fit more parameters simultaneously.***

But there are still small biases. We have room for improvement.

Summary

- ☐ Missing neutrinos and ISR/BS photon are reconstructed by kinematical reconstruction.

  - • $P_{z, \gamma_{ISR}}$ cannot be reconstructed precisely, but it is useful to reduce miss combination and hard ISR events.

- ☐ $\omega - \Omega$ distributions, which called optimal variables, can be reconstructed. One rejects outliers events and fit form factors.

→ ***Comparable results with semi-leptonic analysis. More parameters can be fitted simultaneously.***

- ☐ There are still small biases. (Goodness of fit is also not so great. It is discussed in backup slides)

→ *There is room for improvement* (e.g. take ISR effects into account for the fitting form factor with *template method*)

Backup

Top EW Couplings Study

☐ Top quark is the heaviest particle in the SM. Its large mass implies that it is strongly coupled to the mechanism of electroweak symmetry breaking (EWSB)

→ **Top EW couplings are good probes for New physics behind EWSB**

$$\mathcal{L}_{\text{int}} = \sum_{v=\gamma,Z} g^v \left[V_l^v \bar{t} \gamma^l (F_{1V}^v + F_{1A}^v \gamma_5) t + \frac{i}{2m_t} \partial_\nu V_l \bar{t} \sigma^{l\nu} (F_{2V}^v + F_{2A}^v \gamma_5) t \right]$$
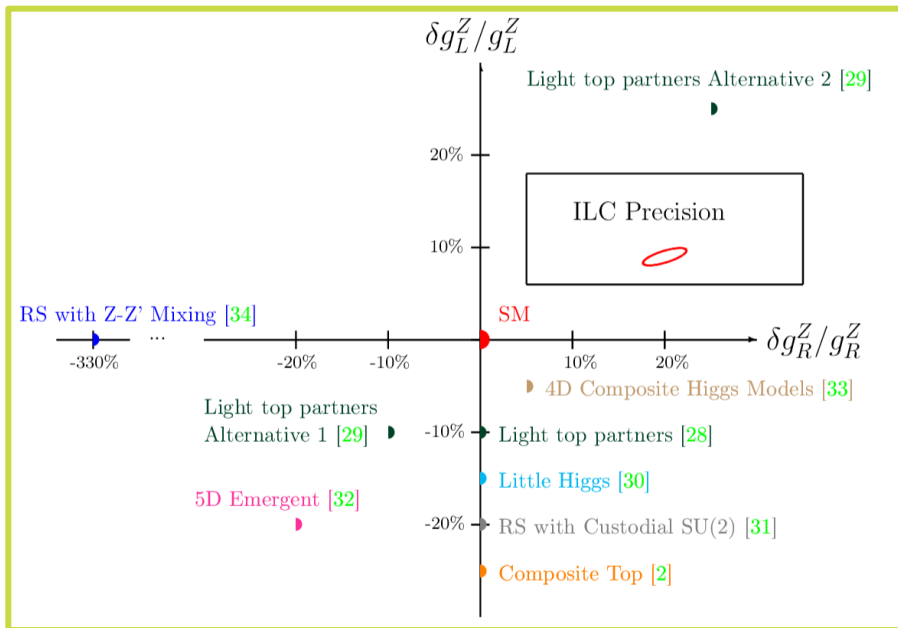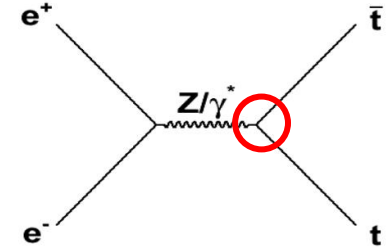
In new physics models, such as composite models, the predicted deviation of coupling constants, g_L^Z, g_R^Z $(= F_{1V}^Z \mp F_{1A}^Z)$ from SM is typically 10 %

Di-leptonic State of the top pair production

Top pair production has three different final states:

- Fully-hadronic state $\left(e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b}q\bar{q}q\bar{q}\right)$ 46.2 %

- Semi-leptonic state $\left(e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b}q\bar{q}l\nu\right)$ 43.5%

- **Di-leptonic state** $\left(e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b}l\nu l\nu\right)$ **10.3%**
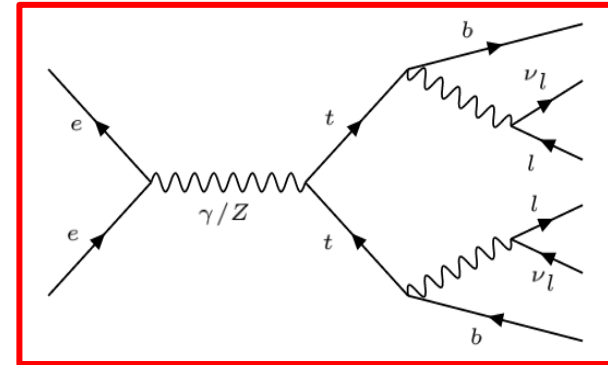
Advantage

- More observables be computed

→ Higher intrinsic sensitivity to the form factors, in principle.

Difficulty

- Two missing neutrinos

- Lower statistics : 6 times less events than the semi-leptonic state

  ((2/3 x 43.5 %) / (4/9 x 10.3 %) = ~ 6.3)

Pre-selection

The quality cut is necessary to reject the b-jet miss-assignment events when we don't use the b-charge reconstruction. The cut might be also effective to reject background events.

We use only two loose constraints, called **Pre-selection**, before the kinematical reconstruction of top quark, which is useful to shorten the CPU time.

- **1 isolated μ^- and 1 isolated μ^+**

- **1 (or 2) jet has high b-tag value obtained by the LCFI Plus (b-tag1 > 0.8 or b-tag2 > 0.8)**

Other constraints that can be considered :

Thrust value, Visible energy, Mass of $\mu^- \mu^+$, …

Pre-selection : Cut table

250 fb^{-1} (-0.8, +0.3) Left	Initial	$\mu^+\mu^-$	b-tag1>0.8 or b-tag2>0.8
Signal $bb\mu\mu\nu\nu$	2961	2725 (e = 92.0%)	2374 (e = 80.2%)
$bbll\nu\nu$ (except $bb\mu\mu\nu\nu$)	23609	387	335
$bbl\nu qq$	104114	40	31
$qqll$ (ZZ)	91478	13800	2519
ll (weight = 4)	212274 ($\rightarrow$849096)	74961 ($\rightarrow$ 299844)	90 ($\rightarrow$ 360)
$l\nu l\nu$ (WW) (weight = 4)	377058 ($\rightarrow$ 1508232)	1884 ($\rightarrow$ 7536)	3 ($\rightarrow$ 12)
$lll\nu l\nu$ (llWW)	3021	947	19

Pre-selection : Cut table

250 fb^{-1} (+0.8, -0.3) Right	initial	$\mu^+\mu^-$	b-tag1>0.8 or b-tag2>0.8
Signal *bbμμνν*	1255	1162 (e = 92.6%)	1040 (e = 82.9%)
bbllνν **(except** *bbμμνν* **)**	10181	160	138
bblνqq	45053	18	12
qqll **(ZZ)**	46344	6980	1237
ll **(weight = 4)**	161371 (→64524)	57916 (→ 231664)	61 (→ 244)

Cut table

250 fb⁻¹ (-0.8,+0.3) Right	initial	$\mu^+\mu^-$	b-tag1>0.8 or b-tag2>0.8		$q_{min} < 3$ & $\left| P_{z,\gamma} \right| < 50$ GeV	
Signal $bb\mu\mu\nu\nu$ **(True)**	1255	1162 (e = 92.6 %)	874 (84.0%)	(e = 82.9%)	437 (94.2%)	(e = 37.0%)
Signal $bb\mu\mu\nu\nu$ **(Miss)**			166 (16.0%)		27 (5.8%)	
$bbll\nu\nu$ **(except** $bb\mu\mu\nu\nu$ **)**	10181	160	138		30	
$bbl\nu qq$	45053	18	12		0	
$qqll$ **(ZZ)**	46344	6980	1237		6	
ll **(weight = 4)**	161371 (→ 64524)	57916 (→ 231664)	61 (→ 244)		0	

Helicity Angles

All final state particles including two neutrinos can be calculated. The 9 helicity angles which are related to the ttZ/γ vertex can be computed.

$$\theta_t, \theta_{W^+}^{t \text{ frame}}, \phi_{W^+}^{t \text{ frame}}, \theta_{\mu^+}^{W^+ \text{ frame}}, \phi_{\mu^+}^{W^+ \, frame}, \theta_{W^-}^{\bar{t} \, frame}, \phi_{W^-}^{\bar{t} \, frame}, \theta_{\mu^-}^{W^- \, frame}, \phi_{\mu^-}^{W^- \, frame}$$

(G. L. Kane, G. A. Ladinsky, C.-P. Yuan, Phys.Rev. D45 (1992) 124-141)

Parameters of Likelihood function

Breit-Wigner function of mass of top and W

$$BW(m) \propto \frac{1}{1 + \left(\dfrac{m - m_0}{m_0 \Gamma_0}\right)^2}$$

$$m_{t,0} = m_{\bar{t},0} = 174, m_{W^+,0} = m_{W^-,0} = 80.4, \Gamma_0 = 5$$

Gaussian function of Beam energy spread

$$Gaus(E_{total}) \propto \exp\left[-\left(\frac{E_{total} - 500}{0.39}\right)^2\right]$$

Energy resolution of b-jets
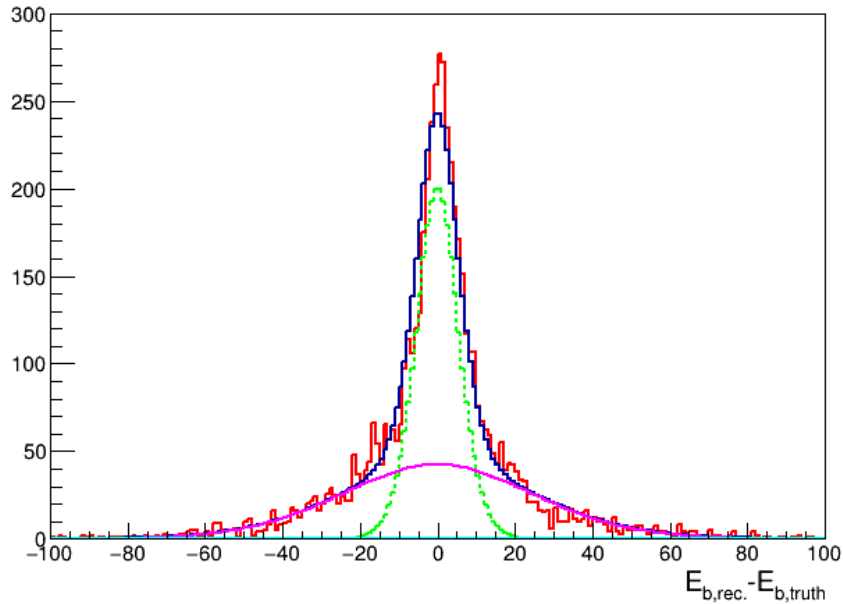
$$G\left(\sigma_j, K; E_b^{\text{Measurement}}\right)$$

$$\propto \exp\left[-\left(\frac{E_b^{\text{Measurement}} - E_b^{\text{MC}}}{\sigma_j * \left(E_b^{\text{Measurement}}\right)^K}\right)^2\right]$$

Define the resolution function R as

$$R = c_1 G_1 + c_2 G_2 + c_3 G_3$$

Results of fit :

$$c_1 = 0.50, \sigma_{j,1} = 0.77, K_1 = 0.45$$

$$c_2 = 0.48, \sigma_{j,2} = 6.4, K_2 = 0.31$$

$$c_3 = 0.02, \sigma_{j,3} = 4.7, K_3 = 0.69$$

Matrix Element Method

We assume that full matrix squared, $|M|^2$, includes up to quadratic terms of the form factors, hence the expected number of events also includes up to quadratic terms;

$$|M|^2 = \left(1 + \sum_i \omega_i \delta F_i + \sum_{ij} \widetilde{\omega}_{ij} \delta F_i \delta F_j \right) |M|^2_{\text{SM}}$$

$$N = \left(1 + \sum_i \Omega_i \delta F_i + \sum_{ij} \widetilde{\Omega}_{ij} \delta F_i \delta F_i \right) N_{\text{SM}}$$

where δF_i is deference of the form factor from SM.

Goodness of Fit

The confidence level is just computed from δF^{Fit} (or $\chi^2(\delta F^{\text{Fit}})$)

→ **Need to assess goodness of fit in another way**

Reminder of our assumption for the Matrix Element Method

$$|M|^2 = \left(1 + \sum_i \omega_i \delta F_i + \sum_{ij} \widetilde{\omega}_{ij} \delta F_i \delta F_j\right) |M|^2_{\text{SM}}, N = \left(1 + \sum_i \Omega_i \delta F_i + \sum_{ij} \widetilde{\Omega}_{ij} \delta F_i \delta F_i\right) N_{\text{SM}}$$

→ One can define PDF as $f(\delta F) = \dfrac{\left(1 + \sum_i \omega_i \delta F_i + \sum_{ij} \widetilde{\omega}_{ij} \delta F_i \delta F_j\right)}{\left(1 + \sum_i \Omega_i \delta F_i + \sum_{ij} \widetilde{\Omega}_{ij} \delta F_i \delta F_i\right)} f_{\text{SM}}$ where $f_{\text{SM}} = \dfrac{|M|^2_{\text{SM}}}{N_{\text{SM}}}$ is PDF of SM.

Expected value of ω_i and $\widetilde{\omega}_{ij}$ for given δF can be computed from the PDF

$$< \omega_i > (\delta F) = \int \omega_i f(\delta F) d\omega d\widetilde{\omega}, \qquad < \widetilde{\omega}_{ij} > (\delta F) = \int \widetilde{\omega}_{ij} f(\delta F) d\omega d\widetilde{\omega}$$

$< \omega_i > (\delta F), < \widetilde{\omega}_{ij} > (\delta F)$ should be close to $< \omega_i >_{\text{data}}, < \widetilde{\omega}_{ij} >_{\text{data}}$ if our assumption is correct.

Goodness of Fit

Define $\chi^2_{\text{GoF},i}(\delta F)$ and $\tilde{\chi}^2_{\text{GoF},ij}(\delta F)$ to assess the Goodness of Fit.

$$\chi^2_{\text{GoF},i}(\delta F) = \frac{\left(<\omega_i>_{\text{data}} - <\omega_i>(\delta F)\right)^2}{<\omega_i^2>_{\text{data}} - <\omega_i>^2_{\text{data}}}, \qquad \tilde{\chi}^2_{\text{GoF},ij}(\delta F) = \frac{\left(<\tilde{\omega}_{ij}>_{\text{data}} - <\tilde{\omega}_{ij}>(\delta F)\right)^2}{<\tilde{\omega}_{ij}^2>_{\text{data}} - <\tilde{\omega}_{ij}>^2_{\text{data}}}$$

chi2	tF_1V^g	tF_1V^Z	tF_1A^g	tF_1A^Z	tF_2V^g	tF_2V^Z	Re tF_2A^g	Re tF_2A^Z	Im tF_2A^g	Im tF_2A^Z
	1.6	8.8	1	0.18	0.42	0.45	0.1	0.00058	0.085	0.062

tchi2	tF_1V^g	tF_1V^Z	tF_1A^g	tF_1A^Z	tF_2V^g	tF_2V^Z	Re tF_2A^g	Re tF_2A^Z	Im tF_2A^g	Im tF_2A^Z
tF_1V^g	6.4	3.7	0.026	5.2	0.11	3.3	0.042	0.0016	0.0018	0.012
tF_1V^Z		4.4	5.2	0.24	3.3	0.3	0.0016	0.028	0.012	0.00029
tF_1A^g			10	5.3	2.9	2	0.00011	0.0025	0.12	0.65
tF_1A^Z				9.5	2	2.8	0.0025	0.00022	0.65	0.17
tF_2V^g					19	19	1.6	1.6	0.42	0.42
tF_2V^Z						19	1.6	1.6	0.42	0.42
Re tF_2A^g							9.2	9.2	0.014	0.33
Re tF_2A^Z								9.2	0.057	1
Im tF_2A^g									9.2	9.2
Im tF_2A^Z										9.2

Table of $\chi^2_{\text{GoF},i}$, $\tilde{\chi}^2_{\text{GoF},ij}$ of Left polarization events

Some of $\chi^2_{GoF,i}$, $\tilde{\chi}^2_{GoF,ij}$ have very large values !

Goodness of Fit : Outliers

Large χ^2_{GoF} implies that our assumption might be wrong.

However, there is another possibility of reason → **Outliers**
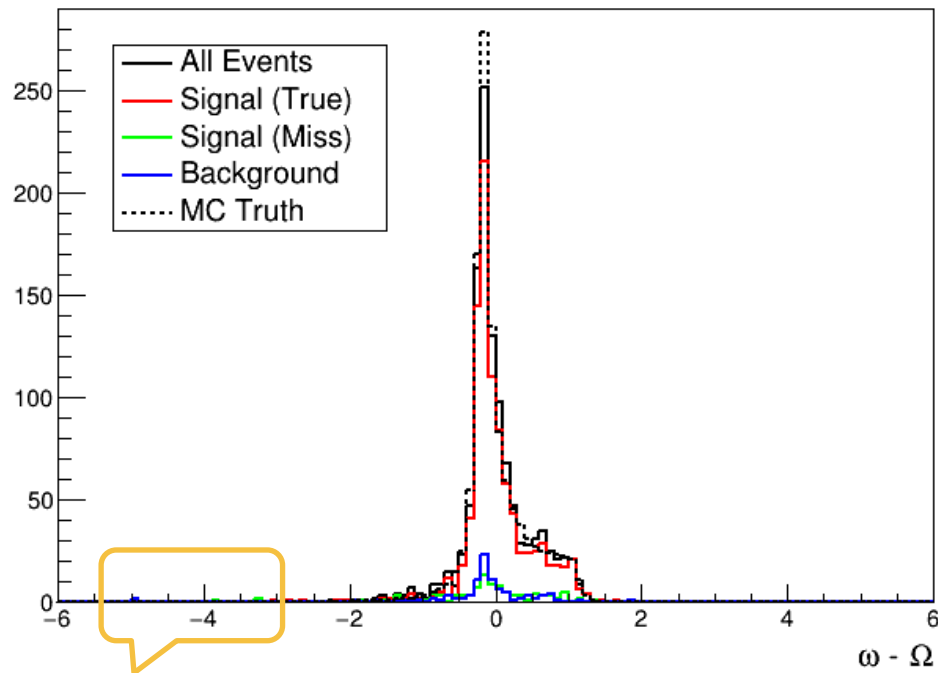
$\omega - \Omega$ distribution for δF^Z_{1V} of Left polarization events (same as P16.)

A few events are distributed far from other events. It can be caused by detector effects, ISR effects, etc.

→ *These events might be outliers and induce so large χ^2_{GoF}*

Goodness of Fit

Reject events which have too large(small) $\omega, \widetilde{\omega}$:

$$|\omega_i - \Omega_i| > 10\sigma[\omega_i^{SM}], \qquad |\widetilde{\omega}_{ij} - \widetilde{\Omega}_{ij}| > 10\sigma[\omega_{ij}^{SM}]$$

Criteria are selected *very preliminary*.

Efficiency cost is only 1.6%(0.8%) for Left(Right) polarization events.

Left table:

chi2	tF_1V^g	tF_1V^Z	tF_1A^g	tF_1A^Z	tF_2V^g	tF_2V^Z	Re tF_2A^g	Re tF_2A^Z	Im tF_2A^g	Im tF_2A^Z
chi2	1.6	8.8	1	0.18	0.42	0.45	0.1	0.00058	0.085	0.062

tchi2	tF_1V^g	tF_1V^Z	tF_1A^g	tF_1A^Z	tF_2V^g	tF_2V^Z	Re tF_2A^g	Re tF_2A^Z	Im tF_2A^g	Im tF_2A^Z
tF_1V^g	6.4	3.7	0.026	5.2	0.11	3.3	0.042	0.0016	0.0018	0.012
tF_1V^Z		4.4	5.2	0.24	3.3	0.3	0.0016	0.028	0.012	0.00029
tF_1A^g			10	5.3	2.9	2	0.00011	0.0025	0.12	0.65
tF_1A^Z				9.5	2	2.8	0.0025	0.00022	0.65	0.17
tF_2V^g					19	19	1.6	1.6	0.42	0.42
tF_2V^Z						19	1.6	1.6	0.42	0.42
Re tF_2A^g							9.2	9.2	0.014	0.33
Re tF_2A^Z								9.2	0.057	1
Im tF_2A^g									9.2	9.2
Im tF_2A^Z										9.2

Right table:

chi2	tF_1V^g	tF_1V^Z	tF_1A^g	tF_1A^Z	tF_2V^g	tF_2V^Z	Re tF_2A^g	Re tF_2A^Z	Im tF_2A^g	Im tF_2A^Z
chi2	0.62	1.8	0.46	0.16	0.087	0.084	0.015	0.01	0.021	0.59

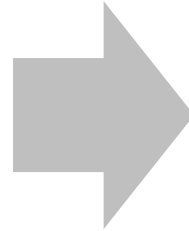| tchi2	tF_1V^g	tF_1V^Z	tF_1A^g	tF_1A^Z	tF_2V^g	tF_2V^Z	Re tF_2A^g	Re tF_2A^Z	Im tF_2A^g	Im tF_2A^Z
tF_1V^g	2.1	1.5	0.2	1.5	0.0073	1.1	0.34	0.25	0.022	0.0038
tF_1V^Z		1.3	1.5	0.039	1.1	0.054	0.25	0.32	0.0038	0.018
tF_1A^g			3.7	1.2	3.5	2	0.69	0.71	0.13	0.00053
tF_1A^Z				3.4	2	3.2	0.71	0.7	0.00053	0.1
tF_2V^g					11	11	0.18	0.18	0.0006	0.0006
tF_2V^Z						11	0.18	0.18	0.0006	0.0006
Re tF_2A^g							3.9	3.9	0.00068	0.18
Re tF_2A^Z								3.9	0.15	0.61
Im tF_2A^g									3.9	3.9
Im tF_2A^Z										3.9

Most of large $\chi^2_{\mathrm{GoF},i}, \widetilde{\chi}^2_{\mathrm{GoF},ij}$ become much better. It implies such large values are induced by outliers. Although some still have large values (~ 11), one may reduce them changing criteria for outliers.

Some of $\widetilde{\chi}^2_{\mathrm{GoF},ij}$ are still 3-4. We suppose it comes from ISR effects and detector effect

Preliminary Results without Outliers

Results of 10 parameters multi-fit

$$
\begin{bmatrix}
\mathcal{R}e\ \delta\tilde{F}_{1V}^{\gamma} & -0.0138 \pm 0.0132 \\
\mathcal{R}e\ \delta\tilde{F}_{1V}^{Z} & +0.0284 \pm 0.0229 \\
\mathcal{R}e\ \delta\tilde{F}_{1A}^{\gamma} & +0.0171 \pm 0.0183 \\
\mathcal{R}e\ \delta\tilde{F}_{1A}^{Z} & +0.0537 \pm 0.0285 \\
\mathcal{R}e\ \delta\tilde{F}_{2V}^{\gamma} & -0.0847 \pm 0.0401 \\
\mathcal{R}e\ \delta\tilde{F}_{2V}^{Z} & +0.1132 \pm 0.0642 \\
\mathcal{R}e\ \delta\tilde{F}_{2A}^{\gamma} & -0.0160 \pm 0.0239 \\
\mathcal{R}e\ \delta\tilde{F}_{2A}^{Z} & -0.0539 \pm 0.0408 \\
\mathcal{I}m\ \delta\tilde{F}_{2A}^{\gamma} & +0.0428 \pm 0.0265 \\
\mathcal{I}m\ \delta\tilde{F}_{2A}^{Z} & +0.0222 \pm 0.0372
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
\mathcal{R}e\ \delta\tilde{F}_{1V}^{\gamma} & -0.0148 \pm 0.0129 \\
\mathcal{R}e\ \delta\tilde{F}_{1V}^{Z} & +0.0232 \pm 0.0226 \\
\mathcal{R}e\ \delta\tilde{F}_{1A}^{\gamma} & +0.0140 \pm 0.0184 \\
\mathcal{R}e\ \delta\tilde{F}_{1A}^{Z} & +0.0309 \pm 0.0286 \\
\mathcal{R}e\ \delta\tilde{F}_{2V}^{\gamma} & -0.0736 \pm 0.0371 \\
\mathcal{R}e\ \delta\tilde{F}_{2V}^{Z} & +0.0564 \pm 0.0601 \\
\mathcal{R}e\ \delta\tilde{F}_{2A}^{\gamma} & -0.0059 \pm 0.0226 \\
\mathcal{R}e\ \delta\tilde{F}_{2A}^{Z} & -0.0377 \pm 0.0389 \\
\mathcal{I}m\ \delta\tilde{F}_{2A}^{\gamma} & +0.0403 \pm 0.0238 \\
\mathcal{I}m\ \delta\tilde{F}_{2A}^{Z} & +0.0007 \pm 0.0343
\end{bmatrix}
$$

The results of fit becomes also slightly better. We will optimize criteria taking outliers into account.