Symmetry and geometry in generalized Higgs sector

~ Finiteness of oblique corrections v.s. perturbative unitarity ~

Yoshiki Uchida
(Nagoya U.)

Collaborators：
Ryo Nagai, Masaharu Tanabashi, Koji Tsumura,,
(Tohoku U.) (Nagoya U.) (Kyoto U.)

Introduction

We believe that the SM is not complete

Hierarchy problem? Dark Matter?, etc…

→ <u>Beyond the SM is needed !</u>

Correct understanding of the EWSB sector is the key for NP search

<Roles of Higgs boson in SM>

I. Higgs unitalize W_L scattering amplitude at tree level (tree level unitarity)

$$\mathcal{M}_{W_L W_L \to W_L W_L} \simeq \frac{s+t}{v^2}\left(1-(\kappa_V^h)^2\right) \quad (\ \kappa_V^h = 1 \ \text{ in SM})$$

II. Higgs cancels the divergence in oblique corrections

Peskin Takeuchi
Phys. Rev. Lett. 65 (1990) 964

$$S \simeq \frac{1}{12\pi}\left(1-(\kappa_V^h)^2\right)\ln\frac{\Lambda^2}{\mu^2}$$

Introduction

We believe that the SM is not complete

Hierarchy problem? Dark Matter?, etc…

→ Beyond the SM is needed !

Correct unde NP search

＜Roles of Hig

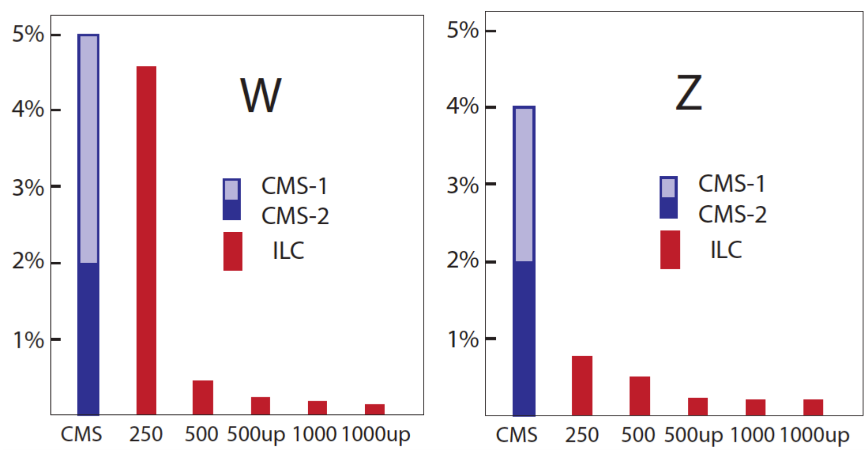
I. Higgs unitali e level unitarity)

($\kappa_V^h = 1$ in SM)

II. Higgs cancel

Peskin Takeuchi
Rev. Lett. 65 (1990) 964

κ_V^h will be measured in a great accuracy



M. E. Peskin arXiv:1312.4974

What happens if κ_V^h deviate from 1?

If κ_V^h deviate from 1, NP is needed to ensure perturbative unitarity and the finiteness of oblique correction.

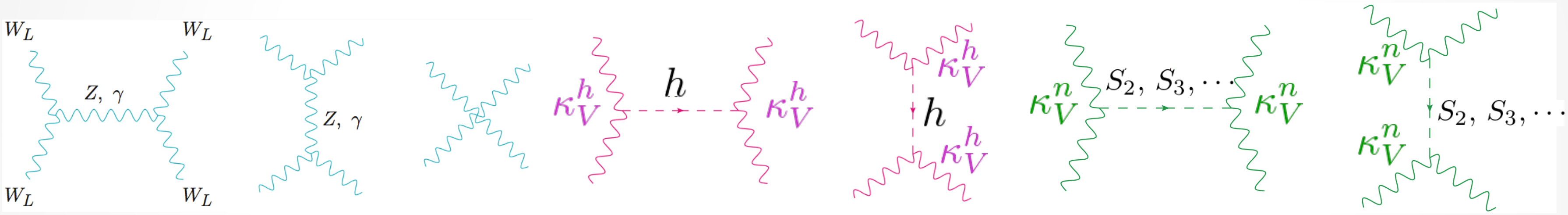
Examples

Ex.) neutral singlet extension w/ custodial sym.

$$U = \exp\left(i\frac{\pi^a}{v}\frac{\tau^a}{2}\right)$$

$$\mathcal{L} = \left(\frac{v^2}{4} + \frac{v}{2}\kappa_V^h h + \frac{v}{2}\sum_{n=2}^{N}\kappa_V^n S_n\right)\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U] + \mathcal{L}_{kinetic} - V(h, S_n)$$

I. New particles should ensure the tree level unitarity



$$\mathcal{M}_{W_L W_L \to W_L W_L} \simeq \frac{s+t}{v^2}\left(1 - (\kappa_V^h)^2 - \sum_{n=2}^{N}(\kappa_V^n)^2\right)$$

J F Gunion, H E Haber, J Wudka Phys. Rev. D 43 904 (1991)

unitarity sum rules

$$1 - (\kappa_V^h)^2 - \sum_{n=2}^{N}(\kappa_V^n)^2 = 0$$

II. New particles should cancel the divergence in oblique corrections

$$S \simeq \frac{1}{12\pi}\left(1 - (\kappa_V^h)^2 - \sum_{n=2}^{N}(\kappa_V^n)^2\right)\ln\frac{\Lambda^2}{\mu^2}$$

finiteness conditions

$$1 - (\kappa_V^h)^2 - \sum_{n=2}^{N}(\kappa_V^n)^2 = 0$$

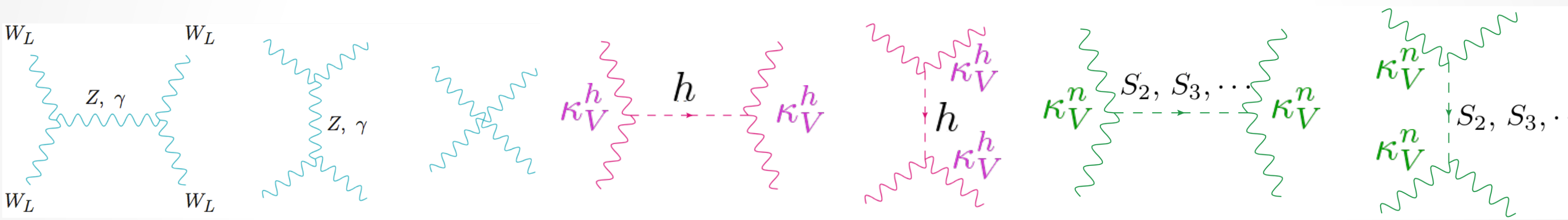
Examples

Ex.) neutral singlet extension w/ custodial sym.

$$U = \exp\left(i\frac{\pi^a}{v}\frac{\tau^a}{2}\right)$$

$$\mathcal{L} = \left(\frac{v^2}{4} + \frac{v}{2}\kappa_V^h h + \frac{v}{2}\sum_{n=2}^{N}\kappa_V^n S_n\right)\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U] + \mathcal{L}_{kinetic} - V(h, S_n)$$

I. New particles should ensure the tree level unitarity



$\mathcal{M}_{W_L W}$ … N … $V)$ … $1)$

unitarity sum rules

$$1 - (\kappa_V^h)^2 - \sum_{n=2}^{N}(\kappa_V^n)^2 = 0$$

Is there any relationship between

unitarity sum rules

&

finiteness conditions

?

II. I… gence in oblique corrections

$$S \simeq \frac{1}{12\pi}\left(1 - (\kappa_V^h)^2 - \sum_{n=2}^{N}(\kappa_V^n)^2\right)\ln\frac{\Lambda^2}{\mu^2}$$

finiteness conditions

$$1 - (\kappa_V^h)^2 - \sum_{n=2}^{N}(\kappa_V^n)^2 = 0$$

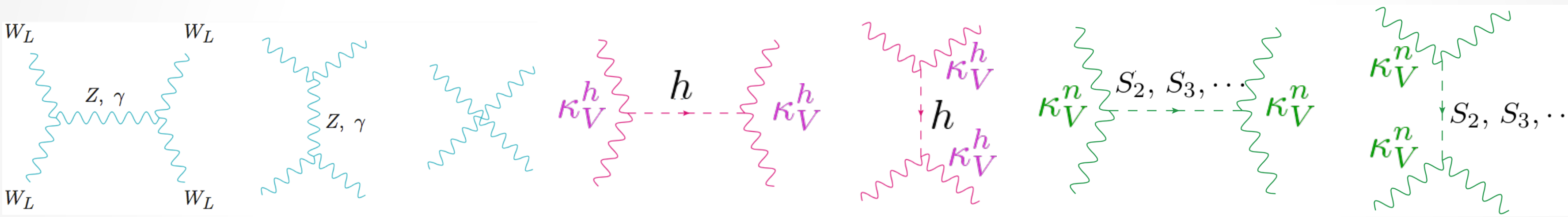
Examples

Ex.) neutral singlet extension w/ custodial sym.

$$U = \exp\left(i\frac{\pi^a}{v}\frac{\tau^a}{2}\right)$$

$$\mathcal{L} = \left(\frac{v^2}{4} + \frac{v}{2}\kappa_V^h h + \frac{v}{2}\sum_{n=2}^{N}\kappa_V^n S_n\right)\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U] + \mathcal{L}_{kinetic} - V(h, S_n)$$

I. New particles should ensure the tree level unitarity



$\mathcal{M}_{W_L W}$ ... $\sum^{N}$ ... $)$

unitarity sum rules

$$1 - (\kappa_V^h)^2 - \sum_{n=2}^{N}(\kappa_V^n)^2 = 0$$

Is there any relationship between

unitarity sum rules

&

finiteness conditions

?

II. I... gence in oblique corrections

$$S \simeq \frac{1}{12\pi}\left(1 - (\kappa_V^h)^2 - \sum(\kappa_V^n)^2\right)\ln\frac{\Lambda^2}{\cdots}$$

Yes, in particular models

finiteness conditions

$$1 - (\kappa_V^h)^2 - \sum_{n=2}^{N}(\kappa_V^n)^2 = 0$$

Tree level unitarity & EWPT are deaply related

Tree level unitarity

$\Downarrow$

S,T,U parameters' 1-loop finiteness

R Nagai, M Tanabashi, K Tsumura
Phys. Rev. D 91, 034030 (2015)

Neutral singlet Extension of SM
w/o custodial sym.

Unitarity sum rules

$$4 - 3\frac{v_Z^2}{v^2} - \sum_n^N (\kappa_V^n)^2 = 0 \,, \quad \frac{v_Z^2}{v^2} - \frac{v^2}{v_Z^2}\sum_n^N (\kappa_V^n)^2 = 0 \,, \quad \cdots$$

$$\Leftrightarrow \quad 1 - \sum_n^N (\kappa_V^n)^2 - \frac{1}{2}\sum_{n,m}^N (\kappa_V^{nm})^2 = 0$$

linear combination

S parameter's divergence (1-loop level)

$$S \simeq \frac{1}{12}\left(1 - \sum_n^N (\kappa_V^n)^2 - \frac{1}{2}\sum_{n,m}^N (\kappa_V^{nm})^2\right)\ln\frac{\Lambda^2}{\mu^2}$$

Tree level unitarity & EWPT
are deaply related

Tree level unitarity

$\Downarrow$

S,T,U parameters' 1-loop finiteness

R Nagai, M Tanabashi, K Tsumura
Phys. Rev. D 91, 034030 (2015)

Neutral singlet
Extension of SM
w/o custodial sym.

Unitarity sum rules

$$4 - 3\frac{v_Z^2}{v^2} - \sum_n^N (\kappa_V^n)^2 = 0 \ , \quad \frac{v_Z^2}{v^2} - \frac{v^2}{v_Z^2}\sum_n^N (\kappa_V^n)^2 = 0 \ , \quad \cdots$$

$$\Leftrightarrow \quad 1 - \sum_n^N (\kappa_V^n)^2 - \frac{1}{2}\sum_{n,m}^N (\kappa_V^{nm})^2 = 0$$

linear combination

S parameter's divergence (1-loop level)

$$S \simeq \frac{1}{12}\left(1 - \sum_n^N (\kappa_V^n)^2 - \frac{1}{2}\sum_{n,m}^N (\kappa_V^{nm})^2\right) \ln\frac{\Lambda^2}{\mu^2}$$

Tree level unitarity & EWPT
are deaply related

Tree level unitarity

$$\Downarrow$$

S,T,U parameters' 1-loop finiteness

R Nagai, M Tanabashi, K Tsumura
Phys. Rev. D 91, 034030 (2015)

How about the arbitrary model ?

Unitarity sum rules

$$4 - 3\frac{v_Z^2}{v^2} - \sum_n^N (\kappa_V^n)^2 = 0 \ , \quad \frac{v_Z^2}{v^2} - \frac{v^2}{v_Z^2}\sum_n^N (\kappa_V^n)^2 = 0 \ , \quad \cdots$$

$$\Leftrightarrow \quad 1 - \sum_n^N (\kappa_V^n)^2 - \frac{1}{2}\sum_{n,m}^N (\kappa_V^{nm})^2 = 0$$

linear combination

S parameter's divergence (1-loop level)

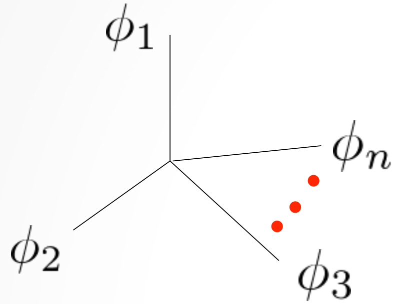
$$S \simeq \frac{1}{12}\left(1 - \sum_n^N (\kappa_V^n)^2 - \frac{1}{2}\sum_{n,m}^N (\kappa_V^{nm})^2\right) \ln\frac{\Lambda^2}{\mu^2}$$

Goal

tree level unitarity $\overset{?}{\Rightarrow}$ S,U parameter 1-loop finiteness

If we focus on the internal space of scalar fields···

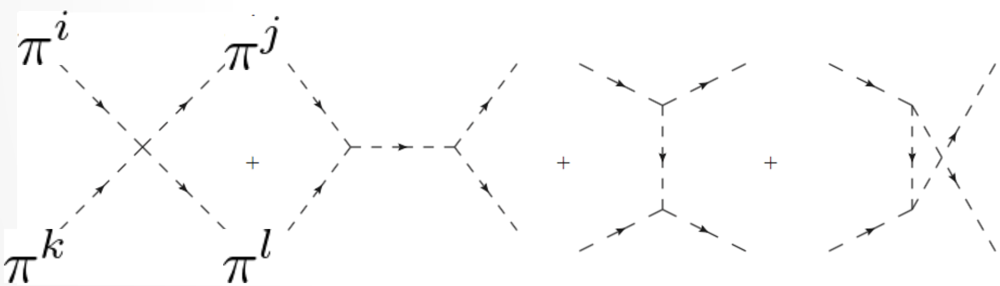
Alonso et al, Phys. Lett. B754 (2016)



$$\mathcal{L} = \frac{1}{2}\underline{g_{ij}(\phi)}(\partial_\mu\phi)^i(\partial^\mu\phi)^j - V(\phi)$$

regard as metric tensor

① Unitarity-violating amplitudes can be written as Riemann tensor

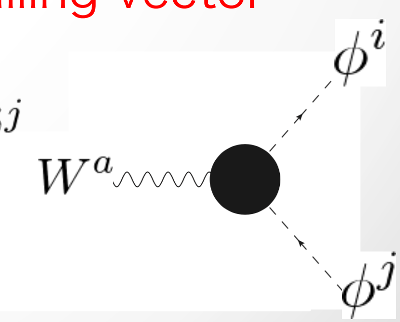


$$= -i(t\,\mathcal{R}_{ijkl} + u\,\mathcal{R}_{iljk})$$

② Charged and neutral current can be written as Killing vector

Global sym. = isometry of the scalar manifold

$(w_a^i)_{;j}$



w_a^i : Killing vector for SU(2)_L symmetry (a=1~3)

y^i : Killing vector for U(1)_Y symmetry

Goal

tree level unitarity $\overset{?}{\Rightarrow}$ S,U parameter 1-loop finiteness

S and U parameter can be written in terms of charged and neutral currents

$$S_{\rm div} = -\frac{1}{6\pi}\sum_{i,j} w^{j}_{3;i} y^{i}_{;j} \times \ln(\Lambda/\mu)$$

Killing vector

$$U_{\rm div} = -\frac{1}{6\pi}\sum_{i,j}\left((w^{j}_{1})_{;i}(w^{i}_{1})_{;j} - (w^{j}_{3})_{;i}(w^{i}_{3})_{;j}\right) \times \ln(\Lambda/\mu)$$

Goal

tree level unitarity $\overset{?}{\Rightarrow}$ S,U parameter 1-loop finiteness

S and U parameter can be written in terms of charged and neutral currents

$$S_{\text{div}} = -\frac{1}{6\pi}\sum_{i,j} w^{j}_{3;i}\, y^{i}_{;j} \times \ln(\Lambda/\mu)$$

$$U_{\text{div}} = -\frac{1}{6\pi}\sum_{i,j}\Big((w^{j}_{1})_{;i}(w^{i}_{1})_{;j} - (w^{j}_{3})_{;i}(w^{i}_{3})_{;j} \Big) \times \ln(\Lambda/\mu)$$

In terms of geometric description…

Goal

$$w^{j}_{3;i}\, y^{i}_{;j} \overset{?}{\propto} \mathcal{R}^{k}_{lmn}$$

$$(w^{j}_{1})_{;i}(w^{i}_{1})_{;j} - (w^{j}_{3})_{;i}(w^{i}_{3})_{;j} \overset{?}{\propto} \mathcal{R}^{k}_{lmn}$$

Goal

tree level unitarity $\overset{?}{\Rightarrow}$ S,U parameter 1-loop finiteness

S and U parameter can be written in terms of charged and neutral currents

$$S_{\text{div}} = -\frac{1}{6\pi}\sum_{i,j} w^j_{3;i} y^i_{;j} \times \ln(\Lambda/\mu)$$

$$U_{\text{div}} = -\frac{1}{6\pi}\sum_{i,j}\left((w^j_1)_{;i}(w^i_1)_{;j} - (w^j_3)_{;i}(w^i_3)_{;j}\right) \times \ln(\Lambda/\mu)$$

Using Killing equation, we derive the general formula.

$$w^j_{3;i} y^i_{;j} = \frac{1}{2}\epsilon_{3ab} w^k_c w^l_3 (w^j_b)_{;i}(\mathcal{R}^i_{jkl}) + \frac{1}{2}\epsilon_{3bc} w^k_b w^l_c (y^j)_{;i}(\mathcal{R}^i_{jkl})$$

$$(w^j_1)_{;i}(w^i_1)_{;j} - (w^j_3)_{;i}(w^i_3)_{;j}$$
$$= -\frac{1}{2}\epsilon_{1bc} w^k_b w^l_c (w^i_1)_{;j}(\mathcal{R}^i_{jkl}) + \frac{1}{2}\epsilon_{3bc} w^k_b w^l_c (w^i_3)_{;j}(\mathcal{R}^i_{jkl})$$

Goal

tree level unitarity $\overset{?}{\Rightarrow}$ S,U parameter 1-loop finiteness

S and U parameter can be written in terms of charged and neutral currents

$$S_{\rm div} = -\frac{1}{6\pi}\sum_{i,j} w^j_{3;i} y^i_{;j} \times \ln(\Lambda/\mu)$$

$$U_{\rm div} = -\frac{1}{6\pi}\sum_{i,j}\Big((w^j_1)_{;i}(w^i_1)_{;j} - (w^j_3)_{;i}(w^i_3)_{;j} \Big) \times \ln(\Lambda/\mu)$$

Using Killing equation, we derive the general formula.

$$w^j_{3;i} y^i_{;j} = \frac{1}{2}\epsilon_{3ab} w^k_c w^l_3 (w^j_b)_{;i} (\mathcal{R}^i_{jkl}) + \frac{1}{2}\epsilon_{3bc} w^k_b w^l_c (y^j)_{;i} (\mathcal{R}^i_{jkl})$$

$$(w^j_1)_{;i}(w^i_1)_{;j} - (w^j_3)_{;i}(w^i_3)_{;j} = -\frac{1}{2}\epsilon_{1bc} w^k_b w^l_c (w^i_1)_{;j} (\mathcal{R}^i_{jkl}) + \frac{1}{2}\epsilon_{3bc} w^k_b w^l_c (w^i_3)_{;j} (\mathcal{R}^i_{jkl})$$

Goal

tree level unitarity $\overset{?}{\Rightarrow}$ S,U parameter 1-loop finiteness

S and U parameter can be written in terms of charged and neutral currents

$$S_{\text{div}} = -\frac{1}{\ldots}\sum w_{3;i}^{j} y_{;j}^{i} \times \ln(\Lambda/\mu)$$

If tree level unitary is satisfied
(scalar manifold is flat)

$\Downarrow$

S, U parameter is 1-loop finite
(product of killing vectors = 0)

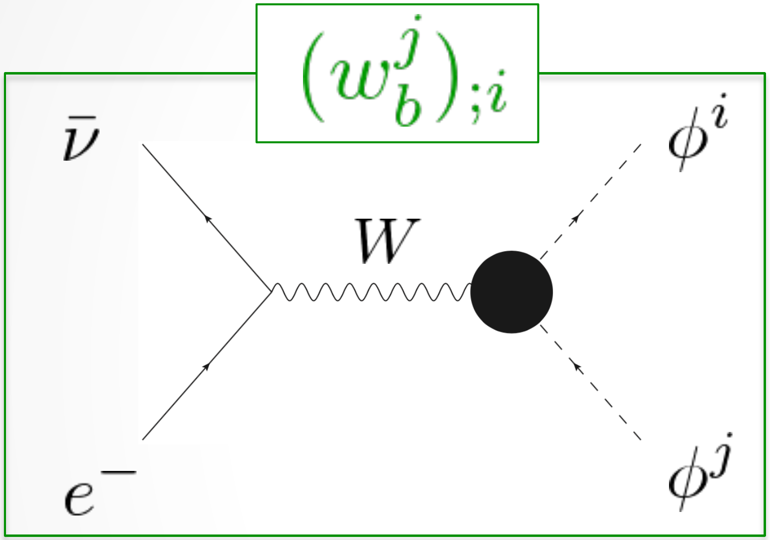
Using

$w_{3;i}^{j} y_{;j}^{i}$ … $\mathcal{R}^{i}_{jkl})$

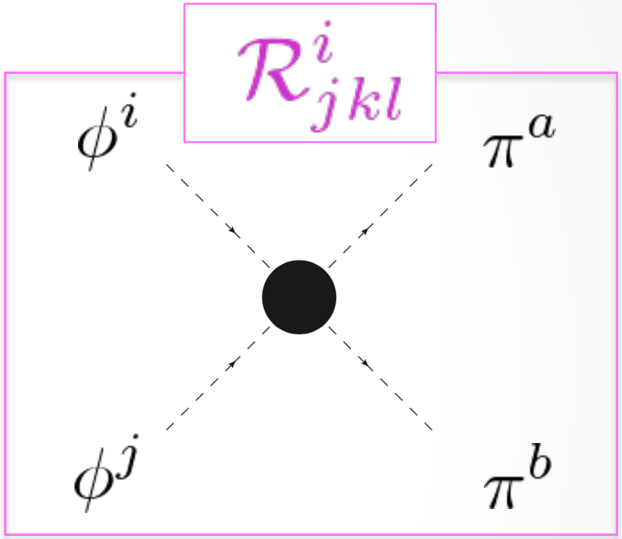
$$(w_1^j)_{;i}(w_1^i)_{;j} - (w_3^j)_{;i}(w_3^i)_{;j}$$
$$= -\frac{1}{2}\epsilon_{1bc} w_b^k w_c^l (w_1^i)_{;j} (\mathcal{R}^i_{jkl}) + \frac{1}{2}\epsilon_{3bc} w_b^k w_c^l (w_3^i)_{;j} (\mathcal{R}^i_{jkl})$$

Interpretation

$$w^j_{3;i}y^i_{;j} = \frac{1}{2}\epsilon_{3ab}w^k_c w^l_3 (w^j_b)_{;i}(\mathcal{R}^i_{jkl}) + \cdots$$



×



ϕ^i : scalar field (ex. $H, A, H^{\pm}, \pi, \cdots$)

For keeping the consistency with EWPT, It is not necessary for all the components of Riemann tensor to be zero.

<interesting scenario>

Tree level unitarity is broken in certain amplitudes with keeping the consistency with EWPT.

Summary

- Tree level unitarity and S,T,U parameters' 1-loop finiteness is deeply related

tree level unitary $\Longrightarrow$ S,T,U parameter 1-loop finiteness

In "neutral singlet extension of SM"

- Dose the relationship still hold in the models with arbitral Higgs sector?

tree level unitary $\Longrightarrow$ S,U parameter 1-loop finiteness

In arbitral Higgs sector

- By listing the Riemann tensors contributing the S parameter, we can provide the list of amplitudes preferred to be small to keep S,U parameter << 1.

Back Up

What is the most general form of the scalar sector ?

If we assume SM matter contents ⋯

$$\Phi = \begin{pmatrix} \pi^+ \\ \frac{v+h+\pi^0}{\sqrt{2}} \end{pmatrix}$$

① linear form (SMEFT)

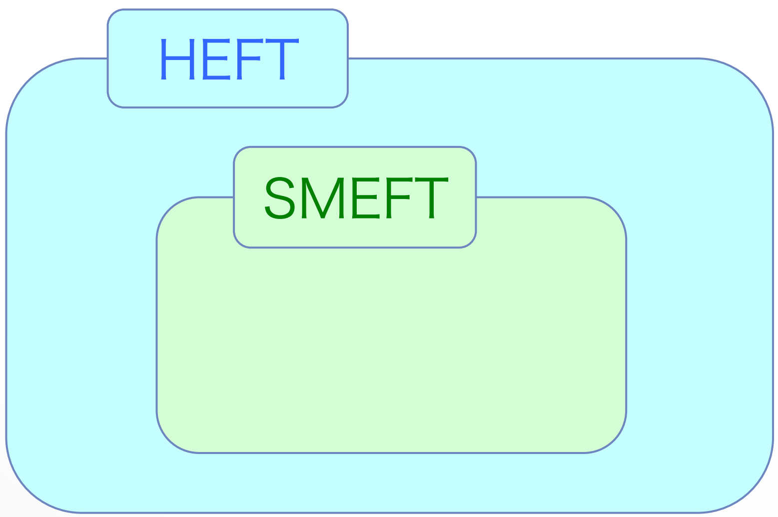
$$\mathcal{L} = (D_\mu \Phi)^\dagger D^\mu \Phi + \mu^2 |\Phi|^2 - \lambda |\Phi|^4 + \sum_{d \geq 5} c_i \mathcal{O}_i(\Phi)$$

More general

② non-linear form (HEFT)

$$\mathcal{L} = \frac{v^2}{4} F(h) \mathrm{tr}[(D_\mu U)^\dagger (D^\mu U)] + \frac{1}{2} \partial_\mu h \partial^\mu h - V(h) + \cdots$$

$F(h)$: arbitrary function of h $\quad F_{\mathrm{SM}}(h) = \left(1 + \frac{h}{v}\right)^2 \quad U = \exp\left(i \frac{\pi^a}{v} \frac{\tau^a}{2}\right)$



What is the most general form of the scalar sector ?

If we assume SM matter contents …

① linear form (SMEFT)

$$\Phi = \begin{pmatrix} \pi^+ \\ \frac{v+h+\pi^0}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L} = (D_\mu\Phi)^\dagger D^\mu\Phi + \mu^2|\Phi|^2 - \lambda|\Phi|^4 + \sum_{d\geq 5} c_i \mathcal{O}_i(\Phi)$$

More general

② non-linear form (HEFT)

$$\mathcal{L} = \frac{v^2}{4}F(h)\mathrm{tr}[(D_\mu U)^\dagger(D^\mu U)] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h) + \cdots$$

$F(h)$: arbitrary function of h $\quad F_{\rm SM}(h) = \left(1+\frac{h}{v}\right)^2 \quad U = \exp\left(i\frac{\pi^a}{v}\frac{\tau^a}{2}\right)$

If we consider the most general Higgs sector (including new particle)

$$\boxed{\mathcal{L} = \frac{1}{2}g_{ij}(\phi)(D_\mu\phi)^i(D^\mu\phi)^j - V(\phi)}$$

ϕ^i : scalar field

Ex.) SM ($\phi^i = h, \pi^a$)

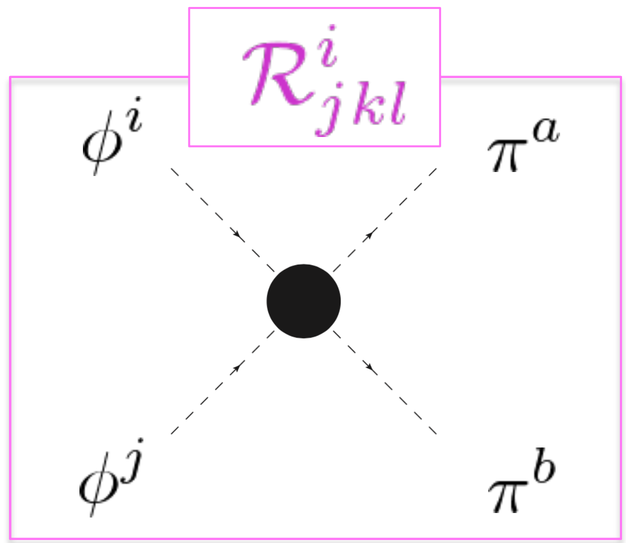
$$g_{ij}(\phi) = \begin{pmatrix} \hat{g}_{ab} & 0 \\ 0 & 1 \end{pmatrix} \qquad \hat{g}_{ab} = \left(1+\frac{h}{v}\right)^2\left[\delta^{ab} - \frac{1}{3v^2}\{(\vec{\pi}\cdot\vec{\pi})\delta^{ab} - \pi^a\pi^b\} + \cdots\right]$$

Ex.) neutral singlet extension w/ custodial sym. ($\phi^i = h(=S_0), S_1, S_2\cdots, \pi^a$)

$$g_{ij}(\phi) = \begin{pmatrix} \hat{g}_{ab} & 0 \\ 0 & \delta_{nm} \end{pmatrix} \qquad \hat{g}_{ab} = \left(1+2\sum_{n=1}^{N}\kappa_n\frac{S_n}{v}\right)\left[\delta^{ab} - \frac{1}{3v^2}\{(\vec{\pi}\cdot\vec{\pi})\delta^{ab} - \pi^a\pi^b\} + \cdots\right]$$

Interpretation

$$w^j_{3;i}\, y^i_{;j} = \frac{1}{2}\epsilon_{3ab} w^k_c w^l_3 (w^j_b)_{;i} (\mathcal{R}^i_{jkl}) + \cdots$$



These amplitudes are preferred to be zero to keep S,U parameter << 1

Even if the unitarity-violating amplitude are nonzero,
it is (at least) consistent with the bound on S,U parameter.

$$w^j_{3;i}y^i_{;j} = \frac{1}{2}\epsilon_{3ab}w^k_c w^l_3 (w^j_b)_{;i}(\mathcal{R}^i_{jkl}) + \frac{1}{2}\epsilon_{3bc}w^k_b w^l_c (y^j)_{;i}(\mathcal{R}^i_{jkl})$$

<derivation>

In Riemann Normal Coordinate

Solution of Killing eq. $(w^j_a)_{;k;l} = \mathcal{R}_{ilkj}w^i_a$

$$w^i_a = \overline{w}^i_a + \overline{w}^i_{a,j}\phi^j + \frac{1}{3}\overline{\mathcal{R}}^i_{jkl}\overline{w}^l_a\phi^j\phi^k + \cdots \qquad \cdots①$$

Commutation relation of Killing vector

$$w^i_a(y^j)_{,i} - y^i(w^j_a)_{,i} = 0 \qquad \cdots②$$

$$w^i_a(w^j_b)_{,i} - w^i_b(w^j_a)_{,i} = \epsilon_{abc}w^j_c \qquad \cdots③$$

Substituting ① into ②, ③ and comparing the coefficient of ϕ we can get some formulas about $[T_a, T_Y]_j{}^i$ where $(T_a)_j{}^i = \overline{w}^i_{a,j}$, $(T_Y)_j{}^i = \overline{y}^i_{,j}$

Substituting these formula into

$$\mathrm{tr}(T_3T_Y) = \frac{1}{2}\epsilon_{3bc}\mathrm{tr}([T_c, T_Y]T_b) + \frac{1}{2}\epsilon_{3bc}\overline{w}^k_b\overline{w}^l_c\overline{\mathcal{R}}^i_{jkl}(T_Y)_i{}^j$$

and writing in the covariant form, we get above formula.