Probing CP violating Higgs sectors via the precision measurement of coupling constants

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[In preparation]

Contents

- 1. Introduction
- 2. Two Higgs doublet model with CP violating effects
- 3. Numerical analysis
- 4. Summary

Introduction

- Higgs boson which is predicted in the Standard Model(SM) was detected at the Large Hadron Collider(LHC).
- Phenomena beyond the SM have been reported.
 - Dark matter

- Neutrino oscillations
- Baryon asymmetry of the Universe (BAU).

The CP violating effect of the SM is too small.

[G. R. Farrar and M. E. Shaposhnikov, Phys. Rev. D 50, 774 (1994)]

- On the other hand, the Higgs sector is still unknown.
 - The number of the Higgs field?
 - The Higgs field is elementary or composed?
 - Dynamics of the electroweak symmetry breaking (EWSB) ?
- Phenomena beyond the SM can be related to the extended Higgs sector.
 - → We can introduce new CP violating effects by extended Higgs sector.

Motivation

- We can measure the CP violating effects by following methods:
 - Electric dipole moment (EDM)
 - Features of new particles
 - Angular distribution of decay products [Prof.Daniel Jeans's talk on Monday]
 - ...

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However, EDM and new particles have not been detected yet.

- In this talk, we discuss how to test the CP violating effects by precision measurements of the SM-like Higgs boson couplings.

 Higgs boson couplings will be precisely measured.
- ❖ We focus on two Higgs doublet model(2HDM) with CP violating effects and discuss the deviation of the Higgs couplings from the SM between the CP conserving case and the CP violating case.

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2HDM with CP violating effects

Tree-level potential of 2HDM with softly broken Z₂ symmetry $\Phi_l = \begin{pmatrix} \omega_l^{\scriptscriptstyle \top} \\ \frac{1}{\sqrt{2}} (h_l + iz_l) \end{pmatrix} \quad (l = 1, 2)$ $V = \mu_1^2 |\Phi_1|^2 + \mu_2^2 |\Phi_2|^2 - (\mu_3^2) \Phi_1^{\dagger} \Phi_2) + h.c.$ $+\frac{1}{2}\lambda_{1}|\Phi_{1}|^{4}+\frac{1}{2}\lambda_{2}|\Phi_{2}|^{4}+\lambda_{3}|\Phi_{1}|^{2}|\Phi_{2}|^{2}+\lambda_{4}|\Phi_{1}^{\dagger}\Phi_{2}|^{2}+\left\{\frac{1}{2}\lambda_{5}\Phi_{1}^{\dagger}\Phi_{2}\right)^{2}+h.c.\right\}$

Vacuum expectation value

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v_1 \end{pmatrix}$$
 and $\langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v_2 \end{pmatrix}$

 $\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ \frac{1}{6}v_1 \end{pmatrix}$ and $\langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ \frac{1}{6}v_2 \end{pmatrix}$ (One of the complex parameters(ξ) can be absorbed by redefinition of the phases in fields.) by redefinition of the phases in fields.)

Stationary conditions

$$\frac{\partial V}{\partial h_1} = 0, \frac{\partial V}{\partial h_2} = 0 \text{ and } \frac{\partial V}{\partial z_2} = 0.$$

$$\frac{\partial V}{\partial h_1} = 0, \frac{\partial V}{\partial h_2} = 0 \text{ and } \frac{\partial V}{\partial z_2} = 0.$$

$$\begin{cases}
\mu_1^2 = \frac{v_2}{v_1} \operatorname{Re}(\mu_3^2) - \frac{1}{2} (\lambda_1 v_1^2 + \lambda_{345} v_2^2) & \mu_2^2 = \frac{v_1}{v_2} \operatorname{Re}(\mu_3^2) - \frac{1}{2} (\lambda_2 v_2^2 + \lambda_{345} v_1^2) \\
2 \operatorname{Im}(\mu_3^2) = v_1 v_2 \operatorname{Im}(\lambda_5)
\end{cases}$$

Complex parameters

$$\xi, \operatorname{Im}(\mu_3^2), \operatorname{Im}(\lambda_5) \Longrightarrow \operatorname{Im}(\lambda_5)$$



$$\operatorname{Im}(\lambda_5)$$

2HDM with CP violating effects

NGB and Charged Higgs boson $\tan \beta = \frac{v_2}{v_3}$ [S. Davidson and H. E. Haber, Phys. Rev. D 72, 035004 (2005)]

$$\begin{pmatrix} \hat{\phi}_1 \\ \hat{\phi}_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \qquad \hat{\phi_1} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h_1' + iG^0) \end{pmatrix}, \ \hat{\phi_2} = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2' + ih_3') \end{pmatrix}$$

Mass matrix (h_1', h_2', h_3')

$$(\lambda_{345} = \lambda_3 + \lambda_4 + \text{Re}[\lambda_5], M^2 \equiv \frac{\text{Re}[\mu_3^2]}{\sin\beta\cos\beta})$$

$$\sin 2(\beta - \tilde{\alpha})(\tilde{m}_h^2 - \tilde{m}_H^2) \qquad -\frac{v^2}{2}\text{Im}\lambda_5\sin(2\beta)$$

$$\cos^2(\beta - \tilde{\alpha}) + \tilde{m}^2\sin^2(\beta - \tilde{\alpha}) \qquad v^2\text{Im}\lambda_5\cos(2\beta)$$

$$\mathcal{M}^{2} = \begin{pmatrix} \tilde{m}_{h}^{2} \sin^{2}(\beta - \tilde{\alpha}) + \tilde{m}_{H}^{2} \cos^{2}(\beta - \tilde{\alpha}) \\ \frac{1}{2} \sin 2(\beta - \tilde{\alpha})(\tilde{m}_{h}^{2} - \tilde{m}_{H}^{2}) \\ -\frac{v^{2}}{2} \operatorname{Im} \lambda_{5} \sin(2\beta) \end{pmatrix}$$

$$\mathcal{M}^{2} = \begin{pmatrix} \tilde{m}_{h}^{2} \sin^{2}(\beta - \tilde{\alpha}) + \tilde{m}_{H}^{2} \cos^{2}(\beta - \tilde{\alpha}) & \frac{1}{2} \sin 2(\beta - \tilde{\alpha})(\tilde{m}_{h}^{2} - \tilde{m}_{H}^{2}) & -\frac{v^{2}}{2} \operatorname{Im} \lambda_{5} \sin(2\beta) \\ \frac{1}{2} \sin 2(\beta - \tilde{\alpha})(\tilde{m}_{h}^{2} - \tilde{m}_{H}^{2}) & \tilde{m}_{h}^{2} \cos^{2}(\beta - \tilde{\alpha}) + \tilde{m}_{H}^{2} \sin^{2}(\beta - \tilde{\alpha}) & -\frac{v^{2}}{2} \operatorname{Im} \lambda_{5} \cos(2\beta) \\ -\frac{v^{2}}{2} \operatorname{Im} \lambda_{5} \sin(2\beta) & -\frac{v^{2}}{2} \operatorname{Im} \lambda_{5} \cos(2\beta) & \tilde{m}_{A}^{2} \end{pmatrix}$$

(In the CP conserving limit, \tilde{m}_h , \tilde{m}_H , \tilde{m}_A and $\tilde{\alpha}$ parameters are the mass eigenvalues and mixing angle for CP-even states.)

$$\begin{pmatrix} h'_{1} \\ h'_{2} \\ h'_{3} \end{pmatrix} = R \begin{pmatrix} H_{1} \\ H_{2} \\ H_{3} \end{pmatrix}, \quad R^{T} \mathcal{M}^{2} R = \mathcal{M}^{2}_{\text{diag}} = \text{diag}(m_{H_{1}}^{2}, m_{H_{2}}^{2}, m_{H_{3}}^{2}) \quad \text{(H}_{1} \text{ is the SM-like Higgs.)}$$

$$\begin{pmatrix} R^{T} R = I \end{pmatrix}$$

Independent parameters

$$v(=246 \text{GeV}), m_{H_1} = m_h(125 \text{GeV}), m_{H^{\pm}}, \tilde{m}_H, \tilde{m}_A, \tan \beta, \sin(\beta - \tilde{\alpha}), M^2, \text{Im} \lambda_5$$

The SM-like Higgs boson couplings

❖ We impose Z₂ symmetry on 2HDM in order to avoid FCNC at tree-level.

| | Φ_1 | Φ_2 | Q_L | L_L | u_R | d_R | e_R |
|---------|----------|----------|-------|-------|-------|-------|-------|
| Type-I | + | _ | + | + | = | = | _ |
| Type-II | + | _ | + | + | _ | + | + |
| Type-X | + | _ | + | + | - | - | + |
| Type-Y | + | - | + | + | _ | + | 3-1 |

SM-like Higgs boson couplings

$$\mathcal{L}_{hVV}^{\text{2HDM}} = \underline{R_{11}} \left(g_W^{\text{SM}} W_{\mu}^+ W^{-\mu} + \frac{1}{2} g_Z^{\text{SM}} Z_{\mu} Z^{\mu} \right) H_1$$

 H_1 : Higgs boson (125GeV)

f: quark or charged lepton

V: W or Z boson

$$I_u = 1/2,$$

 $I_d = I_e = -1/2$

$$\mathcal{L}_{hff}^{ ext{2HDM}} = -g_f^{ ext{SM}} ar{f}(\underline{c_f^s + i \gamma_5 c_f^p}) f H_1 \underbrace{ egin{pmatrix} c_f^s = R_{11} + R_{21} \xi_f \ c_f^p = (-2I_f) R_{31} \xi_f \end{bmatrix} }$$

| | ξ_u | ξ_d | ξ_e |
|---------|---------------|---------------|---------------|
| Type-I | $+\cot \beta$ | $+\cot \beta$ | $+\cot \beta$ |
| Type-II | $+\cot \beta$ | $-\tan \beta$ | $-\tan \beta$ |
| Type-X | $+\cot \beta$ | $+\cot \beta$ | $-\tan \beta$ |
| Type-Y | $+\cot \beta$ | $-\tan \beta$ | $+\cot \beta$ |

Scaling factor for
$$H_1VV$$
: $\kappa_V = \frac{g_{hVV}^{2HDM}}{g_{hVV}^{SM}} = R_{11}$

We use the scaling factor and the ratio of decay rate for following numerical analysis.

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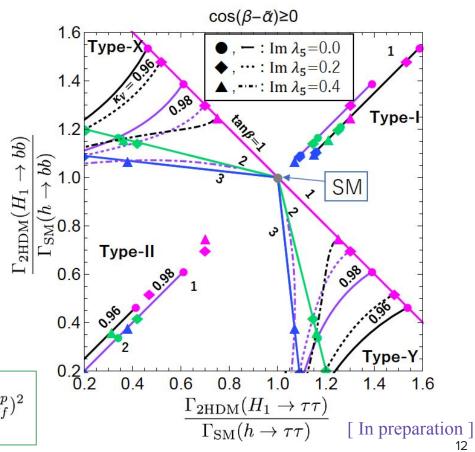
Input parameters

$$egin{array}{lll} v &= 246 \; {
m GeV}, \ m_h &= 125 \; {
m GeV}, \ ilde{m}_H &= 200 \; {
m GeV}, \ ilde{m}_A &= 250 \; {
m GeV} \end{array}$$

(The numerical results don't depend on M and m_{H^+} .)

In the case of Imλ₅ values(=0.2, 0.4), we can treat tilde mass parameters as mass eigenvalues.

$$\kappa_V = rac{g_{hVV}^{2HDM}}{g_{hVV}^{SM}} = R_{11}$$
 , $rac{\Gamma(h o far{f})_{
m 2HDM}}{\Gamma(h o far{f})_{
m SM}} \simeq (c_f^s)^2 + (c_f^p)^2$



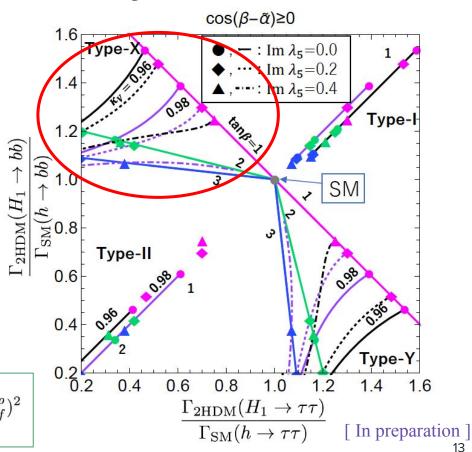
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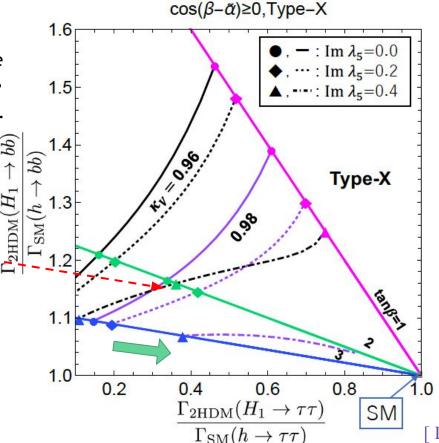
$$\kappa_V=rac{g_{hVV}^{2HDM}}{g_{hVV}^{SM}}=R_{11}$$
 , $rac{\Gamma(h o far{f})_{
m 2HDM}}{\Gamma(h o far{f})_{
m SM}}\simeq (c_f^s)^2+(c_f^p)^2$



When there are CP violating effects in the Higgs potential, Higgs boson couplings are different from ones in CP conserving case.



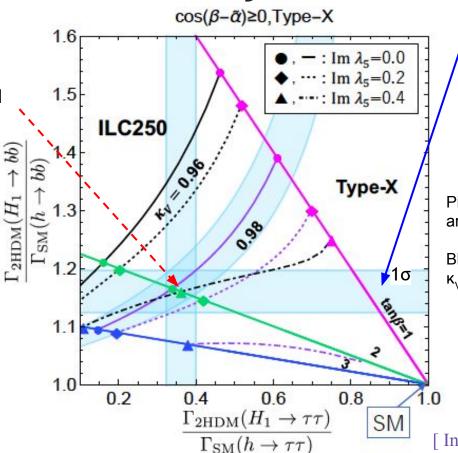
For instance, we focus on the ratio at green triangle mark.



In preparation] ₁₄

All sensitivity regions are on $\tan \beta = 2$ and $\kappa_{\text{V}} = 0.98$ and $\tan \lambda_5 = 0$ by experiments...

The ratios and the scaling factors are favored CP conserving case.



 κ_{V} : 0.38 % κ_{τ} : 1.9 % κ_{b} : 1.8 % κ_{c} : 2.4 % ILC250 (2ab⁻¹)

[K. Fujii et al., arXiv:1710.07621]

Pink, green and blue are $tan\beta=1$, 2 and 3.

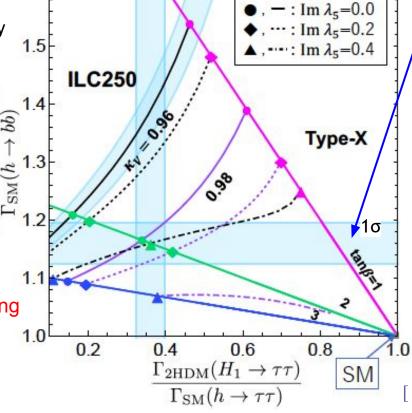
Black and purple are κ_{V} =0.96 and 0.98.

[In preparation] ₁₅

If sensitivity region for hVV is on $\kappa_{V}=0.96$ by experiments, CP conserving 2HDM cannot explain the ratios and the scaling factors.

In the CP violating case, the scaling factors and the ratios can be explained.

In principle, we can test the CP violating effects by precision measurements of SM-like Higgs boson couplings.



cos(β-ã)≥0,Type-X

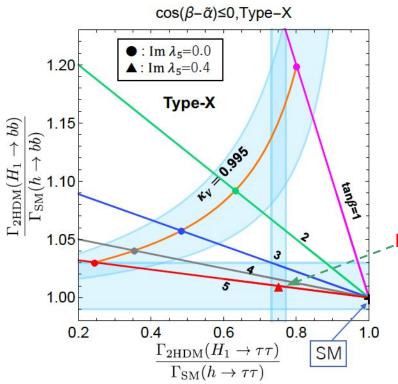
 $\kappa_{V}: 0.38\%$ $\kappa_{\tau}: 1.9 \%$ $\kappa_b : 1.8 \%$ $\kappa_{\rm c}$: 2.4 % ILC250 (2ab-1)

[K. Fujii et al., arXiv:1710.07621]

Pink, green and blue are $tan\beta=1$, 2 and 3.

Black and purple are κ_{v} =0.96 and 0.98.

In preparation] 16



 $\kappa_V : 0.2 \% \\ \kappa_{\tau} : 1 \% \\ \kappa_b : 1 \%$

The accuracy can be achieved with ILC250(8ab⁻¹).

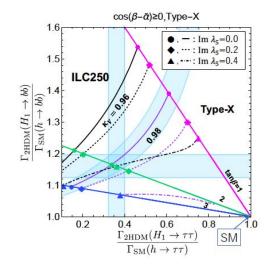
Red triangle mark isn't excluded by EDM experiment.

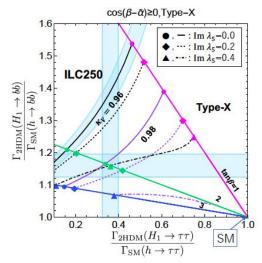
(tan8=5, κ_V =0.995 and Im λ_s =0.4)

CP violating effects which aren't excluded by EDM can be tested by precision measurements of SM-like Higgs boson couplings.
[In preparation]
₁₇

Summary

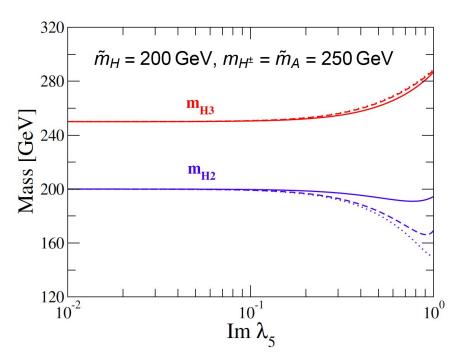
- CP violating effects can be introduced into the Higgs potential by extended Higgs sector.
- We discuss how to test the CP violating effects by precision measurements of SM-like Higgs boson couplings.
- In fact, the deviation of the Higgs boson couplings depends on CP violating effects. We might be able to measure the effects by the precision measurements of the Higgs boson couplings.





Back up

2HDM with CP violating effects

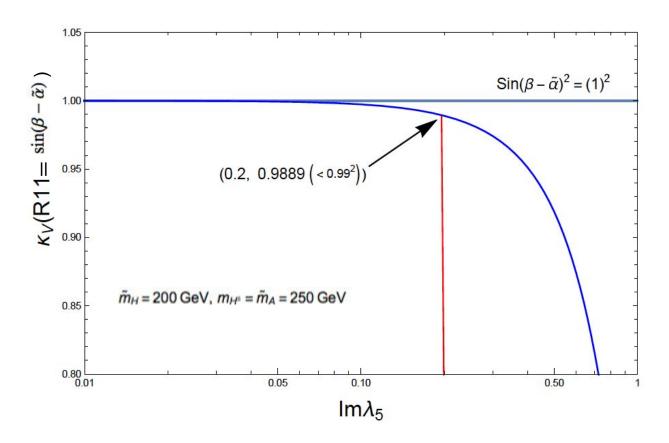


The solid, dashed and dotted curves correspond to $\tan \beta = 2$, 5 and 10, respectively.

 \bullet If Im λ_5 -0.2 and 0.4, $\tilde{m}_H, \ \tilde{m}_A$ is about $m_{H2}, \ m_{H3}$.

$$\begin{pmatrix} h'_1 \\ h'_2 \\ h'_3 \end{pmatrix} = R \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}, \quad R^T \mathcal{M}^2 R = \mathcal{M}^2_{\text{diag}} = \text{diag}(m_{H_1}^2, m_{H_2}^2, m_{H_3}^2)$$

2HDM with CP violating effects



Electric dipole moment $P(\vec{S}) = \vec{S}, P(\vec{E}) = -\vec{E},$

$$P(\vec{S}) = \vec{S}, \quad P(\vec{E}) = -\vec{E}$$

$$T(\vec{S}) = -\vec{S}, \quad T(\vec{E}) = \vec{E}.$$

$$\mathcal{H} = -d\frac{\vec{S}}{|\vec{S}|} \cdot \vec{E} - \mu \frac{\vec{S}}{|\vec{S}|} \cdot \vec{B}$$

$$\varepsilon_{+} = -\mu B - dE$$

$$\mathcal{E}_{+} = \mu B + dE,$$

$$\mathcal{E}_{-} = \mu B - dE$$

$$\mathcal{E}_{-} = \mu B - dE$$

$$\mathcal{E}_{-} = \frac{2dE}{h}$$

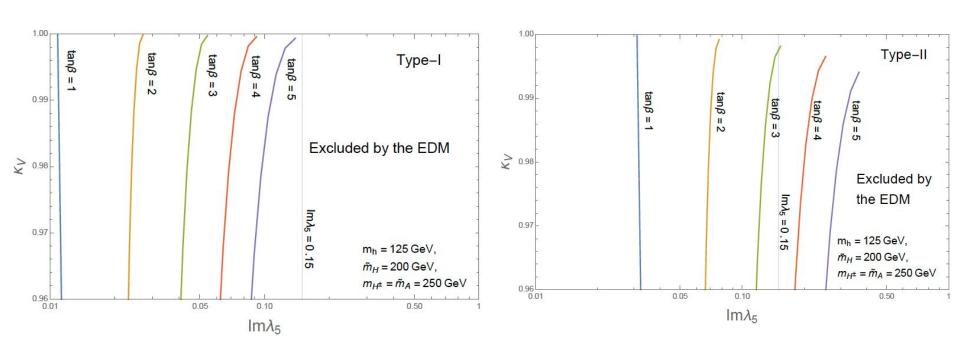
Constraints by EDM [K. Cheung, J. S. Lee, E. Senaha and P. Y. Tseng, JHEP 1406, 149 (2014)]

$$\begin{split} -\mathcal{L} \supset & -g_{H_1VV}^{SM} \xi_V^{H_1} H_1 V_{\mu} V^{\mu} + \sum_{f=u,d,e} \frac{m_f}{v} \sum_{i=1,2,3} \left(\xi_f^{H_i} \bar{f} f H_i + i \tilde{\xi}_f^{H_i} \bar{f} \gamma_5 f H_i \right) \\ & + \frac{\sqrt{2}}{v} \left[V_{ud} \bar{u} (m_d \xi_d P_R + m_u \xi_u P_L) dH^+ + m_e \xi_e \bar{v} P_R eH^+ + H.c. \right], \end{split}$$

$$|\tilde{\xi}_u^{H_1}| \lesssim 7 \times 10^{-3} \text{ (I)}, \qquad 2 \times 10^{-2} \text{ (II)}, \qquad 3 \times 10^{-2} \text{ (X)}, \qquad 6 \times 10^{-3} \text{ (Y)}$$

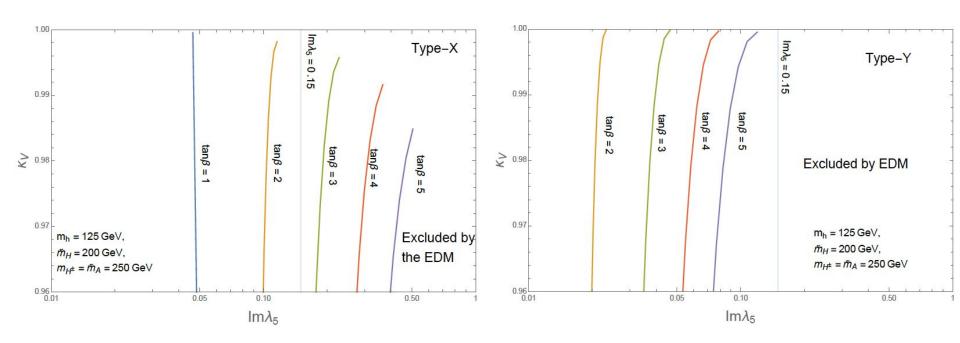
EDM constraints

[K. Cheung, J. S. Lee, E. Senaha and P. Y. Tseng, JHEP 1406, 149 (2014)]



EDM constraints

[K. Cheung, J. S. Lee, E. Senaha and P. Y. Tseng, JHEP 1406, 149 (2014)]



• ρ parameter [G. C. Joshi, M. Matsuda and M. Tanimoto, Phys. Lett. B 341, 53 (1994)]

$$\begin{split} \Delta \rho &= \frac{3\alpha}{16\pi \cos^2 \theta_w} \sum_{i=1}^3 \frac{O_{1i}^2}{m_Z^2 - m_W^2} L(m_{Hi}^2, m_{Href}^2) \\ &+ \frac{\alpha}{16\pi \sin^2 \theta_w m_W^2} \Biggl(\sum_{i=1}^3 (1 - O_{1i})^2 F(m_{Hi}^2, m_{Hch}^2) - \frac{1}{2} \sum_{\substack{i,j,k=1\\i \neq i,j \neq k,k \neq i}}^3 O_{1i}^2 F(m_{Hj}^2, m_{Hk}^2) \Biggr), \end{split}$$

where

$$F(x, y) = \frac{x+y}{2} - \frac{xy}{x-y} \log \frac{x}{y}, \quad L(x, y) = F(x, m_Z^2) - F(x, m_W^2) + F(y, m_W^2) - F(y, m_Z^2).$$

$$\to \Delta \rho = O(10^{-4})$$

• $\xi \sim O(10^{-2})$ to get sufficient baryon asymmetry

[A. G. Cohen, D. Kaplan, and A. Nelson, Annu. Rev. Nucl. Part. Sci. 43, 27 (1993)]

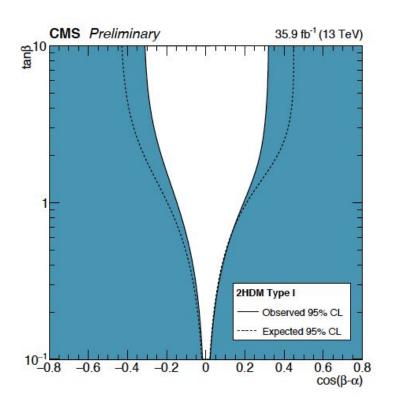
Sensitivity for **k**

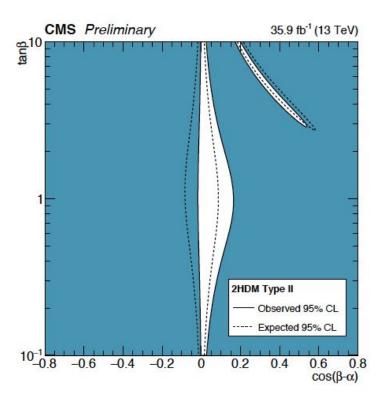
[S. Dawson et al., arXiv:1310.8361]

| Facility | LHC | HL-LHC | ILC500 | ILC500-up | ILC1000 | ILC1000-up | CLIC | TLEP (4 IPs) |
|---|----------|-----------|-----------|-------------|------------------|--------------------|-------------------|--------------|
| $\sqrt{s} \; (\mathrm{GeV})$ | 14,000 | 14,000 | 250/500 | 250/500 | 250/500/1000 | 250/500/1000 | 350/1400/3000 | 240/350 |
| $\int \mathcal{L}dt \ (\mathrm{fb}^{-1})$ | 300/expt | 3000/expt | 250 + 500 | 1150 + 1600 | 250 + 500 + 1000 | 1150 + 1600 + 2500 | 500 + 1500 + 2000 | 10,000+2600 |
| κ_{γ} | 5 - 7% | 2 - 5% | 8.3% | 4.4% | 3.8% | 2.3% | -/5.5/<5.5% | 1.45% |
| κ_g | 6 - 8% | 3-5% | 2.0% | 1.1% | 1.1% | 0.67% | 3.6/0.79/0.56% | 0.79% |
| κ_W | 4 - 6% | 2-5% | 0.39% | 0.21% | 0.21% | 0.2% | 1.5/0.15/0.11% | 0.10% |
| κ_Z | 4-6% | 2-4% | 0.49% | 0.24% | 0.50% | 0.3% | 0.49/0.33/0.24% | 0.05% |
| κ_{ℓ} | 6 - 8% | 2-5% | 1.9% | 0.98% | 1.3% | 0.72% | 3.5/1.4/<1.3% | 0.51% |
| $\kappa_d = \kappa_b$ | 10-13% | 4-7% | 0.93% | 0.60% | 0.51% | 0.4% | 1.7/0.32/0.19% | 0.39% |
| $\kappa_u = \kappa_t$ | 14-15% | 7-10% | 2.5% | 1.3% | 1.3% | 0.9% | 3.1/1.0/0.7% | 0.69% |

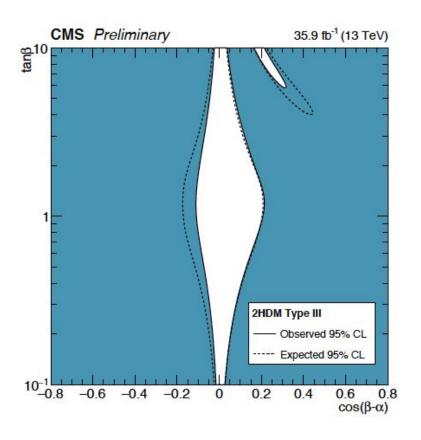
| | ILC250 κ fit | (2ab ⁻¹) |
|----------------|---------------------|-------------------------------------|
| g(hbb) | 1.8 | |
| g(hcc) | 2.4 | |
| g(hgg) | 2.2 | FIV. F |
| g(hWW) | 1.8 | [K. Fujii et al., arXiv:1710.07621] |
| $g(h\tau\tau)$ | 1.9 | |
| g(hZZ) | 0.38 | |

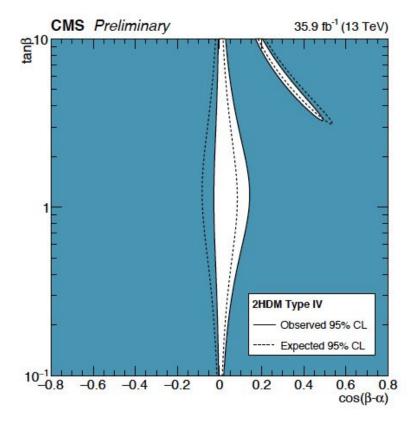
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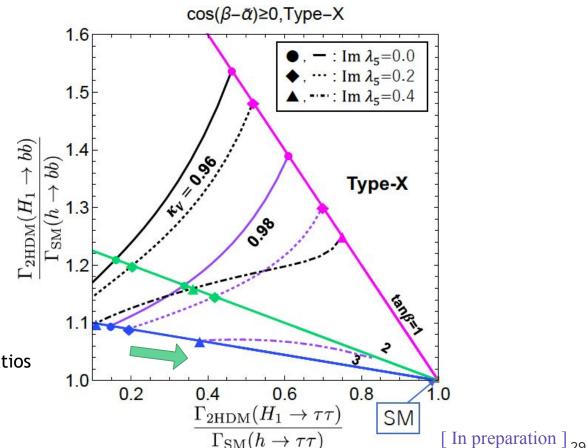




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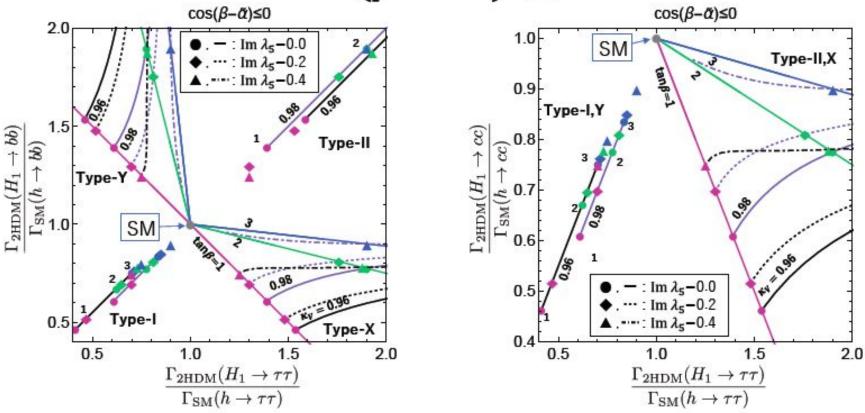




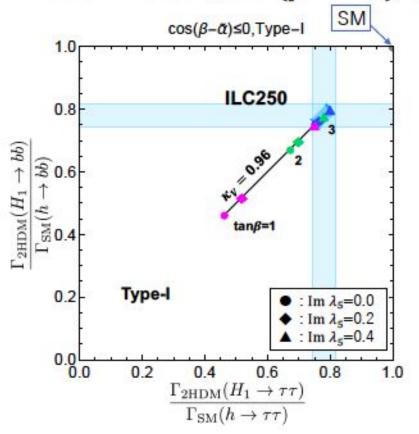


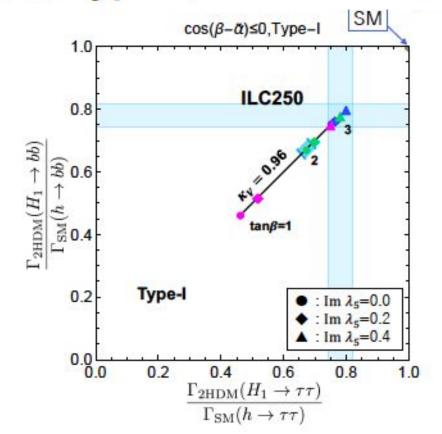
(When ${\rm Im}\lambda_5$ become large, the ratios approach $\,\kappa_V^2 + (1-\kappa_V^2)\xi_f^2.$)

$\cos(\beta - \alpha) \leq 0$

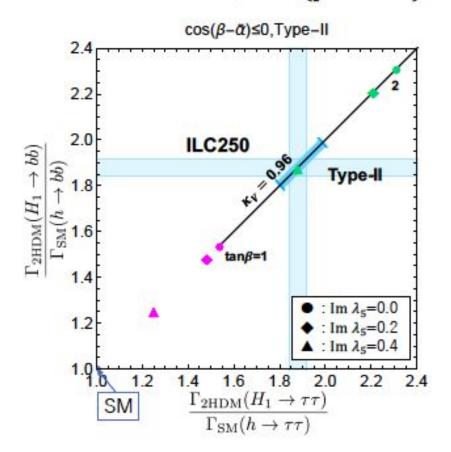


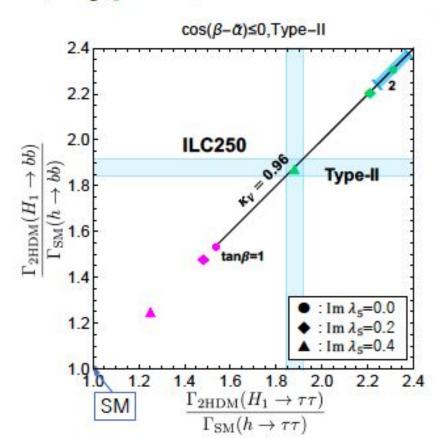
$hbb-hττ (cos(β - α) \le 0, Type-I)$



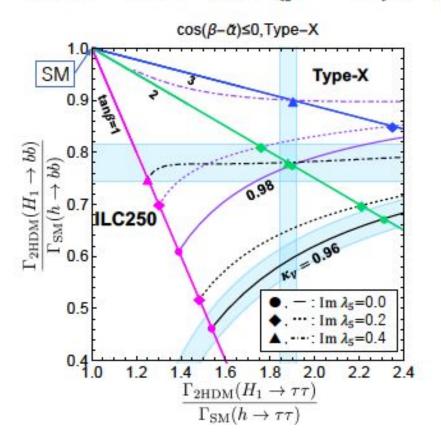


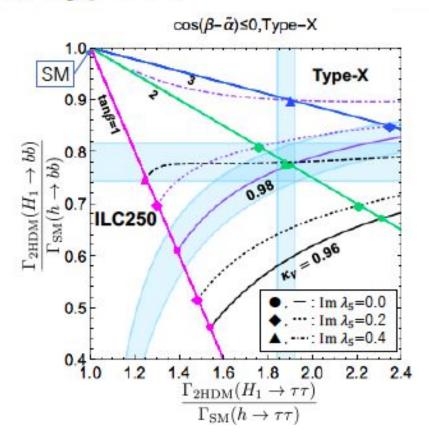
$hbb-hττ (cos(β - α) \le 0, Type-II)$



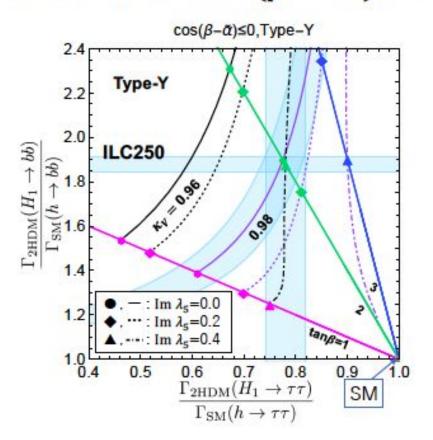


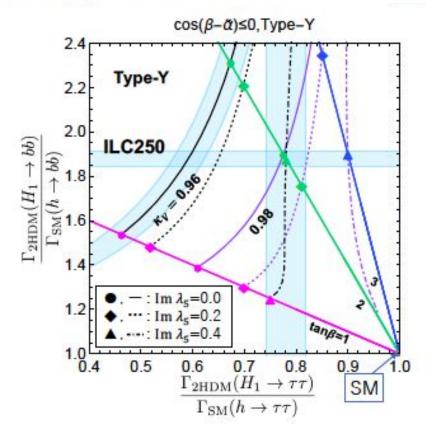
hbb- hττ (cos(β - α) ≤ 0, Type-X)



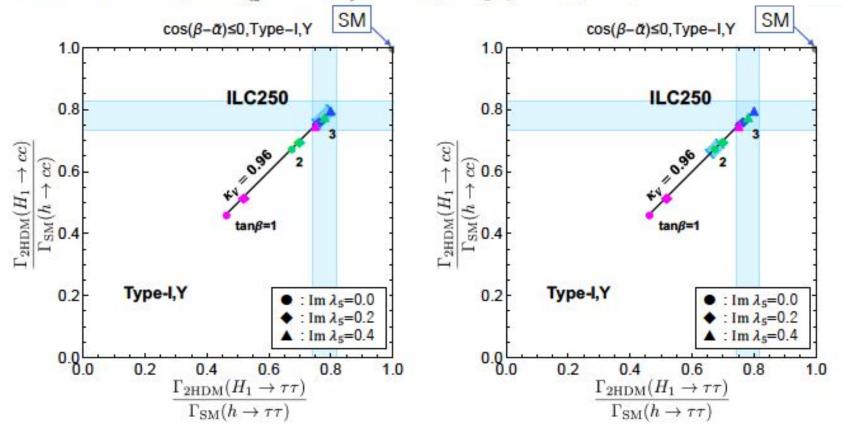


$hbb-hττ (cos(β - α) \le 0, Type-Y)$

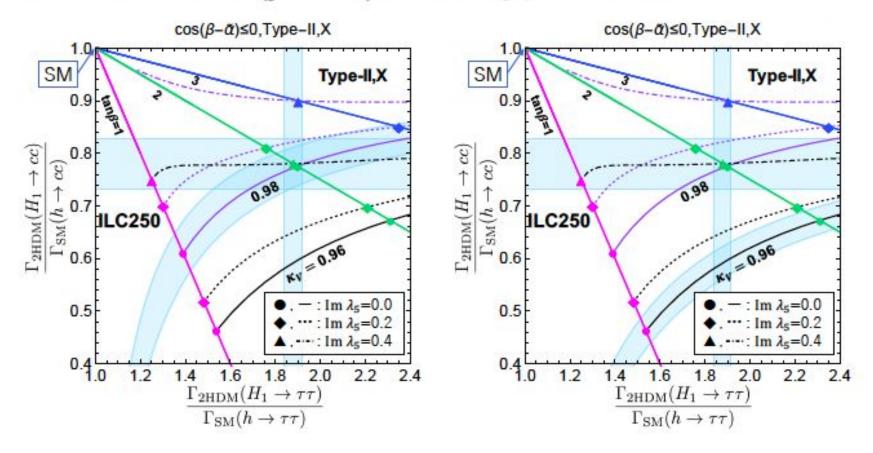




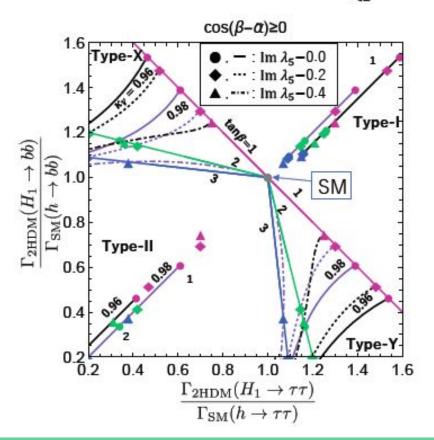
hcc- hττ $(cos(β - α) \le 0$, Type-I,-Y)

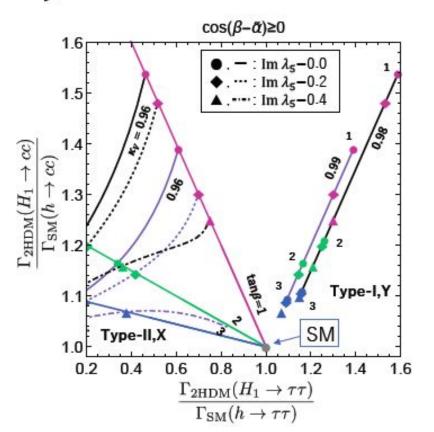


$hcc-h\tau\tau \left(\cos(\beta-\alpha)\leq 0, \text{ Type-II,-X}\right)$

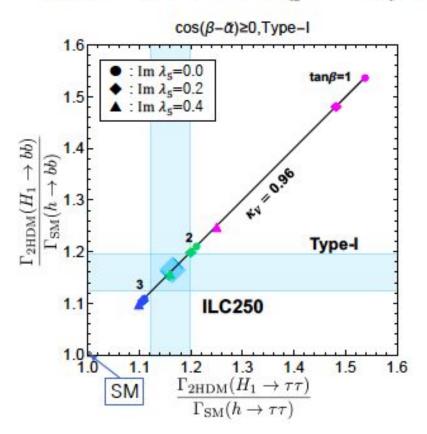


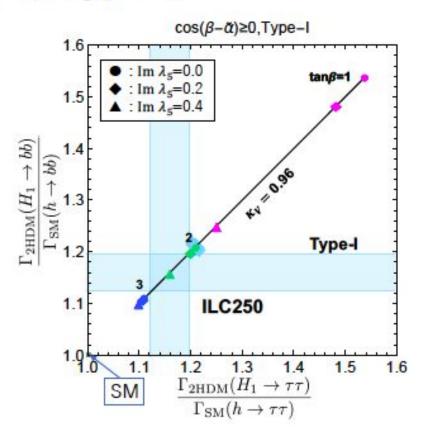
$\cos(\beta - \alpha) \ge 0$



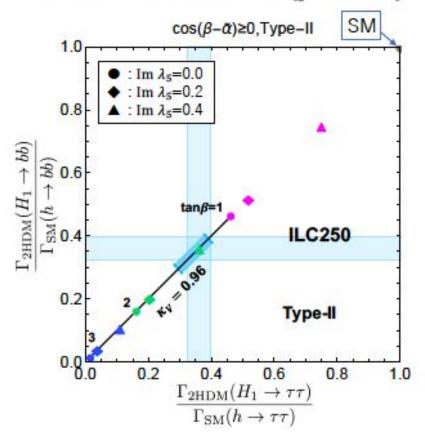


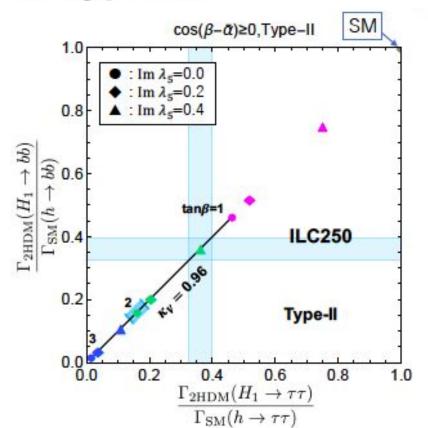
hbb- hττ (cos(β - α) ≥ 0, Type-I)



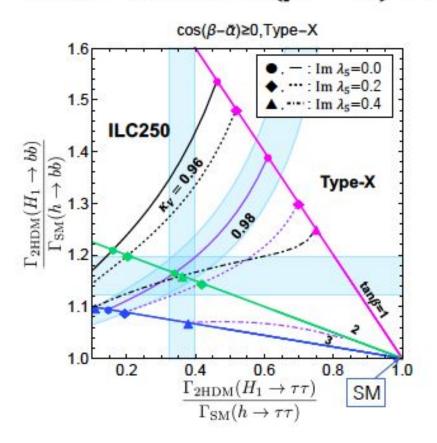


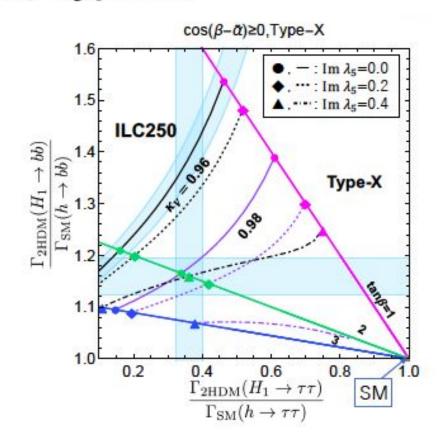
hbb- hττ (cos(β - α) ≥ 0, Type-II)



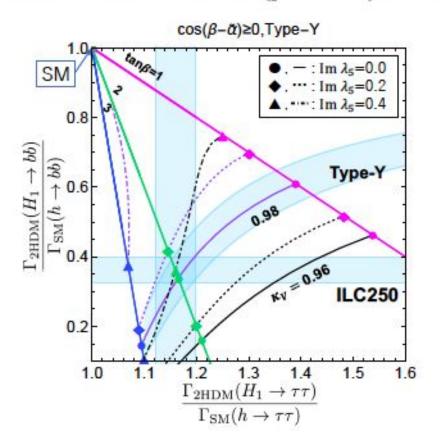


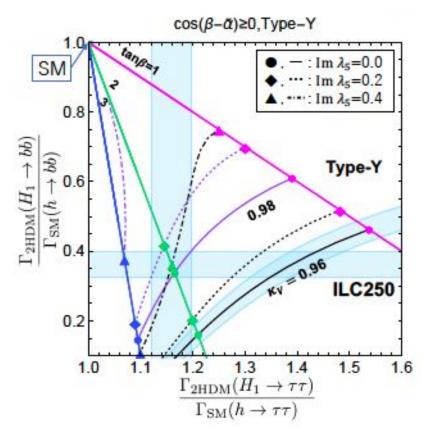
hbb- hττ (cos(β - α) ≥ 0, Type-X)



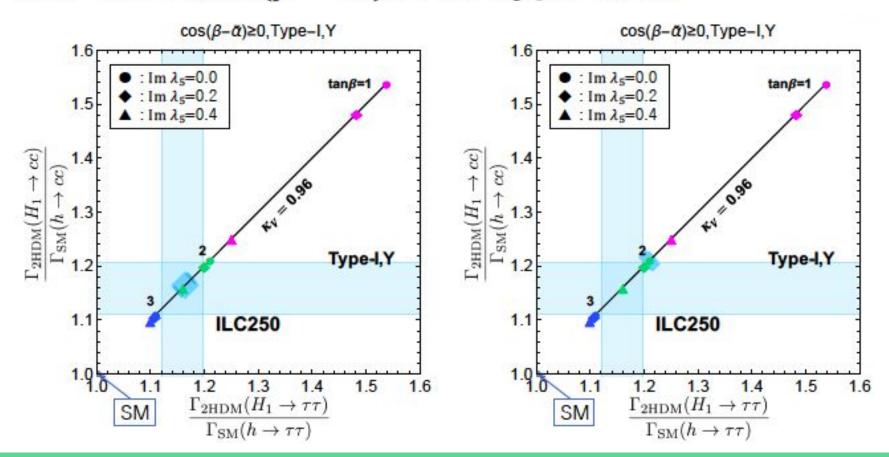


hbb- hττ (cos(β - α) ≥ 0, Type-Y)





$hcc-h\tau\tau \left(\cos(\beta-\alpha)\geq 0, \text{Type-I,-Y}\right)$



$hcc-hττ (cos(β - α) \ge 0, Type-II,-X)$

