

Branching ratio measurement of $h \rightarrow \mu^+ \mu^-$ at the ILC

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Abstract

We study the prospects of measurement of the branching ratio of $h \rightarrow \mu^+ \mu^-$ at the International Linear Collider (ILC). The study is performed at center-of-mass energies of 250 GeV and 500 GeV, using fully-simulated MC samples with the International Large Detector (ILD) model. For both center-of-mass energies, the two final states $q\bar{q}h$ and $\nu\bar{\nu}h$ have been analyzed. For an integrated luminosity of 2000 fb⁻¹ at 250 GeV and 4000 fb⁻¹ at 500 GeV, corresponding to the H20 running scenario as well as its staged version, the precision on $\sigma \times \text{BR}(h \rightarrow \mu^+ \mu^-)$ is estimated.

1 Introduction

The investigation of the Higgs boson is one of the most important research topics in recent particle physics. In the Standard Model (SM), the Yukawa coupling between matter fermions and the Higgs boson is proportional to the fermion's mass. If we observe any deviations from this proportionality, it is an indication of new physics beyond the SM.

In this study, we focus on the $h \rightarrow \mu^+ \mu^-$ channel. This is a very challenging analysis because in the SM the branching ratio of $h \rightarrow \mu^+ \mu^-$ is estimated to be very small: 2.2×10^{-4} for the mass of the Higgs boson of 125 GeV [1]. However this channel is still important, because the mass of the muon has a small uncertainty unlike quarks which typically have large theoretical uncertainties from QCD, which means that this channel will be a suitable probe for the precise measurement. We can study not only the muon-Yukawa coupling itself, but also the relation between mass and coupling using the coupling ratios of second and third generation leptons (κ_μ/κ_τ), and second generation lepton and quark (κ_μ/κ_c) to understand the mass generation mechanism.

In this study, we estimate the precision expected for the measurement of $\sigma \times \text{BR}(h \rightarrow \mu^+ \mu^-)$ at the ILC based on full simulation of the ILD detector concept. Actually, this channel has been studied several times under various settings in linear colliders physics [2–7], but all studies except Ref. [7] have been performed at a center-of-mass energy (\sqrt{s}) of 1 TeV or higher. In addition, the studies in Refs. [5] and [7] are based on the mass of Higgs boson of 120 GeV. In Ref. [7] for example, the precision of $\sigma \times \text{BR}(h \rightarrow \mu^+ \mu^-)$ has been estimated to be 91% at $\sqrt{s} = 250$ GeV with 250 fb⁻¹, assuming Higgs mass of 120 GeV and Silicon Detector (SiD) concept for the ILC. In this study, on the other hand, we focus on $\sqrt{s} = 250$ GeV and 500 GeV, assuming a Higgs mass of 125 GeV for the first time. This study will give the prospects for measuring this rare decay channel at lower center-of-mass energies.

At the Large Hadron Collider (LHC), the $h \rightarrow \mu^+ \mu^-$ decay is explored using pp collision data. The latest results are shown in Ref. [8] by ATLAS and in Ref. [9] by CMS. They also have studied the prospects at the HL-LHC, ATLAS projects $\sim 21\%$ precision on the signal strength with 3000 fb⁻¹ data [10], while the CMS estimate is $\sim 10\%$ for the phase-II detector upgrade [11]. However, all measurements at the LHC are for the cross section times branching ratio $\sigma \times \text{BR}$. At the ILC on the other hand, most of the measurements are $\sigma \times \text{BR}$, but it is possible to measure the total cross section σ itself by using the recoil technique. By combining $\sigma \times \text{BR}$ and σ measurements, we can extract absolute numbers for the branching ratios without model dependencies. We can also measure the Higgs total width at the ILC, thus we can extract absolute coupling constants [12].

The Higgs production cross section as a function of \sqrt{s} at the ILC is shown in Figure 1, together with corresponding Feynman diagrams. In this study, we assume the so-called “H20” running scenario,

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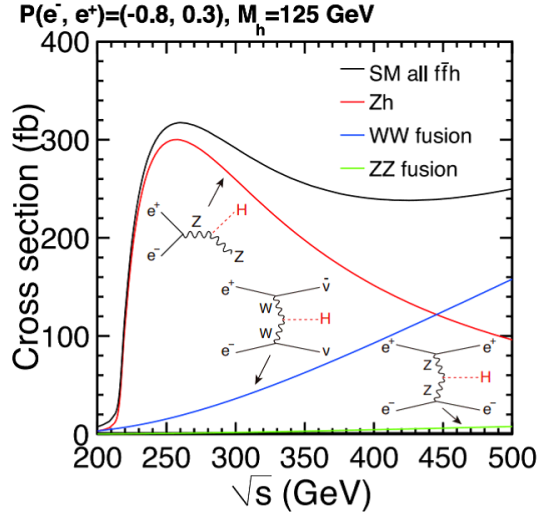


Figure 1: The Higgs production cross section as a function of \sqrt{s} [13].

Table 1: The expected number of signal events assuming H20 scenario. The symbols L and R mean the combination of beam polarization of electrons and positrons; L: left-handed, $(e^-, e^+) = (-80\%, +30\%)$, R: right-handed, $(e^-, e^+) = (+80\%, -30\%)$.

250 GeV	$q\bar{q}h$	$\nu\bar{\nu}h$
L	61.7 (1350 fb $^{-1}$)	22.5 (1350 fb $^{-1}$)
R	14.1 (450 fb $^{-1}$)	4.2 (450 fb $^{-1}$)
500 GeV	$q\bar{q}h$	$\nu\bar{\nu}h$
L	24.6 (1600 fb $^{-1}$)	57.5 (1600 fb $^{-1}$)
R	16.4 (1600 fb $^{-1}$)	7.9 (1600 fb $^{-1}$)

45 accumulating 2000 fb $^{-1}$ at 250 GeV and 4000 fb $^{-1}$ at 500 GeV with actual beam polarization sharing [13,
46 14]. The expected number of signal events are summarized in Table 1. We analyze in total 8 channels as
47 listed in Table 1.

48 Recently, the “staging” running scenario which starts from 250 GeV operation has been proposed [15].
49 We will discuss the prospects with the staging scenario in Section 4.

50 2 Analysis

51 We use fully-simulated MC samples with the ILC detector model which have been generated in the
52 context of ILC Technical Design Report [3]. We use all available MC samples at 250 GeV and 500 GeV,
53 in total $\sim 8 \times 10^7$ MC events.

54 The analyzes are structured in the same way in all channels. First, a pair of well-reconstructed
55 oppositely charged muons consistent with $h \rightarrow \mu^+\mu^-$ are selected. Then, the rest of the event is subject
56 to a procedure to remove the $\gamma\gamma \rightarrow$ low P_t hadron overlay and a further, channel-specific selection as a
57 last step of the event selection, a boosted decision tree is applied for each channel. In this proceedings
58 contribution, we give as an example the details for 500 GeV with $q\bar{q}h$ final state and left-handed beam
59 polarization. For simplicity, this channel is described as qqh500-L.

60 2.1 $h \rightarrow \mu^+\mu^-$ Selection

61 We apply the so-called IsolatedLeptonTagger [16] to select $h \rightarrow \mu^+\mu^-$ candidate from $e^+e^- \rightarrow q\bar{q}h \rightarrow$
62 $q\bar{q}\mu^+\mu^-$ topology. In this tagger several variables are used to identify isolated leptons. For the isolated
63 muon tagging, we require the following conditions: $E_{\text{CAL}}/|p| < 0.5$, $E_{\text{yoke}} > 0.5$ GeV, $|d_0/\sigma(d_0)| < 5$,

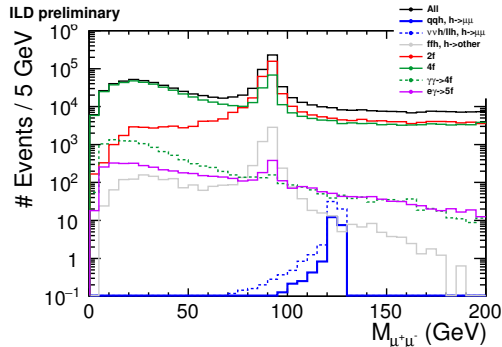


Figure 2: The $M_{\mu^+\mu^-}$ distribution before applying the cut to $M_{\mu^+\mu^-}$ (qqh500-L).

64 $|z_0/\sigma(z_0)| < 5$, $|p| > 10$ GeV, and MVA cut > 0.7 , where E_{CAL} and E_{yoke} are the energy deposits in
 65 the calorimeter and yoke, p is the track momentum, $d_0(z_0)$ is the impact parameter in the $xy(rz)$ -plane,
 66 $\sigma(d_0)$ ($\sigma(z_0)$) is the measured error of $d_0(z_0)$, respectively. The final MVA cut is a parameter to check
 67 the isolation from other activities. This tagger can also be used for isolated electrons, but for this muon
 68 selection we adjust the variables to make this tagger only behave as the isolated muon tagger. Thus, the
 69 isolated electrons will not be included in the $h \rightarrow \mu^+\mu^-$ candidate category.

70 We apply cuts only related to the $h \rightarrow \mu^+\mu^-$ candidate as the general event selection. Since the
 71 signal events always have $h \rightarrow \mu^+\mu^-$ activities, we can use the general selection as the common cuts for
 72 all analyses. We require following conditions sequentially to $h \rightarrow \mu^+\mu^-$ candidate:

- 73 1. exactly one μ^+ and one μ^- ,
- 74 2. $0.5 < \chi^2/\text{Ndf}(\mu^\pm) < 1.5$,
- 75 3. $|d_0(\mu^\pm)| < 0.02$ mm, $|d_0(\mu^-) - d_0(\mu^+)| < 0.02$ mm,
- 76 4. $|z_0(\mu^\pm)| < 0.5$ mm, $|z_0(\mu^-) - z_0(\mu^+)| < 0.5$ mm,
- 77 5. $\sigma(M_{\mu^+\mu^-}) < 1$ GeV for 500 GeV and < 0.5 GeV for 250 GeV,
- 78 6. $100 < M_{\mu^+\mu^-} < 130$ GeV,
- 79 7. $\cos\theta_{\mu^+\mu^-} < 0.55$ for 500 GeV and < -0.4 for 250 GeV,

80 where χ^2/Ndf is the parameter of how much a track fitted well divided by the number of degrees of
 81 freedom of track fit, $\sigma(M_{\mu^+\mu^-})$ is the event-by-event mass resolution, $\theta_{\mu^+\mu^-}$ is the angle between μ^+ and
 82 μ^- , respectively. The second and fifth cuts are requiring very well measured tracks and muons, while
 83 third and fourth cuts are requiring prompt muons to avoid muons from τ lepton decay. The sixth and
 84 seventh cuts are used to select only $h \rightarrow \mu^+\mu^-$ candidates. Figure 2 shows the $M_{\mu^+\mu^-}$ spectrum before
 85 applying the sixth cut.

86 2.2 $Z \rightarrow q\bar{q}$

87 In the remaining particles after the selection of $h \rightarrow \mu^+\mu^-$ candidate, it is expected that it only contains
 88 the activities of $Z \rightarrow q\bar{q}$ and no isolated leptons. We again use the IsolatedLeptonTagger [16] to the
 89 remaining particles to count the number of isolated leptons and use for vetoing. However at 500 GeV,
 90 we also have non-negligible contributions from $\gamma\gamma \rightarrow$ low P_t hadron overlay [17]. To remove these
 91 contributions, we use the exclusive k_T clustering algorithm [18, 19] with a generalized jet radius of 1.0.
 92 We require 4 jets to allow hard gluon emission from each quark. Any particles not included in these 4
 93 jets are removed since these are most likely coming from $\gamma\gamma \rightarrow$ low P_t hadrons background. After this,
 94 we use the Durham clustering algorithm [20] to force the remaining particles into 2 jets, and consider
 95 this as the $Z \rightarrow q\bar{q}$ candidate.

96 We additionally apply dedicated cuts to select $Z \rightarrow q\bar{q}$ candidate and reject background events. For
 97 qqh500-L we apply the following cuts sequentially:

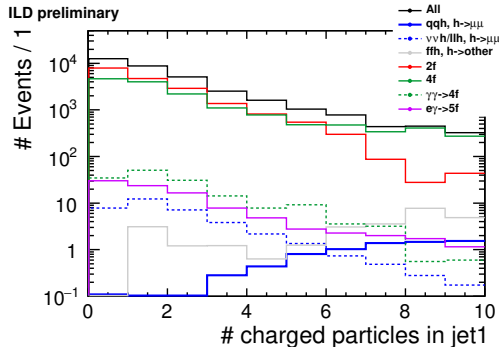


Figure 3: The distribution of the number of charged particles in the most energetic jet (qqh500-L).

Table 2: Cut table of qqh500-L.

	$e^+e^- \rightarrow q\bar{q}h$ $h \rightarrow \mu^+\mu^-$	$e^+e^- \rightarrow \nu\bar{\nu}h/\ell\bar{\ell}h$ $h \rightarrow \mu^+\mu^-$	$e^+e^- \rightarrow f\bar{f}h$ $h \rightarrow \text{other}$	$e^+e^- \rightarrow 2f$	$e^+e^- \rightarrow 4f$	$\gamma\gamma \rightarrow 4f$	$e^\pm\gamma \rightarrow 5f$
No cut	24.6	64.1	4.12×10^5	4.22×10^7	4.59×10^7	3.36×10^5	2.29×10^5
# μ^\pm	22.8	59.7	6455.1	1.31×10^6	1.01×10^6	1.49×10^4	5752.4
χ^2/Ndf	22.6	59.1	6396.6	1.21×10^6	9.24×10^5	1.31×10^4	5369.9
d_0	22.5	58.8	6338.4	1.18×10^6	8.51×10^5	1.13×10^4	4978.7
z_0	22.5	58.7	6332.1	1.17×10^6	8.45×10^5	1.12×10^4	4952.9
$\sigma(M_{\mu^+\mu^-})$	22.1	58.3	6269.1	8.03×10^5	8.15×10^5	1.11×10^4	4890.5
$M_{\mu^+\mu^-}$	21.5	56.6	166.0	3.83×10^4	2.96×10^4	360.5	372.3
$\cos\theta_{\mu^+\mu^-}$	21.5	56.6	121.3	2.43×10^4	2.81×10^4	359.9	371.5
veto	21.2	52.8	115.1	2.38×10^4	2.08×10^4	218.5	126.4
# jet	21.2	36.6	113.8	1.88×10^4	1.68×10^4	159.5	101.9
# charged	18.4	1.5	97.7	627.4	3056.3	12.9	14.0
M_{jj}	17.3	0.1	87.8	193.8	2298.5	4.8	9.6

1. veto: require no isolated leptons in the remaining particles after selecting $h \rightarrow \mu^+\mu^-$ candidate,
2. jet clustering successful,
3. after the Durham clustering, each jet should contain at least 4 charged particles,
4. $60 < M_{jj} < 160$ GeV,

where M_{jj} is the invariant mass of the two jets. The third cut is applied to reject 3-prong τ decay events, while the fourth cut is selecting $Z \rightarrow q\bar{q}$ candidate. Figure 3 shows the distribution of number of charged particles in jet1 before applying third cut, where jet1 denotes a jet which has higher jet energy between two jets. Table 2 shows the cut table of qqh500-L.

After all cuts mentioned above, we perform multivariate analysis for further background rejection. We use gradient boosted decision tree method (BDTG) which is included in TMVA package in ROOT [21,22]. For qqh500-L, we use the following 7 variables: thrust, $\cos\theta_h$, charge $\times \cos\theta_{\mu^+}$, charge $\times \cos\theta_{\mu^-}$, E_{leading} , $E_{\text{subleading}}$, and M_{jj} , where θ_h is the polar angle of the reconstructed Higgs boson using $h \rightarrow \mu^+\mu^-$ candidate, $\theta_{\mu^+}(\theta_{\mu^-})$ is the polar angle of $\mu^+(\mu^-)$, $E_{\text{leading}}(E_{\text{subleading}})$ is the first(second) largest energy between two muons of $h \rightarrow \mu^+\mu^-$ candidate, respectively. Figure 4 shows the distribution of $E_{\text{subleading}}$ as an example of the input variables to BDTG. Figure 5 shows the result of the BDTG analysis. We apply a cut of $\text{BDTGoutput} > 0.65$. The remaining signal events N_S after this cut are 11.2 while background events N_B are 422.

3 Results

Figure 6 shows the $M_{\mu^+\mu^-}$ spectrum after all cuts mentioned in the previous sections. We can clearly see several spikes in the background distribution, due to the limited MC statistics for SM background. Therefore, the result will be significantly affected by statistical fluctuations. To improve this, we apply a toy MC technique to estimate the final uncertainties.

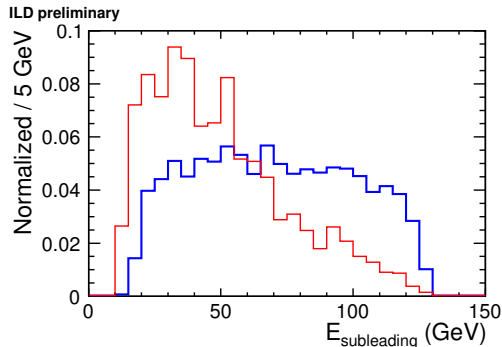


Figure 4: Spectrum of $E_{\text{subleading}}$ as the input to BDTG analysis (qqh500-L). Blue shows signal and red shows background, both histograms are normalized to 1.

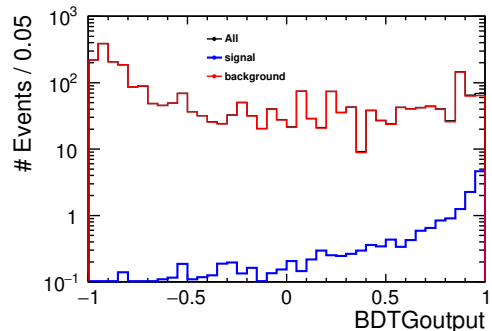


Figure 5: Distribution of the BDTGoutput (qqh500-L).

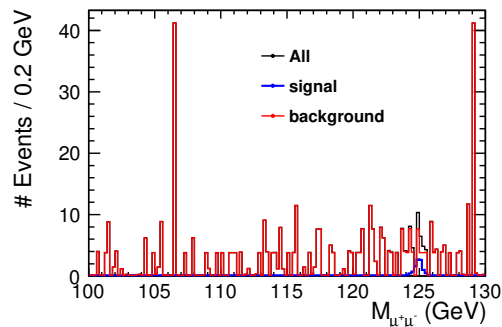


Figure 6: Spectrum of $M_{\mu^+\mu^-}$ after all cuts (qqh500-L).

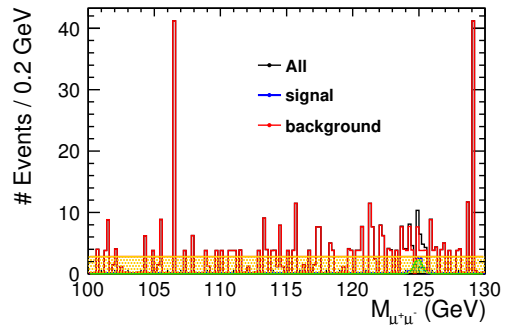


Figure 7: Similar to Figure 6 but results of the fitting are added. Green shows the fitting result for signal using f_S and yellow shows for background using f_B .

As a first step, analytic functions are fitted to the relevant signal and background histograms. We use a normalized Gaussian as the signal fitting function f_S and a constant as the background fitting function f_B . Figure 7 shows the result of fitting to $M_{\mu^+\mu^-}$ spectrum using f_S and f_B .

Then we perform pseudo-experiments using the obtained f_S and f_B . In one pseudo-experiment, the number of pseudo-events are determined by $N_S(N_B)$ with Poisson fluctuation. Figure 8 shows an example of one pseudo-experiment. We use the function $f \equiv Y_S f_S + Y_B f_B$ as the fitting function where Y_S is the signal event yield, Y_B is the background event yield, and both are free parameters in the fit. The purple curve in Figure 8 shows the result of fitting using f for sum of the pseudo-data.

We repeat pseudo-experiments for 200000 times and obtain Y_S distribution and pull distribution. From the Gaussian fit to the Y_S distribution, we obtain the mean value of 10.93 ± 0.01 and the width of 5.227 ± 0.008 . The resulting precision for $\sigma \times \text{BR}(h \rightarrow \mu^+\mu^-)$ is estimated to be 47.8%. The pull is defined as $(Y_S - Y_{\text{true}})/\Delta Y_S$, where ΔY_S is the fitting error of Y_S and Y_{true} is corresponding to the number of pseudo-data determined as N_S with Poisson fluctuation. If there are no biases in the fitting, the pull distribution should have the mean of ~ 0 and width of ~ 1 . However, we obtain the mean of -0.071 ± 0.002 and width of 0.779 ± 0.001 from Gaussian fitting to pull distribution. This result indicates that there are some biases included in our analysis. In addition, we find asymmetric distribution for the Y_S and pull distribution. The reason is under investigation.

In a similar way, we have analyzed all channels listed in Table 1. The results are summarized in Table 3. By combining all 250 GeV results, we can obtain 25.0% combined precision on the cross section

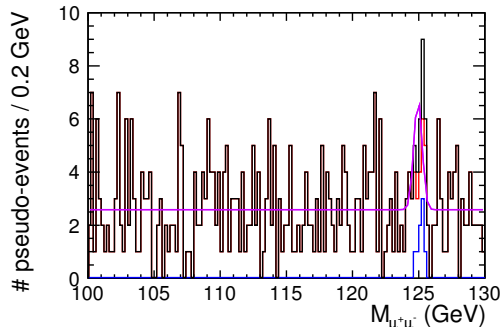


Figure 8: One example of pseudo-experiment (qgh500-L). Blue(red) shows pseudo-signal(background) data, black is the sum of blue and red, purple is the fitting results to black using function f , respectively.

Table 3: Summary of the precision of $\sigma \times \text{BR}(h \rightarrow \mu^+ \mu^-)$.

250 GeV	$q\bar{q}h$	$\nu\bar{\nu}h$
L	30.0%	123.5%
R	52.5%	125.4%
500 GeV	$q\bar{q}h$	$\nu\bar{\nu}h$
L	47.8%	39.2%
R	52.1%	71.5%

139 times branching ratio $\sigma \times \text{BR}$. This result is much better than SiD results [7] with statistical scaling
 140 extrapolation ($\sim 39\%$ for left-handed 250 GeV $q\bar{q}h$ and $\nu\bar{\nu}h$ channels). Together with the 500 GeV results,
 141 the combined precision is estimated to be 17.5%. This is comparable to ATLAS HL-LHC prospects [10],
 142 but worse than CMS HL-LHC prospects [11] due to the statistics of number of signal events. However
 143 as we explained in Section 1, we can extract absolute couplings together with other measurements at the
 144 ILC without model dependencies, while LHC results always have model dependencies.

145 4 Further Study

146 After the LCWS2017, we have studied the case of staging scenario [15], and investigated further im-
 147 provements. In the staging scenario, the beam polarization sharing for 250 GeV is changed from H20
 148 scenario [15]. The expected number of signal events are summarized in Table 4.

149 We apply the same analysis procedure except the optimization of BDTGoutput cut and the way of toy
 150 MC. The BDTGoutput cut mentioned at the end of Section 2 was not optimized. We have studied the
 151 optimum cut on BDTGoutput together with the result of the toy MC procedure, and adopted the best
 152 case as the optimum result. Furthermore, the function $f = Y_S f_S + Y_B f_B$ using the fitting to pseudo-data,
 153 we fix Y_B as N_B which is the number of remaining background after BDTGoutput cut. Since we have
 154 found that Y_B can be determined more precisely compare to Y_S and its precision is $\sim 5\%$ or better, it is
 155 possible to fix Y_B .

156 The new results are summarized in Table 5. The combined precision for 250 GeV is estimated to be
 157 20.5%, and all combined result is 15.4%. The combined 250 GeV result is relatively $\sim 20\%$ improved
 158 from the result in Section 3. The all combined result is also relatively $\sim 10\%$ improved.

159 In summary, we studied the prospects of the branching ratio measurement of $h \rightarrow \mu^+ \mu^-$ at the
 160 ILC assuming ILD detector model in the H20 running scenario as well as in its staged version. The
 161 combined precision using all 250 GeV results is estimated to be 20.5% for $\sigma \times \text{BR}(h \rightarrow \mu^+ \mu^-)$, which
 162 presents a considerable improvement with respect to a previous study at 250 GeV. Together with the
 163 500 GeV results, the combined precision improves to 15.4%, which is similar to the HL-LHC prospects.
 164 We are planning to analyze $e^+ e^- \rightarrow \ell^+ \ell^- h$ channel, and work on more background rejection for further
 165 improvement.

Table 4: The expected number of signal events at $\sqrt{s} = 250$ GeV assuming the staging scenario.

	$q\bar{q}h$	$\nu\bar{\nu}h$
L	41.1 (900 fb ⁻¹)	15.0 (900 fb ⁻¹)
R	28.1 (900 fb ⁻¹)	8.4 (900 fb ⁻¹)

Table 5: Summary of the precision of $\sigma \times \text{BR}(h \rightarrow \mu^+\mu^-)$ with further study and staging scenario.

250 GeV	$q\bar{q}h$	$\nu\bar{\nu}h$
L	32.5%	108.6%
R	28.1%	110.4%
500 GeV	$q\bar{q}h$	$\nu\bar{\nu}h$
L	44.5%	37.0%
R	49.5%	74.5%

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