Top electroweak couplings study using di-leptonic state at $\sqrt{s}$ = 500 GeV, ILC with the Matrix Element Method

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Status

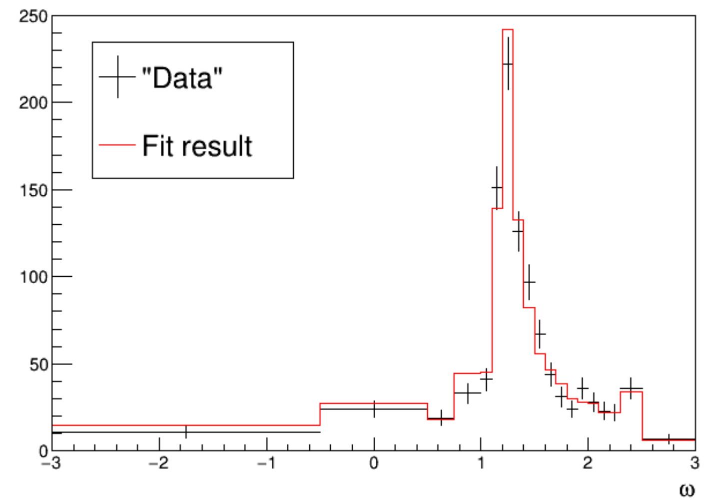
- Writing thesis
- Optimization of criteria for the quality cut

Improved method : Binned likelihood analysis

Estimate the number of events in each bin of the ω distribution described as function of δF, $N_b(\delta F)$, from the full MC simulation. Fit $N_b(\delta F)$ to the *"data"* using the following $\chi^2(\delta F)$.

$$\chi^2(\delta F) = \sum_{b=1}^{N_{\mathrm{bin}}} \frac{\left(n_b^{\mathrm{Data}} - N_b(\delta F)\right)^2}{n_b^{\mathrm{Data}}}$$

where n_b^{Data} is the number of events in bin b of the *"data"*.



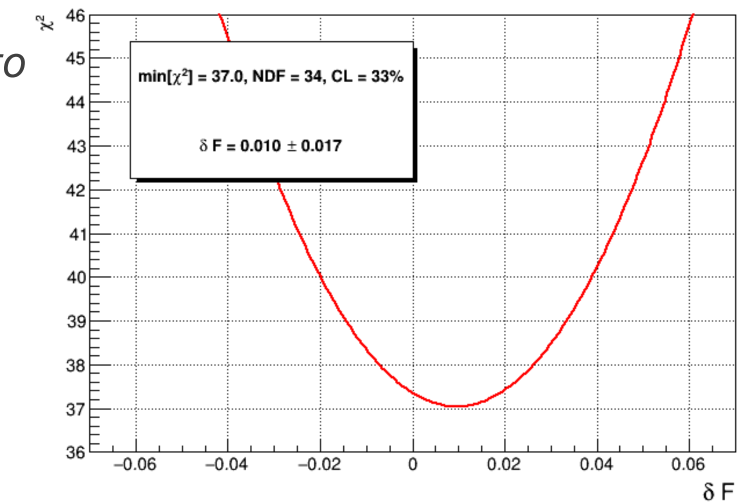
ω distribution for δF_{1V}^Z of Left polarization events

This method is by construction unbiased if the full MC simulation describes the "data" and one can use $\chi^2(\delta F)$ to assess the goodness of fit.

Example : Result of 1 parameter fit

$$\delta \tilde{F}_{1V}^Z = 0.010 \pm 0.017 \; (\mathrm{CL} = 33\%)$$

For the multi-parameter fit, more statistics of the full MC simulation is required.



$\chi^2(\delta \tilde{F}_{1V}^Z)$ function

Improved method : Binned likelihood analysis

$\chi^2(\delta F)$ is defined as following;

$$\chi^2(\delta F) = \sum_{b=1}^{N_{\mathrm{bin}}} \frac{\left(n_b^{\mathrm{Data}} - N_b(\delta F)\right)^2}{n_b^{\mathrm{Data}}}$$

$N_b(\delta F)$ is obtained from the very large full MC simulation changing δF, which is called ***the template method***. However, it can be also obtained by ***the re-weighting method***

$$N_b(\delta F) = \frac{n^{\mathrm{Data}}}{N^{\mathrm{MC\ Simulation}}} \sum_{e \in b} 1 * \frac{|M|^2(\delta F)}{|M|^2_{\mathrm{SM}}}$$

$$= \frac{n^{\mathrm{Data}}}{N^{\mathrm{MC\ Simulation}}} \sum_{e \in b} \left(1 + \sum \omega_i^{\mathrm{Truth}} \delta F_i + \sum \widetilde{\omega}_{ij}^{\mathrm{Truth}} \delta F_i \delta F_j\right)$$

where $\omega_i^{\mathrm{Truth}}$ and $\widetilde{\omega}_{ij}^{\mathrm{Truth}}$ are the optimal variables at MC truth level. Only one simulation is needed if one uses this method.

Since $\widetilde{\omega}^{\mathrm{Truth}}$ is a coefficient of $O(\delta F^2)$ and δF is so small, we use only $\omega_i^{\mathrm{Truth}}$ for now.

Improved method : Binned likelihood analysis

In the definition of $\chi^2(\delta F)$, we assume the deviation is $\sqrt{n_b}$. So the n_b must be large (>10).

The following likelihood function can be used even if n_b is small.

$$-2\log L(\delta F) = -2\sum_{b=1}^{N_{bin}} \left(\begin{array}{c} n_b^{\mathrm{Data}} \ln\left(1 + \sum_i o_{b,i}\delta F_i + \sum_{ij} \tilde{o}_{b,ij}\delta F_i \delta F_j\right) \\ -N_b^{\mathrm{SM}} \left(\sum_i o_{b,i}\delta F_i + \sum_{ij} \tilde{o}_{b,ij}\delta F_i \delta F_j\right) \end{array} \right)$$

where $o_{b,i} = \frac{1}{N_b^{SM}} \sum_{e\in b} \omega_i^{\mathrm{Truth}}$, $\tilde{o}_{b,ij} = \frac{1}{N_b^{SM}} \sum_{e\in b} \widetilde{\omega}_{ij}^{\mathrm{Truth}}$.

This definition is more precise because we don't use any assumptions and it is also by construction unbiased. However we cannot assess the goodness of fit from the likelihood function.

Matrix Element Method

We assume that full matrix squared, $|M|^2$, includes up to quadratic terms of the form factors, hence the expected number of events also includes up to quadratic terms;

$$|M|^2 = \left(1 + \sum_i \omega_i \delta F_i + \sum_{ij} \widetilde{\omega}_{ij} \delta F_i \delta F_j\right) |M|^2_{\mathrm{SM}}$$

$$N = \left(1 + \sum_i \Omega_i \delta F_i + \sum_{ij} \widetilde{\Omega}_{ij} \delta F_i \delta F_i\right) N_{\mathrm{SM}}$$

where δF_i is deference of the form factor from SM.

Matrix Element Method

Matrix element method is based on the maximum likelihood method and a likelihood function is written by $|M|^2$ and N;

$$-2\log L(\delta F) = \chi^2(\delta F) =$$

$$-2\left(\sum_{e=1}^{N_{\mathrm{event}}} \log\left(1 + \sum_i \omega_i(\Phi_e)\delta F_i + \sum_{ij} \widetilde{\omega}_{ij}(\Phi_e)\delta F_i \delta F_j\right) - N_{\mathrm{event}} \log\left(1 + \sum_i \Omega_i \delta F_i + \sum_{ij} \widetilde{\Omega}_{ij}\delta F_i \delta F_j\right)\right)$$

where Φ_e is helicity angles which have sensitivity for the form factors. $\chi^2(\delta F)$ is scaled to 0 at $\delta F = 0$.

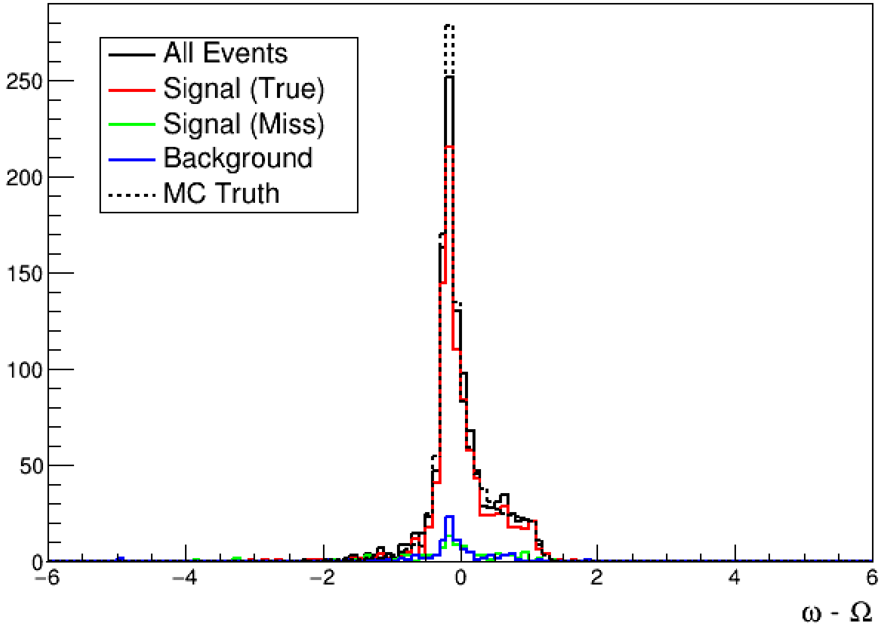
If we use the information of yields with Poisson distribution, the second term can be replaced as $N_{\mathrm{event}}\left(\sum_i \Omega_i \delta F_i^{\mathrm{SM}} + \sum_{ij} \widetilde{\Omega}_{ij}\delta F_i^{\mathrm{SM}} \delta F_j^{\mathrm{SM}}\right)$

Matrix Element Method

What we must do to fit the form factors correctly is to reconstruct ω_i correctly.

Indeed the results of fit are related with ω_i and Ω_i which are called ***optimal variables***

- $\delta F_i^{\mathrm{Fit}} \simeq \frac{<\omega_i - \Omega_i>}{<{\omega_i}^2>}$
- Covariance matrix, V_{ij} : $V_{ij}^{-1} \simeq N_{\mathrm{event}} < \omega_i \omega_j >$

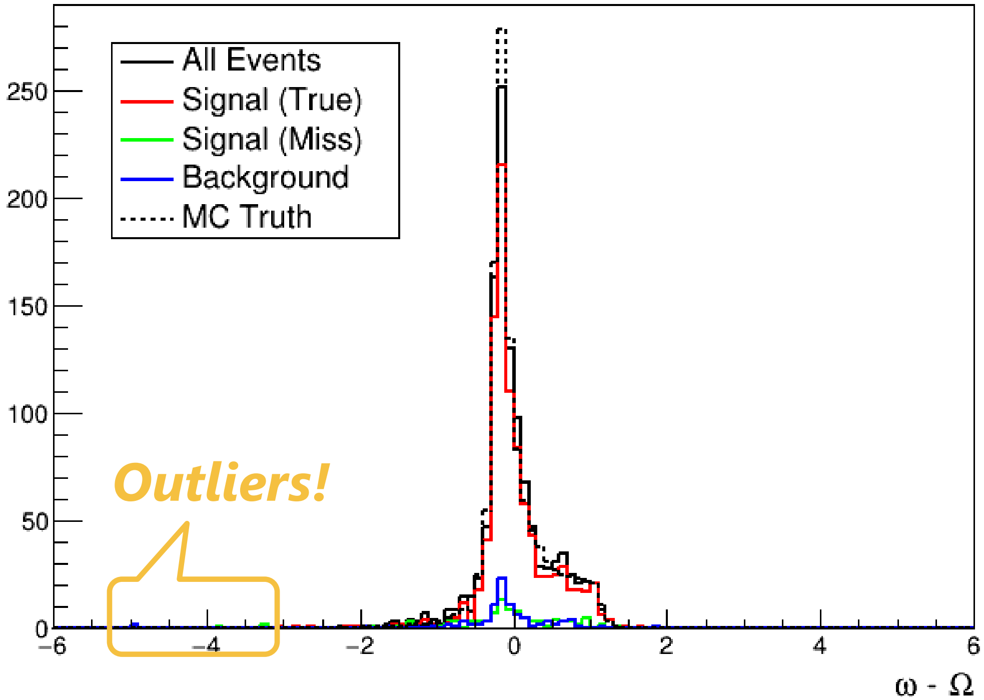


$\omega - \Omega$ distribution for δF_{1V}^{Z} of Left polarization events

Reconstructed (All Events) are similar with MC Truth

Outliers

A few events are distributed far from other events. It can be caused by detector effects and ISR effects, in other wards they are badly reconstructed events.



$\omega - \Omega$ distribution for δF_{1V}^{Z} of Left polarization events (same as last slide)

These events easily induce biases on results of fit. → ***Outliers***

We fit $\omega - \Omega$ distribution within a region not including *outliers*. Efficiency cost is only 1.6%(0.8%) for Left(Right) polarization events.

Preliminary Results without Outliers

Results of 10 parameters multi-fit

$$\begin{bmatrix} \mathcal{R}e\ \delta\tilde{F}_{1V}^{\gamma} & -0.0148 \pm 0.0129 \\ \mathcal{R}e\ \delta\tilde{F}_{1V}^{Z} & +0.0232 \pm 0.0226 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{\gamma} & +0.0140 \pm 0.0184 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{Z} & +0.0309 \pm 0.0286 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{\gamma} & -0.0736 \pm 0.0371 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{Z} & +0.0564 \pm 0.0601 \\ \mathcal{R}e\ \delta\tilde{F}_{2A}^{\gamma} & -0.0059 \pm 0.0226 \\ \mathcal{R}e\ \delta\tilde{F}_{2A}^{Z} & -0.0377 \pm 0.0389 \\ \mathcal{I}m\ \delta\tilde{F}_{2A}^{\gamma} & +0.0403 \pm 0.0238 \\ \mathcal{I}m\ \delta\tilde{F}_{2A}^{Z} & +0.0007 \pm 0.0343 \end{bmatrix}$$

Efficiency : 35 %

CL(*) : 35 % (* It is discussed in backup slides)

This precision is comparable with semi-leptonic state analysis considering difference of statistics. ***One can fit more parameters simultaneously.***

But there are still small biases. We have room for improvement → ***Next Slide***