



PROBING 6D OPERATORS
AT FUTURE E-E+ COLLIDERS

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Based on

[W.H. Chiu, S.C. Leung, TL, K.F. Lyu and L.T. Wang, arXiv: 1711.04046 (JHEP 2018)]
[TL, K.F. Lyu, J. Ren, H.X. Zhu, arXiv: 1803.04359]



EFT at e+e- Colliders

In an EFT, the leading effects of BSM physics above the EW scale can be parametrized by a set of 6D operators

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^2} O_i$$

Given their model independence, the sensitivity of probing these operators can serve as a nice measure of the performance of the currently proposed e-e+ colliders

A large amount of such studies have been pursued for future lepton colliders in literatures

[See M. Peskin's talk]



The 6D Operators to Analyze

$$\mathcal{O}_{WW} = g^2 |H|^2 W_{\mu\nu}^a W^{a,\mu\nu}$$

$$\mathcal{O}_{WB} = gg' H^\dagger \sigma^a H W_{\mu\nu}^a B^{\mu\nu}$$

$$\mathcal{O}_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{O}_{3W} = g \frac{\epsilon_{abc}}{3!} W_\mu^{a\nu} W_\nu^{b\rho} W_\rho^{a\mu}$$

$$\mathcal{O}_T = \frac{1}{2} (H^\dagger \overset{\leftrightarrow}{D}_\mu H)^2$$

$$\mathcal{O}_H = \frac{1}{2} (\partial_\mu |H|^2)^2$$

$$\mathcal{O}_6 = \lambda |H^\dagger H|^3$$

$$\mathcal{O}_L^{(3)l} = (i H^\dagger \sigma^a \overset{\leftrightarrow}{D}_\mu H) (\bar{L}_L \gamma^\mu \sigma^a L_L)$$

$$\mathcal{O}_{LL}^{(3)l} = (\bar{L}_L \gamma_\mu \sigma^a L_L) (\bar{L}_L \gamma^\mu \sigma^a L_L)$$

$$\mathcal{O}_L^l = (i H^\dagger \overset{\leftrightarrow}{D}_\mu H) (\bar{L}_L \gamma^\mu L_L)$$

$$\mathcal{O}_R^e = (i H^\dagger \overset{\leftrightarrow}{D}_\mu H) (\bar{l}_R \gamma^\mu l_R)$$

$e^+ e^- \rightarrow Zh$

unpolarized 240 GeV

$$\begin{aligned} \frac{\Delta\sigma}{\sigma_0} = & 0.225 \frac{c_{WW}}{\Lambda^2} + 0.0554 \frac{c_{WB}}{\Lambda^2} + 0.0164 \frac{c_{BB}}{\Lambda^2} - 0.0500 \frac{c_T}{\Lambda^2} - 0.0606 \frac{c_H}{\Lambda^2} \\ & + 0.627 \frac{c_L^{(3)l}}{\Lambda^2} + 0.264 \frac{c_{LL}^{(3)l}}{\Lambda^2} + 0.891 \frac{c_L^l}{\Lambda^2} - 0.781 \frac{c_R^e}{\Lambda^2} - 0.00106 \frac{c_6}{\Lambda^2} \end{aligned}$$

These operators contribute by renormalizing the fields,
shifting the EW parameters, or introducing new vertices

[See S. Jung's talk]



The 6D Operators to Analyze

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unpolarized 240 GeV

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$e^+ e^- \rightarrow W^+ W^-$

unpolarized 240 GeV

$$\begin{aligned} \frac{\Delta\sigma}{\sigma_0} = & -0.0287 \frac{c_{WB}}{\Lambda^2} + 0.170 \frac{c_T}{\Lambda^2} - 0.0741 \frac{c_L^{(3)l}}{\Lambda^2} + 0.338 \frac{c_{LL}^{(3)l}}{\Lambda^2} \\ & - 0.0282 \frac{c_L^l}{\Lambda^2} - 0.0194 \frac{c_R^e}{\Lambda^2} + 0.000696 \frac{c_{3W}}{\Lambda^2} \end{aligned}$$



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Six operators contribute to representative EWPT observables

$$\begin{aligned} \frac{\Delta \Gamma_Z}{\Gamma_Z} = & -0.0112 \frac{c_{WB}}{\Lambda^2} + 0.079 \frac{c_T}{\Lambda^2} - 0.121 \frac{c_L^{(3)l}}{\Lambda^2} + \\ & 0.158 \frac{c_{LL}^{(3)l}}{\Lambda^2} - 0.0113 \frac{c_L^l}{\Lambda^2} - 0.0113 \frac{c_R^e}{\Lambda^2} \end{aligned}$$

Important to utilize all of these observables for a sensible sensitivity analysis



Despite the work in literatures, we would like to make a comparative study on the sensitivity performance at CEPC, FCC-ee and ILC

— helpful for optimizing the machine design and the operating scenario choice



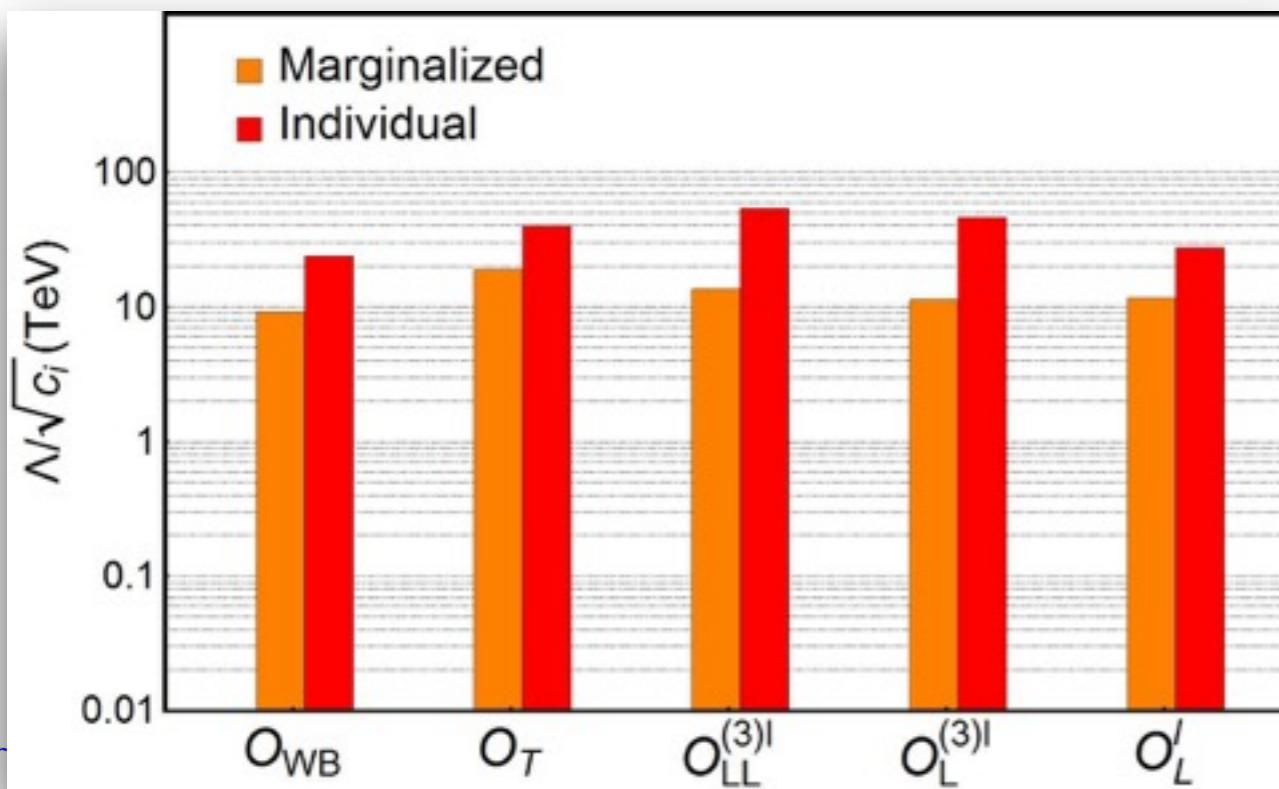
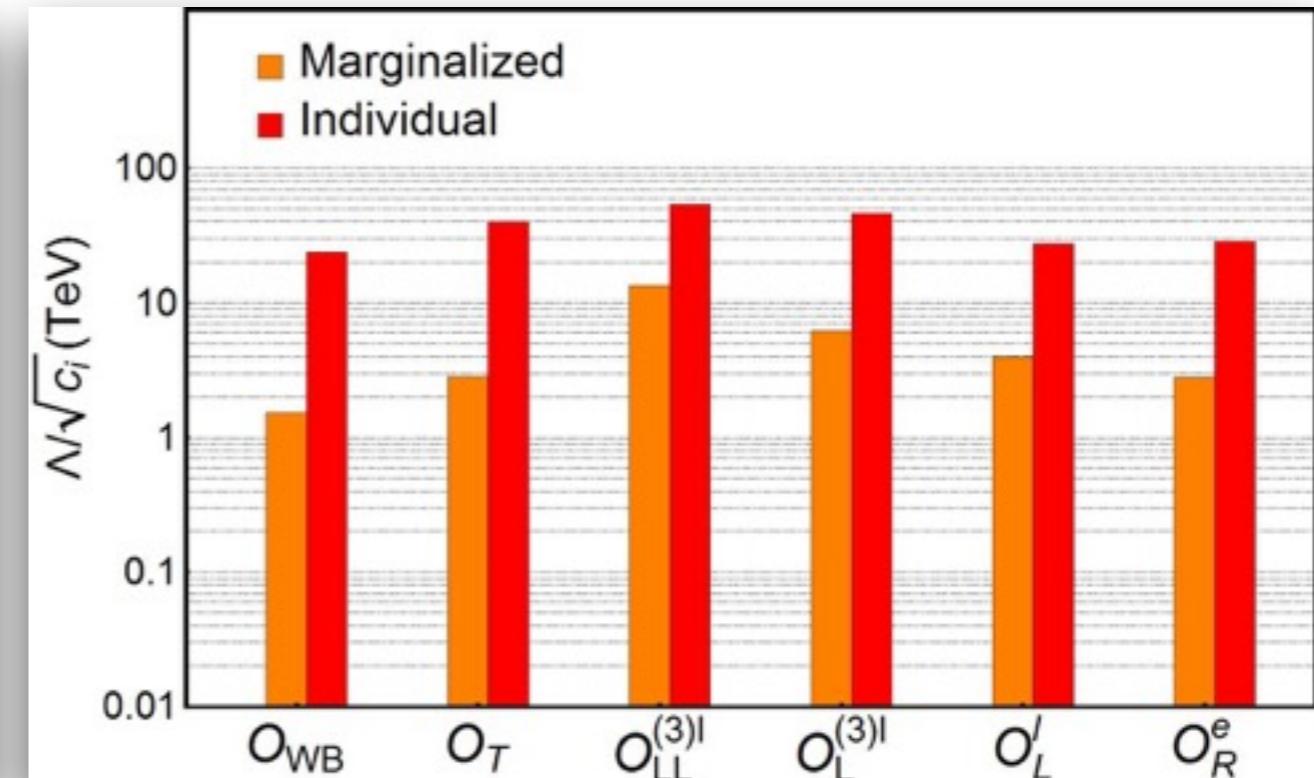
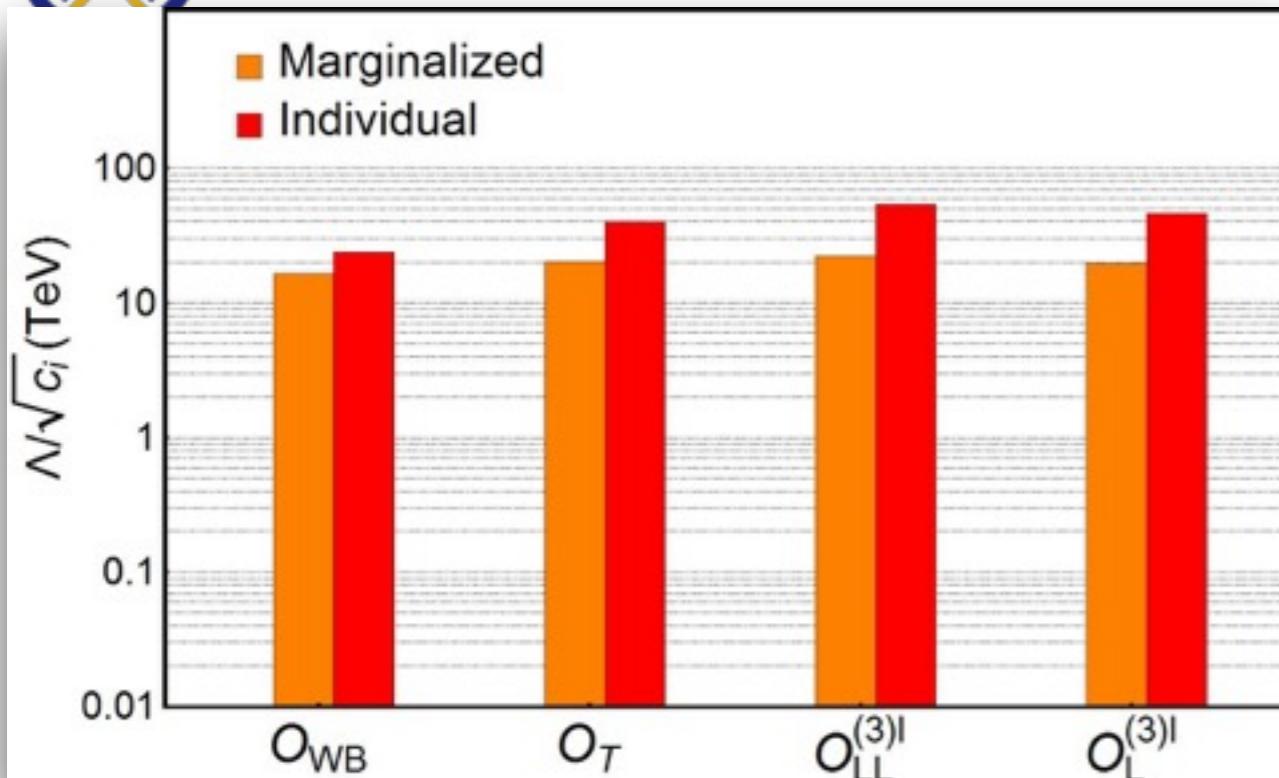
Operating Scenarios and Projected Precisions

Observables	ILC		FCC-ee		CEPC	
$\sigma(Zh)$	2.0% [25]	250GeV,2ab $^{-1}$	0.5% [35]	240GeV,5ab $^{-1}$	0.5% [6]	240GeV,5ab $^{-1}$
	4.2% [25]	500GeV,4ab $^{-1}$	-	-	-	-
	3.89% [5]	250GeV,2ab $^{-1}$	0.97% [19]	350GeV,1.5ab $^{-1}$	2.86% [19]	240GeV,5ab $^{-1}$
	1.45% [5]	500GeV,4ab $^{-1}$	-	-	-	-
	15.0% [5]	500GeV,4ab $^{-1}$	-	-	-	-
	0.0200% [40]	250GeV,2ab $^{-1}$	0.0136% [40]	240GeV,5ab $^{-1}$	0.0136% [40]	240GeV,5ab $^{-1}$
$\sigma(W^+W^-)$	0.0191% [40]	500GeV,4ab $^{-1}$	-	-	-	-
	0.0013 [4]	☒ CEPC: 240 GeV (5/ab) + Z-pole (150/fb)				
	-	☒ FCC-ee: 240 GeV (5/ab) + 350 GeV (1.5/ab) + W (10/ab) + Z-pole (150/ab)				
	-	☒ ILC: 250 GeV (2/ab) + 500 GeV (4/ab)				
$\Gamma_Z(\text{MeV})$	$\pm 1 \pm 0.21_{\text{in}} [4, 39]$	Z pole,100fb $^{-1}$	$\pm 0.1 \pm 0.08_{\text{th}} \pm 0.065_{\text{in}} [39, 41]$	Z pole,150ab $^{-1}$	$\pm 0.1 \pm 0.08_{\text{th}} \pm 0.13_{\text{in}} [6, 39]$	Z pole, 150fb $^{-1}$
$\sin^2 \theta_{\text{eff}}^{\text{lep}}(10^{-5})$	$\pm 1.3 \pm 1.5_{\text{th}} \pm 2.2_{\text{in}} [4, 39]$	Z pole,100fb $^{-1}$	$\pm 0.3 \pm 1.5_{\text{th}} \pm 1.6_{\text{in}} [39, 41]$	Z pole,150ab $^{-1}$	$\pm 2.3 \pm 1.5_{\text{th}} \pm 2.5_{\text{in}} [6, 39]$	Z pole, 150fb $^{-1}$
m_W (MeV)	$\pm 2.5 \pm 1_{\text{th}} \pm 2.8_{\text{in}} [39, 42]$	250GeV, 2ab $^{-1}$	$\pm 1.2 \pm 1_{\text{th}} \pm 0.91_{\text{in}} [35, 39]$	WW threshold,10ab $^{-1}$	$\pm 3 \pm 1_{\text{th}} \pm 3.8_{\text{in}} [6, 39]$	240GeV,5ab $^{-1}$
A_{θ_1}	0.0083 [33]	250GeV,2ab $^{-1}$	0.0060 [33]	240GeV,5ab $^{-1}$	0.0060 [33]	240GeV,5ab $^{-1}$
$A_{c\theta_1,c\theta_2}$	0.0092 [33]	250GeV,2ab $^{-1}$	0.0067 [33]	240GeV,5ab $^{-1}$	0.0067 [33]	240GeV,5ab $^{-1}$
$A_{\phi}^{(3)}$	0.0092 [33]	250GeV,2ab $^{-1}$	0.0067 [33]	240GeV,5ab $^{-1}$	0.0067 [33]	240GeV,5ab $^{-1}$
$A_{\phi}^{(4)}$	0.0092 [33]	250GeV,2ab $^{-1}$	0.0067 [33]	240GeV,5ab $^{-1}$	0.0067 [33]	240GeV,5ab $^{-1}$

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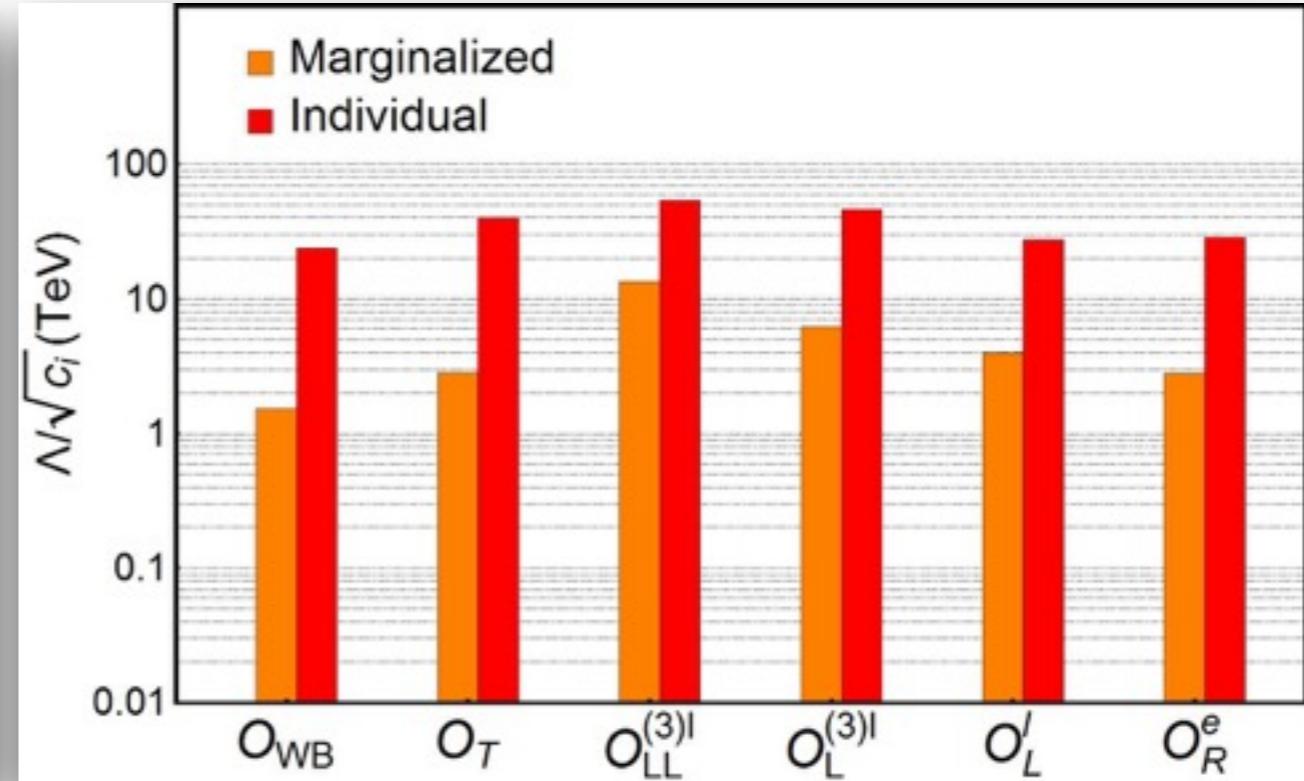
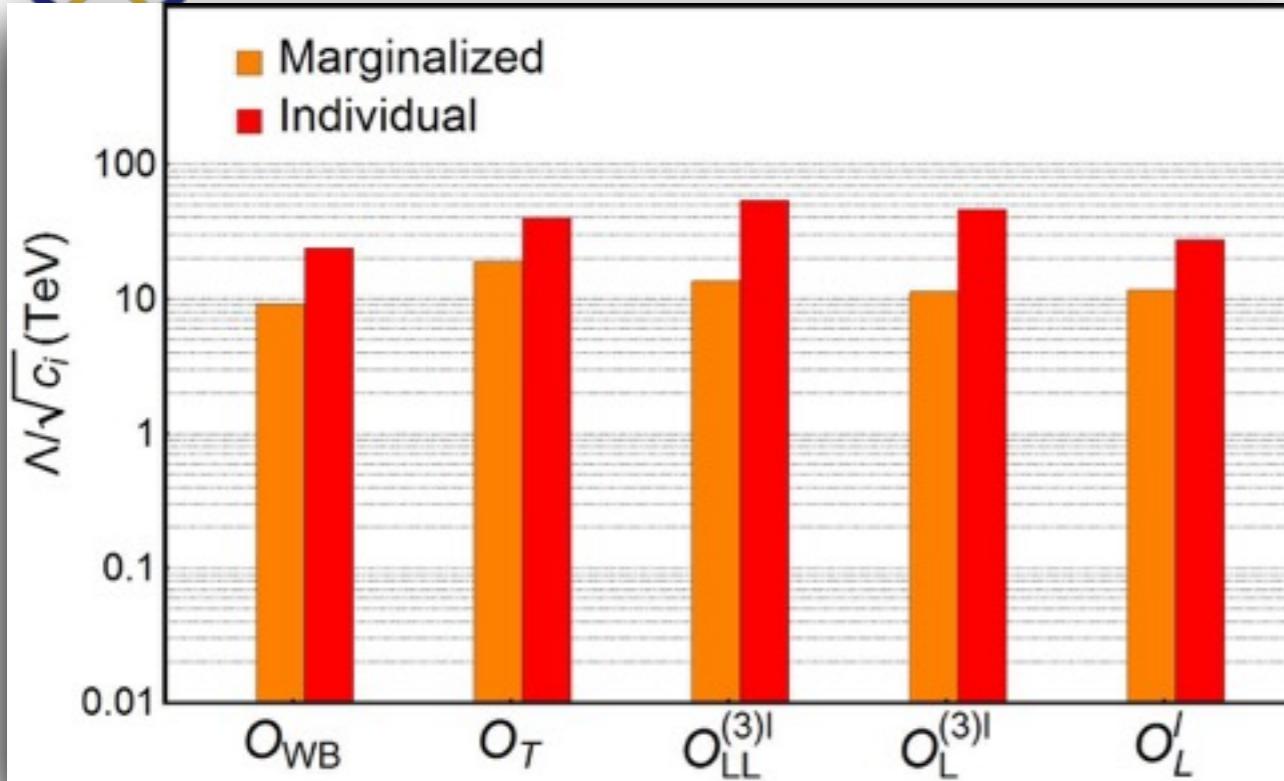
Sensitivities to the Six EW Operators at CEPC



Turning on the sixth operator causes a jump-down of the conservative sensitivity for probing these operators



Sensitivities to the Six EW Operators at CEPC



$$\xi_0 = -1.1c_{WB} + 2c_T - 4c_L^{(3)l} + 4c_{LL}^{(3)l}, \quad \xi_{\pm} = c_L^{(3)l} \pm c_L^l \text{ and } c_R^e,$$

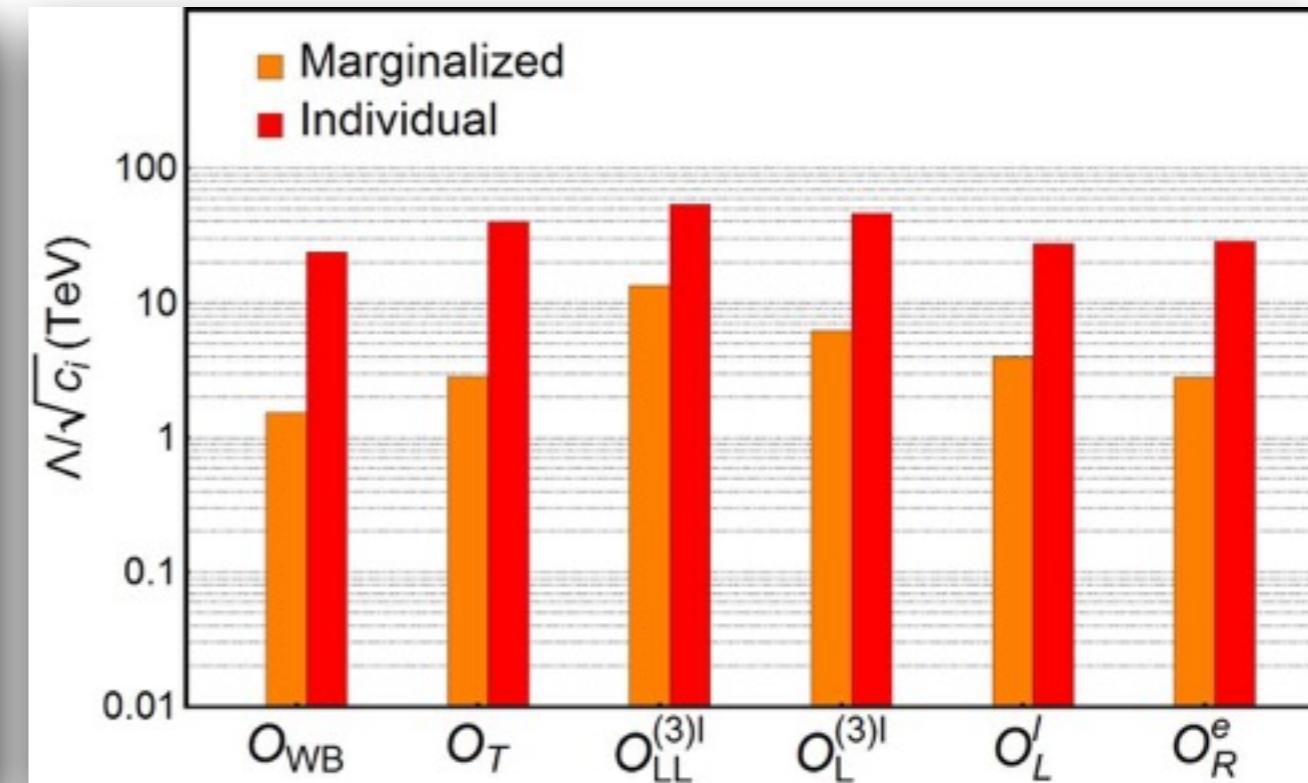
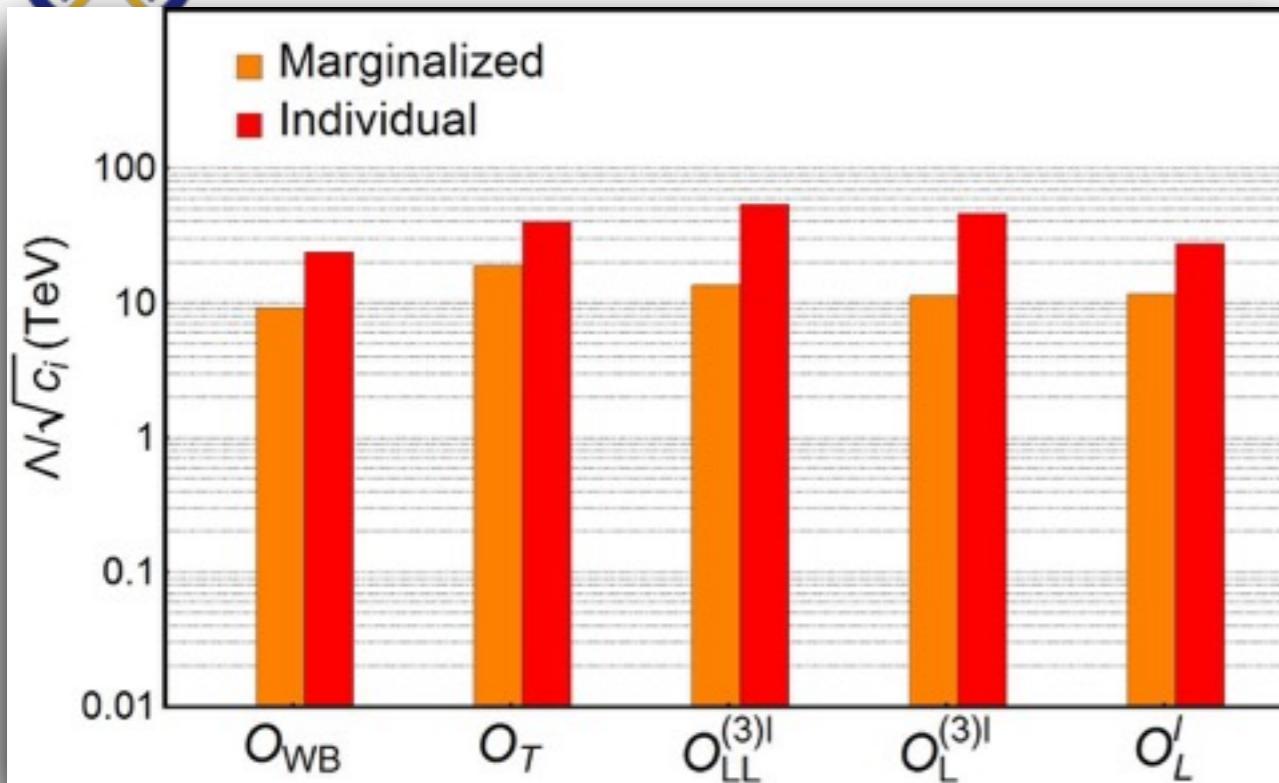
- N_ν . It depends on ξ_0 , ξ_{\pm} and c_R^e .
- A_b and R_b . They only depend on ξ_0 .
- $A_{FB}^{b,\mu}$ and $\sin^2 \theta_{\text{eff}}^{\text{lep}}$. They have the same dependence on ξ_0 , ξ_{\pm} and c_R^e .
- $R_{\mu,\tau}$. They have the same dependence on ξ_0 , ξ_{\pm} and c_R^e .
- Γ_Z and m_W have different dependences on the variables beyond $\xi_{0,\pm}$ and c_R^e .

Seems working well:
six classes of EW observables
vs. six EW operators





Sensitivities to the Six EW Operators at CEPC



$$\xi_0 = -1.1c_{WB} + 2c_T - 4c_L^{(3)l} + 4c_{LL}^{(3)l}, \quad \xi_{\pm} = c_L^{(3)l} \pm c_L^l \text{ and } c_R^e,$$

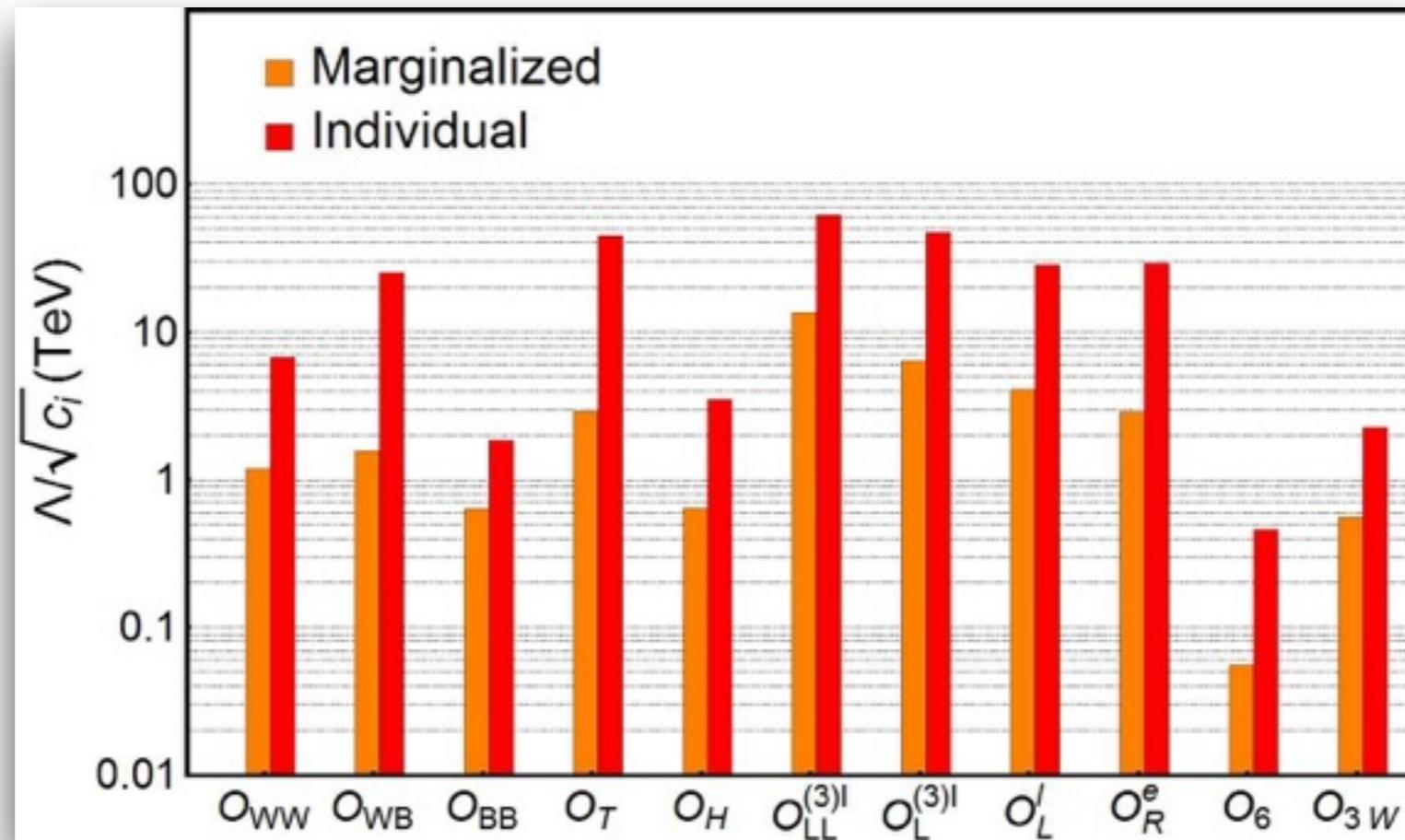
- N_ν . It depends on ξ_0 , ξ_{\pm} and c_R^e .
- A_b and R_b . They only depend on ξ_0 .

Weak observables!

$$\frac{\Delta R_b}{R_b} = 0.00189 \frac{c_{WB}}{\Lambda^2} - 0.00345 \frac{c_T}{\Lambda^2} + 0.00691 \frac{c_L^{(3)l}}{\Lambda^2} - 0.00691 \frac{c_{LL}^{(3)l}}{\Lambda^2}$$



Sensitivities at CEPC



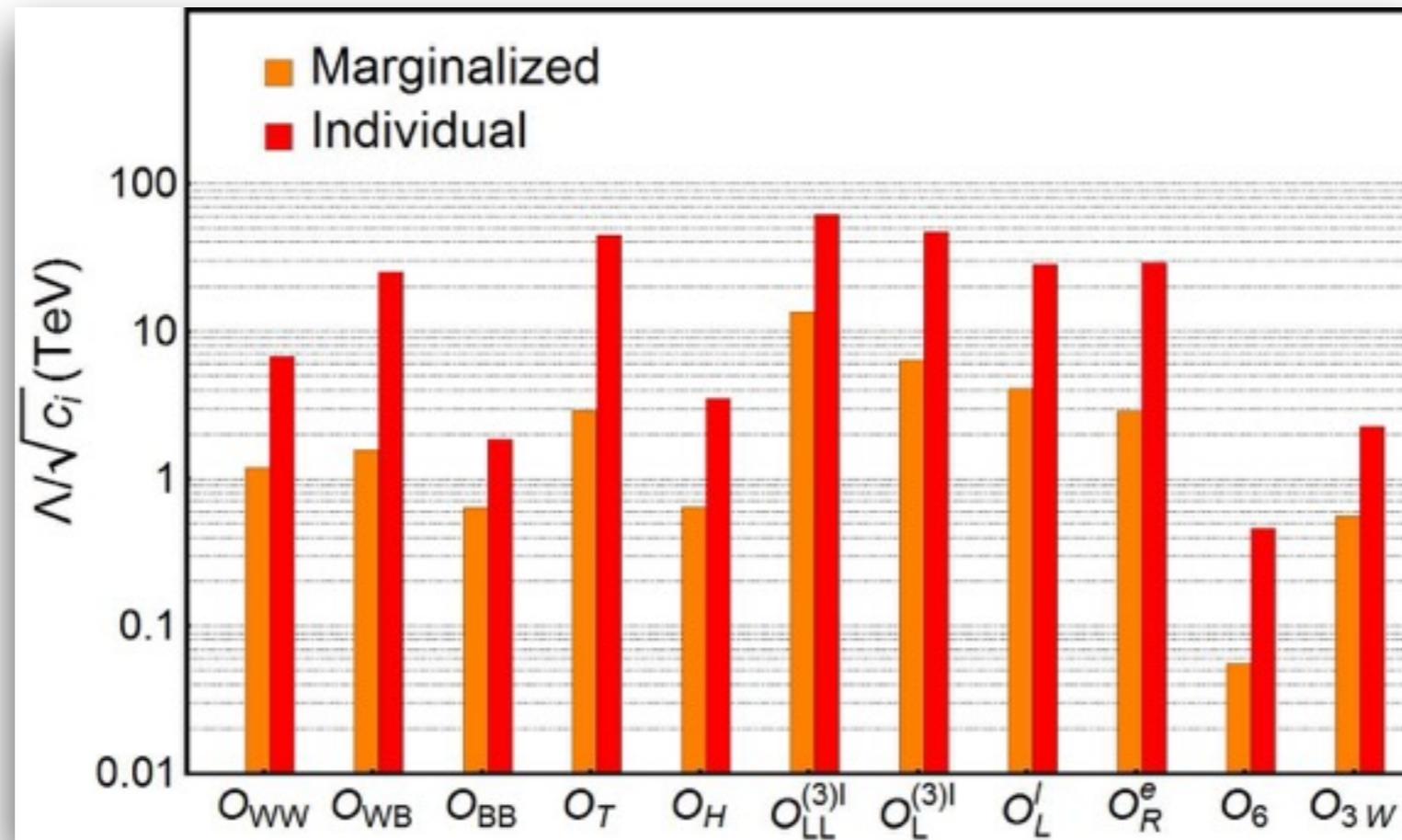
$e^+e^- \rightarrow W^+W^-$
unpolarized 240 GeV

$$\frac{\Delta\sigma}{\sigma_0} = -0.0287 \frac{c_{WB}}{\Lambda^2} + 0.170 \frac{c_T}{\Lambda^2} - 0.0741 \frac{c_L^{(3)l}}{\Lambda^2} + 0.338 \frac{c_{LL}^{(3)l}}{\Lambda^2} - 0.0282 \frac{c_L^l}{\Lambda^2} - 0.0194 \frac{c_R^e}{\Lambda^2} + 0.000696 \frac{c_{3W}}{\Lambda^2}$$

- ▣ A weak probe to O_{3W} (much weaker than the sensitivity obtained in, e.g., arXiv:1704.02333, where the EW operators were assumed to be constrained sufficiently well)



Sensitivities at CEPC



$e^+e^- \rightarrow Zh$
unpolarized 240 GeV

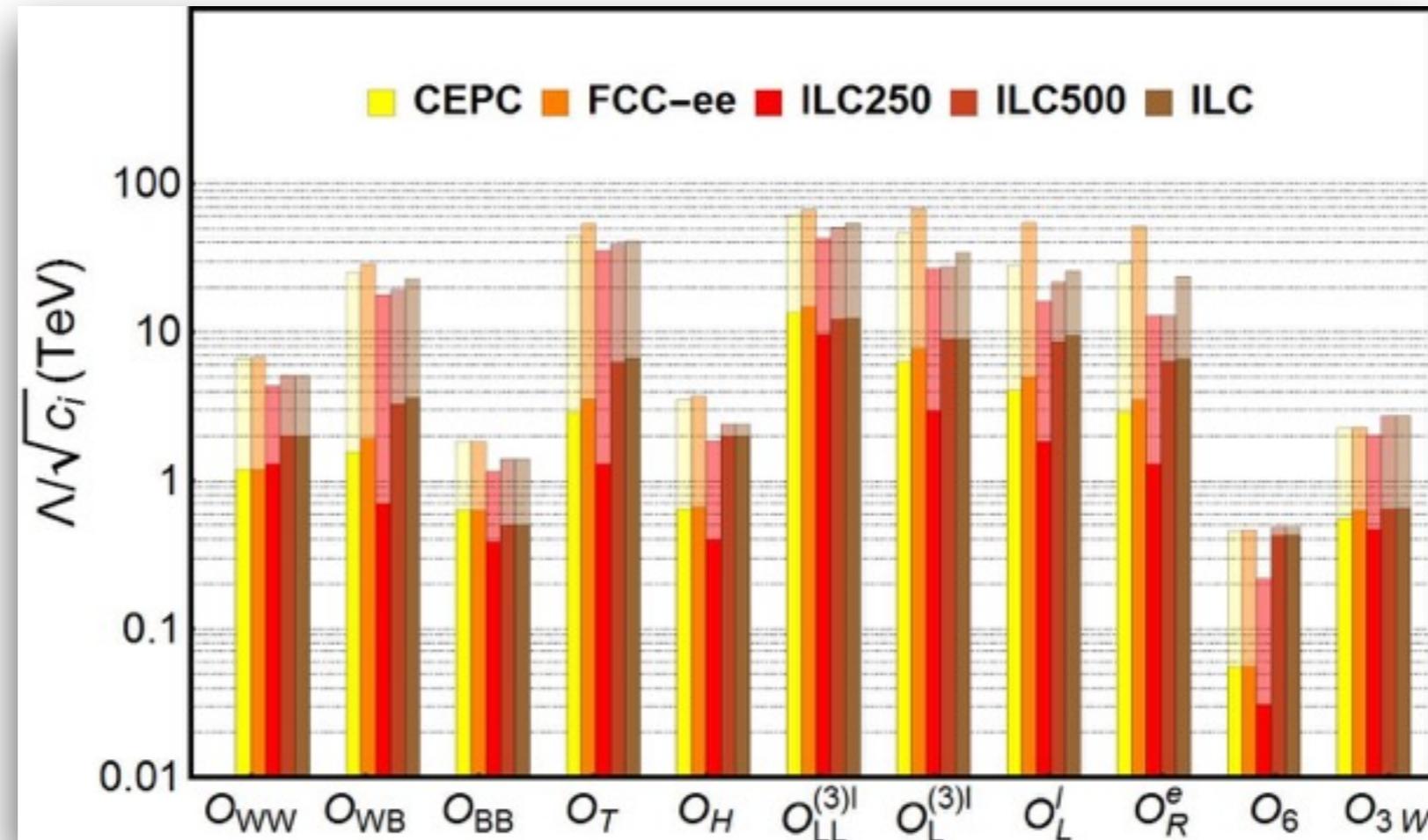
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☒ O_{WW}, O_{BB}, O_H can be probed via single Higgs production

☒ The sensitivity to O_6 is very low!



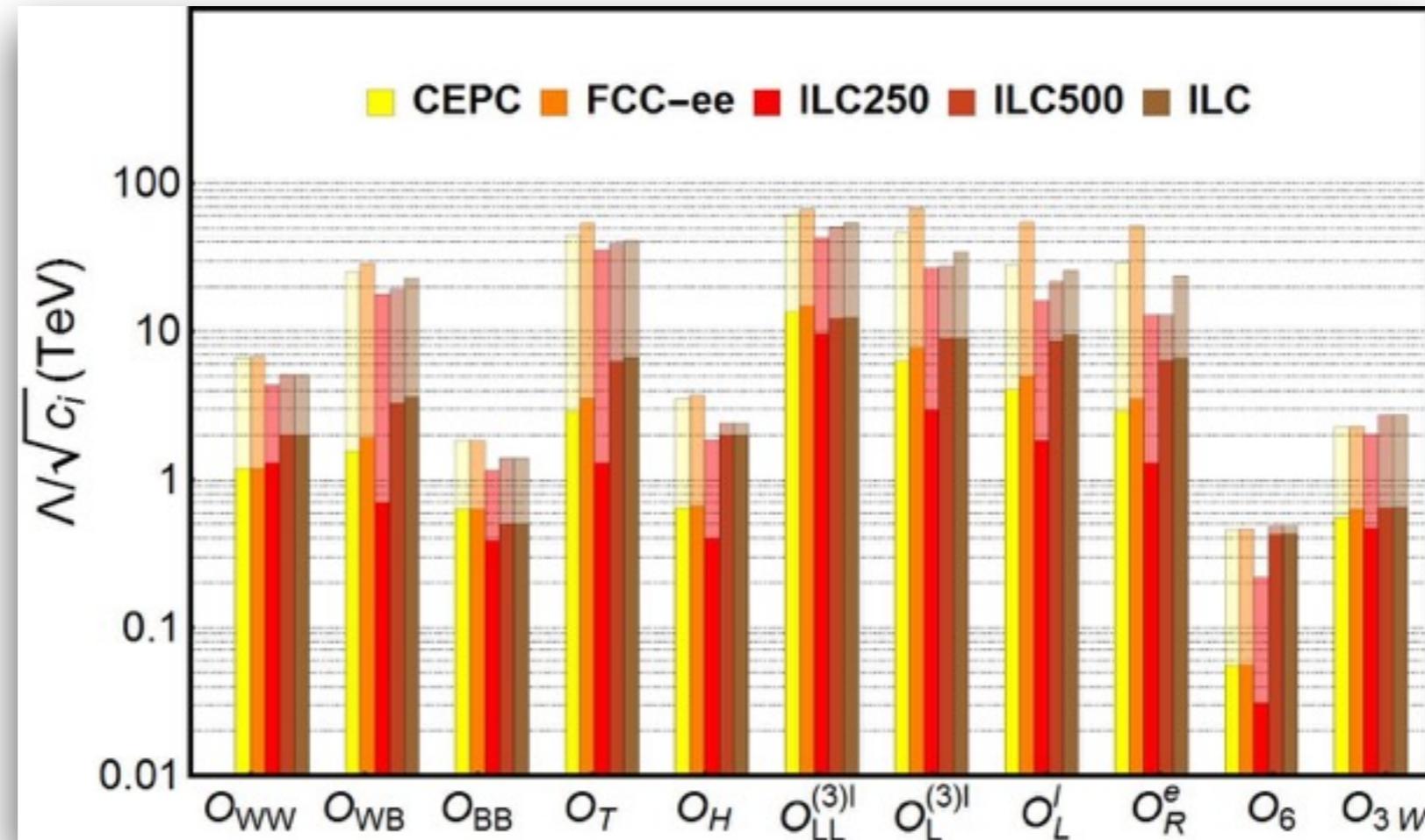
CEPC vs. FCC-ee



- With better precision of EW measurements, and data at WW threshold => the FCC-ee sensitivities are mildly, but universally better than the CEPC ones, in both conservative and optimistic scenarios.



ILC vs. CEPC and FCC-ee



- ☒ ILC250 (+ LEP): small luminosity at 250 GeV and no data at Z-pole => less capable
- ☒ ILC500 (ILC 250 + data at 500 GeV): comparable to the CEPC and FCC- ee sensitivities or even much better (O_6)
- ☒ ILC = ILC500 + Giga Z: slightly improves the sensitivities
- ☒ Note: the beam polarization scenario was oversimplified for the ILC!



Role of ILC at 500 GeV

	\mathcal{O}_{WW}	\mathcal{O}_{WB}	\mathcal{O}_{BB}	\mathcal{O}_T	\mathcal{O}_H	$\mathcal{O}_{LL}^{(3)l}$	$\mathcal{O}_L^{(3)l}$	\mathcal{O}_L^l	\mathcal{O}_R^e	\mathcal{O}_6	\mathcal{O}_{3W}
ILC250	1.30	0.697	0.384	1.29	0.401	9.62	2.92	1.83	1.29	0.0309	0.469
$+\sigma(W^+W^-)$	1.30	2.17	0.386	4.08	0.468	9.63	6.78	6.11	4.08	0.0389	0.523
$+\sigma(Zh)$	1.75	2.21	0.493	4.16	0.897	9.78	6.89	6.21	4.16	0.0895	0.531
$+\sigma(\nu\nu h)$	1.77	2.22	0.498	4.19	1.05	9.83	6.93	6.24	4.18	0.0918	0.534
$+\sigma(Zhh) = \text{ILC500}$	2.01	3.29	0.498	6.34	1.97	12.3	8.90	8.60	6.36	0.428	0.647

- ☒ $\sigma(W^+W^-)$: raises the sensitivities for most of the operators to a level comparable to the CEPC/FCC-ee ones
- ☒ $\sigma(Zh)$ and $\sigma(\nu\nu h)$: raise the sensitivities to O_H
- ☒ $\sigma(Zhh)$: raise the sensitivities to O_6



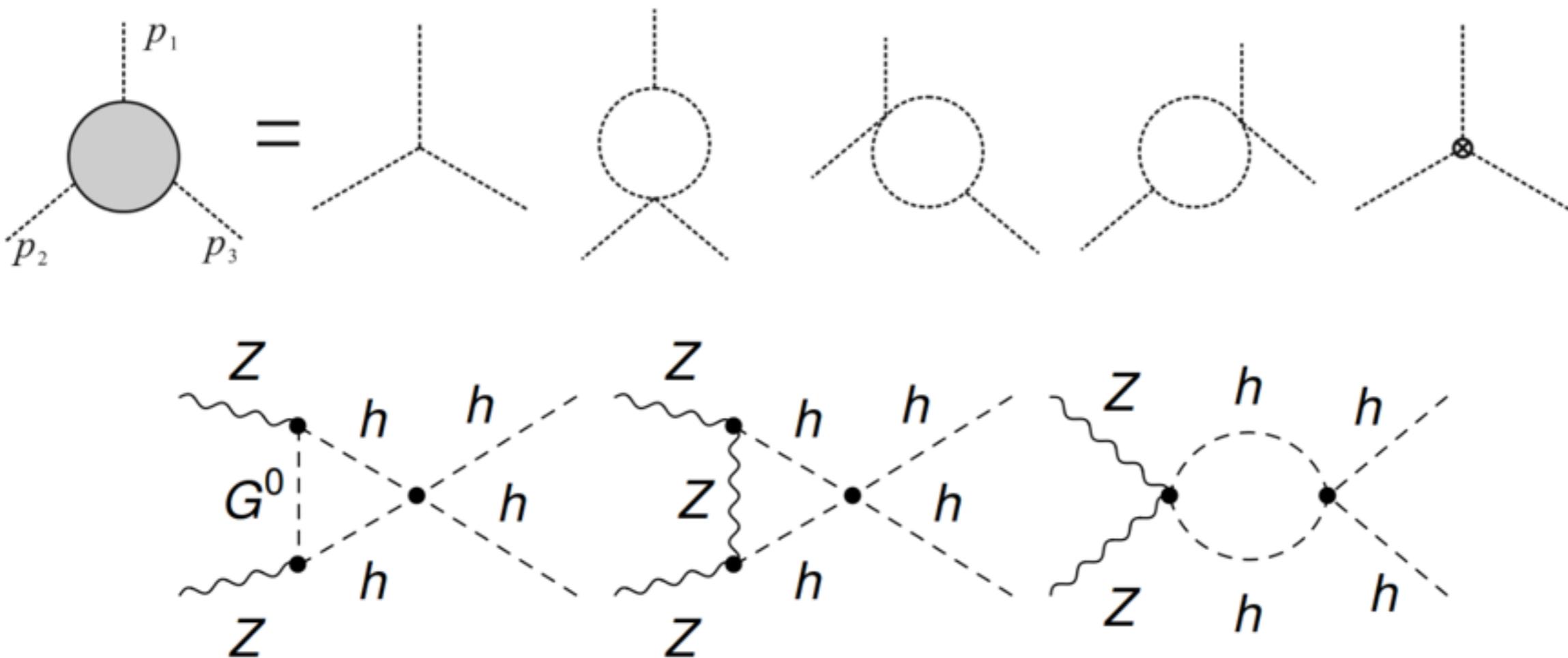
Summary and Others

- ☒ To fully utilize the potential of the e-e+ colliders, it is crucial to further improve the sensitivities to the EW operators (can Gamma_W help?).
- ☒ The ILC250 is less capable in probing the set of operators (in the scenario with fixed beam polarization). But, this can be adequately compensated by the data at 500 GeV
- ☒ The Zhh at ILC500 GeV plays a crucial role in probing O_6 operator/cubic Higgs coupling (how about quartic Higgs coupling?)



Probing Quartic Higgs Coupling

The quartic Higgs coupling contributes to di-Higgs production at loop level by renormalizing the cubic Higgs coupling and modifying the vertex form factors

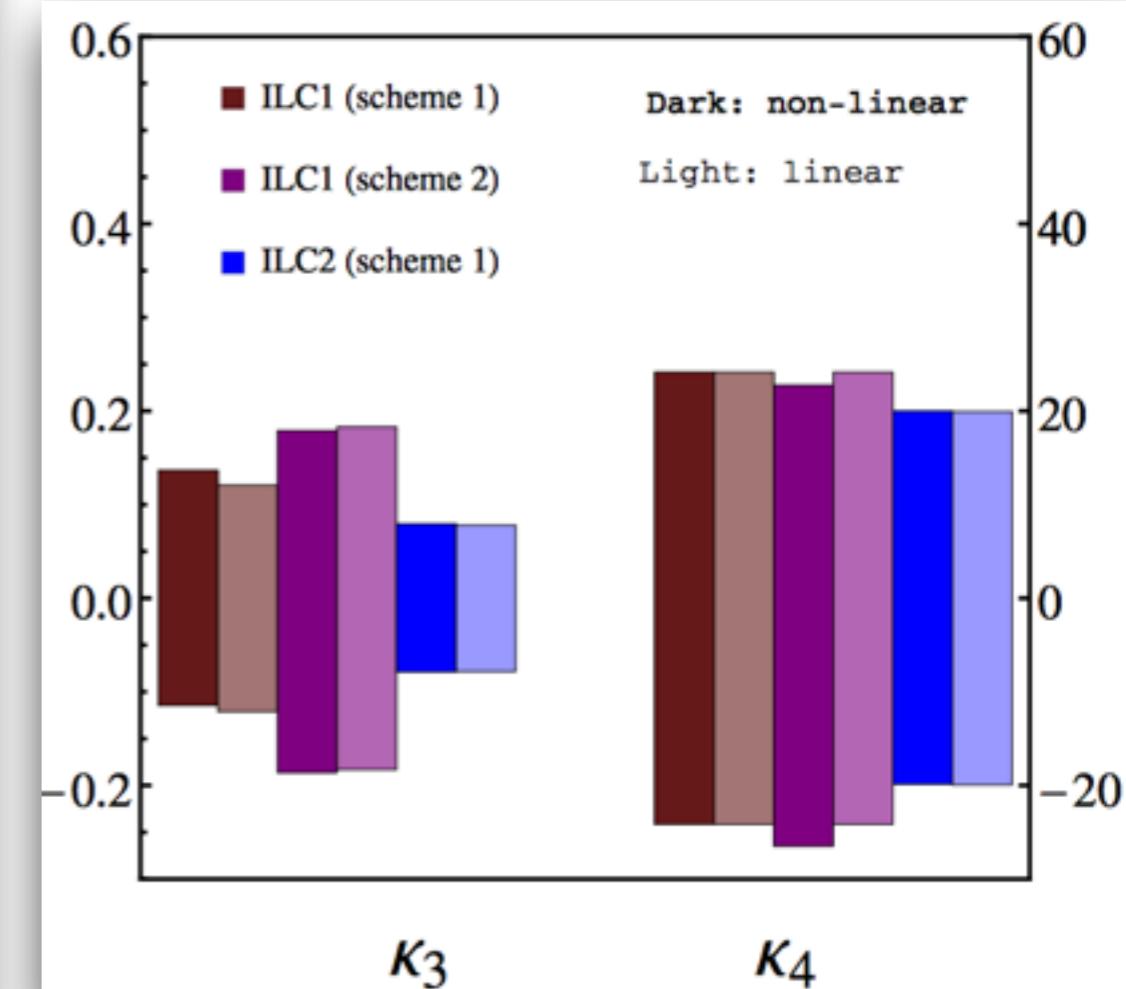
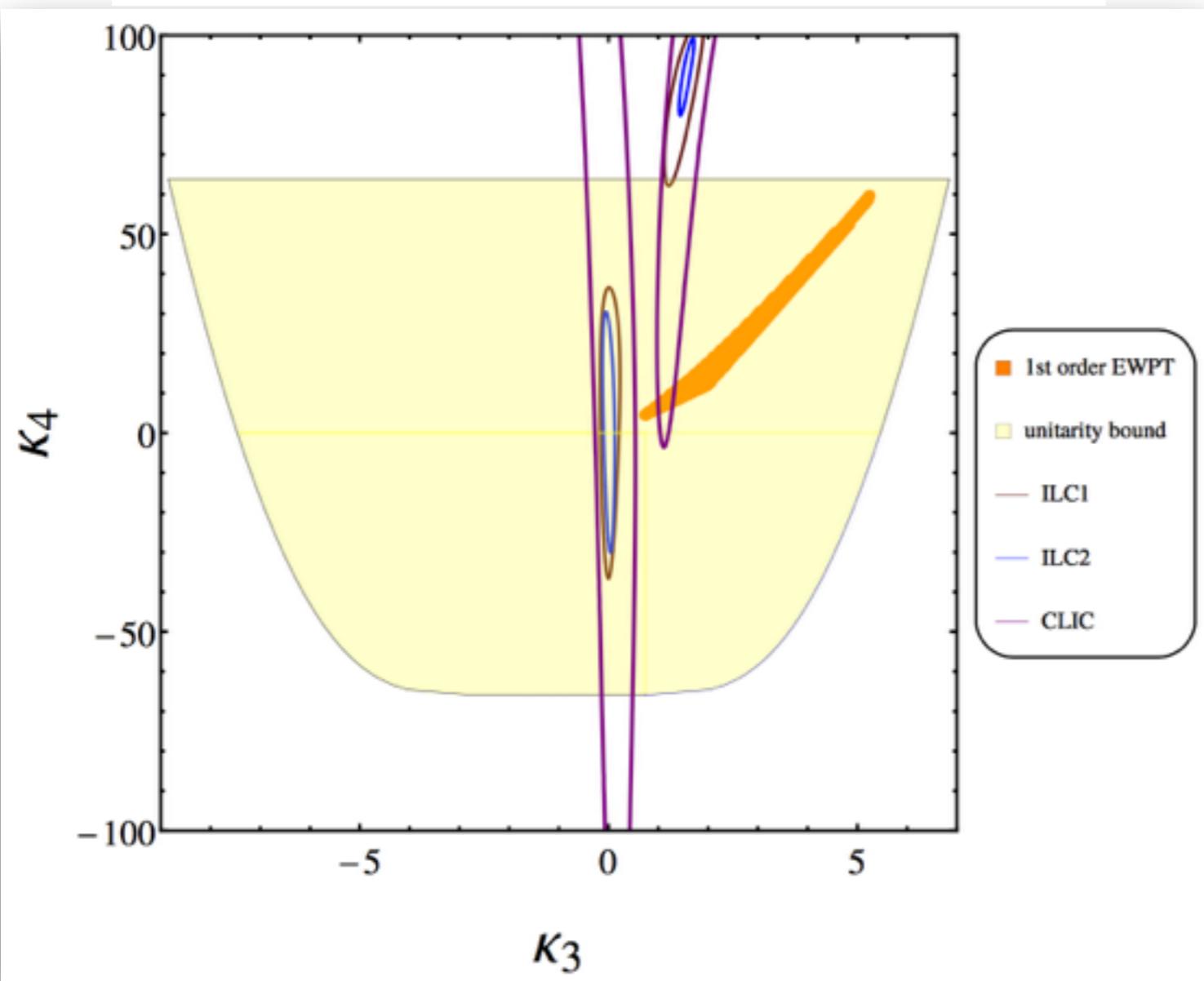




Probing Quartic Higgs Coupling

- ILC1 = ILC (500 GeV, $4 \text{ ab}^{-1} + 1 \text{ TeV}, 2.5 \text{ ab}^{-1}$ [13]);
- ILC2 = ILC (500 GeV, $4 \text{ ab}^{-1} + 1 \text{ TeV}, 8 \text{ ab}^{-1}$ [37]).

$Zhh + \nu\nu hh$



MS bar renormalization scheme with $\mu=125 \text{ GeV}$

Thank you!





Backup - Probing Quartic Higgs Coupling

