

# GRACE for LC

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- What is GRACE system?
  - Full  $O(\alpha)$  electroweak corrections w/ Beam polarization effects
- Recent results for ILC
  - Top-quark physics
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  - Higgs physics
    - Z-Higgs production
    - W-fusion
- Summary

What is GRACE?

# What is GRACE

- GRACE is an automatic system to calculate tree and ELWK-loop cross sections with spin-polarizations based on SM and MSSM.

$$e^+e^- \rightarrow t\bar{t}\gamma$$

Full-ELWK Eur. Phys. J. C **73**, 2400 (2013)

$$e^+e^- \rightarrow e^+e^-\gamma$$

Full-ELWK Phys. Lett. **B740**, 192 (2014)

$$e^+e^- \rightarrow t\bar{t} \text{ W/ beam pol.}$$

Full-ELWK Eur. Phys. J. C **78**, 422 (2018)

# What is GRACE

Model/Process



Feynman Diagrams



Symbolic Codes

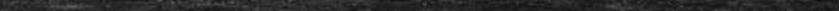
Fortran Codes



Numerical Integration



Histograms  
Events



# What is GRACE

## On-shell renormalization scheme (Kyoto-Scheme)

$m_e$	$0.51099906 \times 10^{-3} \text{ GeV}$	$\Gamma_Z$	2.0 GeV
$m_t$	173.5 GeV	$\alpha$	1/137.0359895
$m_u$	$58.0 \times 10^{-3} \text{ GeV}$	$\sin^2 \theta_W$	$1 - \frac{m_W^2}{m_Z^2}$
$m_c$	1.50 GeV	$m_{\text{Higgs}}$	126.00 GeV
$m_d$	$58.0 \times 10^{-3} \text{ GeV}$	$m_Z$	91.187 GeV
$m_s$	$92.0 \times 10^{-3} \text{ GeV}$	$m_W$	80.22 GeV
$m_b$	4.7 GeV		

<sup>1</sup> K. Aoki et al, Suppl. Prog. Theor. Phys. **73** (1982) 1.

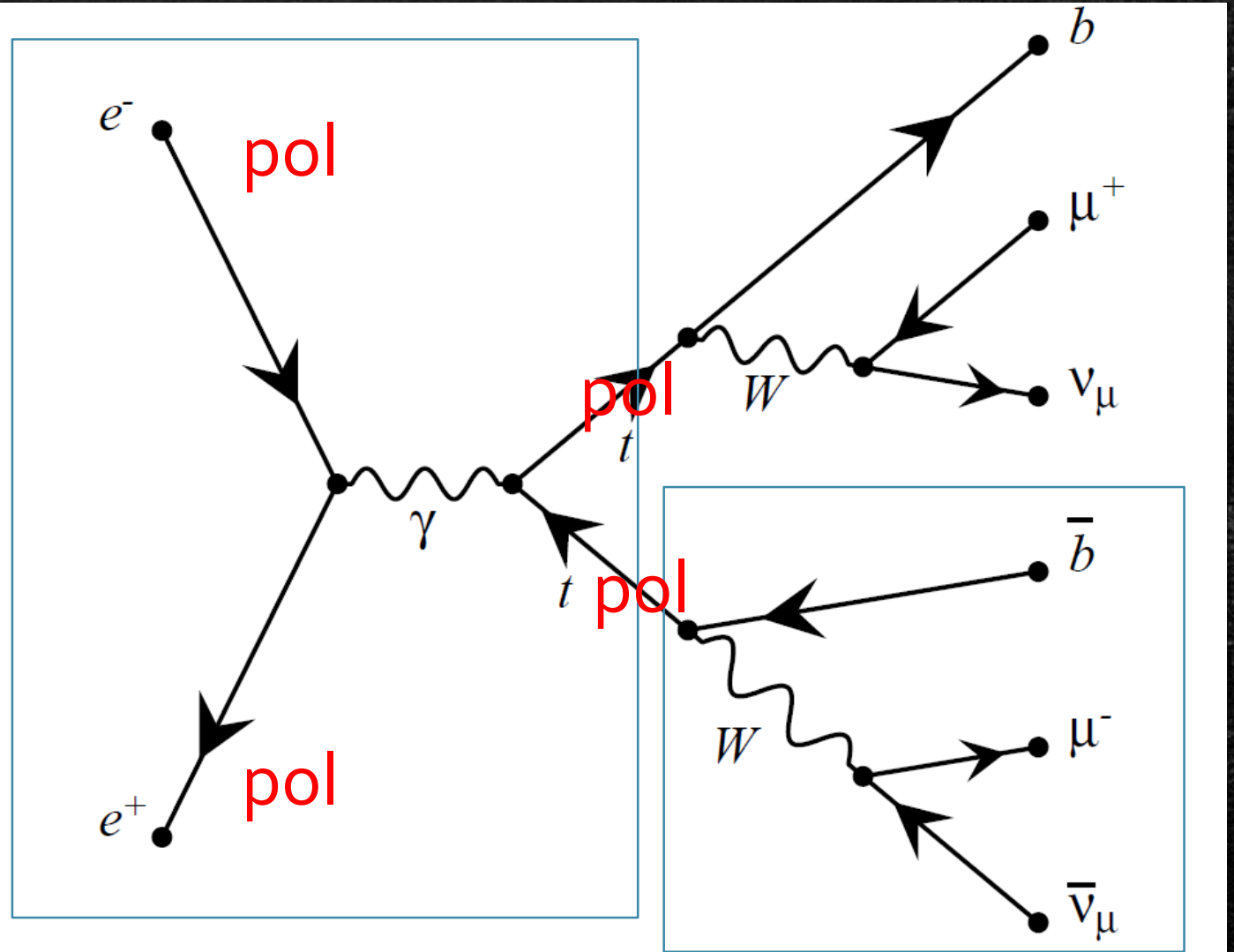
<sup>2</sup> G. Belanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, Y. Shimizu  
Phys, Rept. **430**, 117 (2006)

Recent results:

Top-pair

Narrow width approx.

Double-pole approx.



#diagrams  
Tree ~ 4  
Loop ~ 150

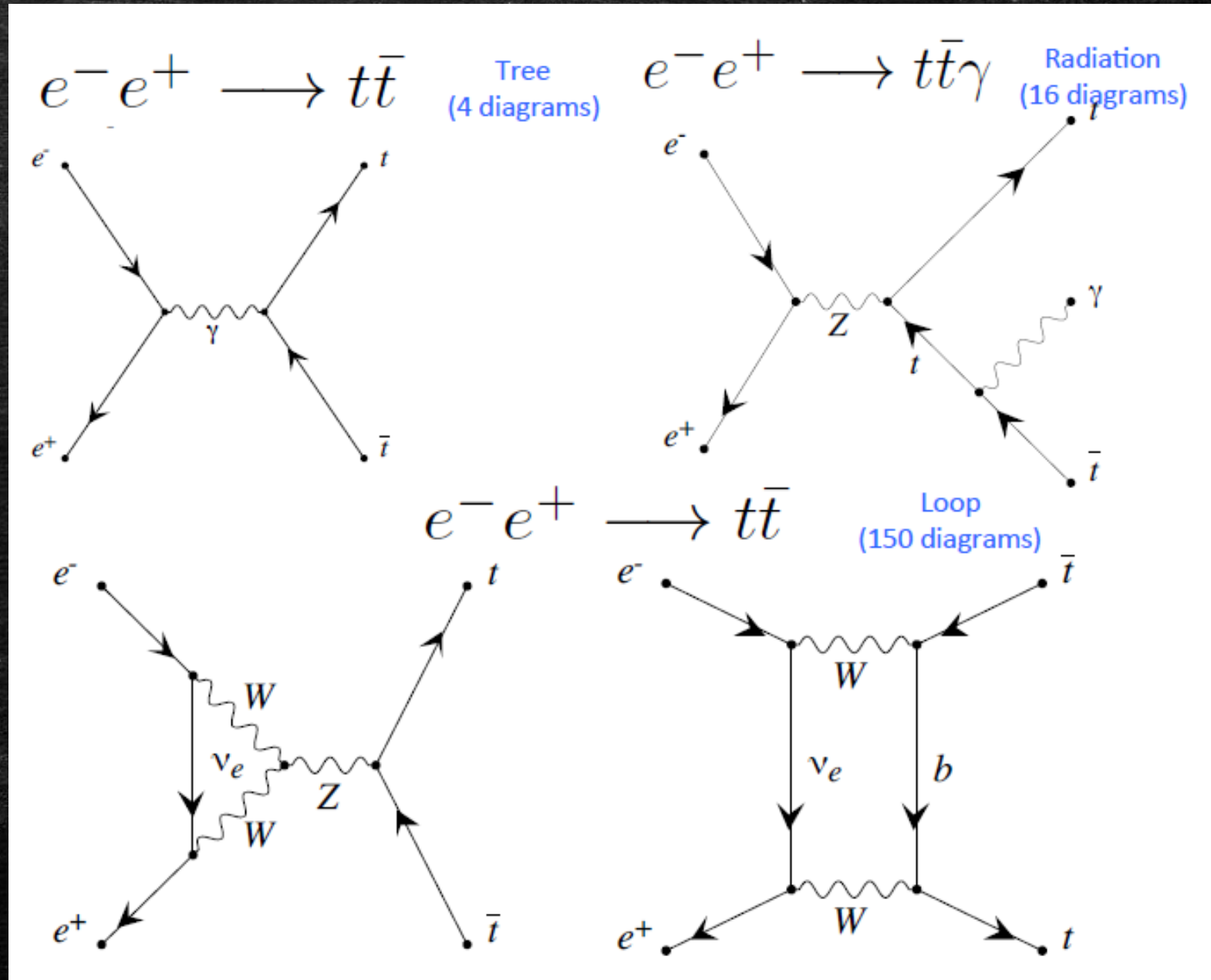
#diagrams  
Tree ~ 2  
Loop ~ 80



Top-pair:  
 $O(\alpha)$  ELWK  
corrections

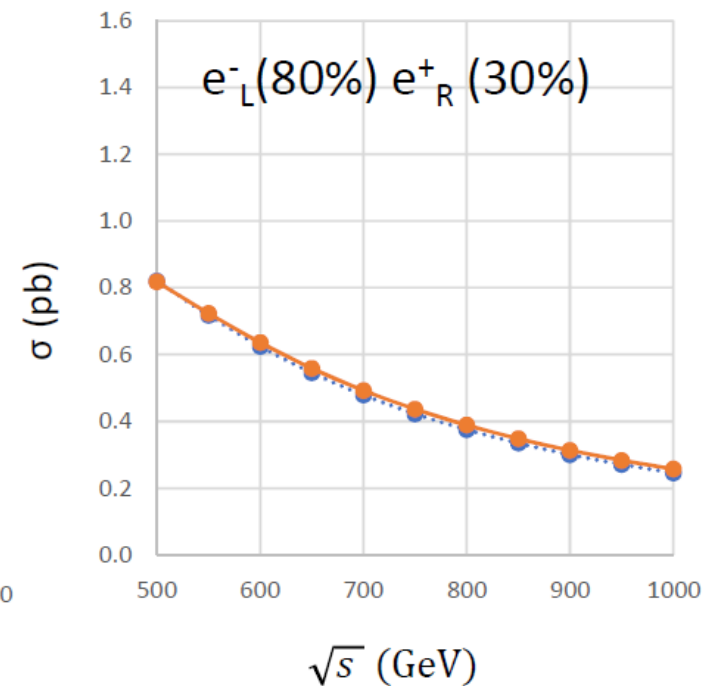
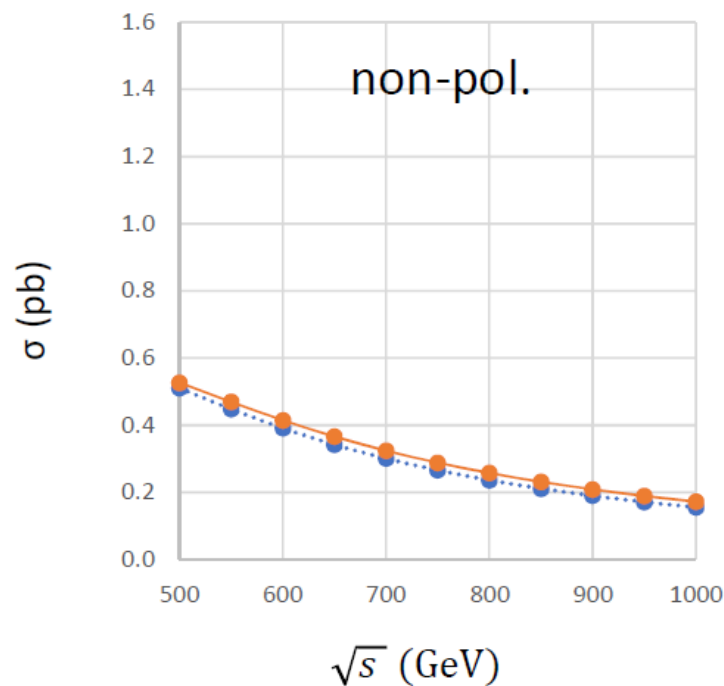
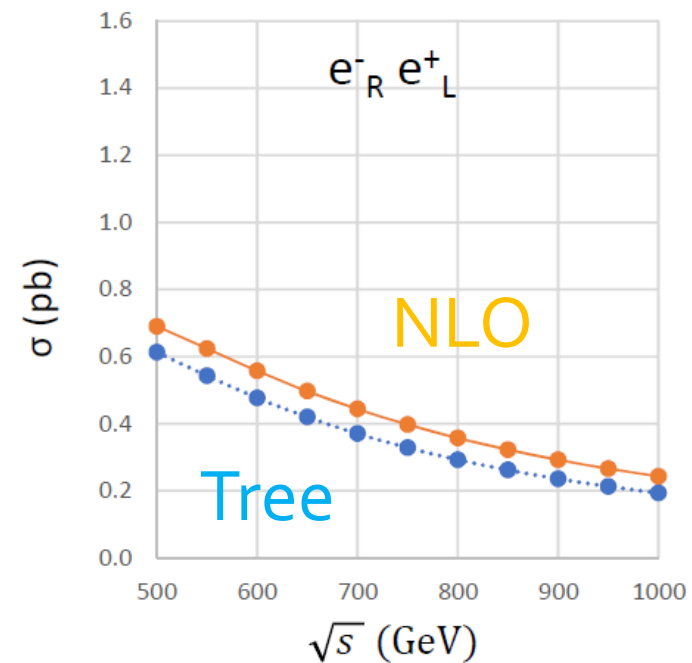
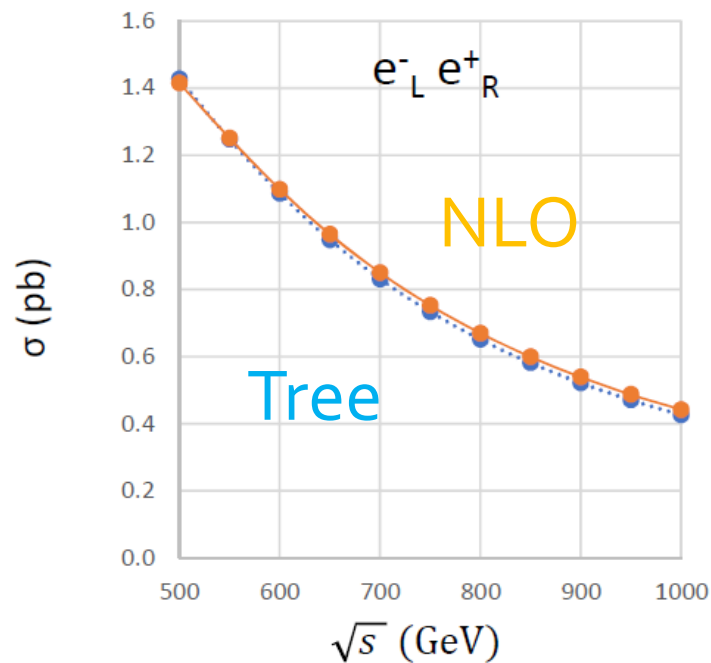
# $O(\alpha)$ ELWK correction

Examples of loop diagrams



# $O(\alpha)$ ELWK correction

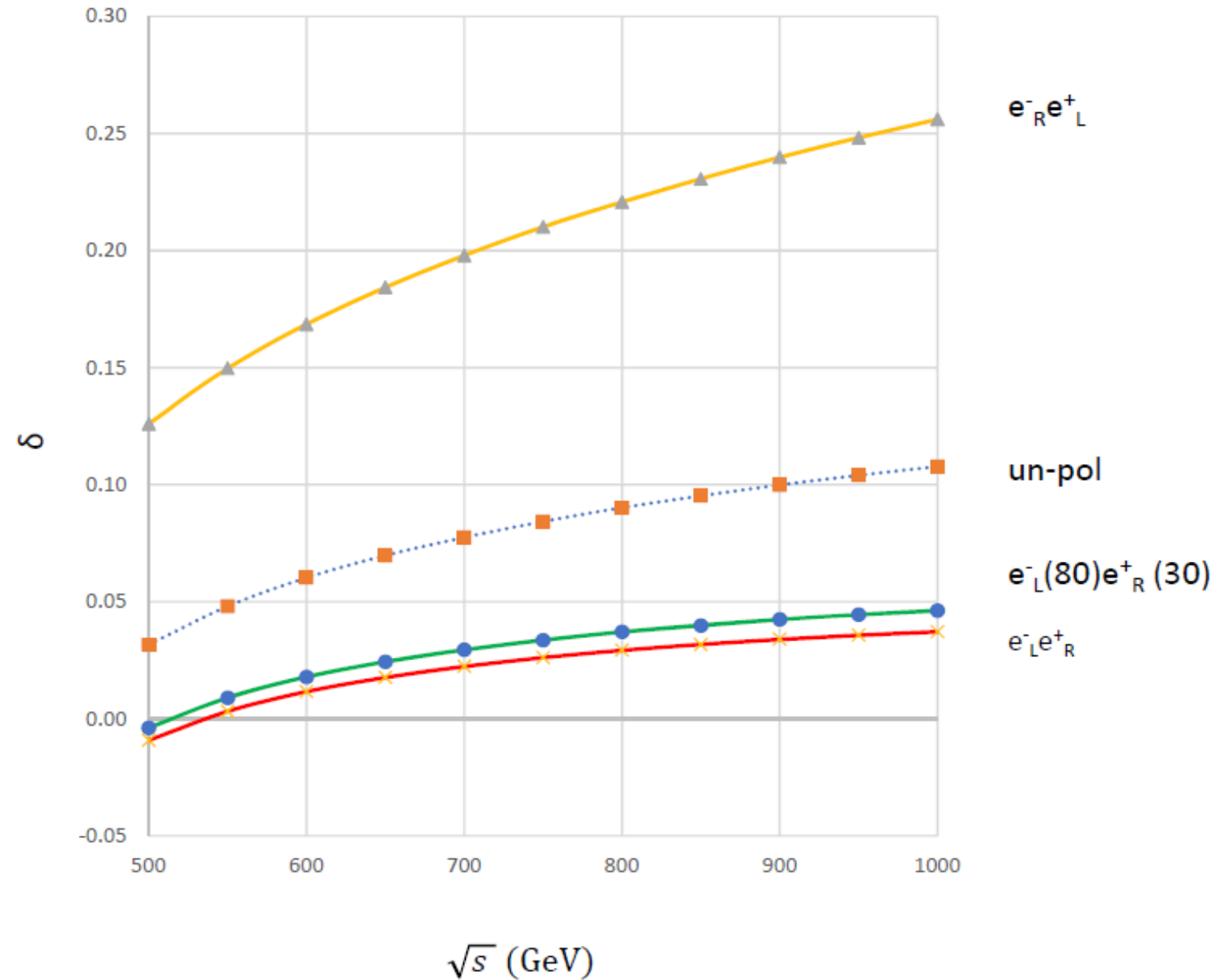
Total cross sections



# $O(\alpha)$ ELWK correction

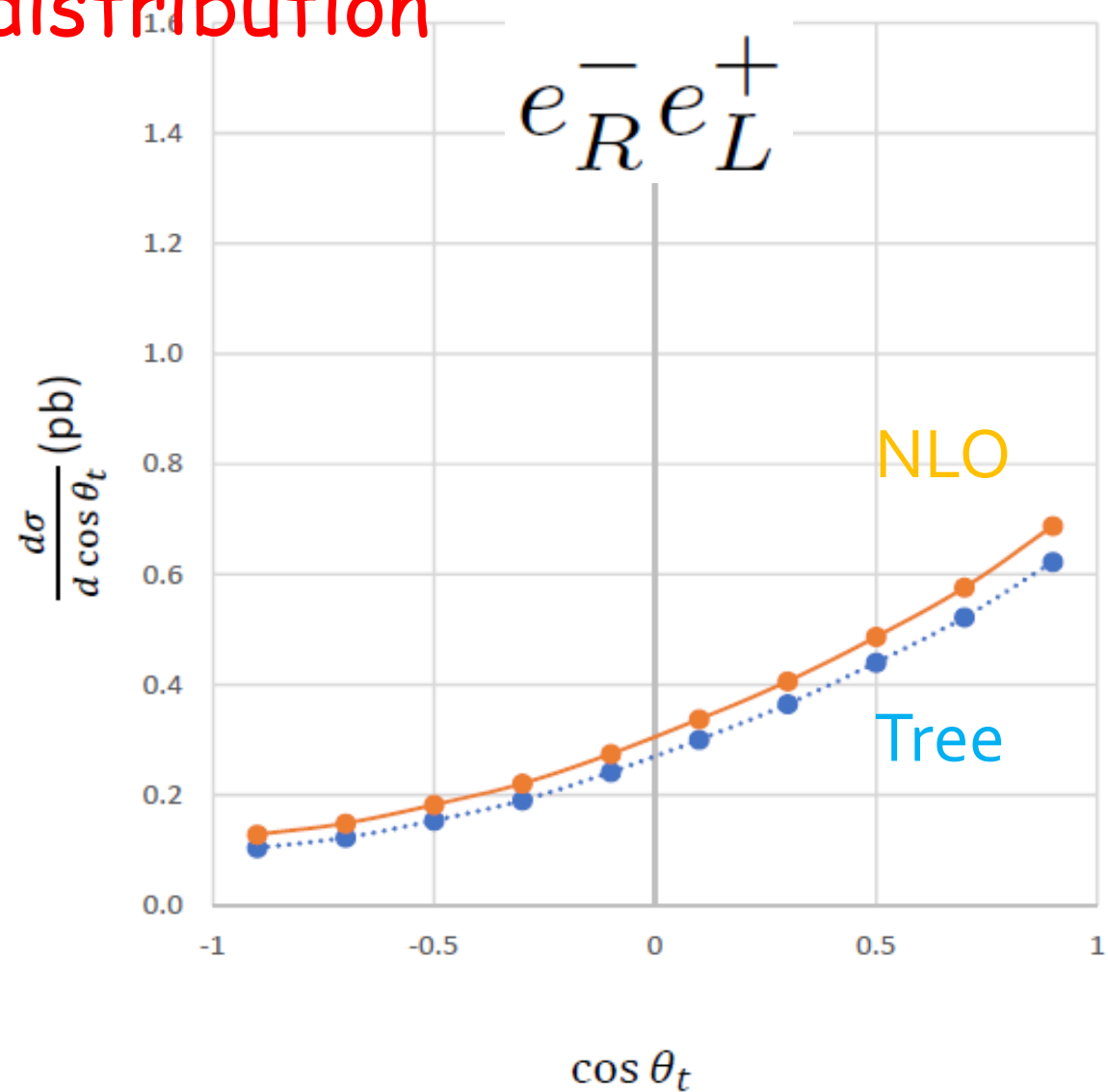
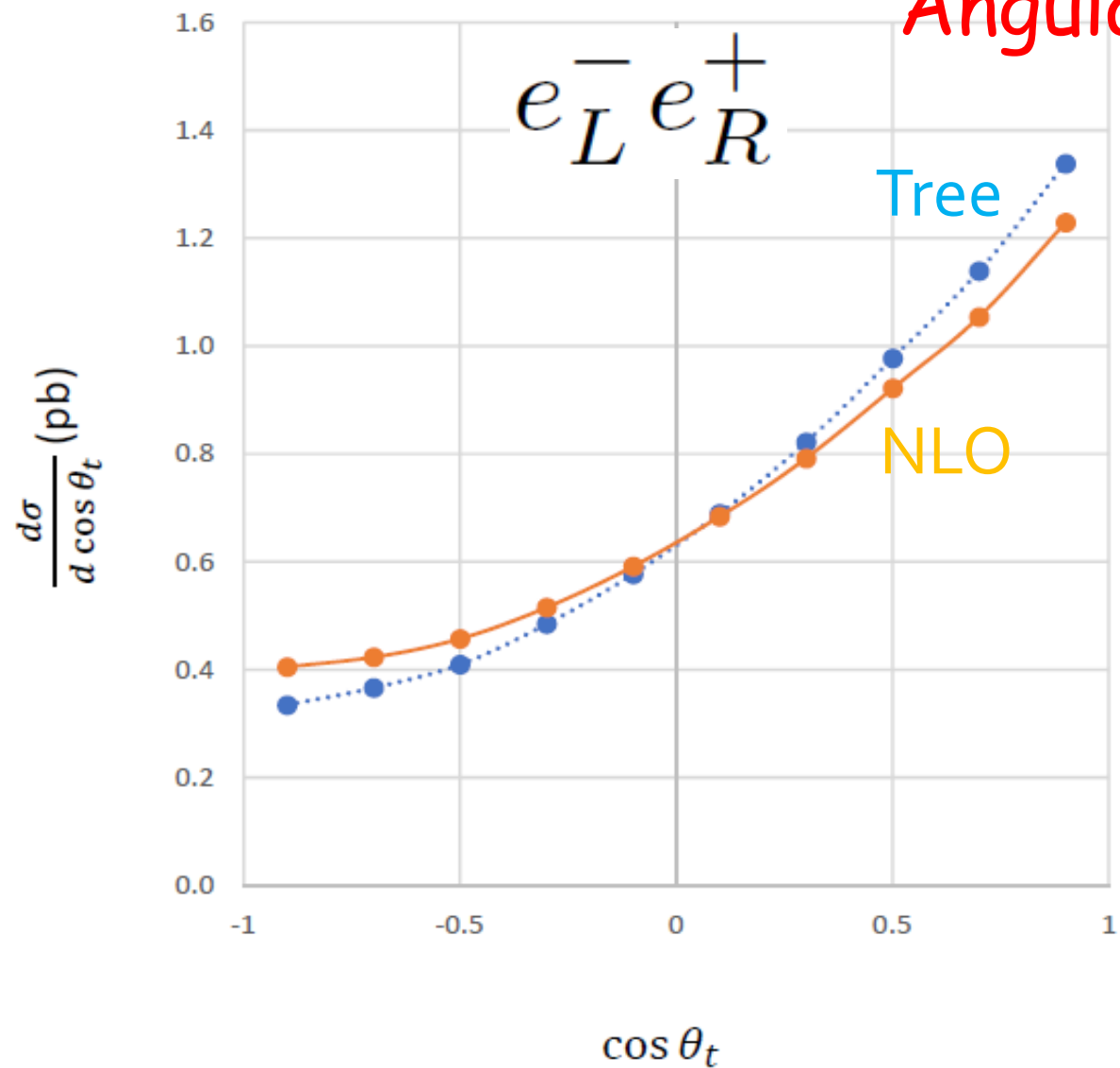
Total cross sections

$$\delta = (\sigma_{NLO} - \sigma_{Tree}) / \sigma_{Tree}$$



# $O(\alpha)$ ELWK correction

Angular distribution

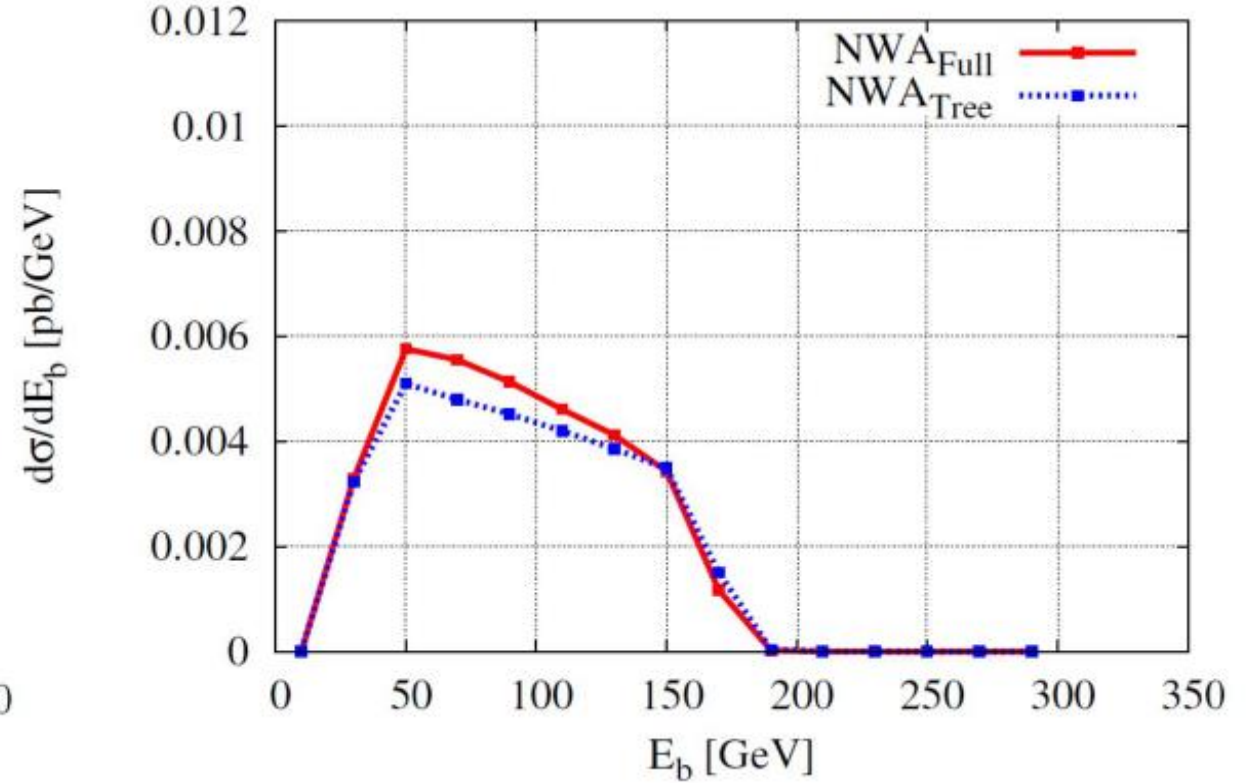
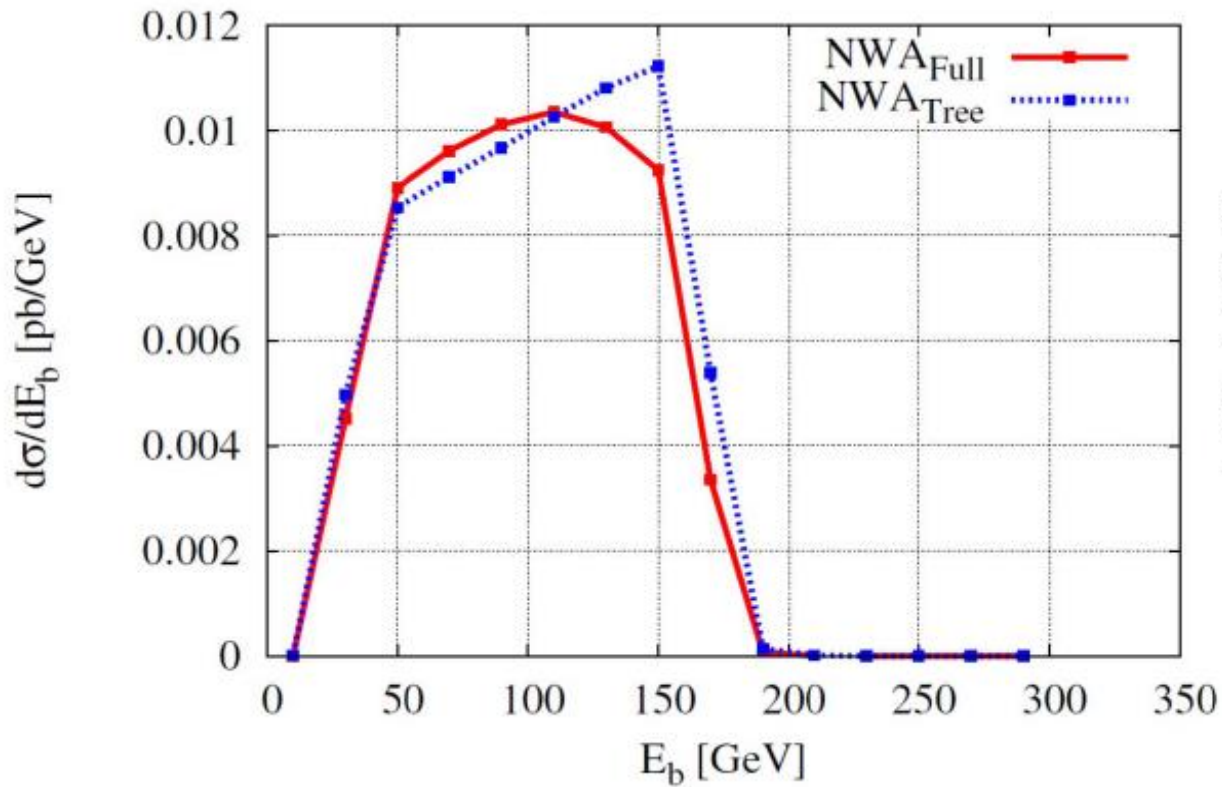


# $O(\alpha)$ ELWK correction

$$e_L^- e_R^+$$

6-body final process

$$e_R^- e_L^+$$

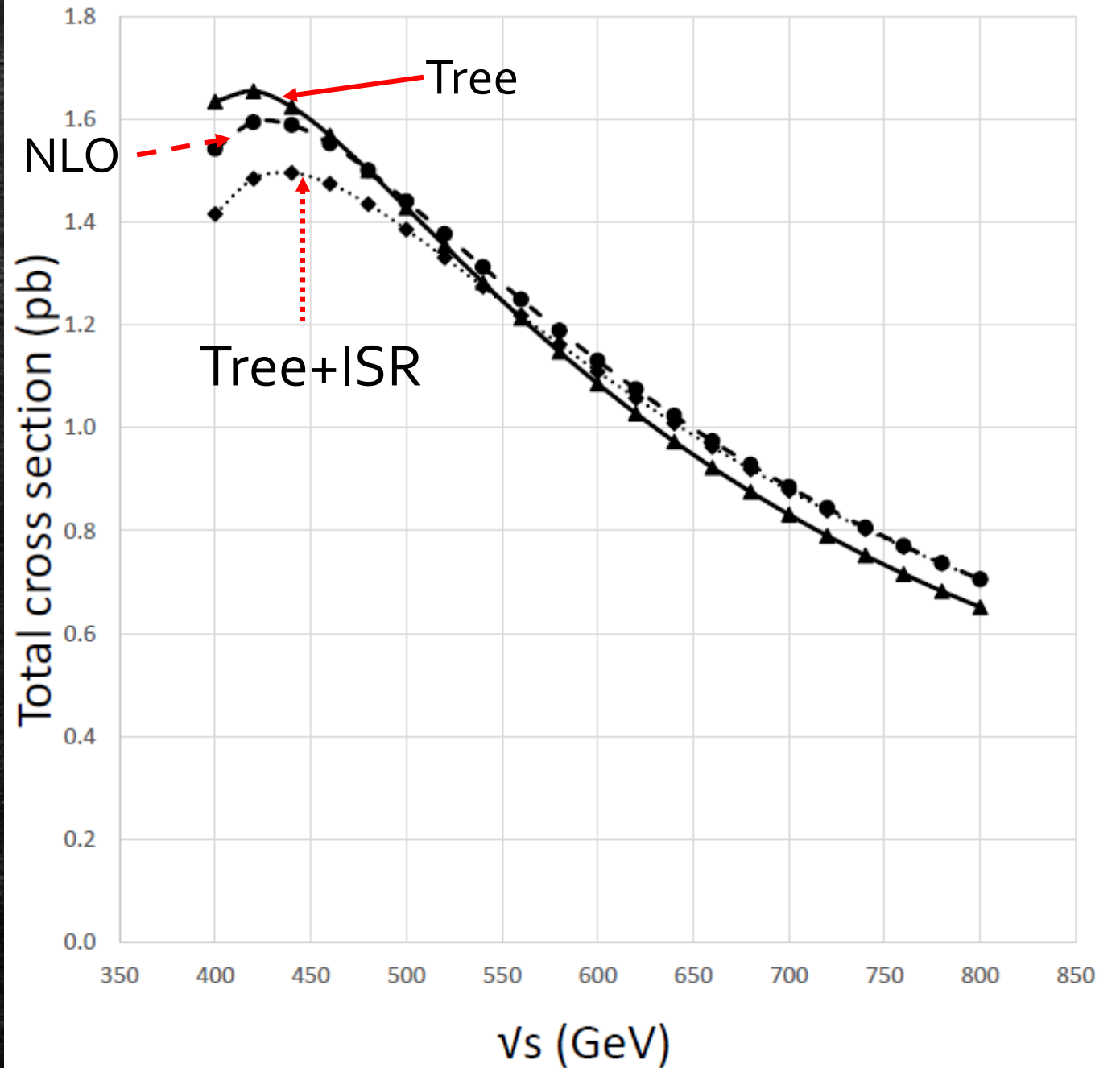


$\bar{b}$ -quark energy

Top-pair:  
ISR re-summation

# ISR re-summation

The ISR correction gives a main correction





# ISR re-summation

$$\sigma_{NLO;fixed} = \sigma_{Loop} + \sigma_{Soft} + \sigma_{Hard} + \sigma_{Tree}$$

$\lambda$

(photon mass)

$k_c$

# ISR re-summation

$$\sigma_{NLO;fixed} = \sigma_{Loop} + \sigma_{Soft} + \sigma_{Hard} + \sigma_{Tree}$$

$$\sigma_{NLO;ISR} = (\sigma_{Loop} - \sigma_{Tree}\delta_{ISL}) + \tilde{\sigma}_{Soft}^{final} + \sigma_{Hard} + \sigma_{ISR}$$

$\lambda$   
(photon mass)

$k_c$

# ISR re-summation

$$\sigma_{NLO;fixed} = \sigma_{Loop} + \sigma_{Soft} + \sigma_{Hard} + \sigma_{Tree}$$

$$\sigma_{NLO;ISR} = (\sigma_{Loop} - \sigma_{Tree}\delta_{ISL}) + \tilde{\sigma}_{Soft}^{final} + \sigma_{Hard} + \sigma_{ISR}$$

$$\sigma_{ISR} = \int_0^{k_c^2/s} dx_1 \int_0^{1-x_1} dx_2 D(x_1, s) D(x_2, s) \sigma_{Tree}(sx_1x_2)$$

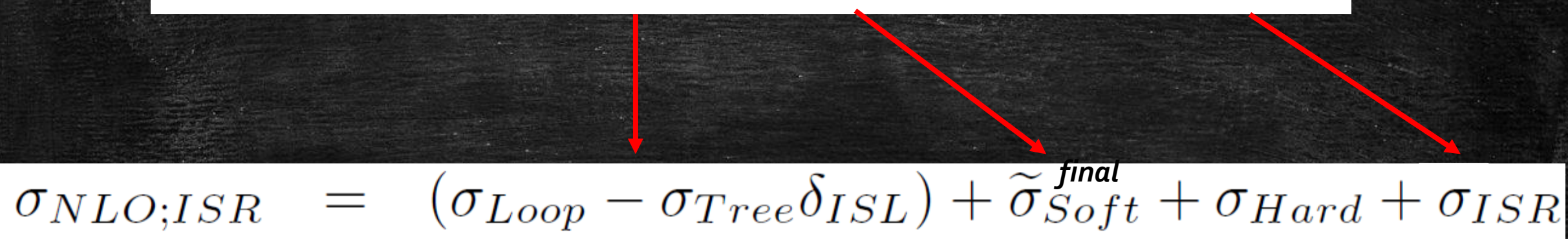
# ISR re-summation

Radiator

$$D(1-x, s)^2 = H(x, s) = \Delta_2 \beta x^{\beta-1} - \Delta_1 \beta \left(1 - \frac{x}{2}\right) + \frac{\beta^2}{8} \left[ -4(2-x) \log x - \frac{1+3(1-x)^2}{x} \log(1-x) - 2x \right]$$

$$\beta = \frac{2\alpha}{\pi} \left( \log \frac{s}{m_e^2} - 1 \right),$$
$$\Delta_2 = 1 + \delta_1 + \delta_2, \quad \Delta_1 = 1 + \delta_1$$
$$\delta_1 = \frac{\alpha}{\pi} \left( \frac{3}{2} L + \frac{\pi^2}{3} - 2 \right), \quad \delta_2 = \left( \frac{\alpha L}{\pi} \right)^2 \left( -\frac{1}{18} L + \frac{119}{72} - \frac{\pi^2}{3} \right)$$

# ISR re-summation

$$\sigma_{NLO;fixed} = \sigma_{Loop} + \sigma_{Soft} + \sigma_{Hard} + \sigma_{Tree}$$


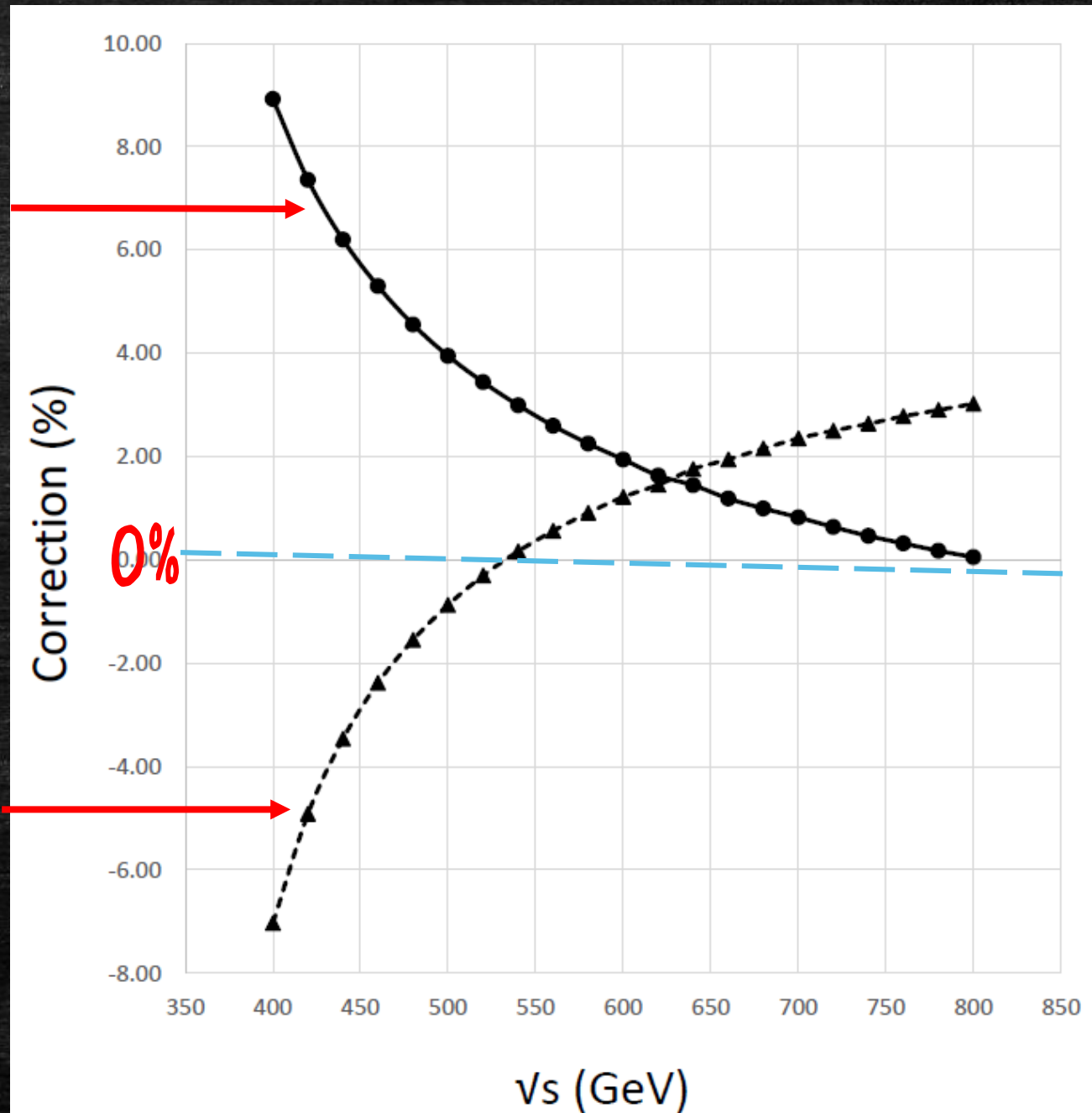
$$\sigma_{NLO;ISR} = (\sigma_{Loop} - \sigma_{Tree}\delta_{ISL}) + \tilde{\sigma}_{Soft}^{final} + \sigma_{Hard} + \sigma_{ISR}$$

$$\delta_{ISL} = \frac{2\alpha}{\pi} \left( - (L - 1) \log \frac{m_e}{\lambda} - \frac{1}{4} L^2 + \frac{3}{4} L + \frac{\pi^2}{3} - 1 \right)$$

# ISR re-summation

$$\delta_{weak} = \frac{\sigma_{NLO;ISR} - \sigma_{ISR}}{\sigma_{ISR}}$$

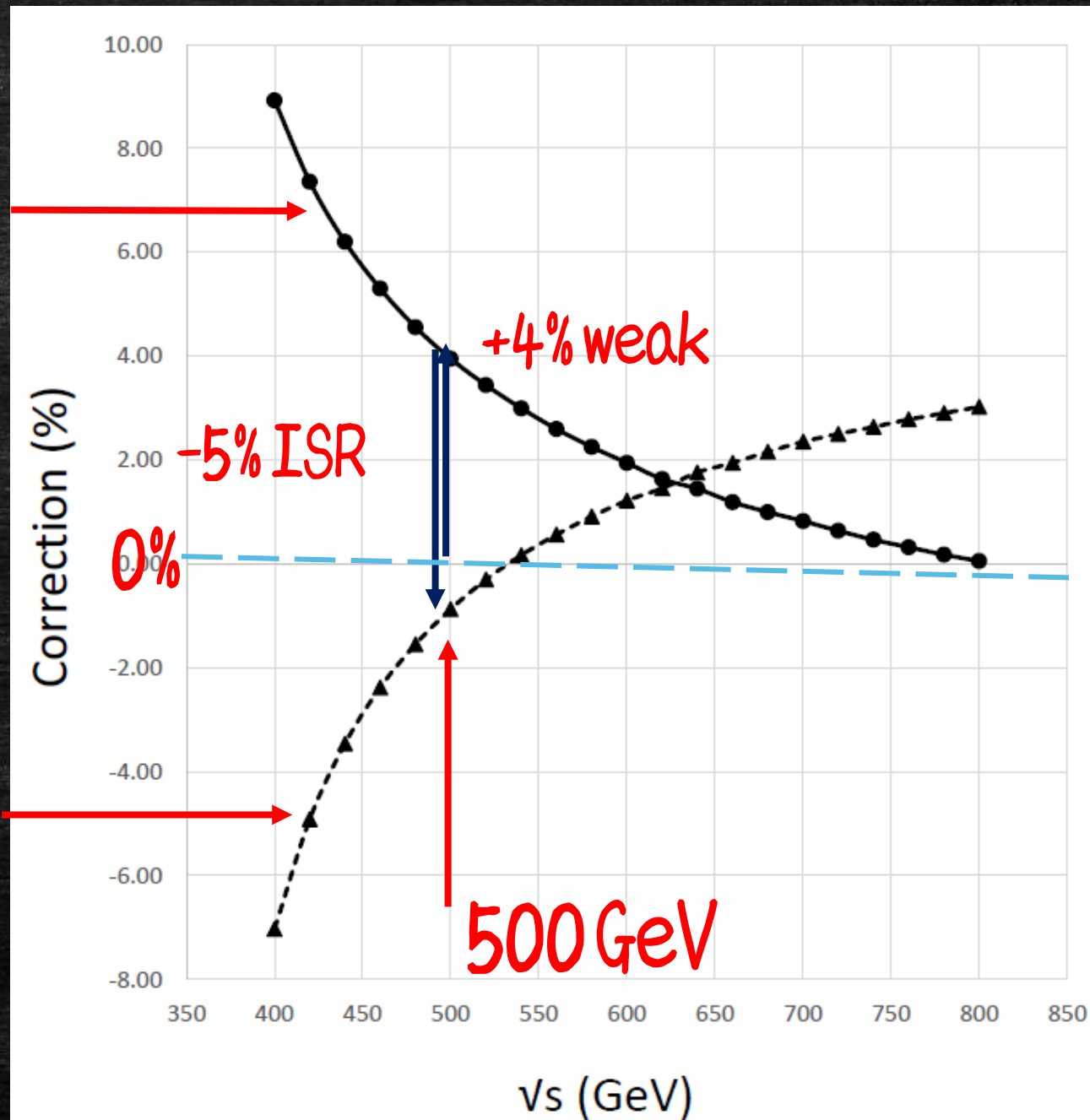
$$\delta_{NLO} = \frac{\sigma_{NLO;fixed} - \sigma_{Tree}}{\sigma_{Tree}}$$



# ISR re-summation

$$\delta_{weak} = \frac{\sigma_{NLO;ISR} - \sigma_{ISR}}{\sigma_{ISR}}$$

$$\delta_{NLO} = \frac{\sigma_{NLO;fixed} - \sigma_{Tree}}{\sigma_{Tree}}$$



# ISR re-summation

## Improved Born

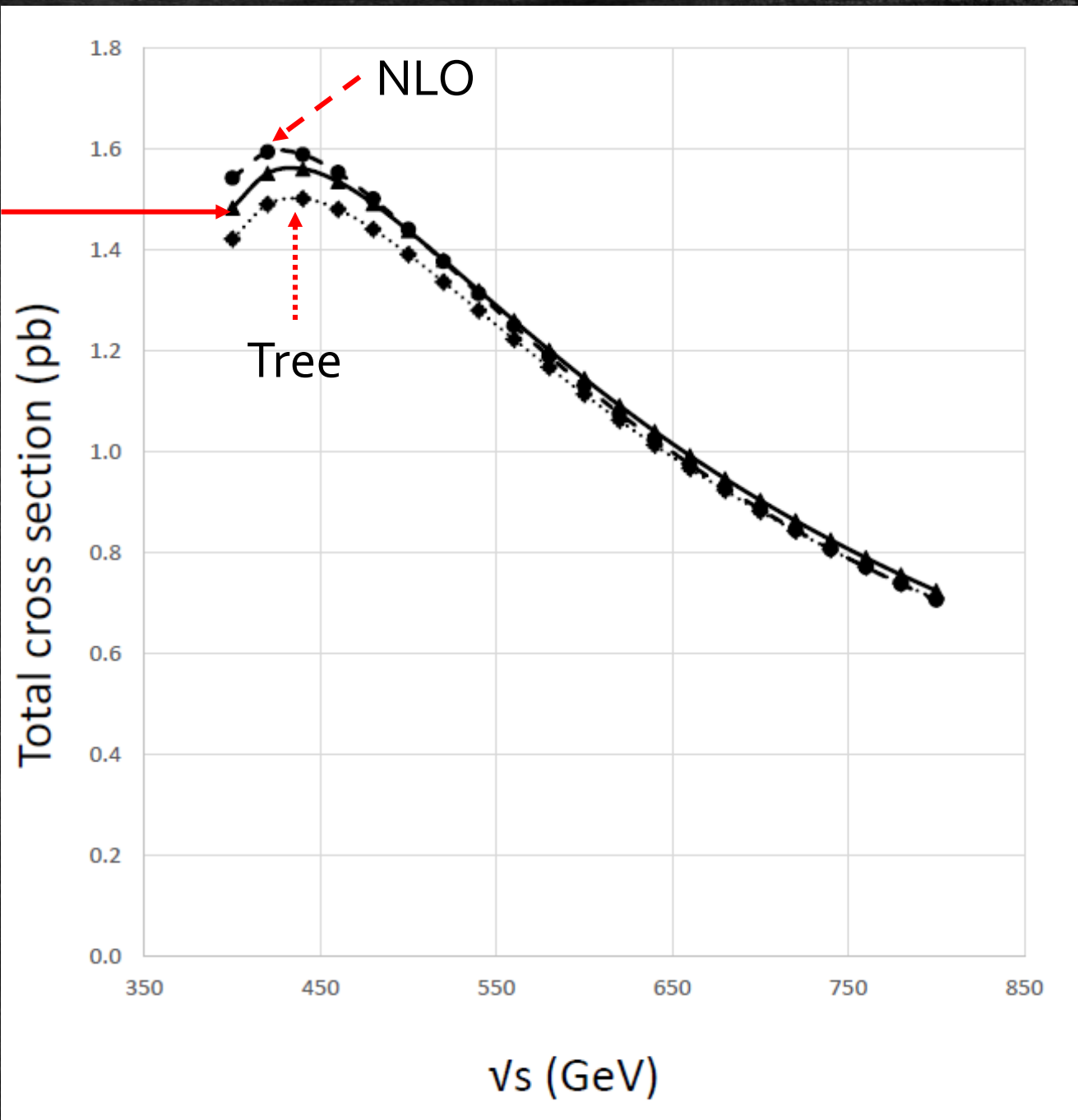
$$\sigma_{imp} = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 D(x_1, s) D(x_2, s)$$

$$\sigma_{Tree}(\alpha(sx_1x_2); sx_1x_2)$$

Improved  
Born

$$\alpha(|q^2|) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log\left(\frac{|q^2|}{\mu^2}\right)}$$

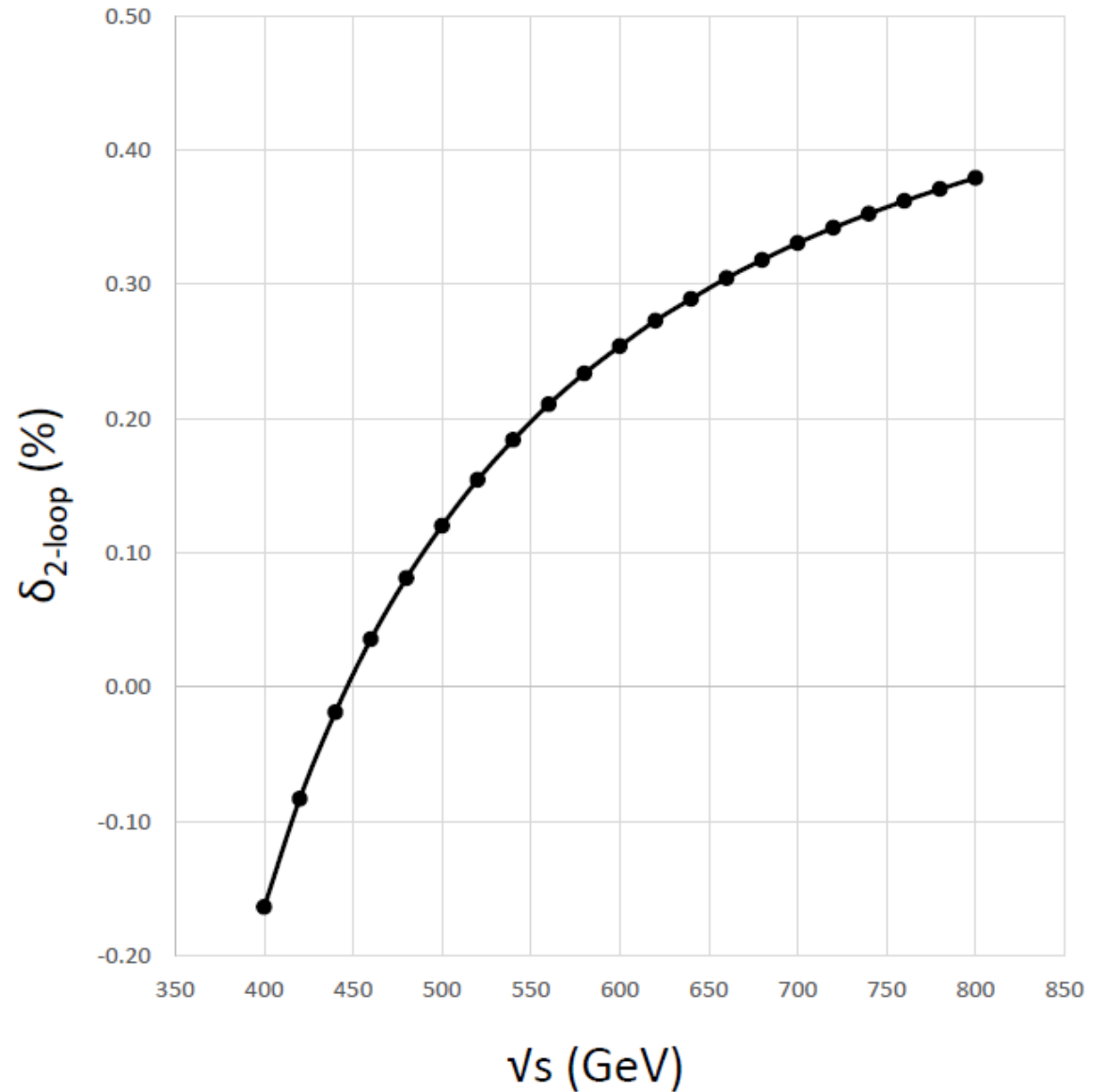
$$\sin^2 \theta_W = \frac{\pi \alpha(|q^2|)}{\sqrt{2} G_F m_W^2}$$





# ISR re-summation

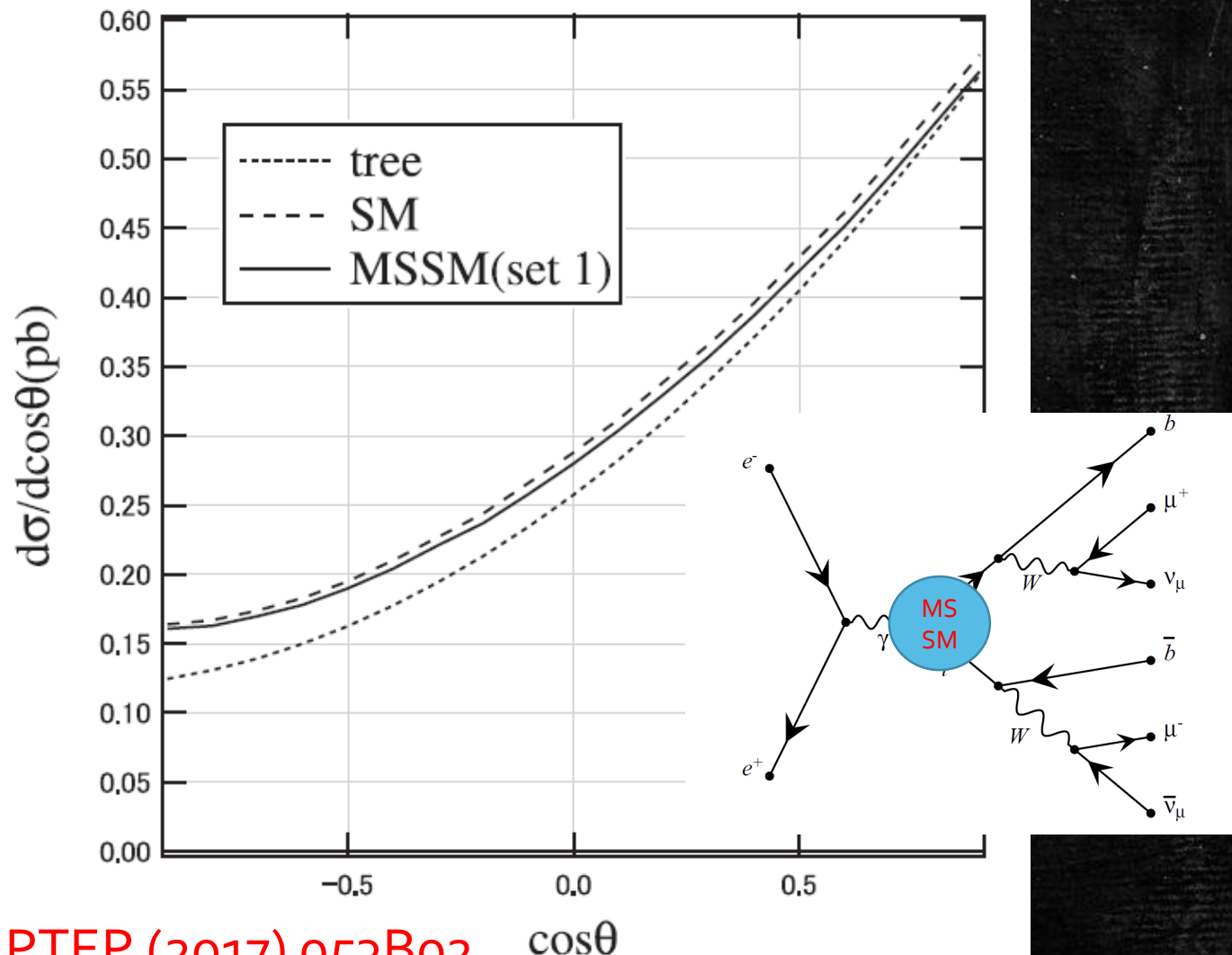
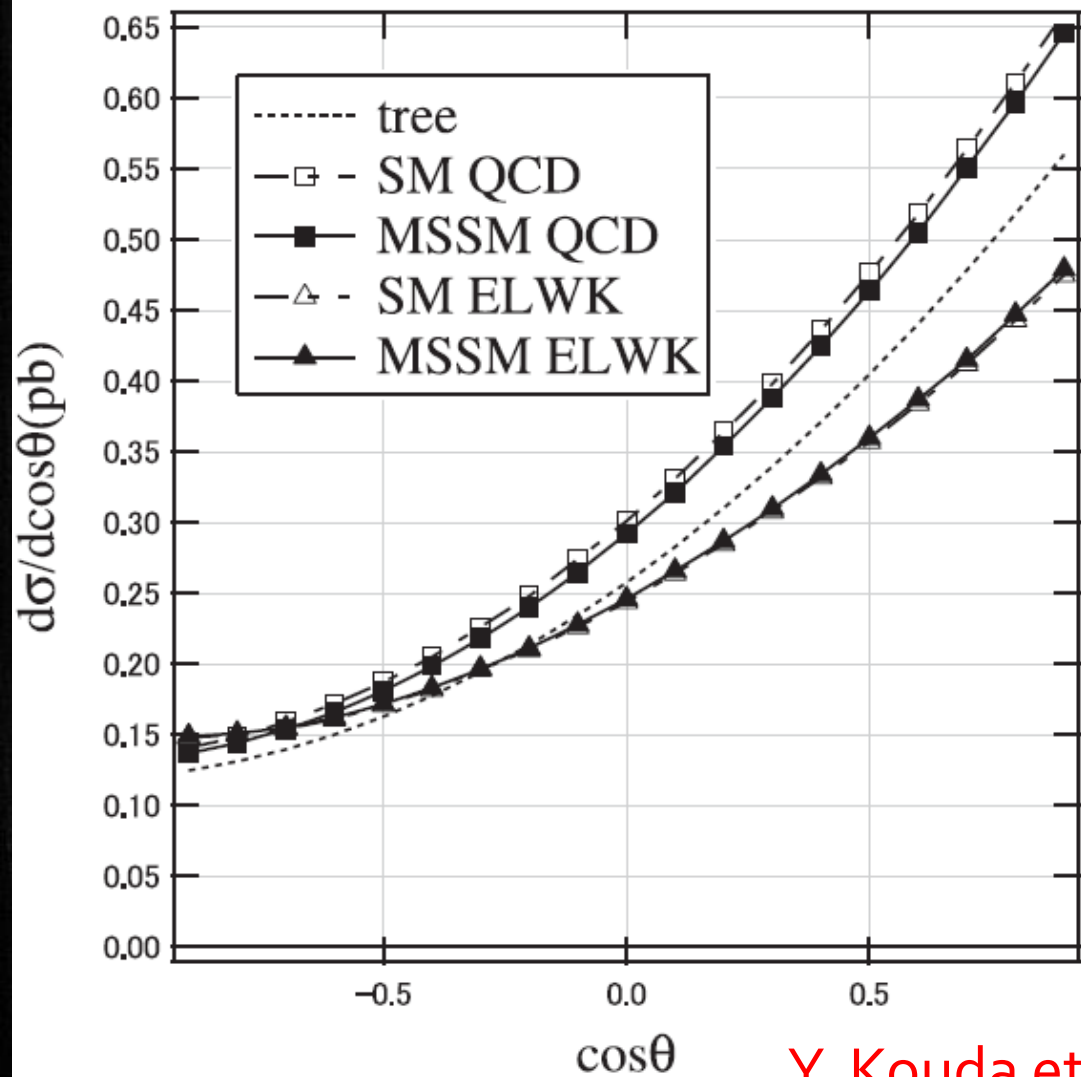
$$\delta_{2-loop} = \frac{\sigma_{ISR} - \sigma_{ISR}^{(1)}}{\sigma_{ISR}^{(1)}}$$



Top-pair:  
MSSM

# MSSM

$\sqrt{s}=500\text{GeV}$  no beam polarization



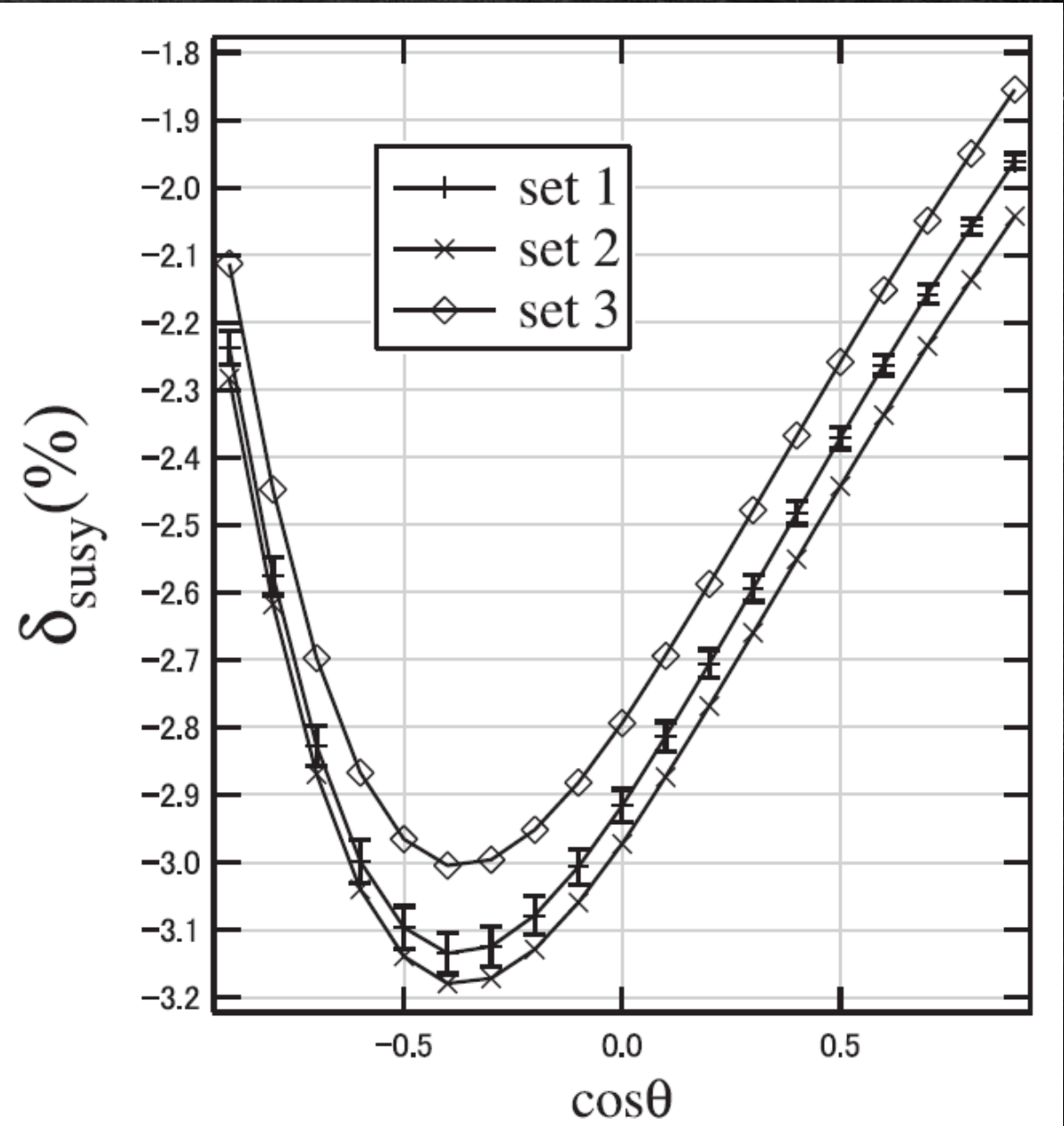
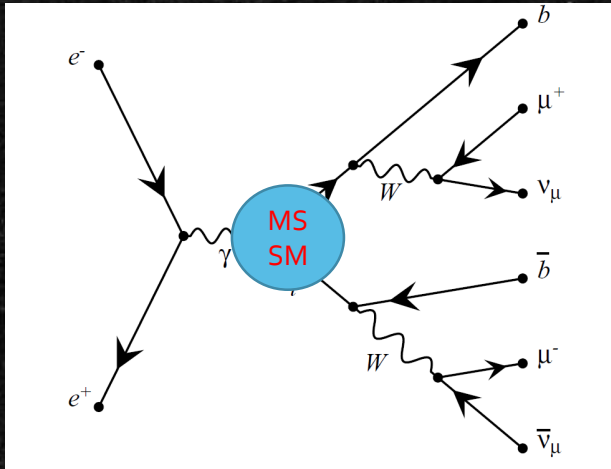
Y. Kouda et al, PTEP (2017) 053B02

# MSSM

$\sqrt{s}=500\text{GeV}$  no beam polarization

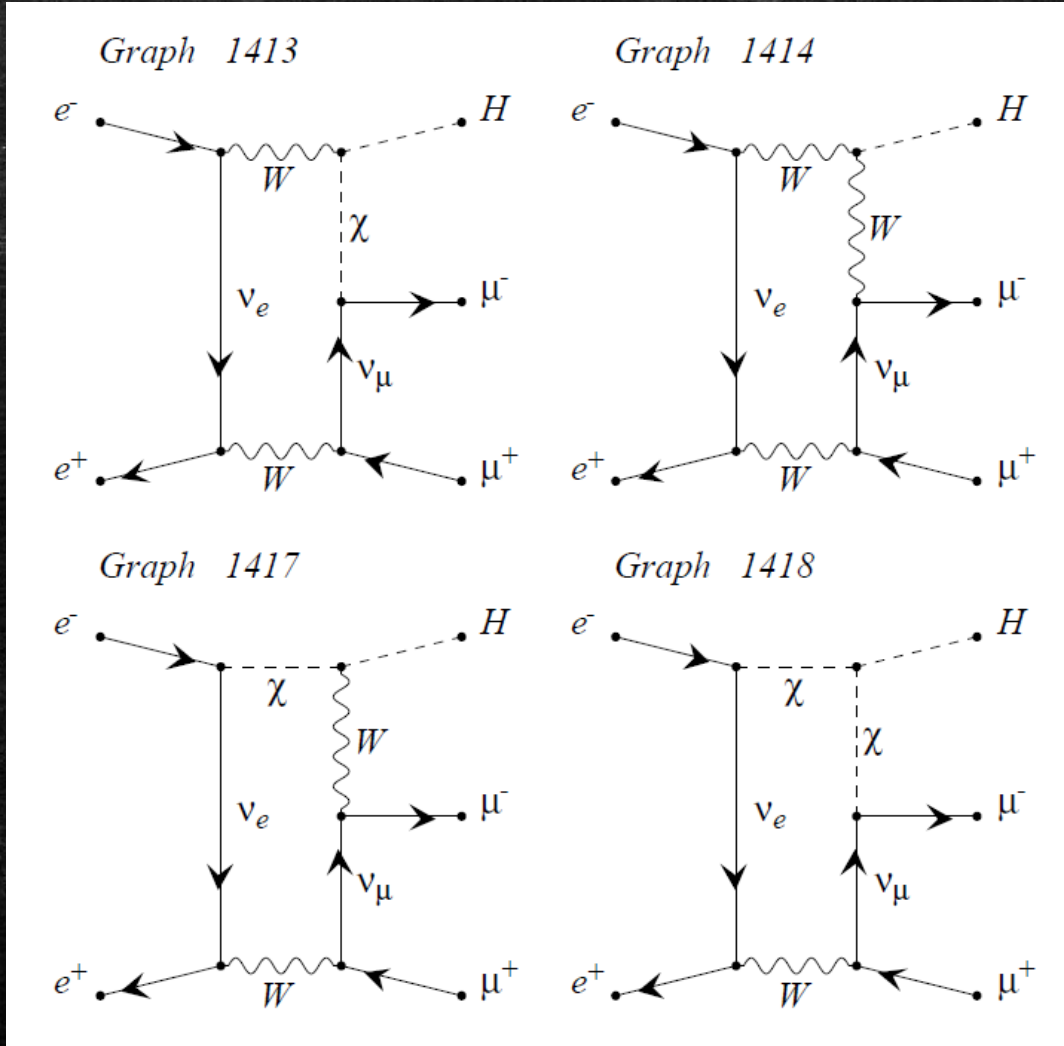
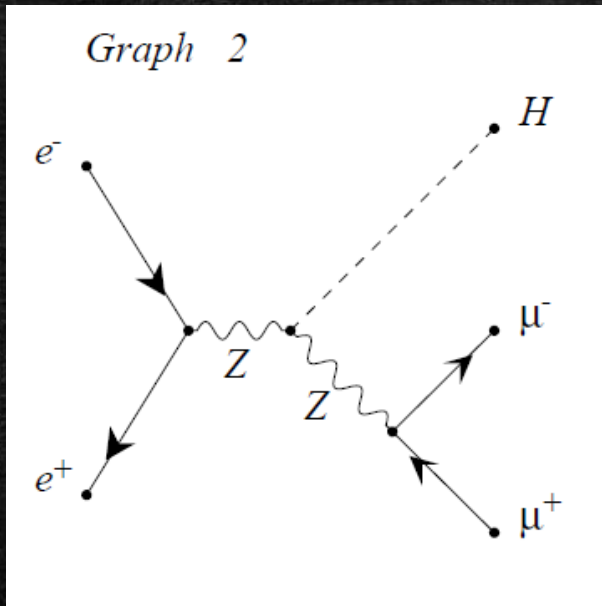
$$\delta_{\text{susy}}^G \equiv \frac{d\sigma_{\text{1loop}}^{\text{MSSM,G}} - d\sigma_{\text{1loop}}^{\text{SM,G}}}{d\sigma_{\text{tree}}}$$

$G = (\text{ELWK or QCD})$



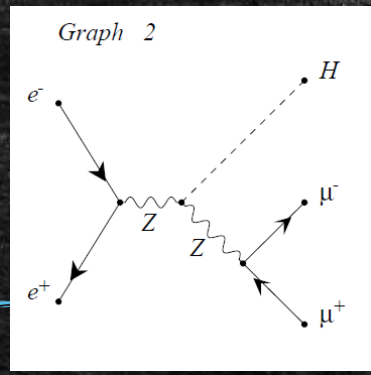
Recent results:  
Higgs productions

# Recent results: $e^+e^- \rightarrow H\mu^+\mu^-$

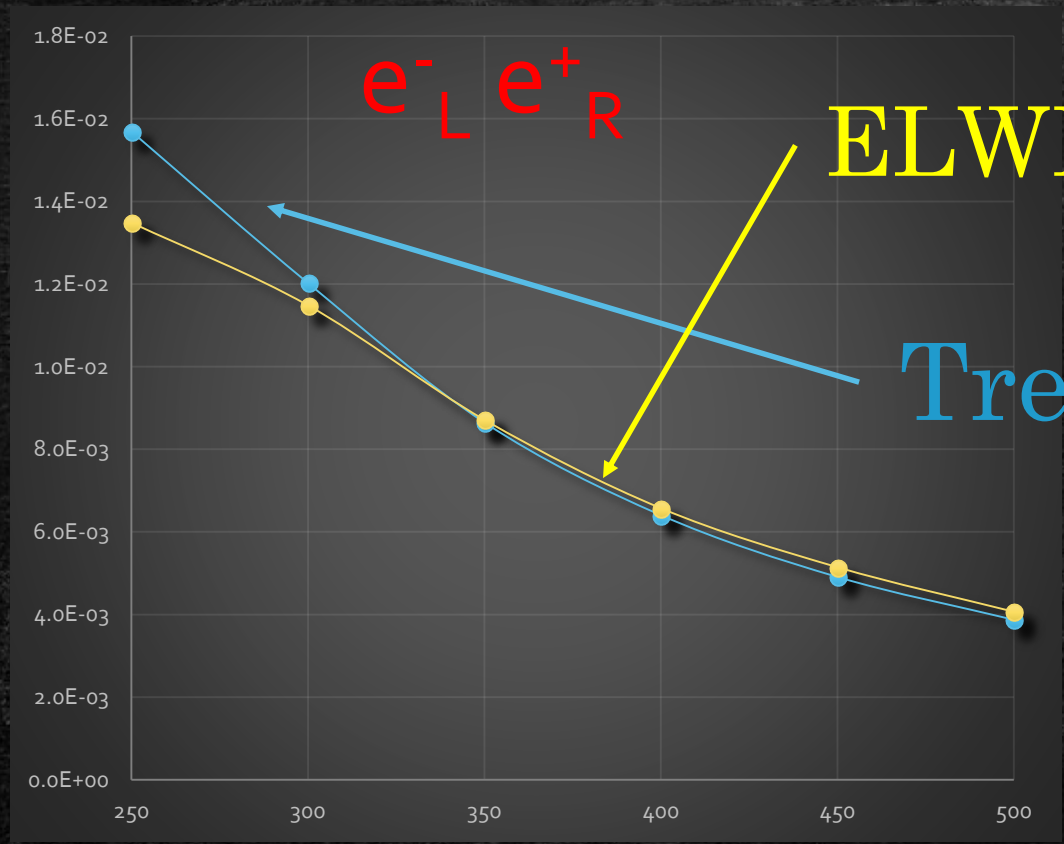


2235 loop diagrams

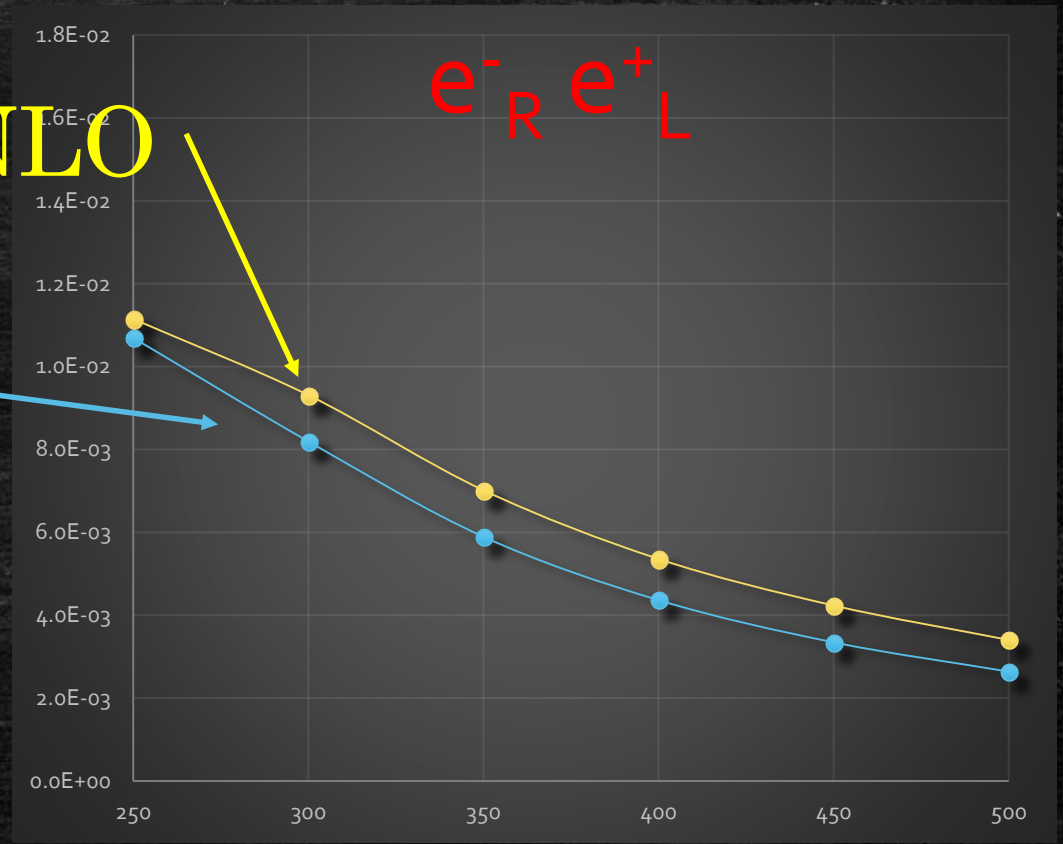
# Recent results: $e^+e^- \rightarrow H\mu^+\mu^-$



Cross section (pb)

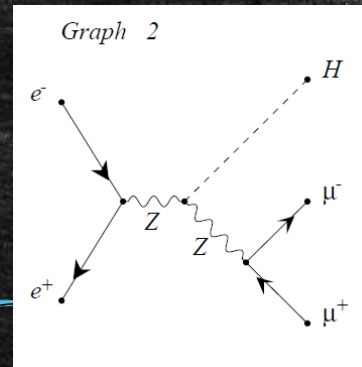


$\sqrt{s}$  GeV



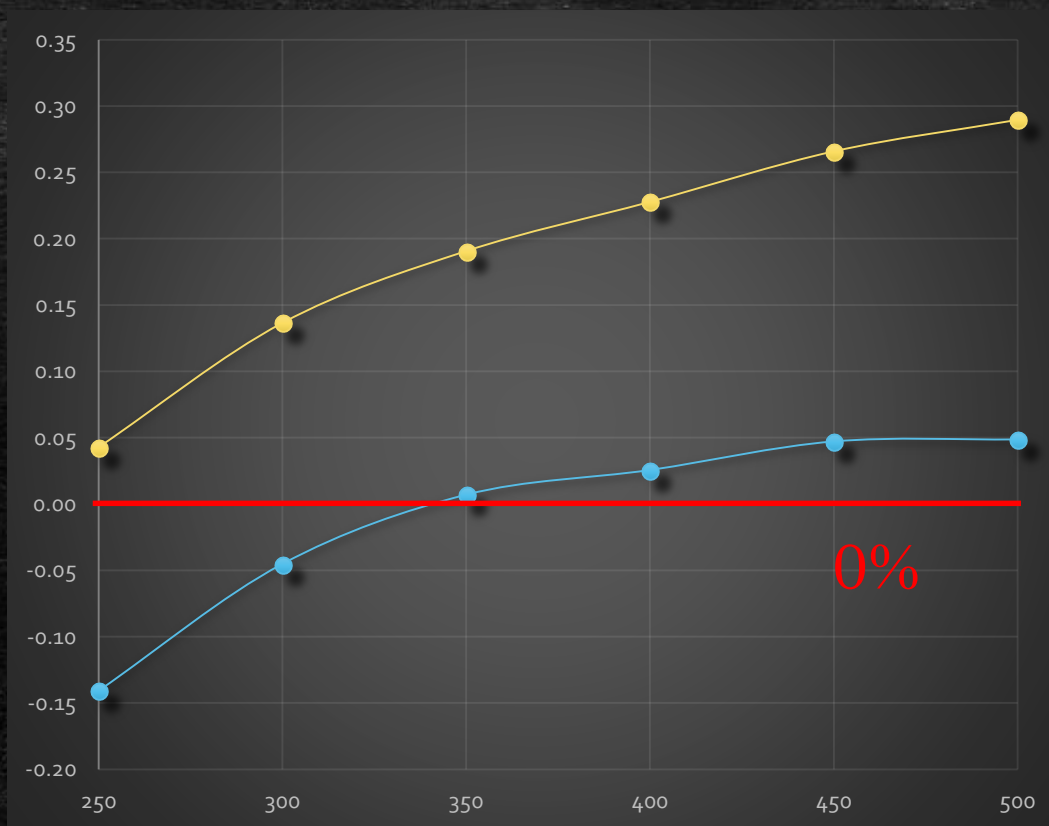
$\sqrt{s}$  GeV

# Recent results: $e^+e^- \rightarrow H\mu^+\mu^-$



$$\delta_{\text{NLO}} = (\sigma_{\text{NLO}} - \sigma_{\text{Tree}}) / (\sigma_{\text{NLO}} + \sigma_{\text{Tree}})$$

$\delta_{\text{NLO}}$



$e^-_R e^+_L$

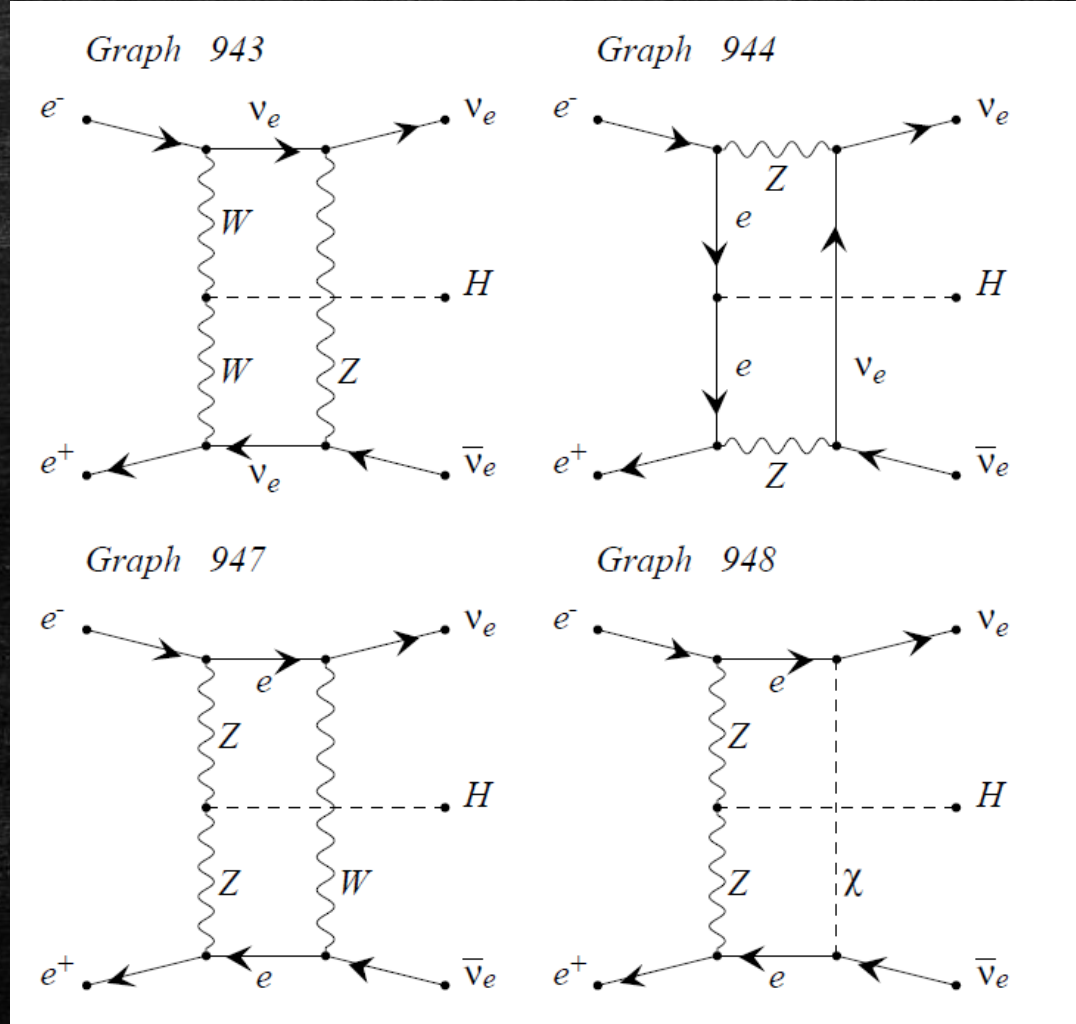
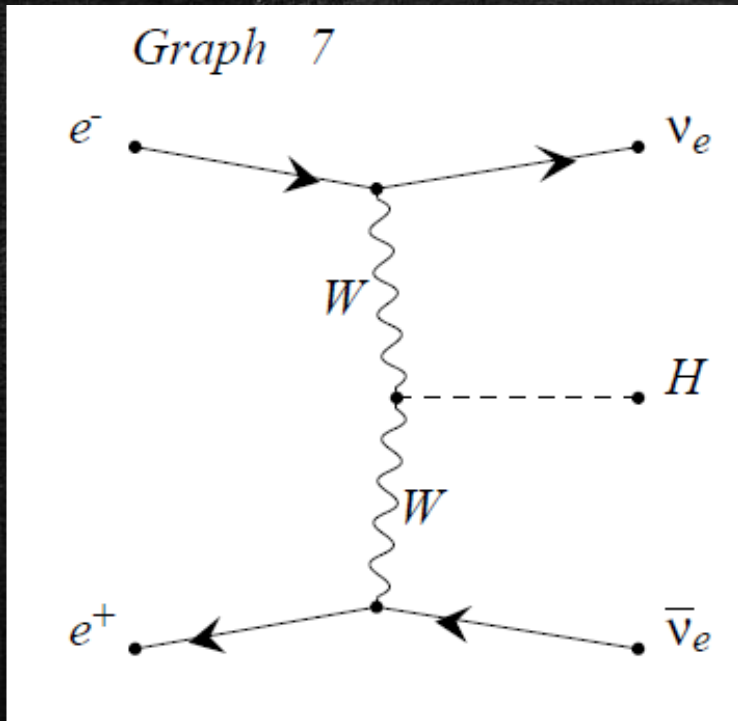
$e^-_L e^+_R$

0%

$\sqrt{s}$  GeV

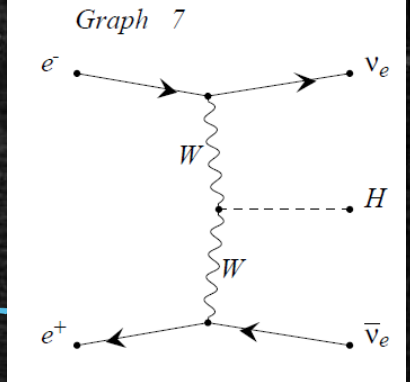


# Recent results: $e^+e^- \rightarrow H\bar{\nu}_e\nu_e$

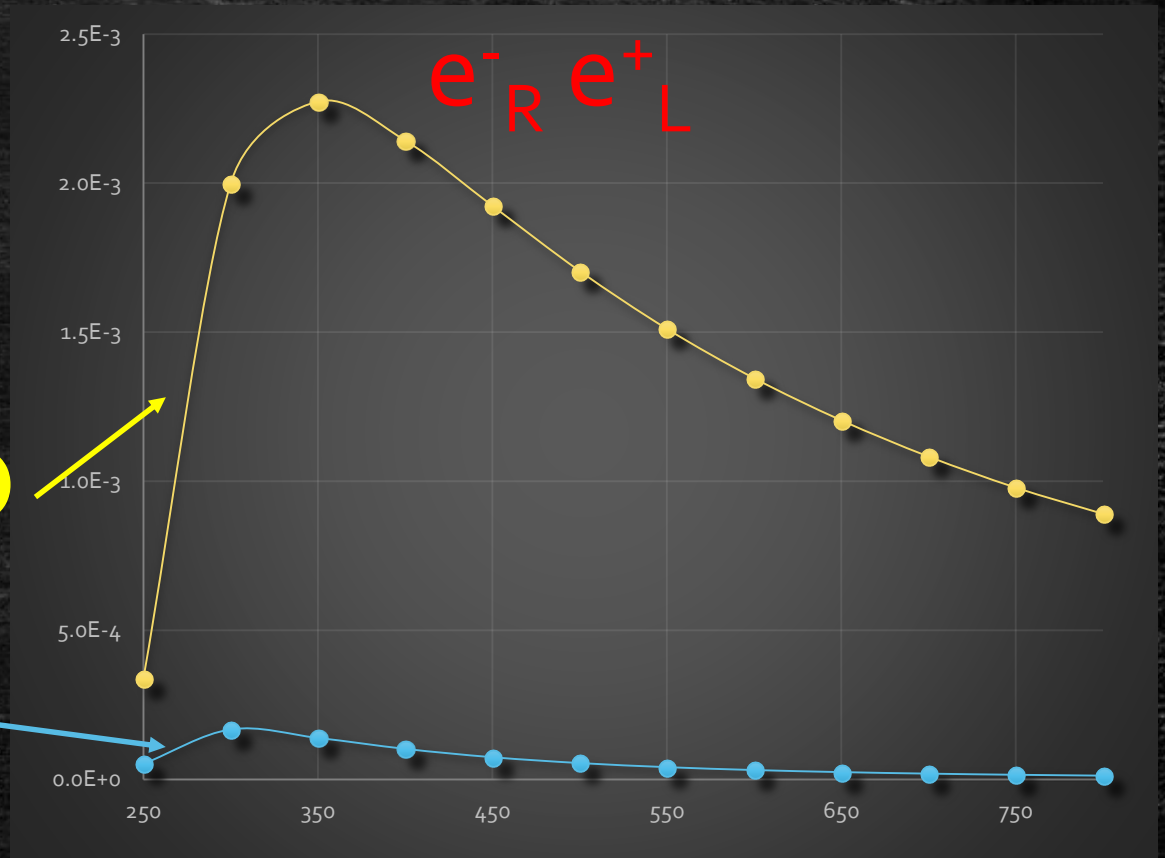
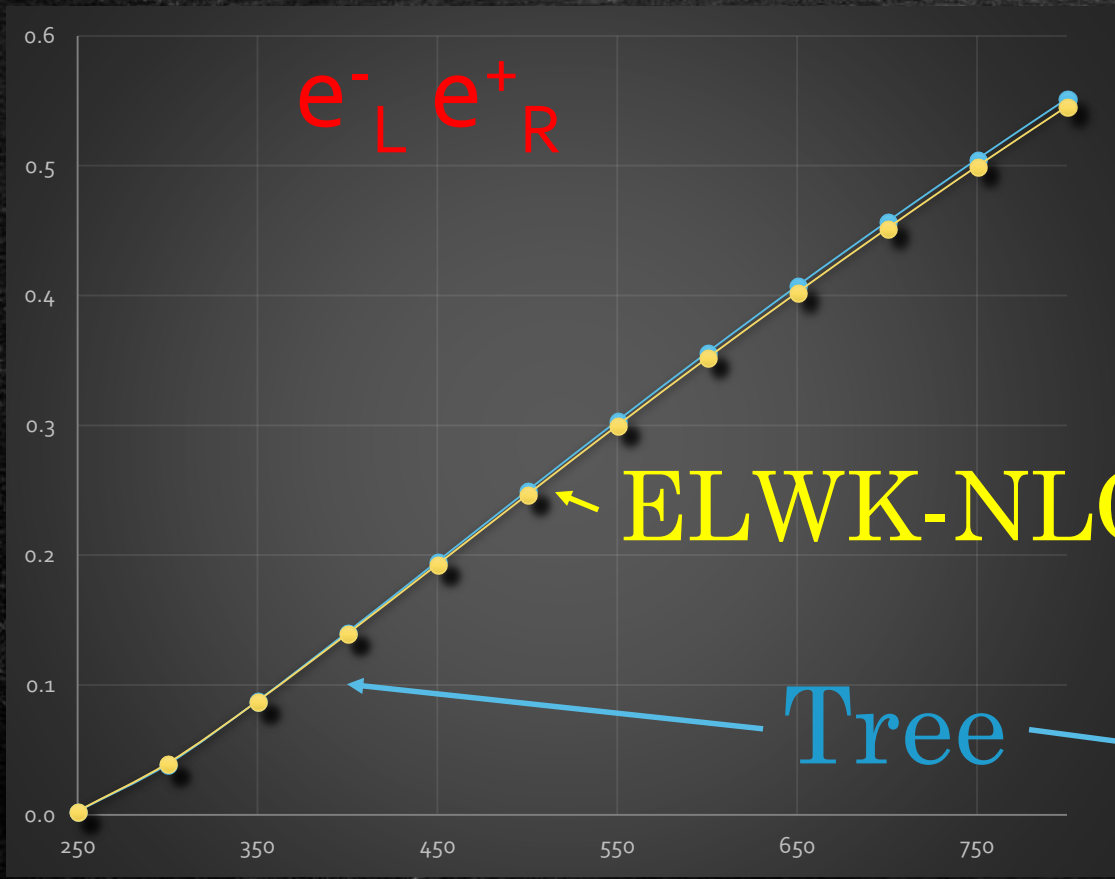


1350 loop diagrams

# Recent results: $e^+e^- \rightarrow H\bar{\nu}_e\nu_e$



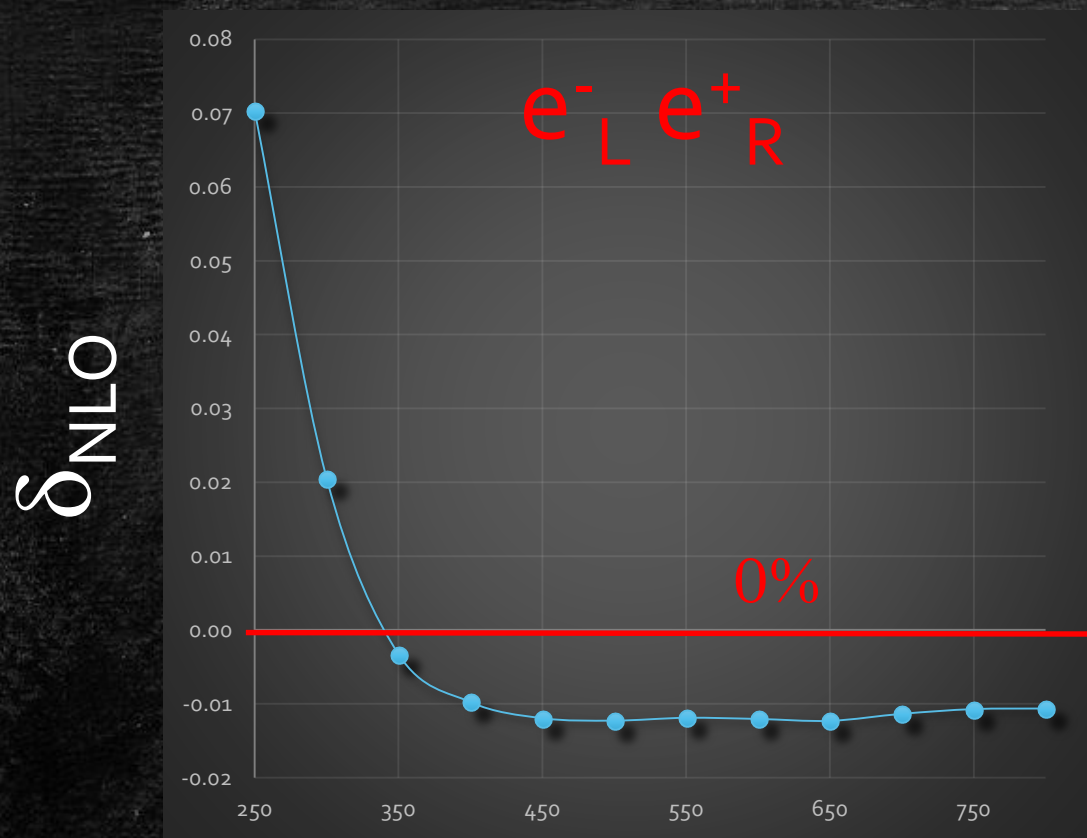
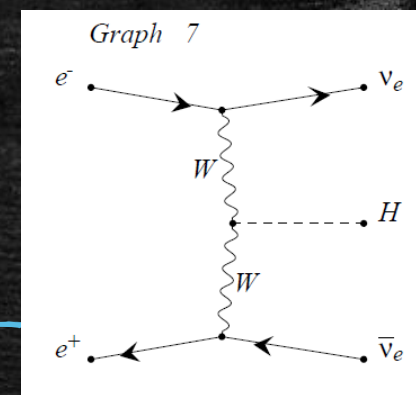
Cross section (pb)



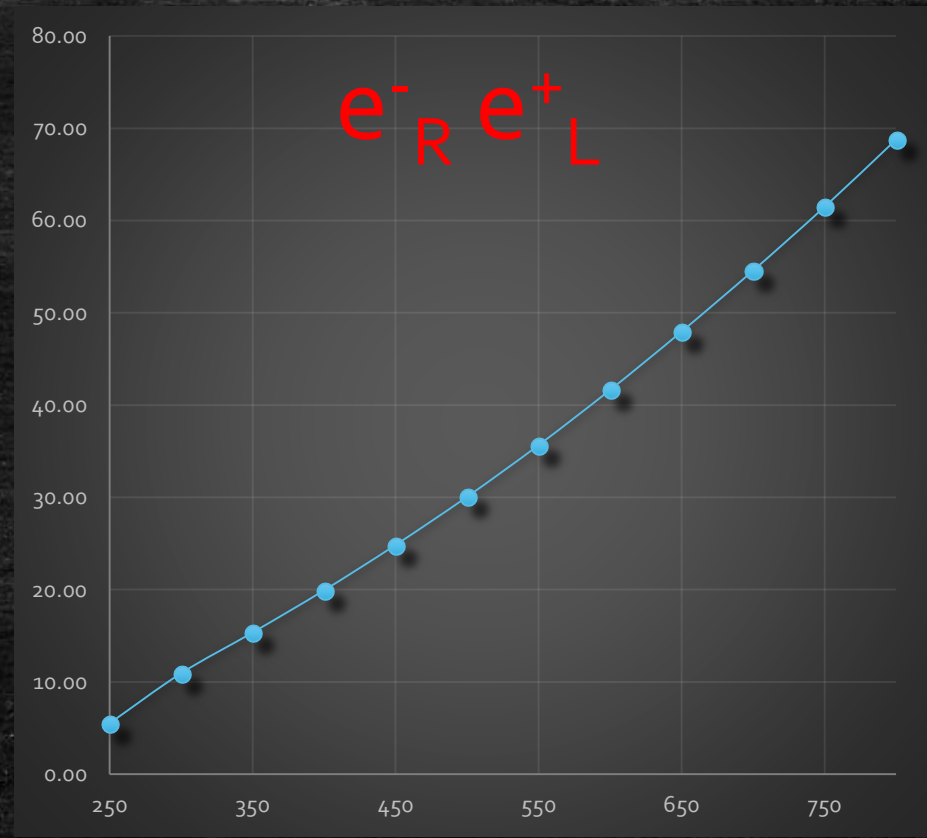
$\sqrt{s}$  GeV

$\sqrt{s}$  GeV

# Recent results: $e^+e^- \rightarrow H\bar{\nu}_e \nu_e$



$\sqrt{s}$  GeV



$\sqrt{s}$  GeV

Summary

# Summary:

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.What is GRACE system?

.Full  $\mathcal{O}(\alpha)$  electroweak corrections on SM/MSSM w/ Beam pol.

.Recent results:

. $e^+e^- \rightarrow tt \rightarrow bbffff \times \psi$ 's

.Z-Higgs production / W-fusion

Goal: Precision Control  $\approx 0.1\%$  @ ILC energies

BP

Set 1				Set 2				Set 3			
$\tilde{\chi}_1^+$	$\tilde{\chi}_2^+$			$\tilde{\chi}_1^+$	$\tilde{\chi}_2^+$			$\tilde{\chi}_1^+$	$\tilde{\chi}_2^+$		
419.9	620.5			508.1	636.8			467.5	626.7		
$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$
218.4	420.0	603.7	620.2	277.9	508.5	603.4	637.1	242.8	467.6	603.6	626.7
$\tilde{\ell}_1$	$\tilde{\ell}_2$	$\tilde{\nu}_\ell$		$\tilde{\ell}_1$	$\tilde{\ell}_2$	$\tilde{\nu}_\ell$		$\tilde{\ell}_1$	$\tilde{\ell}_2$	$\tilde{\nu}_\ell$	
352.5	358.0	349.4		317.8	323.3	313.8		322.8	328.3	318.9	
$\tilde{\tau}_1$	$\tilde{\tau}_2$	$\tilde{\nu}_\tau$		$\tilde{\tau}_1$	$\tilde{\tau}_2$	$\tilde{\nu}_\tau$		$\tilde{\tau}_1$	$\tilde{\tau}_2$	$\tilde{\nu}_\tau$	
228.4	336.3	277.9		283.9	377.1	327.4		320.1	405.3	359.6	
$\tilde{u}_1$	$\tilde{u}_2$	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{u}_1$	$\tilde{u}_2$	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{u}_1$	$\tilde{u}_2$	$\tilde{d}_1$	$\tilde{d}_2$
1719	1739	1740	1740	1720	1739	1740	1741	1720	1739	1740	1741
$\tilde{t}_1$	$\tilde{t}_2$	$\tilde{b}_1$	$\tilde{b}_2$	$\tilde{t}_1$	$\tilde{t}_2$	$\tilde{b}_1$	$\tilde{b}_2$	$\tilde{t}_1$	$\tilde{t}_2$	$\tilde{b}_1$	$\tilde{b}_2$
344.0	2078	899.9	2060.9	1802	2244	1998	2063	279.6	2078	800.0	2061
$\theta_\tau$	$\theta_b$	$\theta_t$		$\theta_\tau$	$\theta_b$	$\theta_t$		$\theta_\tau$	$\theta_b$	$\theta_t$	
0.7970	1.556	1.4502		0.8150	1.376	0.8533		0.8175	1.557	1.456	
$M_1$	$M_2$	$M_3$		$M_1$	$M_2$	$M_3$		$M_1$	$M_2$	$M_3$	
220.0	435.0	2000		280.0	540.0	1500		244.5	489.0	2000	
$\mu = 600, \tan \beta = 30$				$\mu = 600, \tan \beta = 30$				$\mu = 600, \tan \beta = 30$			

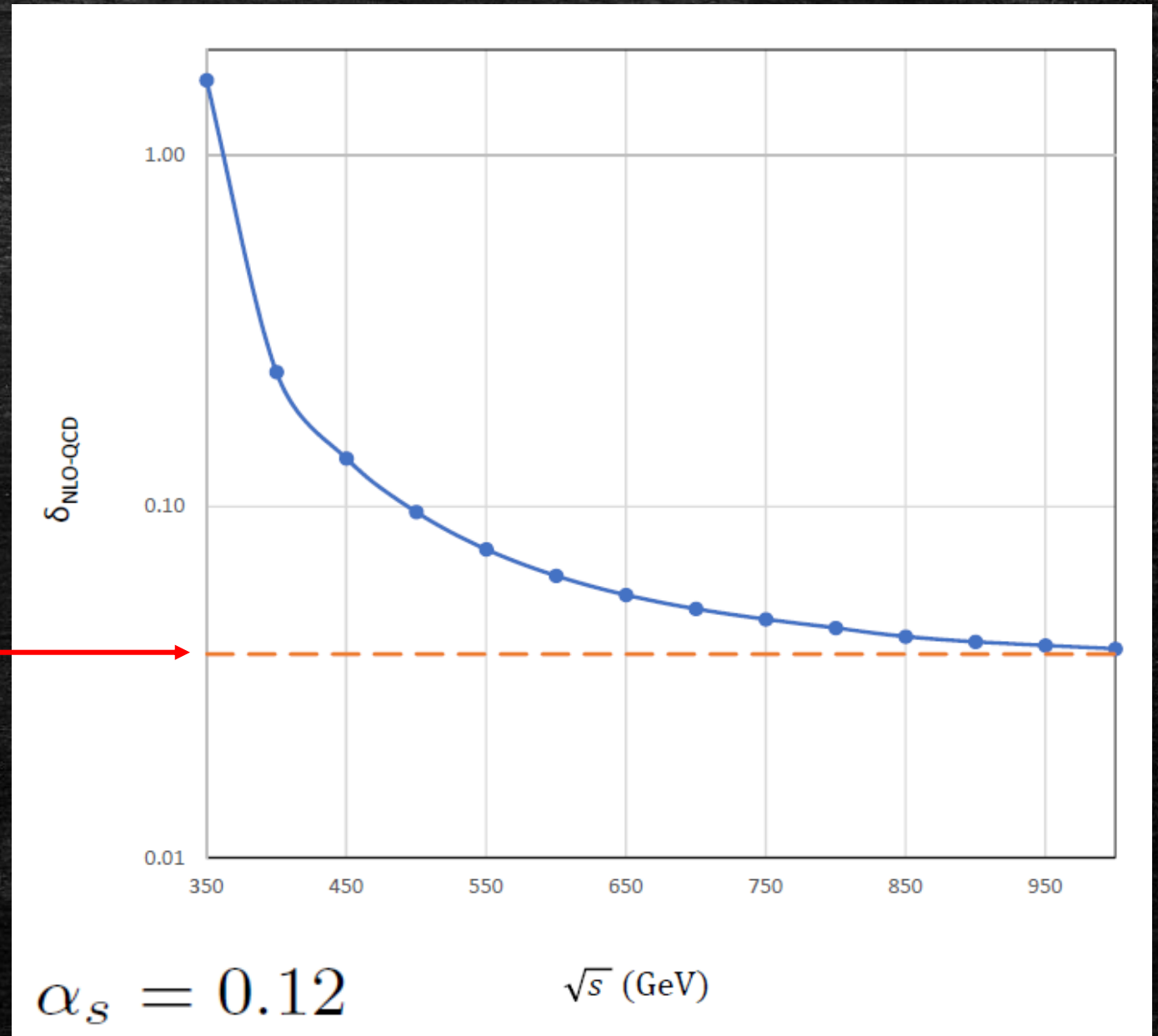
QCD  
corrections



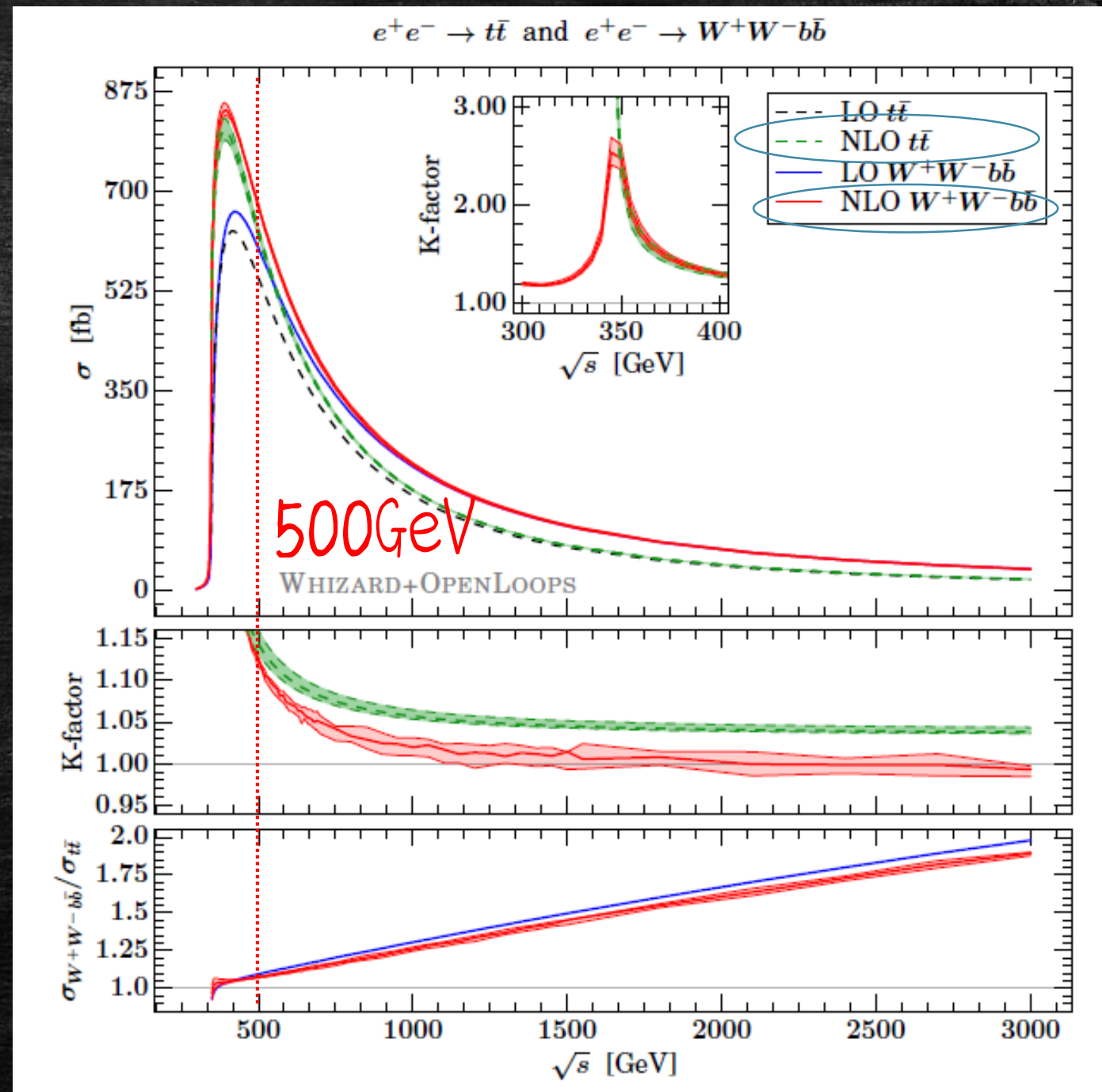
# QCD Correction

Total cross sections w/  
a QCD correction on a  
**t-pair production only**

$$\alpha_s/\pi \simeq 3.8\%$$



# NLO QCD correction w/ double-pole approx



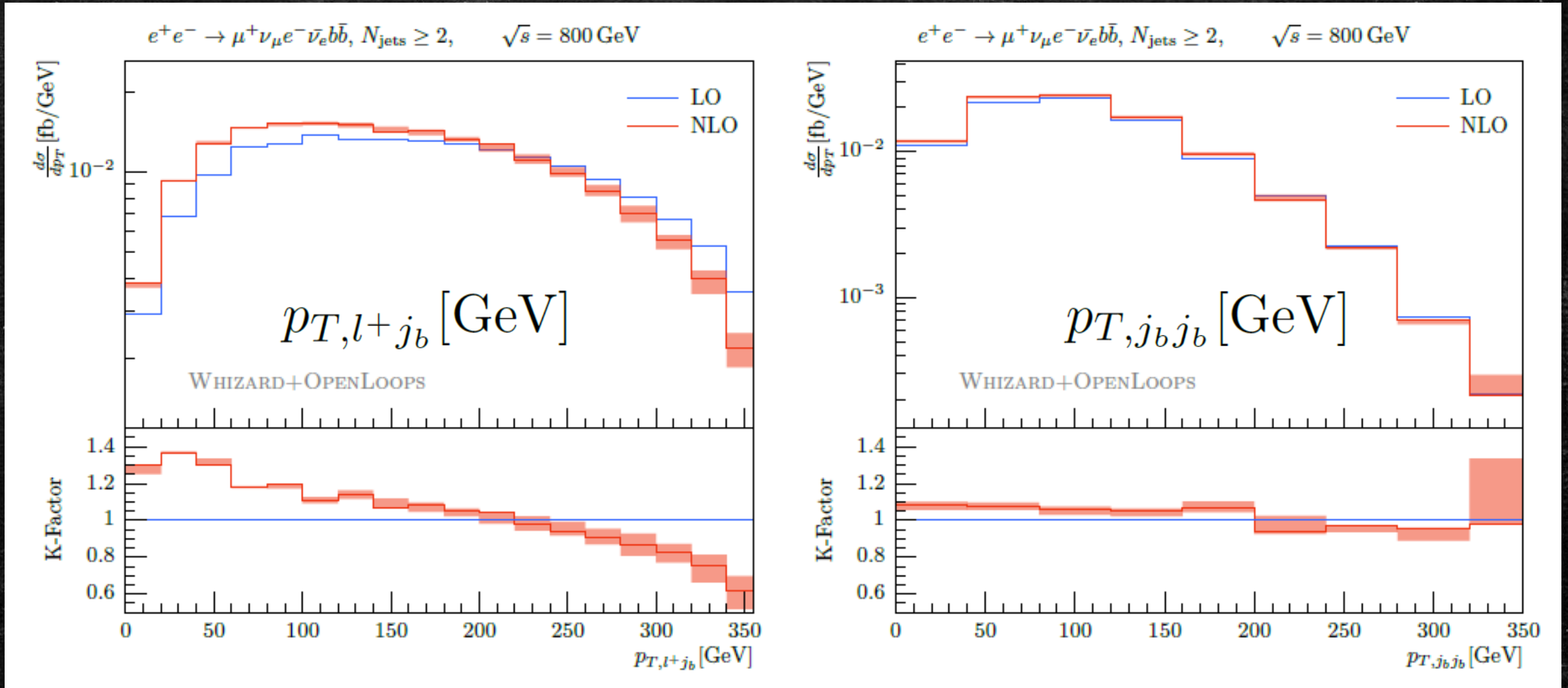
# NLO QCD correction w/ double-pole approx.

$\sqrt{s}$ [GeV]	$e^+e^- \rightarrow t\bar{t}$			$e^+e^- \rightarrow W^+W^-b\bar{b}$		
	$\sigma^{\text{LO}}$ [fb]	$\sigma^{\text{NLO}}$ [fb]	K-factor	$\sigma^{\text{LO}}$ [fb]	$\sigma^{\text{NLO}}$ [fb]	K-factor
500	548.4	$627.4^{+1.4\%}_{-0.9\%}$	1.14	600.7	$675.1^{+0.4\%}_{-0.8\%}$	1.12
800	253.1	$270.9^{+0.8\%}_{-0.4\%}$	1.07	310.2	$320.7^{+1.1\%}_{-0.7\%}$	1.03
1000	166.4	$175.9^{+0.7\%}_{-0.3\%}$	1.06	217.2	$221.6^{+1.1\%}_{-1.0\%}$	1.02
1400	86.62	$90.66^{+0.6\%}_{-0.2\%}$	1.05	126.4	$127.9^{+0.7\%}_{-1.5\%}$	1.01
3000	19.14	$19.87^{+0.5\%}_{-0.2\%}$	1.04	37.89	$37.63^{+0.4\%}_{-0.9\%}$	0.993

		$\sqrt{s} = 800 \text{ GeV}$			$\sqrt{s} = 1500 \text{ GeV}$		
$P(e^-)$	$P(e^+)$	$\sigma^{\text{LO}}$ [fb]	$\sigma^{\text{NLO}}$ [fb]	K-factor	$\sigma^{\text{LO}}$ [fb]	$\sigma^{\text{NLO}}$ [fb]	K-factor
0%	0%	253.7	272.8	1.075	75.8	79.4	1.049
-80%	0%	176.5	190.0	1.077	98.3	103.1	1.049
80%	0%	176.5	190.0	1.077	53.2	55.9	1.049
-80%	30%	420.8	452.2	1.074	124.9	131.0	1.048
-80%	60%	510.7	548.7	1.074	151.6	158.9	1.048
80%	-30%	208.4	224.5	1.077	63.0	66.1	1.049
80%	-60%	240.3	258.9	1.077	72.7	76.3	1.049

B.C. Nejad, et al.,  
JHEP12 (2016) 075

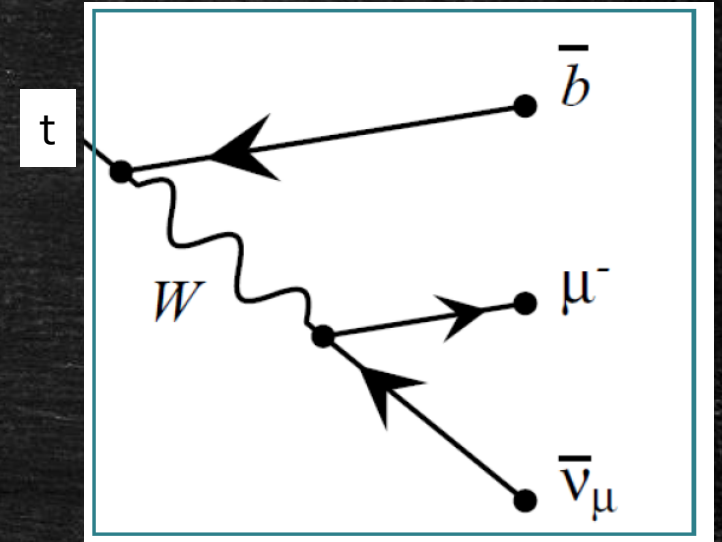
# NLO QCD correction w/ double-pole approx.



# NLO QCD correction for top decay

Semi leptonic

	OCSM
$\Gamma_{\text{lept}}^{(0)} / \text{GeV}$	0.1610645(3)
$\delta_{\text{lept}}^{(1), \text{QCD}} / \%$	-9.379(3)
$\delta_{\text{lept}}^{(1), \text{EW}} / \%$	1.335(2)
$\Gamma_{\text{lept}}^{\text{NLO}} / \text{GeV}$	0.148108(6)



Hadronic

	OCSM
$\Gamma_{\text{hadr}}^{(0)} / \text{GeV}$	0.48319351(5)
$\delta_{\text{hadr}}^{(1), \text{QCD}} / \%$	-5.58(2)
$\delta_{\text{hadr}}^{(1), \text{EW}} / \%$	1.2896(5)
$\Gamma_{\text{hadr}}^{\text{NLO}} / \text{GeV}$	0.46248(9)

