Optimal Variable with Detector Effects for the Top Electroweak Couplings Study

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Outline

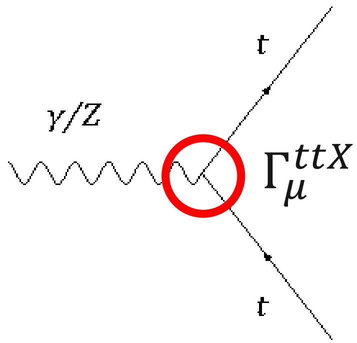
- Introduction
- Setup of simulation
- Signal Reconstruction
- Analysis
- Summary

Introduction

The ttZ/γ couplings

The ttZ/γ couplings are important probes for new physics

(e.g.) Predicted deviation of F or g from SM is ~10 % in composite models.



$$\Gamma_\mu^{ttX}(k^2, q, \bar{q}) = ie\Big[\gamma_\mu\left(F_{1V}^X(k^2) + \gamma_5 F_{1A}^X(k^2)\right) + \frac{\sigma_{\mu\nu}}{2m_t}(q+\bar{q})^\nu\left(iF_{2V}^X(k^2) + \gamma_5 F_{2A}^X(k^2)\right)\Big]\ (X = Z, \gamma)$$

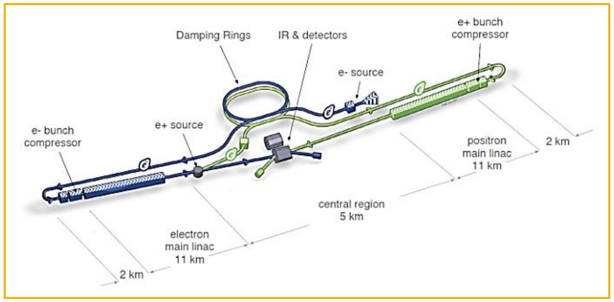
$$(g_L = F_{1V} - F_{1A}, g_R = F_{1V} + F_{1A})$$

- The measurement of the ttZ/γ couplings is difficult in hadron colliders.
- Energy of current lepton colliders (Belle II, etc.) is not enough for $t\bar{t}$ creation.

→ Study at a future lepton collider is needed

Previous Study at the ILC

ILC (International Linear Collider)



The most mature project of a future e^-e^+ collider

Clean data & 250-500 GeV & Polarized beam

→ Suitable for the ttZ/γ measurement

Previous study (Eur.Phys.J. C75 (2015) no.10, 512)

- Signal : Semi-leptonic process at 500 GeV

  $e^-e^+ \to t\bar{t} \to bW^+\bar{b}W^- \to bq\bar{q}\bar{b}l\nu$

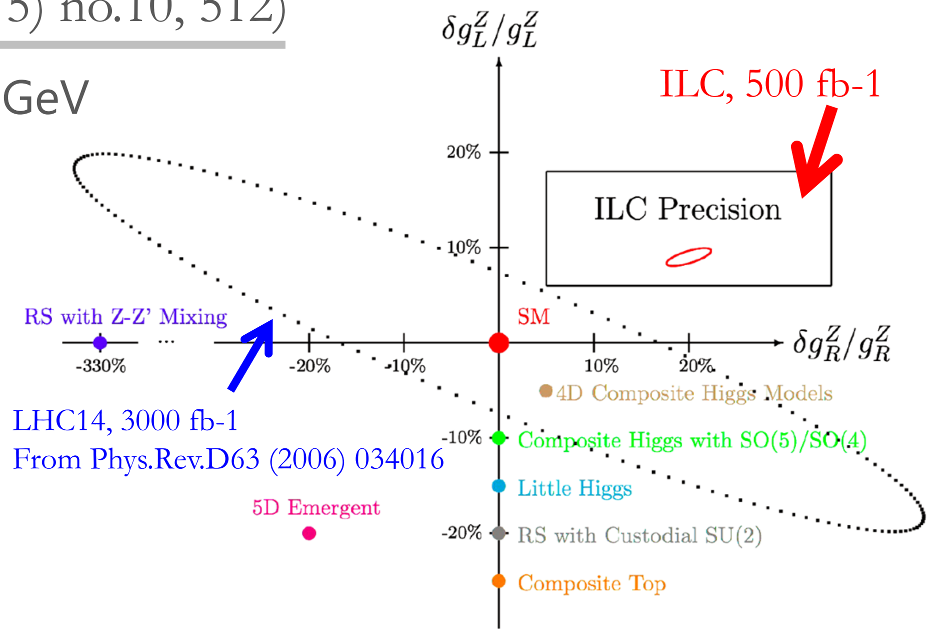
- Beam polarization :

  $(P_{e^-}, P_{e^+}) = (\mp 0.8, \pm 0.3)$

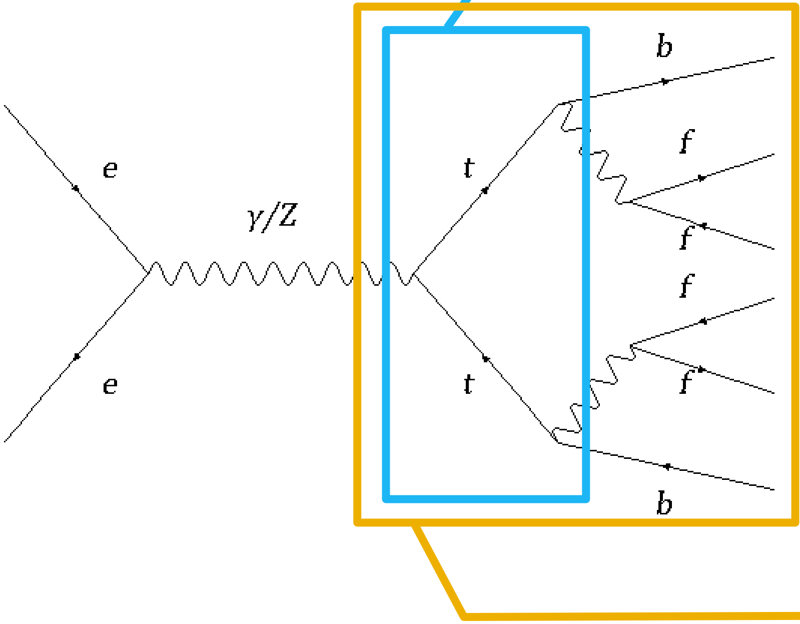
- Observables : A_{FB}, σ

- Parameters : F (form factor) or

  g (coupling constant)



Full Kinematics Analysis



The previous study used A_{FB}, σ

- Obtained from $e^{-}e^{+} \rightarrow t\bar{t}$ process

Decay process has also the information of the ttZ/γ couplings

- Top quark decays before hadronization
- Angular distributions of decay particles depend on the spin of top quark

Full kinematics analysis gives intrinsically higher sensitivities

Full Kinematics Analysis

Matrix element method

Parton level analysis using di-leptonic final state. *arXiv: 1503.04247*

$$
\begin{array}{c}
\begin{array}{cccccccccc}
\mathcal{R}\mathrm{e}\ \delta\tilde{F}_{1V}^{\gamma} & \mathcal{R}\mathrm{e}\ \delta\tilde{F}_{1V}^{Z} & \mathcal{R}\mathrm{e}\ \delta\tilde{F}_{1A}^{\gamma} & \mathcal{R}\mathrm{e}\ \delta\tilde{F}_{1A}^{Z} & \mathcal{R}\mathrm{e}\ \delta\tilde{F}_{2V}^{\gamma} & \mathcal{R}\mathrm{e}\ \delta\tilde{F}_{2V}^{Z} & \mathcal{R}\mathrm{e}\ \delta\tilde{F}_{2A}^{\gamma} & \mathcal{R}\mathrm{e}\ \delta\tilde{F}_{2A}^{Z} & \mathcal{I}\mathrm{m}\ \delta\tilde{F}_{2A}^{\gamma} & \mathcal{I}\mathrm{m}\ \delta\tilde{F}_{2A}^{Z}
\end{array} \\
\begin{bmatrix}
0.0037 & -0.18 & -0.09 & +0.14 & +0.62 & -0.15 & 0 & 0 & 0 & 0 \\
 & 0.0063 & +.14 & -0.06 & -0.13 & +0.61 & 0 & 0 & 0 & 0 \\
 & & 0.0053 & -0.15 & -0.05 & +0.09 & 0 & 0 & 0 & 0 \\
 & & & 0.0083 & +0.06 & -0.04 & 0 & 0 & 0 & 0 \\
 & & & & 0.0105 & -0.19 & 0 & 0 & 0 & 0 \\
 & & & & & 0.0169 & 0 & 0 & 0 & 0 \\
 & & & & & & 0.0068 & -0.15 & 0 & 0 \\
 & & & & & & & 0.0118 & 0 & 0 \\
 & & & & & & & & 0.0069 & -0.17 \\
 & & & & & & & & & 0.0100
\end{bmatrix}
\end{array}
$$

500 GeV, 500 fb^{-1}, $(P_{e^-}, P_{e^+}) = (\pm 0.8, \mp 0.3)$

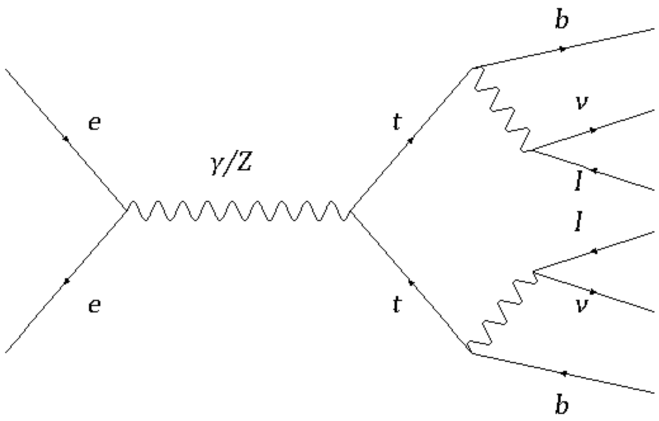
- 10 form factors can be fitted simultaneously at percent level.
- No detector effects, background, hadronization taken account of

Goal of This Study

Goal of this study

Development of the search technique for the anomalous ttZ/γ couplings with the full kinematics analysis based on the ILD full simulation.

- Reconstruction of the di-leptonic process;
  $e^-e^+ \to t\bar{t} \to bW^+\bar{b}W^- \to bl^+\nu\bar{b}l^-\bar{\nu}$
  - The most observables can be obtained



- Analysis with the matrix element method
- Analysis with the binned likelihood method

Setup of simulation

Parameter Setup

Event generator : WHIZARD, Pythia

Detector simulation : Mokka, Marlin

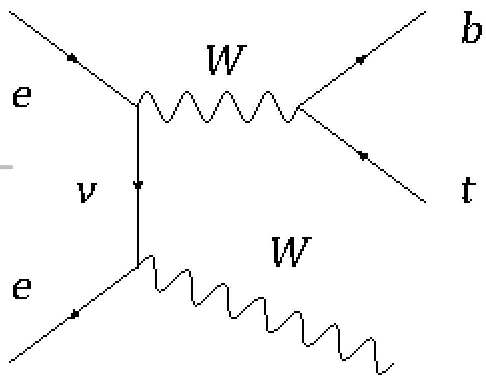
Parameter setup is based on the TDR and DBD.

Center-of-mass energy	$\sqrt{s}$	500 GeV
Beam polarization	(P_{e^-}, P_{e^+})	(-0.8, +0.3) / (+0.8, -0.3) Left / Right
Integrated luminosity	L	250 fb^{-1} / 250 fb^{-1}
Top quark mass	m_t	174 GeV
Other physics parameters		Consistent with SM-LO

Signal and Major Backgrounds

Signal : $e^-e^+ \to bl^+\nu\bar{b}l^-\bar{\nu}$

- Includes the single top production, ZWW etc.
  These are the irreducible background.

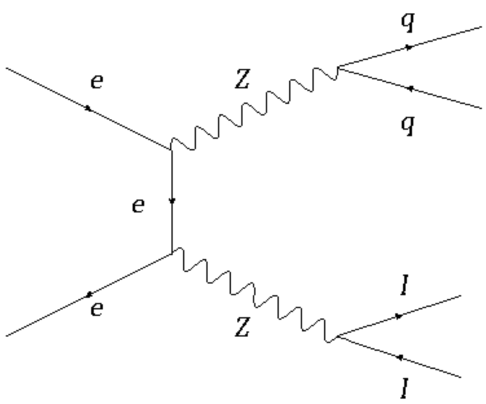


Single top production

Major backgrounds

- $e^-e^+ \to q\bar{q}l^-l^+$ (mainly $e^-e^+ \to ZZ \to q\bar{q}l^-l^+$)
- $e^-e^+ \to bq\bar{q}\bar{b}l\nu$

They might have 2 b-jets and 2 isolated leptons



$e^-e^+ \to ZZ \to q\bar{q}l^-l^+$

Signal Reconstruction

Reconstruction Process

1. Isolated leptons extraction : l^-, l^+
2. $\gamma\gamma \to$ hadrons suppression
3. Jet clustering and b-tagging : $b, \bar{b}$
4. Kinematical reconstruction of the missing neutrinos : $\nu, \bar{\nu}$

First 3 processes are similar with the semi-leptonic analysis.

The 4th process is key for the di-leptonic analysis.

One has to recover the missing neutrinos to reconstruct the top quark and full kinematics of final state.

Reconstruction Process

4. Kinematical Reconstruction

- $\nu, \bar{\nu}$ are not detectable at the ILD detector.
- To recover them, impose the following constraints
  - Initial state constraints : $E_{\text{total}} = 500\ \text{GeV}, \vec{P}_{\text{total}} = \vec{0}\ \text{GeV}$
  - Mass constraints : $m_{t,\bar{t}} = 174\ \text{GeV}, m_{W^{\pm}} = 80.4\ \text{GeV}$
- γ of the ISR/Beamstrahlung deteriorates the initial state condition. Assume the γ is along the beam direction (z-axis).

Unknowns

$\vec{P}_{\nu}, \vec{P}_{\overline{\nu}}, P_{\gamma,z}$: 7

Constraints

$E_{\text{total}}, \vec{P}_{\text{total}},$

$m_t, m_{\bar{t}}, m_{W^+}, m_{W^-}$: 8

Algorithm of the Kinematical Reconstruction

- Introduce 4 free parameters : $\vec{P}_{\nu}, P_{\gamma,z}$

  $\vec{P}_{\bar{\nu}}$ can be computed using the initial momentum constraints

$$\vec{P}_{\bar{\nu}} = -\vec{P}_{\text{vis.}} - \vec{P}_{\nu} - \vec{P}_{\gamma}, \qquad \left(\vec{P}_{\text{vis.}} = \vec{P}_{b} + \vec{P}_{\bar{b}} + \vec{P}_{l^+} + \vec{P}_{l^-}\right)$$

- Define the likelihood function :

$$L_0\left(\vec{P}_{\nu}, P_{\gamma,z}\right) = BW(m_t)BW(m_{\bar{t}})BW(m_{W^+})BW(m_{W^-})Gaus(E_{\text{total}})$$

- To correct the energy resolution of b-jets, add 2 parameters, $E_b, E_{\bar{b}}$, with the resolution functions to L_0 :

$$L\left(\vec{P}_{\nu}, P_{\gamma,z}, E_b, E_{\bar{b}}\right) = L_0 \times Res(E_b, E_b^{\text{meas.}})Res\left(E_{\bar{b}}, E_{\bar{b}}^{\text{meas.}}\right)$$

  Define $q\left(\vec{P}_{\nu}, P_{\gamma,z}, E_b, E_{\bar{b}}\right) = -2\log L + \text{Const.}$

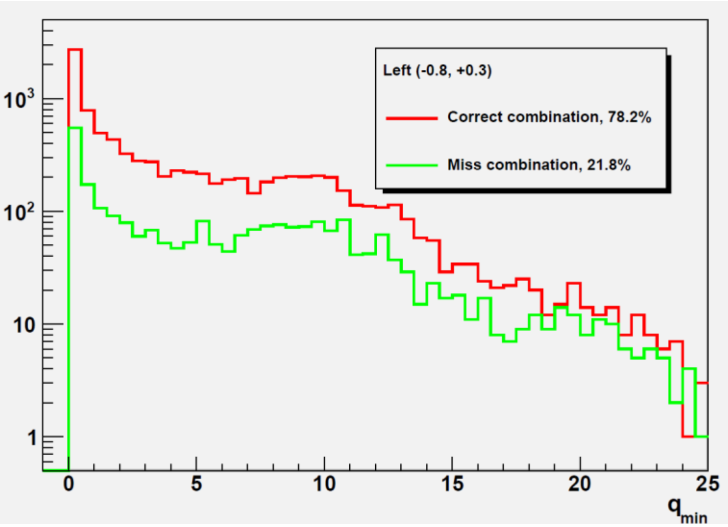
  scaled as the minimum of each component ($BW(m_t)$, etc) is equal to 0

Combination of Lepton and b-jet

Choice of a combination of l and b-jet

There are two candidates for the combination

- Select one having smaller q, defined as $\boldsymbol{q}_{\mathbf{min}}$
- Fraction of correct combination is ~78%

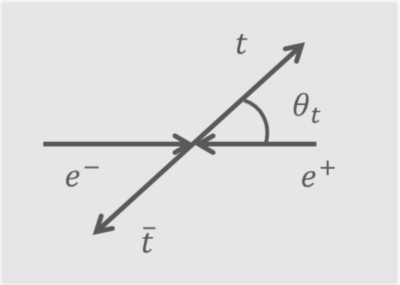


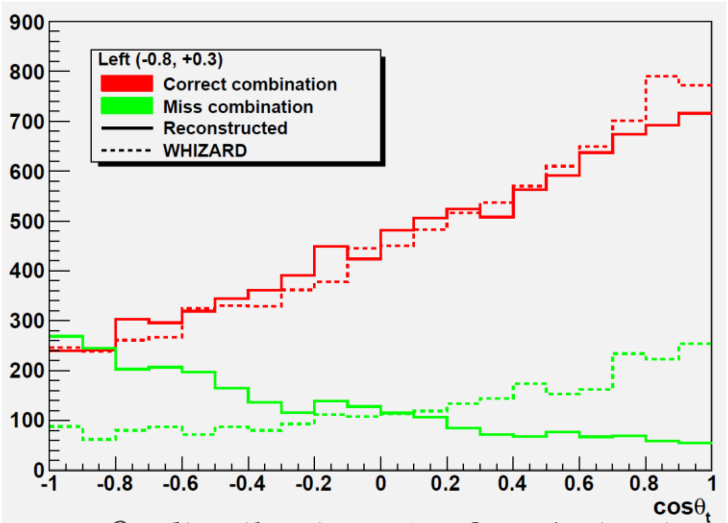
q_{min} distribution (Left polarization)

$\cos\theta_t$ distribution (Rec and MC Truth)

- Correct combination: OK
- Miss combination: Disagree with the MC truth.

Need to estimate an effect of the miss combination for the analysis.





$\cos\theta_t$ distribution (Left polarization)

Event Selection

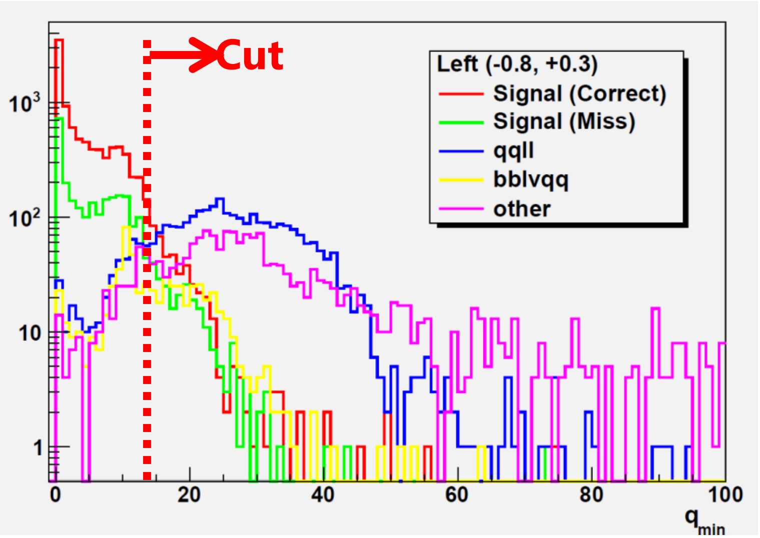
- Quality cut :

  $q_{\min}$ means the quality of reconstruction.
  Useful to suppress the backgrounds.

Criteria are optimized for the significance,

$$S = \frac{N_{\mathrm{signal}}}{\sqrt{N_{\mathrm{signal}} + N_{\mathrm{background}}}}$$

$q_{\min}$ distribution (Left polarization)



Left Polarization Cut Criteria	Signal $blvblv$	All bkg.	$qqll$	$blvbqq$
No cut	26570	2936189	91478	104114
$N_{l^-} = 1$ & $N_{l^+} = 1$	12852	274355	14243	755
b-tag cut	11839	4718	3033	628
Quality cut ($q_{\min} < 13.2$)	11038	851	372	302

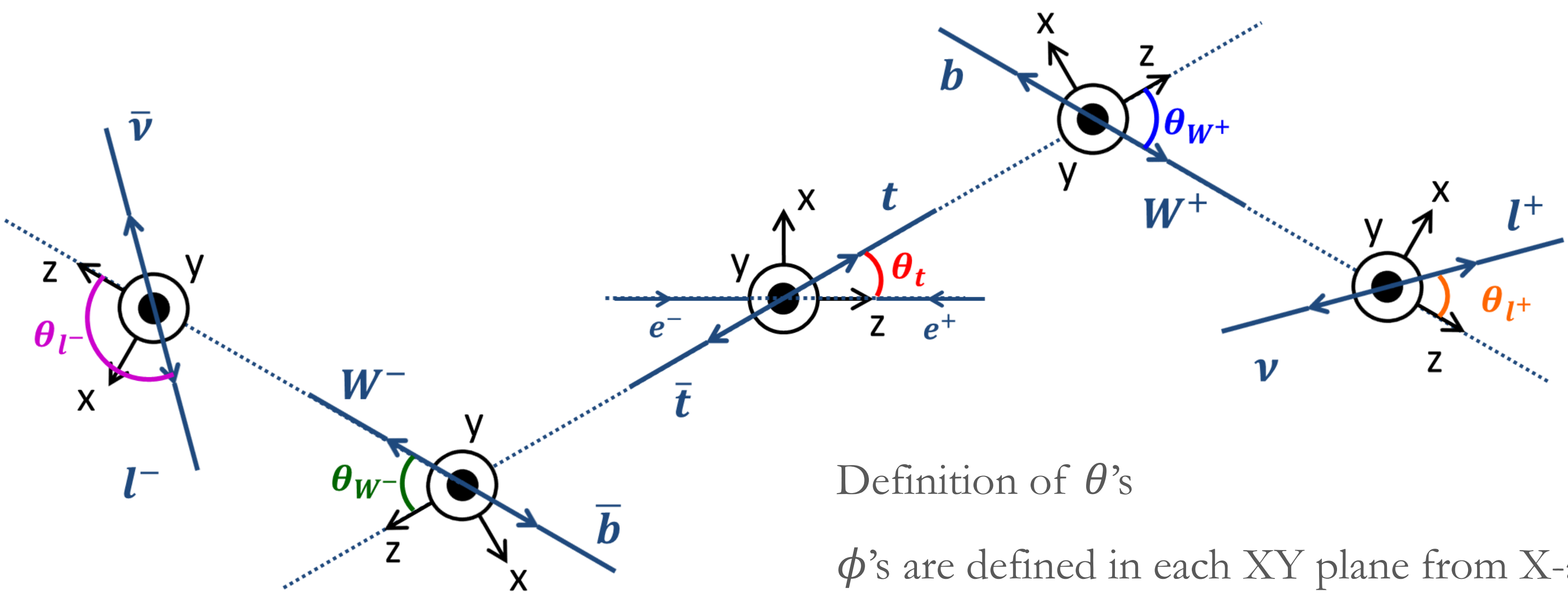
Analysis

The Amplitude of the Di-leptonic Process

The amplitude of the di-leptonic process is a function of 9 angles.

$$|M|^2(\cos\theta_t, \cos\theta_{W^+}, \phi_{W^+}, \cos\theta_{W^-}, \phi_{W^-}, \cos\theta_{l^+}, \phi_{l^+}, \cos\theta_{l^-}, \phi_{l^-}; F)$$

where one uses the narrow width approximation for top quark and W, and the angles are defined at appropriate rest frames.



Definition of θ's

ϕ's are defined in each XY plane from X-axis

The Matrix Element Method

Based of the unbinned likelihood method.

The likelihood function used in the previous study is defined as

$$f(F;\Phi) = \frac{|M|^2(\Phi;F)}{\int |M|^2(\Phi;F)d\Phi}$$

where Φ is the 9 angles.

The reconstructed distribution cannot be described by the function because of the detector, miss combination, background effects.

Solutions

- ☐ Convolution of these effects and the likelihood function.
- ☐ Binned likelihood fit with template method ← **This study**

The Optimal Variables

To handle the 9-dimention space, one introduces the optimal variables;

$$\omega_i(\Phi) = \frac{1}{|M|^2(\Phi; F_{SM})} \cdot \frac{\partial |M|^2(\Phi; F)}{\partial F_i}\Bigg|_{F=F_{SM}}$$

ω_i is the optimal variable in the 9-dimentional phase space for F_i.

Binned likelihood fit of ω distribution gives sensitivity close to MEM.

Estimate F_i dependence of ω_i distribution by the full simulation.

→ **Realistic effects (detector, background…) can be included.**

- Template method : *Straightforward, but heavy CPU time and storage*
- Reweighting method : *Relatively light CPU time and storage*

Reweighting Method

Binned likelihood fit uses $\chi^2(F) = \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data}} - n_i^{\text{Sim.}}(F)}{\sqrt{n_i^{\text{Data}}}} \right)^2$

where $n_i^{Data}, n_i^{\text{Sim.}}$ are number of events in bin_i of data and simulation sample (in this study, n_i^{Data} is another set of simulation sample).

Reweighting Method

Produce a sample with SM value F_{SM}, then change the weight of events.

$$\begin{aligned} n_i^{\text{Sim.}}(F) &= n_i^{\text{Sim.,sig}}(F) + n_i^{\text{Sim.,bkg}} \\ &= n_i^{\text{Sim.,sig}}(F_{SM}) \cdot w_i(F) + n_i^{\text{Sim.,bkg}} \end{aligned}$$

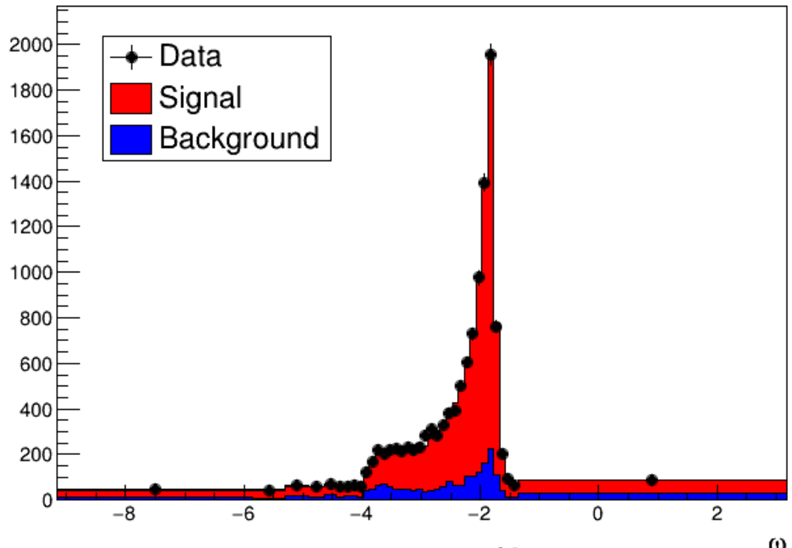
where $w(F)$ is a weight and calculated as

$$w_i(F) = \frac{1}{n_i^{\text{Sim.,sig}}(F_{SM})} \sum_{e \in \text{bin}_i} \frac{|M|^2(\Phi_e; F)}{|M|^2(\Phi_e; F_{SM})}$$

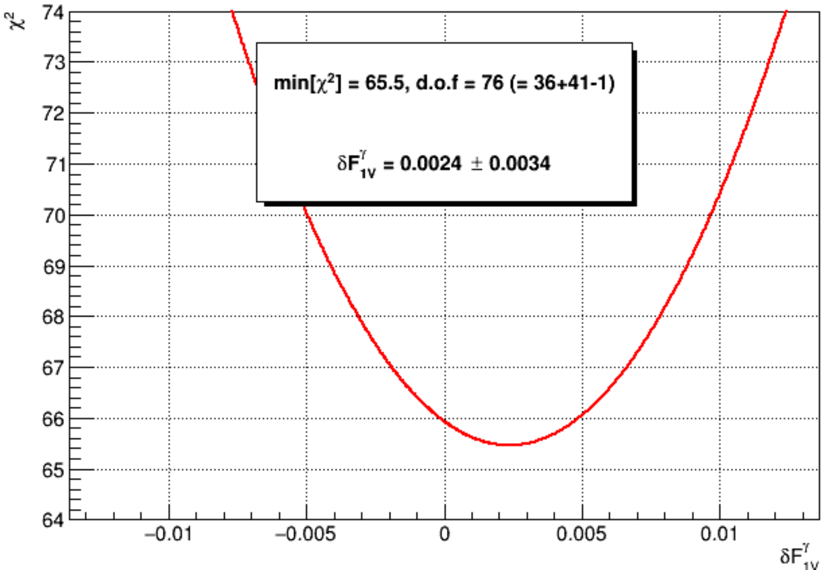
Binned Likelihood Fit

We have done *Single parameter fit for each* F.

(e.g) F_{1V}^{γ} measurement



ω distribution of F_{1V}^{γ} (Left pol.)



$\chi^2(F_{1V}^{\gamma})$ function

Results : $\delta F_{1V}^{\gamma} = 0.0024 \pm 0.0034$ ➔ Consistent with input value

Comparison with previous study

	This study σ_{stat}	Previous (1) $\sigma_{stat} \times \sqrt{6}$	Previous (1) σ_{stat}
F_{1V}^{γ}	± 0.0034	± 0.0049	± 0.002
F_{1V}^{Z}	± 0.0061	± 0.0073	± 0.003
F_{1A}^{γ}	± 0.0082	---	---
F_{1A}^{Z}	± 0.0133	± 0.0171	± 0.007
F_{2V}^{γ}	± 0.0028	± 0.0024	± 0.001
F_{2V}^{Z}	± 0.0049	± 0.0049	± 0.002

	This study σ_{stat}	Previous (2) $\sigma_{stat} \times \sqrt{6}$	Previous (2) σ_{stat}
ReF_{2A}^{γ}	± 0.012	± 0.012	± 0.005
ReF_{2A}^{Z}	± 0.018	± 0.017	± 0.007
ImF_{2A}^{γ}	± 0.011	± 0.015	± 0.006
ImF_{2A}^{Z}	± 0.019	± 0.024	± 0.010

- The precision is consistent with the previous study considering the difference of number of events ($\times \sqrt{6}$).
- If this method is applied for the semi-leptonic process, it's possible that the precision will be improved

(*) Although some results of previous study are from multi-fit, the correlation is small.

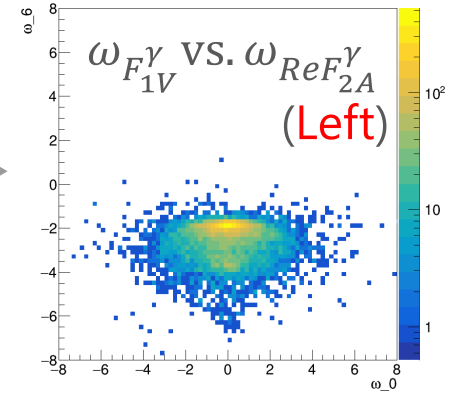
(1) Eur.Phys.J. C75 (2015) no.10, 512

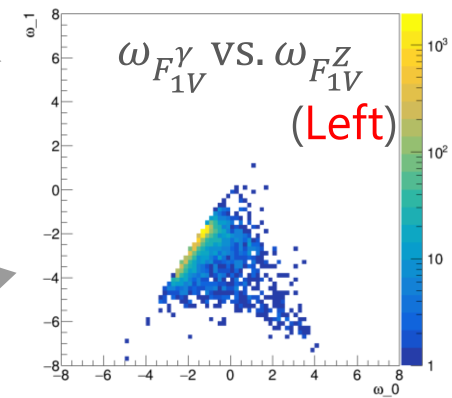
(2) Eur.Phys.J. C78 (2018) no.2, 155

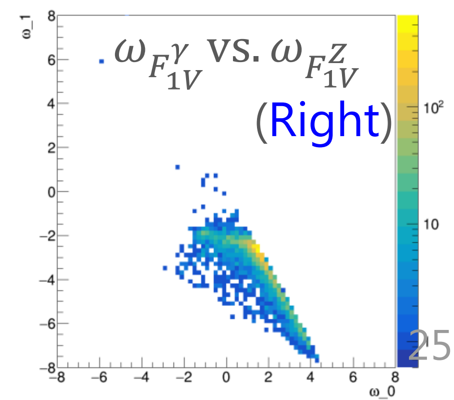
Multi-Parameter fit (Preliminary)

Multi-Parameter fit

- ω's are not correlated (= No shared information)
  → Multi-parameter fit is not necessary.
  (10 Parameters can be separated into top 6, middle 2 and bottom 2)
- ω's are correlated or there is a structure in a 2D histogram
  → Multi-parameter fit with the 2D histogram.
- Different correlation for each beam polarization
  → Two 1D histograms are enough for the multi- parameter fit (not optimal way)







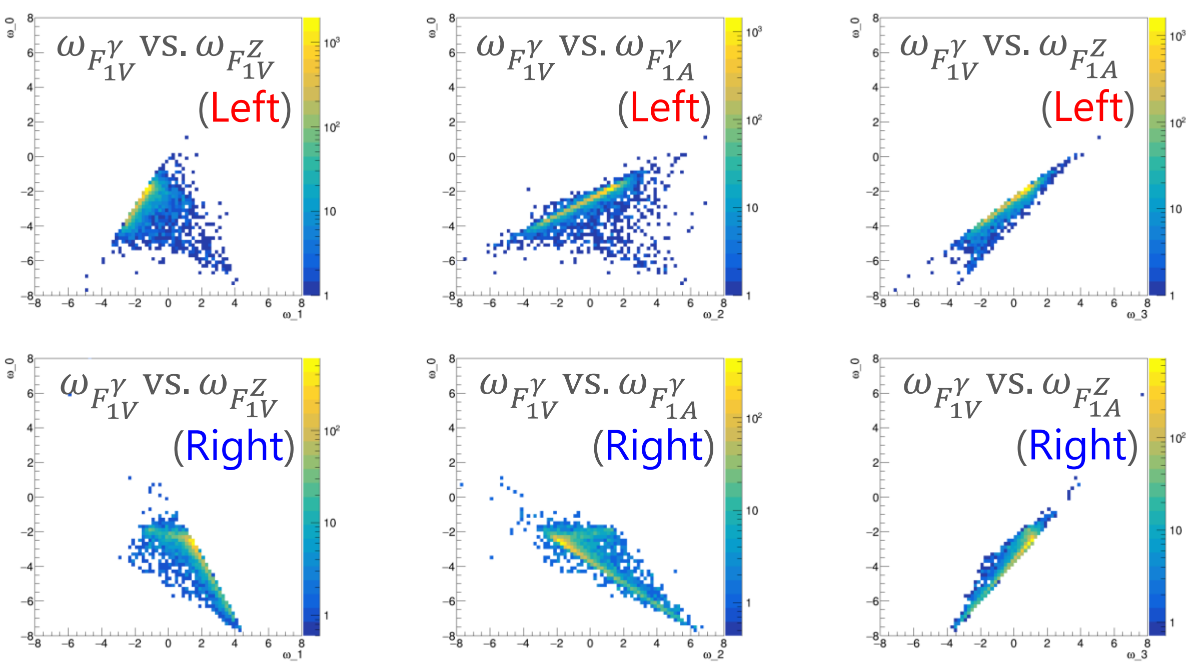
Multi-Parameter fit (Preliminary)

(eg) Use a 1D histogram of ω of F_{1V}^{γ}

Form factor	Multi	Single
F_{1V}^{γ}	± 0.0038	± 0.0034
F_{1V}^{Z}	± 0.0068	± 0.0061
F_{1A}^{γ}	± 0.0099	± 0.0082
F_{1A}^{Z}	± 0.0155	± 0.0133

Correlation matrix

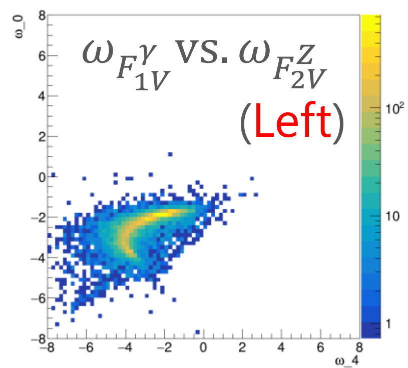
$$\begin{pmatrix} +1.000 & -0.232 & -0.183 & +0.380 \\ -0.232 & +1.000 & +0.328 & -0.123 \\ -0.183 & +0.328 & +1.000 & -0.280 \\ +0.380 & -0.123 & -0.280 & +1.000 \end{pmatrix}$$

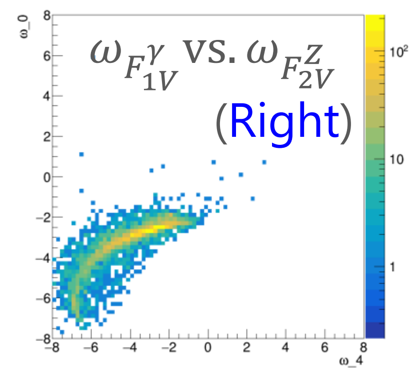


Multi-Parameter fit (Preliminary)

(eg) Use a 2D histogram of $\omega's$ of $F_{1V}^{\gamma}, F_{2V}^{\gamma}$

Form factor	Multi	Single
F_{1V}^{γ}	± 0.0264 ←	± 0.0034
F_{1V}^{Z}	± 0.0437 ←	± 0.0061
F_{1A}^{γ}	± 0.0102	± 0.0082
F_{1A}^{Z}	± 0.0157	± 0.0133
F_{2V}^{γ}	± 0.0210 ←	± 0.0028
F_{2V}^{Z}	± 0.0344 ←	± 0.0049





Correlation matrix

$$\begin{pmatrix} +1.000 & -0.237 & -0.098 & +0.296 & \underline{-0.990} & +0.232 \\ -0.237 & +1.000 & +0.322 & -0.058 & +0.231 & \underline{-0.989} \\ -0.098 & +0.322 & +1.000 & -0.310 & +0.070 & -0.276 \\ +0.296 & -0.058 & -0.310 & +1.000 & -0.246 & +0.034 \\ -0.990 & +0.231 & +0.070 & -0.246 & +1.000 & -0.231 \\ +0.232 & -0.989 & -0.276 & +0.034 & -0.231 & +1.000 \end{pmatrix}$$

Strong constraints on $F_{1V}^{\gamma/Z}, F_{2V}^{\gamma/Z}$ separately cannot be obtained.

→ **Other formalisms are preferred for the multi-parameter fit.**

Summary

Summary

- Development of the search technique for the anomalous ttZ/γ couplings with full kinematics analysis based on the ILD full simulation.
- Reconstructed full kinematics of the di-leptonic process from the kinematical reconstruction.
- Estimated the statistical errors from the binned likelihood fit of the ω distribution which is optimal variable.
- The precision is consistent with the previous study and there's a possibility of improvement if this method is applied for the semi-leptonic state.

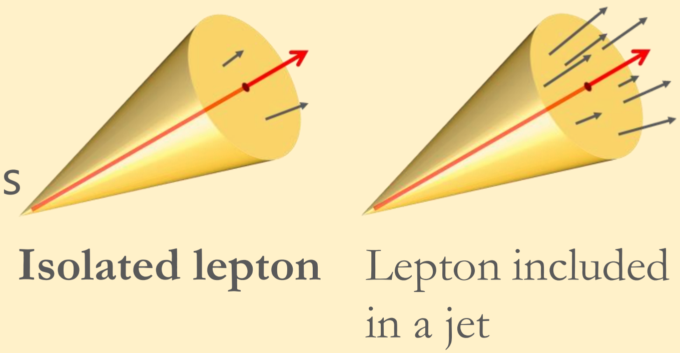
Backup

Reconstruction Process

Reconstruct all final state particles, $bl^+\nu\bar{b}l^-\bar{\nu}$.

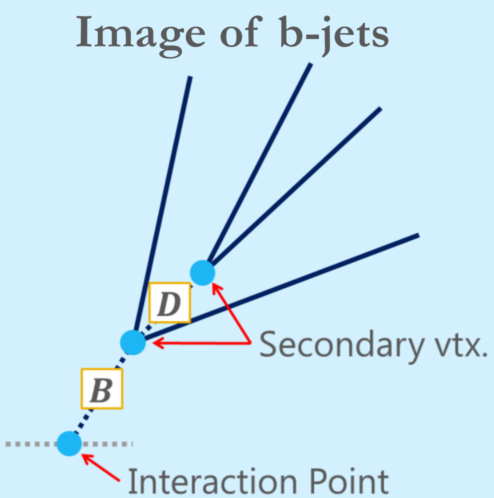
1. Selection of l^+ and l^-

- l^-, l^+ are isolated from other particles
- Extract isolated leptons as final state leptons

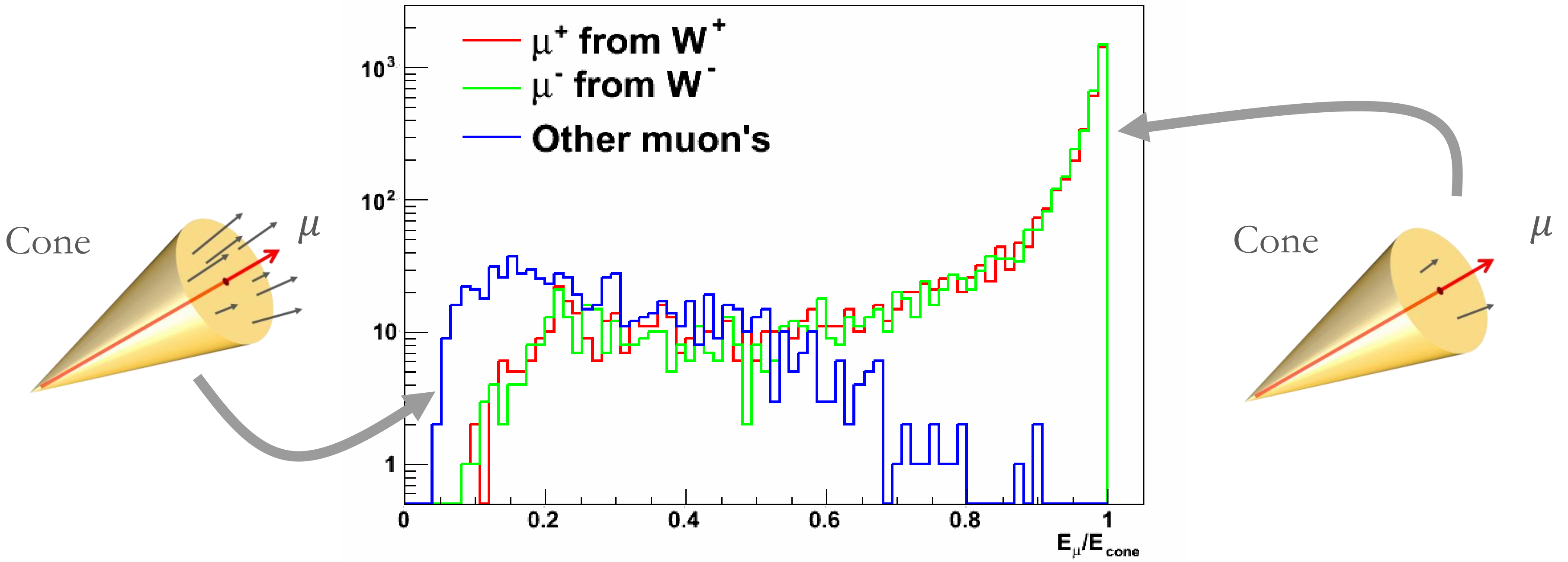


2. Jet clustering and b-tagging

- Cluster jet particles corresponding $b, \bar{b}$
- B, D meson moves ~100 μm before the decay
- Assess the "b-likeness" from the vertex information (such as # of vtx. and distance between IP and vtx.)



Isolated lepton finder



Energy ratio between μ and a cone

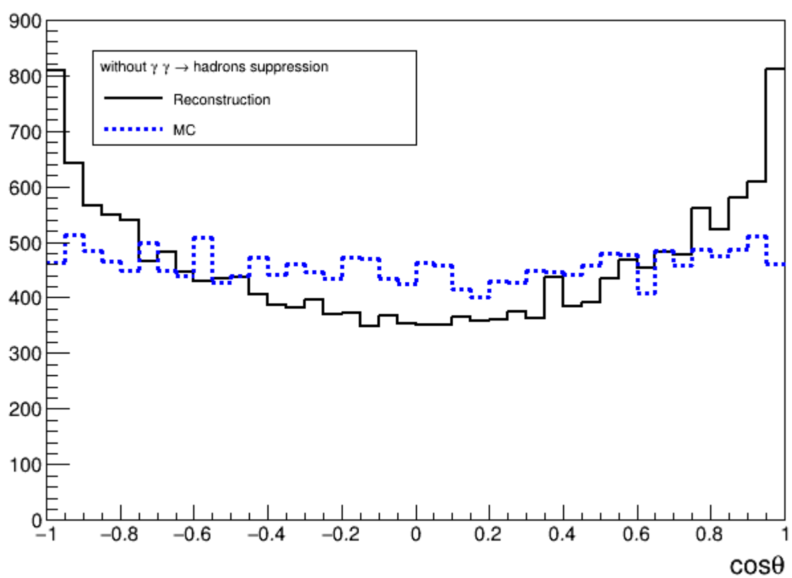
$R = E_\mu / E_{cone}$ is a quantity to evaluate how isolated the muon is.

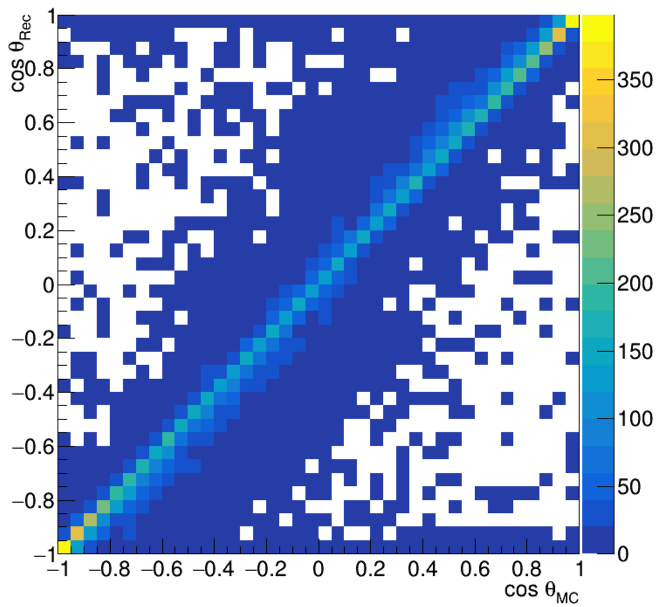
(E_{cone} : total energy of particles in the cone)

μ from W boson is more isolated than other μ

$\gamma\gamma \to$ hadrons rejection

$b, \bar{b}$ are reconstructed from the rest of particles with LCFIPlus





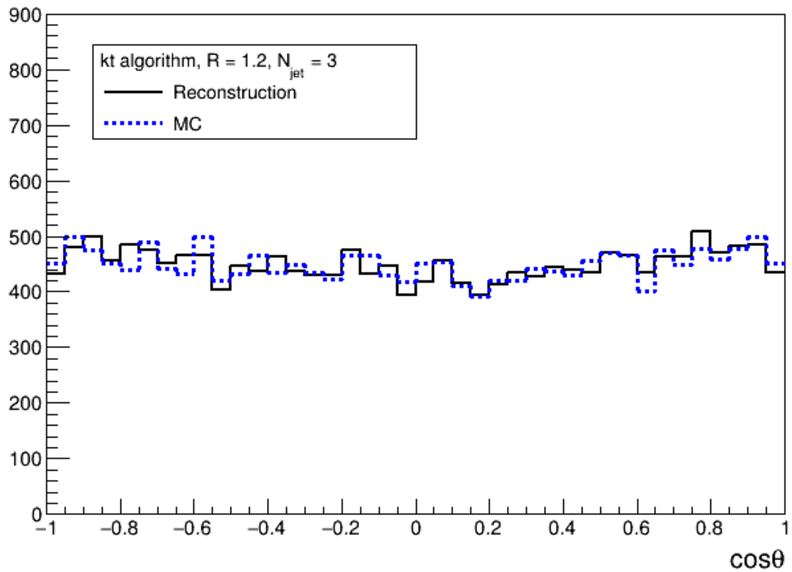
cos θ_{jet} distribution

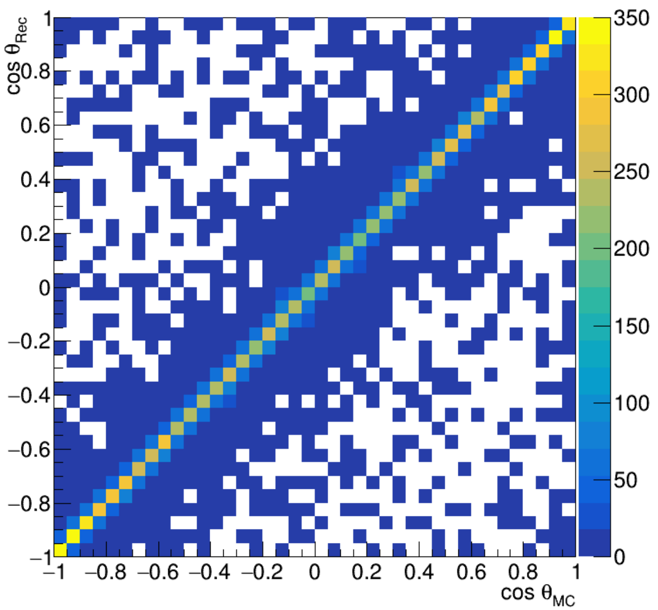
Strongly peaked at very forward region by mistake

$\gamma\gamma \to$ hadrons are emitted along the beam direction

$\gamma\gamma \to$ hadrons rejection

Eliminate particles close to beam direction rather than other particles with kt algorithm.





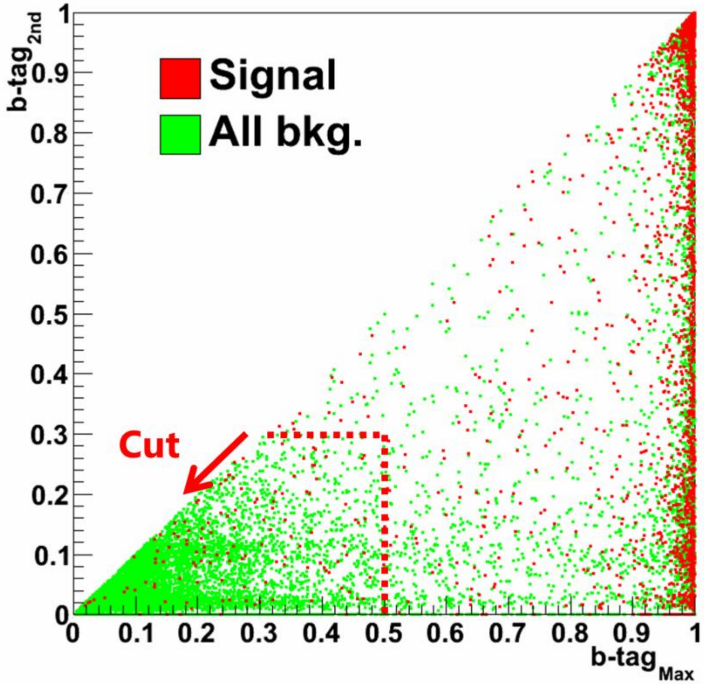
$\cos\theta_{jet}$ distribution

Good agreement between Rec and MC

b-tagging with LCFIPlus

b-tag is TMVA output indicating “b-likeness” of a jet obtained by the LCFIPlus(*).

- $\text{b-tag}_{\text{Max}}$: the largest b-tag
- $\text{b-tag}_{\text{2nd}}$: the 2nd largest b-tag



b-tag value (Left polarization)

- Signal has large $\text{b-tag}_{\text{Max}}$
- Many of bkg. have small $\text{b-tag}_{\text{Max}}$ and $\text{b-tag}_{\text{2nd}}$

$b - tag_{Max} > 0.5$ or $b - tag_{2nd} > 0.3$

(*) A software package of Marlin for the multi-jet analysis.

Kinematical Reconstruction

$$BW(x; m, \Gamma) \propto \frac{1}{1 + \left(\frac{x^2 - m^2}{m\Gamma}\right)^2}$$

$$Gaus(x; \mu, \sigma) \propto \exp\left[-\left(\frac{x - \mu}{\sqrt{2}\sigma}\right)^2\right]$$

Detail definition of L_0 is

$$L_0\left(\vec{P}_\nu, P_{\gamma,Z}\right) = BW(m_t; 174{,}5)BW(m_{\bar{t}}; 174{,}5) \cdot BW(m_{W^+}; 80.4{,}5)BW(m_{W^-}; 80.4{,}5)Gaus(E_{\mathrm{total}}; 500{,}0.39)$$

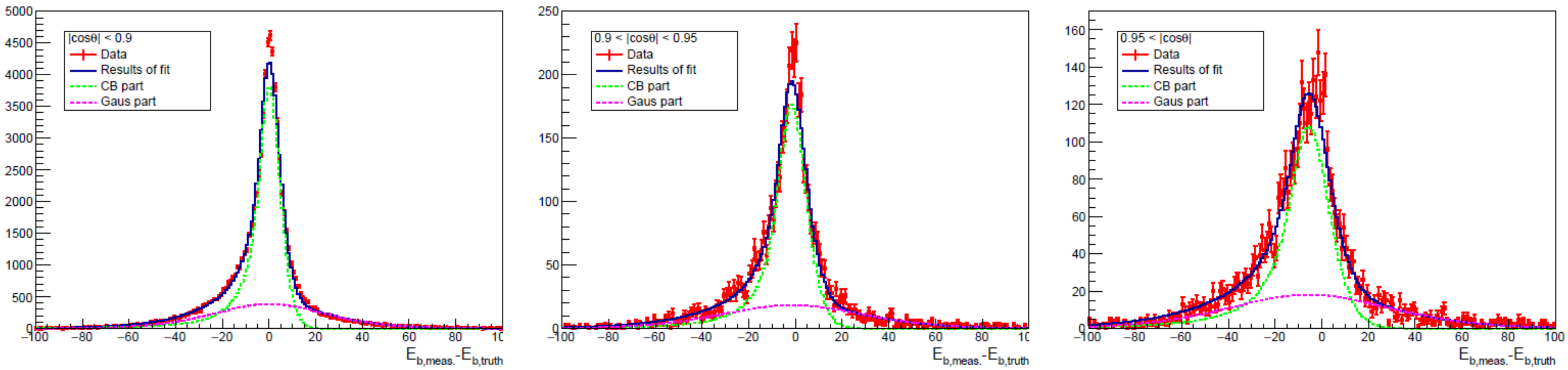
- Larger value for Γ than theoretical value is set because of detector effects
- σ is caused by the Beam energy spread.

Energy resolution of b-jet

Estimate the energy resolution of b-jet with the following $Res(E_b, E_b^{\text{meas.}})$;

$$Res(E_b, E_b^{\text{meas.}}) = (1-f)CB(\Delta E_b; \alpha, n, \mu_{CB}, \sigma_{CB}) + f * Gaus(\Delta E_b; \mu_{Gaus}, \sigma_{Gaus})$$

Divide into 3 regions ; $|\cos\theta| = (0, 0.9), (0.9, 0.95), (0.95, 1)$



(a) $|\cos\theta| < 0.9$ (b) $0.9 < |\cos\theta| < 0.95$ (c) $0.95 < |\cos\theta|$

Crystal Ball function

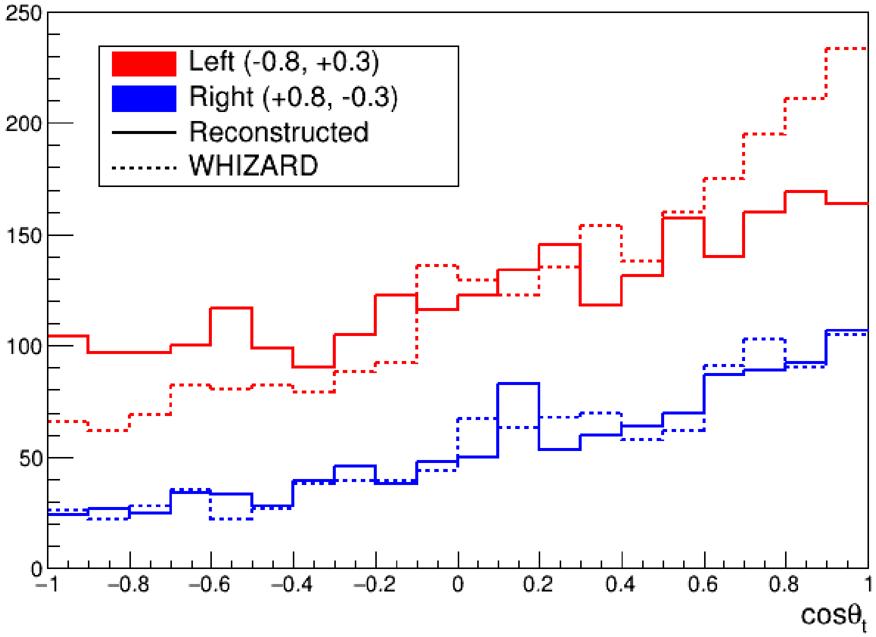
Crystal Ball function consists of a Gaussian core portion and power-law tail.

$$CB(x;\alpha,n,\bar{x},\sigma) = N \cdot \begin{cases} \exp\left(-\frac{(x-\bar{x})^2}{2\sigma^2}\right) & \frac{x-\bar{x}}{\sigma} > -\alpha \\ A \cdot \left(B - \frac{x-\bar{x}}{\sigma}\right)^{-n} & \frac{x-\bar{x}}{\sigma} \leq -\alpha \end{cases}$$

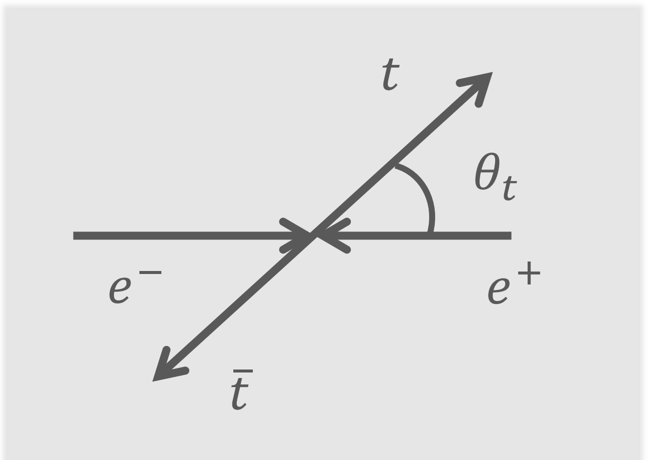
$$\begin{aligned} A &= \left(\frac{n}{|\alpha|}\right)^n \cdot \exp\left(-\frac{|\alpha|^2}{2}\right) \\ B &= \frac{n}{|\alpha|} - |\alpha| \\ N &= \frac{1}{\sigma(C+D)} \\ C &= \frac{n}{|\alpha|} \cdot \frac{1}{n-1} \cdot \exp\left(-\frac{|\alpha|^2}{2}\right) \\ D &= \sqrt{\frac{\pi}{2}}\left(1 + \operatorname{erf}\left(\frac{|\alpha|}{\sqrt{2}}\right)\right) \end{aligned}$$

Results of Reconstruction

Top quark polar angle distribution, $\cos\theta_t$



Definition of θ_t

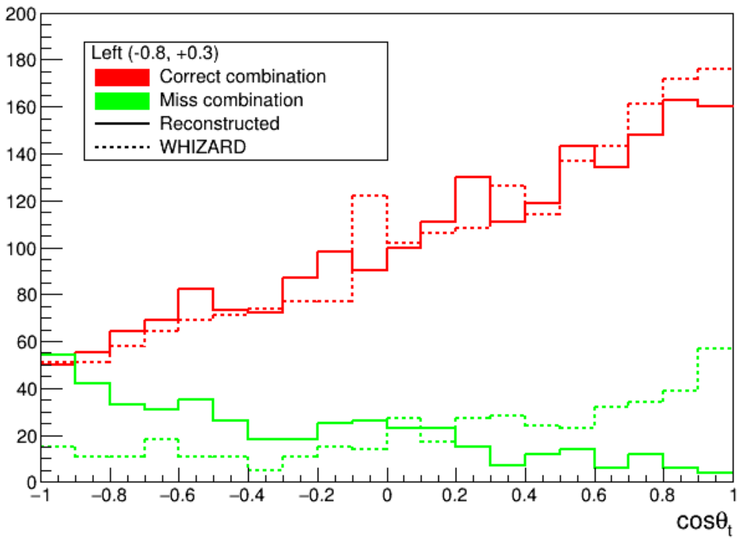


Considerable migration occurs in the Left polarization case

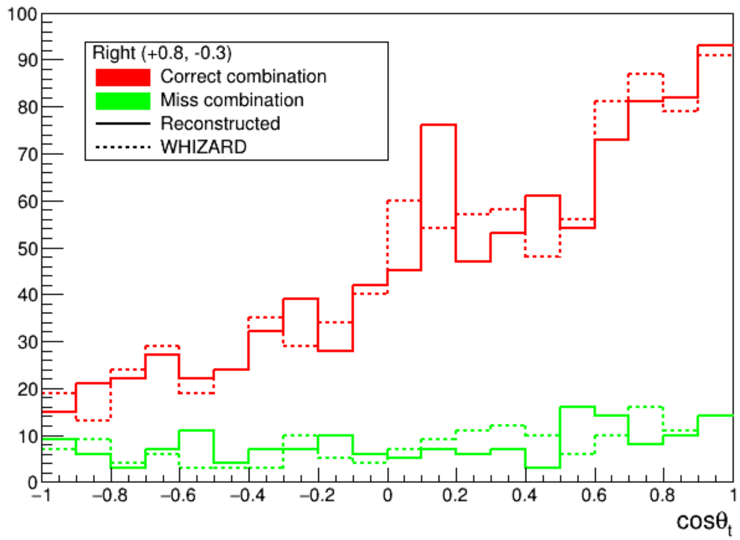
Some events pass from forward to backward because of the miss combination of μ and b-jet.

Dependence from the beam polarization

$\cos\theta_t$ distribution (Left polarization)



$\cos\theta_t$ distribution (Right polarization)



Left polarization

Reconstructed distribution of miss combination is very different from the MC truth.

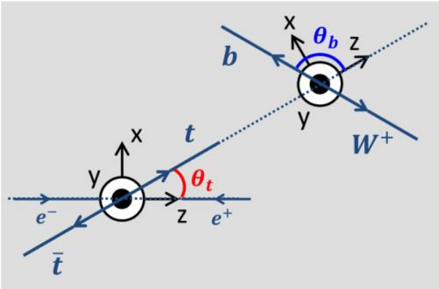
Right polarization

Similar distribution can be reconstructed even when the miss combination is selected.

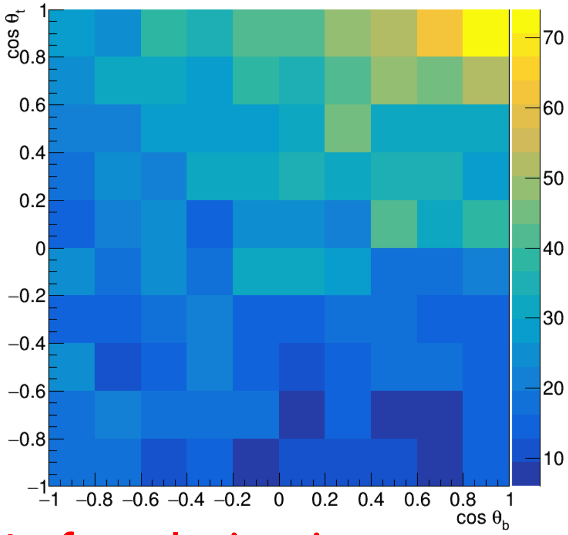
Dependence from the beam polarization

$\cos\theta_b \simeq 1$ → b-jets are energetic

→ **Migration effect is strong**



$\cos\theta_t$ vs. $\cos\theta_b$ (Left polarization)

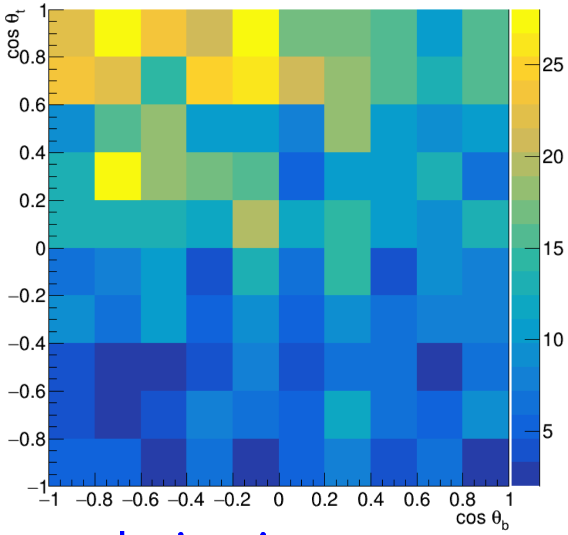


Left polarization

Peak at $\cos\theta_t \simeq 1$ & $\cos\theta_b \simeq 1$

→ Migration is asymmetry

$\cos\theta_t$ vs. $\cos\theta_b$ (Right polarization)



Right polarization

Peak at $\cos\theta_t \simeq 1$ & $\cos\theta_b \simeq -1$

→ Migration is symmetry

Matrix element method

- Based on the unbinned likelihood method. The likelihood function is computed from the amplitude.
  → Full kinematics are used = The most sensitive method in principle.
- Fit results are almost consistent with SM values.
  - ~1.5 σ biases are observed for several form factors

$$\begin{pmatrix} \delta\tilde{F}^{\gamma}_{1V,\mathrm{fit}} \\ \delta\tilde{F}^{Z}_{1V,\mathrm{fit}} \\ \delta\tilde{F}^{\gamma}_{1A,\mathrm{fit}} \\ \delta\tilde{F}^{Z}_{1A,\mathrm{fit}} \\ \delta\tilde{F}^{\gamma}_{2V,\mathrm{fit}} \\ \delta\tilde{F}^{Z}_{2V,\mathrm{fit}} \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2A,\mathrm{fit}} \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{2A,\mathrm{fit}} \\ \mathcal{I}m\ \delta\tilde{F}^{\gamma}_{2A,\mathrm{fit}} \\ \mathcal{I}m\ \delta\tilde{F}^{Z}_{2A,\mathrm{fit}} \end{pmatrix} = \begin{pmatrix} +0.0031 \pm 0.0130 \\ -0.0334 \pm 0.0231 \\ -0.0314 \pm 0.0192 \\ +0.0241 \pm 0.0301 \\ -0.0146 \pm 0.0366 \\ -0.0650 \pm 0.0592 \\ +0.0214 \pm 0.0241 \\ -0.0131 \pm 0.0415 \\ -0.0086 \pm 0.0255 \\ +0.0081 \pm 0.0360 \end{pmatrix}$$

Goodness of fit for the MEM

Expectation value of ω when the fit results are assigned should be equal to mean of reconstructed ω distribution

$$\chi^2_{\mathrm{GoF},k}(\delta F_{\mathrm{fit}}) = \frac{\left(\langle\omega_k\rangle - \Omega_k(\delta F_{\mathrm{fit}})\right)^2}{\left(\langle\omega_k^2\rangle - \langle\omega_k\rangle^2\right)/N_{data}}$$

$$\tilde{\chi}^2_{\mathrm{GoF},kl}(\delta F_{\mathrm{fit}}) = \frac{\left(\langle\widetilde{\omega}_{kl}\rangle - \widetilde{\Omega}_{kl}(\delta F_{\mathrm{fit}})\right)^2}{\left(\langle\widetilde{\omega}_{kl}^2\rangle - \langle\widetilde{\omega}_{kl}\rangle^2\right)/N_{data}}$$

Some χ^2_{GoF} have large value (6~10).

→ Goodness of fit for the MEM is bad.

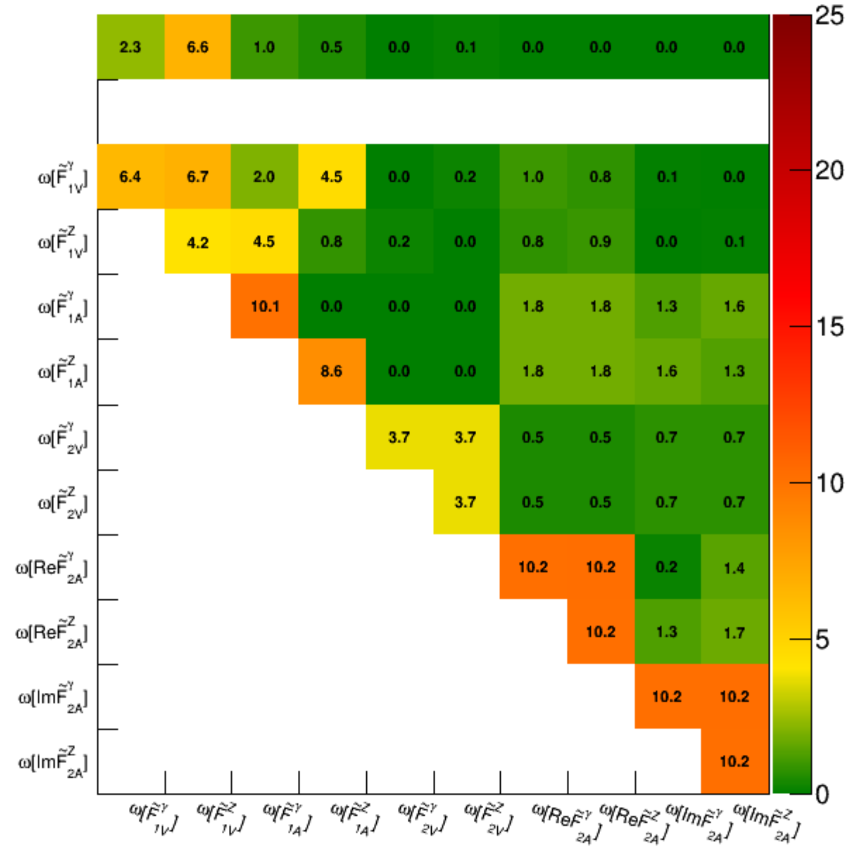


Table of $\chi^2_{\mathrm{GoF}}, \tilde{\chi}^2_{\mathrm{GoF}}$ (Left polarization)

Reweighting (Template-like) Technique

Binned likelihood method : $\chi^2(\delta F) = \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\mathrm{Data}} - n_i^{\mathrm{Sim.}}(\delta F)}{\sqrt{n_i^{\mathrm{Data}}}} \right)^2$

$n_i^{\mathrm{Sim.}}(\delta F)$ is obtained from the large full simulation

Reweighting technique :

Produce a sample using SM value, then change the weight of events.

$$
\begin{aligned}
n_i^{\mathrm{Sim.}}(\delta F) &= n_i^{\mathrm{Sim.,sig}}(\delta F) + n_i^{\mathrm{Sim.,bkg}} \\
&= n_i^{\mathrm{Sim.,sig}}(0)(1\langle\omega\rangle_i \delta F + \langle\widetilde{\omega}\rangle_i \delta F^2) + n_i^{\mathrm{Sim.,bkg}} \\
&\simeq n_i^{\mathrm{Sim.,sig}}(0)(1 + \langle\omega\rangle_i \delta F) + n_i^{\mathrm{Sim.,bkg}}
\end{aligned}
$$

Template technique : Produce many samples using different parameters

Overestimate of goodness of fit

We don't have enough statistics for the background events for now.

$$\chi^2(\delta F) = \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data}} - n_i^{\text{Sim.}}(\delta F)}{\sqrt{n_i^{\text{Data}}}} \right)^2 \rightarrow \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data,Sig.}} - n_i^{\text{Sim.,Sig.}}(\delta F)}{\sqrt{n_i^{\text{Data}}}} \right)^2$$

When $n_i^{\text{Data,Sig.}} = \alpha n_i^{\text{Data}}$ $(\alpha < 1)$

$$\sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data,Sig.}} - n_i^{\text{Sim.,Sig.}}(\delta F)}{\sqrt{n_i^{\text{Data}}}} \right)^2 = \alpha \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data,Sig.}} - n_i^{\text{Sim.,Sig.}}(\delta F)}{\sqrt{n_i^{\text{Data,Sig}}}} \right)^2$$

$$\equiv \alpha \chi^2_{\text{Sig}}$$

$\min\left[\chi^2_{\text{Sig}}\right]$ obeys chi-square distribution of $n.d.f. = N_{bin} - N_{\text{para}}$

→ $\chi^2(\delta F)$ may be $1/\alpha$ times larger if backgrounds are included in $\chi^2(\delta F)$