Study of search technique for anomalous couplings between top quark and gauge particles Z/γ using top pair creation at the ILC

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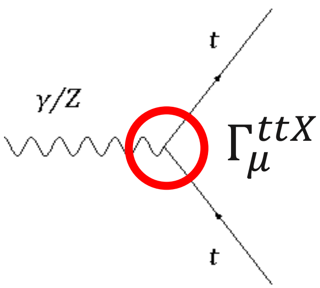
The ttZ/γ couplings study at the ILC

☐ The ttZ/γ couplings are important probes for new physics

(e.g.) Predicted deviation of F or g from SM is ~10 % in composite models.



$$\Gamma_{\mu}^{ttX}(k^2,q,\bar{q}) = ie\left[\gamma_\mu\left(F_{1V}^X(k^2)+\gamma_5 F_{1A}^X(k^2)\right)+\frac{\sigma_{\mu\nu}}{2m_t}(q+\bar{q})^\nu\left(iF_{2V}^X(k^2)+\gamma_5 F_{2A}^X(k^2)\right)\right]$$

$$(X = Z,\gamma)$$

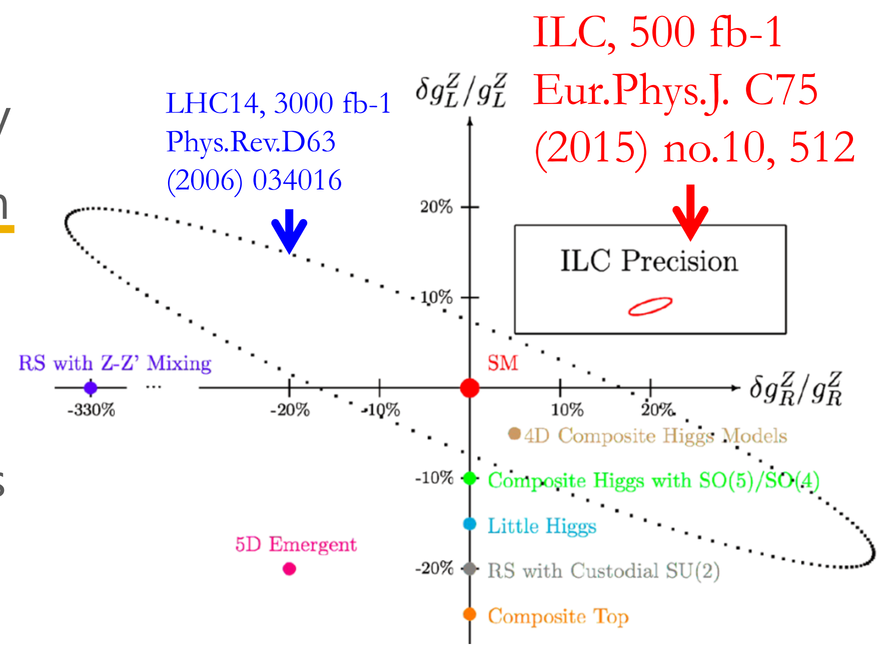
$$(g_L = F_{1V} - F_{1A},\, g_R = F_{1V} + F_{1A})$$

☐ The ILC is suitable for the ttZ/γ study

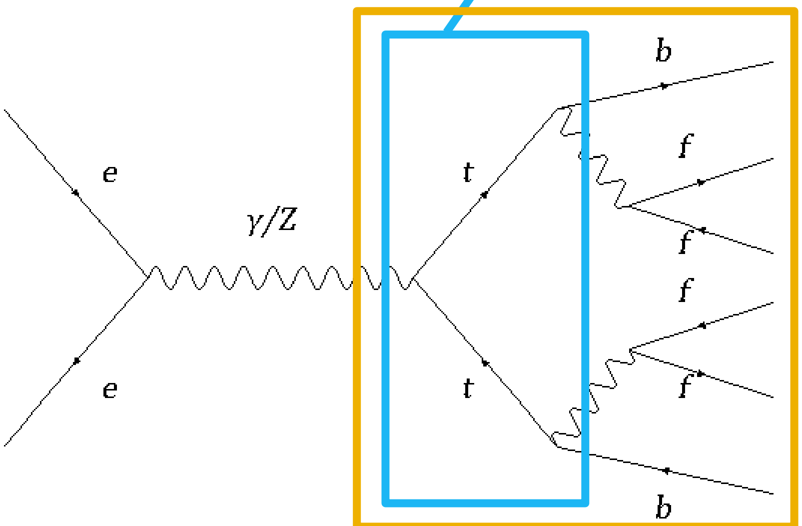
Clean data & 500 GeV & Polarized beam

Expected precision at the ILC

- is much better than the LHC
- allows to distinguish between models



Full angular analysis



The previous study used A_{FB}, σ

- Obtained from $e^- e^+ \rightarrow t\bar{t}$ process

Decay process has also the information of the ttZ/γ couplings

- Top quark decays before hadronization
- Angular distributions of decay particles depend on the spin of top quark

Full angular analysis gives intrinsically higher sensitivities

Goal of this study

Goal of this study

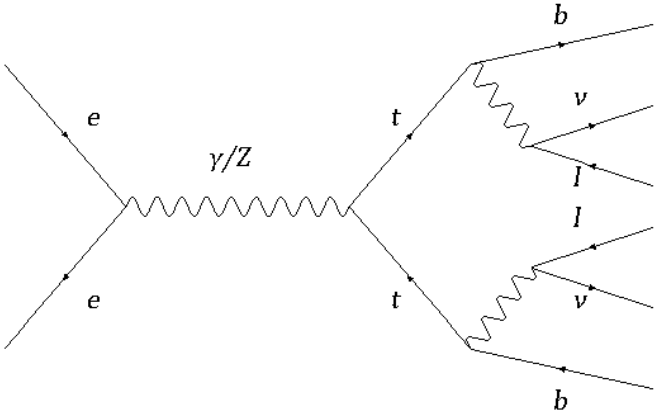
Development of the search technique for the anomalous ttZ/γ couplings with the full angular analysis based on the ILD full simulation.

Progress in FY2017

- Reconstruction of the di-leptonic process;

  $e^-e^+ \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow bl^+\nu\bar{b}l^-\bar{\nu}$

  - The most observables can be obtained
- Analysis with the binned likelihood method



Parameter setup

Event generator : WHIZARD, Pythia

Detector simulation : Mokka, Marlin

Parameter setup is based on the TDR and DBD.

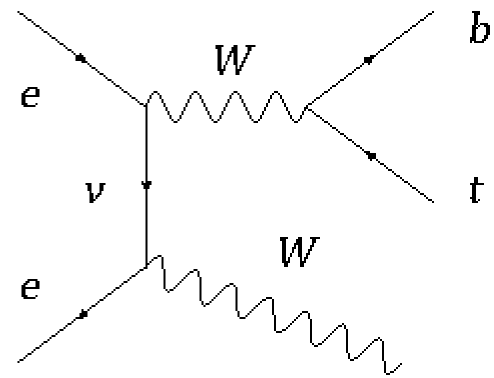
Center-of-mass energy	$\sqrt{s}$	500 GeV
Beam polarization	(P_{e^-}, P_{e^+})	(-0.8, +0.3) / (+0.8, -0.3) Left / Right
Integrated luminosity	L	250 fb^{-1} / 250 fb^{-1}
Top quark mass	m_t	174 GeV
Other physics parameters		Consistent with SM-LO

Signal and major backgrounds

At first, we focus on the $e^- e^+ \to b\bar{b}\mu^-\mu^+\nu\bar{\nu}$

Signal : $e^- e^+ \to b\bar{b}\mu^-\mu^+\nu\bar{\nu}$

- **Both of W's decay to $\mu\nu_\mu$**
  The most accurate to be reconstructed
  in the di-leptonic decay process
- Includes the single top production, ZWW etc.
  These are irreducible backgrounds

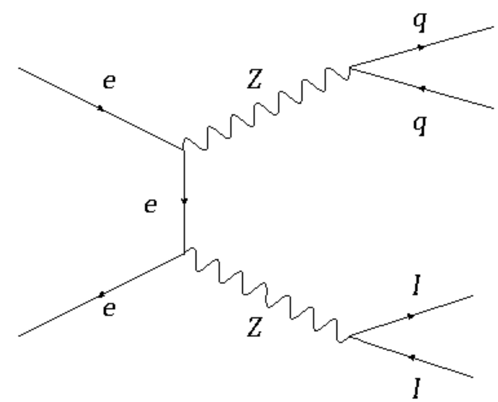


Single top production

Major backgrounds

- $e^- e^+ \to q\bar{q}l^-l^+$ (mainly $e^- e^+ \to ZZ \to q\bar{q}l^-l^+$)
- $e^- e^+ \to b\bar{b}l^-l^+\nu\bar{\nu}$ (except for $b\bar{b}\mu^-\mu^+\nu\bar{\nu}$)

They can have 2 b-jets and 2 isolated muons



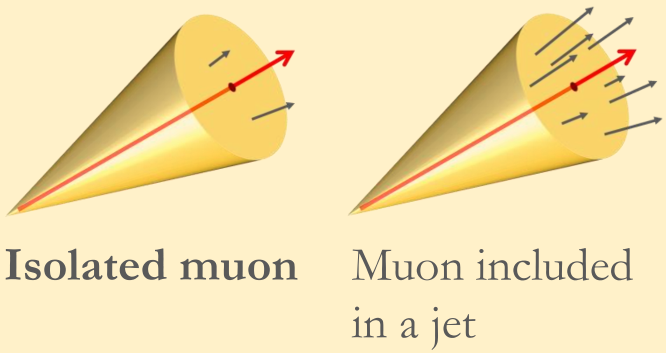
$e^- e^+ \to ZZ \to q\bar{q}l^-l^+$

Reconstruction Process

Reconstruct all final state particles, $b\bar{b}\mu^-\mu^+\nu\bar{\nu}$.

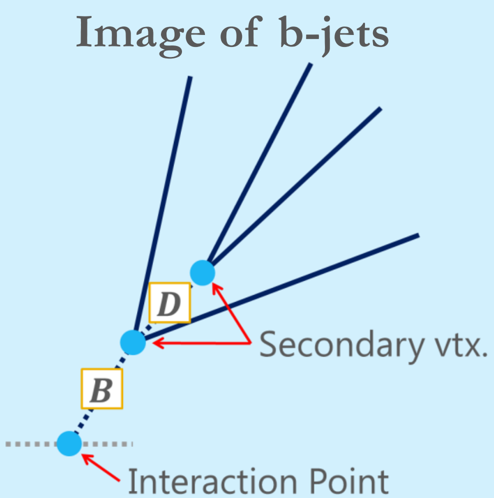
1. Selection of μ^+ and μ^-

- μ^-, μ^+ are isolated from other particles
- Extract isolated muons as final state muons



2. Jet clustering and b-tagging

- Cluster jet particles corresponding to $b, \bar{b}$
- B, D meson moves ~100 μm before the decay
- Assess the "b-likeness" from the vertex information (such as # of vtx. and distance between IP and vtx.)



Reconstruction Process

3. Kinematical Reconstruction

- $\nu, \bar{\nu}$ are not detectable at the ILD detector.
- To recover them, impose the following constraints
  - Initial state constraints : $E_{\mathrm{total}} = 500\ \mathrm{GeV}, \vec{P}_{\mathrm{total}} = \vec{0}\ \mathrm{GeV}$
  - Mass constraints : $m_{t,\bar{t}} = 174\ \mathrm{GeV}, m_{W^{\pm}} = 80.4\ \mathrm{GeV}$
- γ of the ISR/Beamstrahlung deteriorates the initial state condition. Assume the γ is along the beam direction (z-axis).

Unknowns

$\vec{P}_{\nu}, \vec{P}_{\bar{\nu}}, P_{\gamma,z}$: 7

Constraints

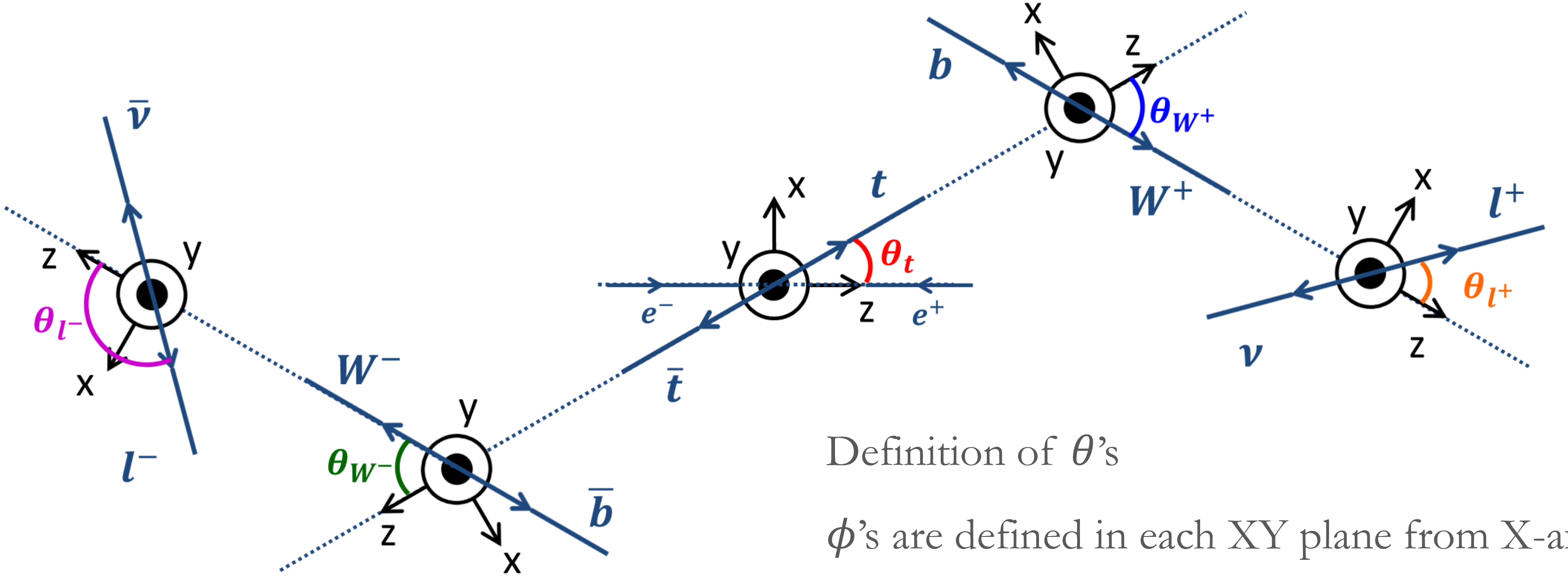
$E_{\mathrm{total}}, \vec{P}_{\mathrm{total}},$

$m_t, m_{\bar{t}}, m_{W^+}, m_{W^-}$: 8

The amplitude of the di-leptonic process

The amplitude of the di-leptonic process is a function of 9 angles.

$$|M|^2(\cos\theta_t, \cos\theta_{W^+}, \phi_{W^+}, \cos\theta_{W^-}, \phi_{W^-}, \cos\theta_{l^+}, \phi_{l^+}, \cos\theta_{l^-}, \phi_{l^-}; F)$$



Definition of θ's

ϕ's are defined in each XY plane from X-axis

It is difficult to handle the 9-dimention phase space

→ Expand the amplitude in the form factors, F

Expansion of the amplitude at SM value

Expand the amplitude in the form factors, F, at SM value :

$$|M|^2(\Phi;F) = \left(1 + \sum_i \omega_i(\Phi)\delta F_i + \sum_{ij} \tilde{\omega}_{ij}(\Phi)\delta F_i \delta F_j\right)|M^{SM}|^2(\Phi;F^{SM})$$

$$\omega_i = \frac{1}{|M|^2(\Phi)} \frac{\partial |M|^2(\Phi)}{\partial F_i}\Big|_{\delta F=0}, \tilde{\omega}_{ij} = \frac{1}{|M|^2(\Phi)} \frac{\partial^2 |M|^2(\Phi)}{\partial F_i \partial F_j}\Big|_{\delta F=0}, \delta F_i = F_i - F_i^{\mathrm{SM}}$$

Φ is a vector of the angles, F is a vector of the form factors.

$\omega, \tilde{\omega}$ are the optimal variables for the form factors

(*) Matrix element method

Use all ω and $\tilde{\omega}$ with unbinned likelihood method.

It is difficult to involve the experimental effects to the likelihood function

Binned likelihood method

Use only ω ignoring the second order of δF

- Prepare ω distribution with large full simulation
- Fit the simulation distribution to a binned "data"(*) using the following χ^2

$$\chi^2(\delta F) = \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data}} - n_i^{\text{Sim.}}(\delta F)}{\sqrt{n_i^{\text{Data}}}} \right)^2$$

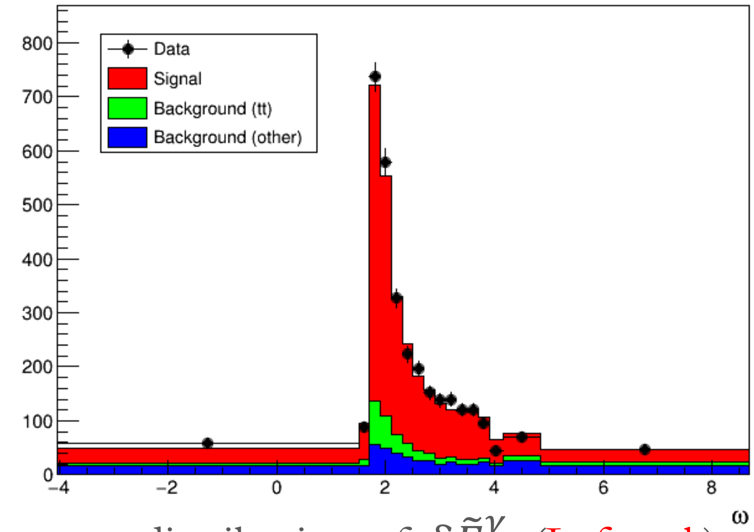
(*) The "data" is also obtained from the full simulation. It will be replaced for real data.

We have done single parameter fit for each F.

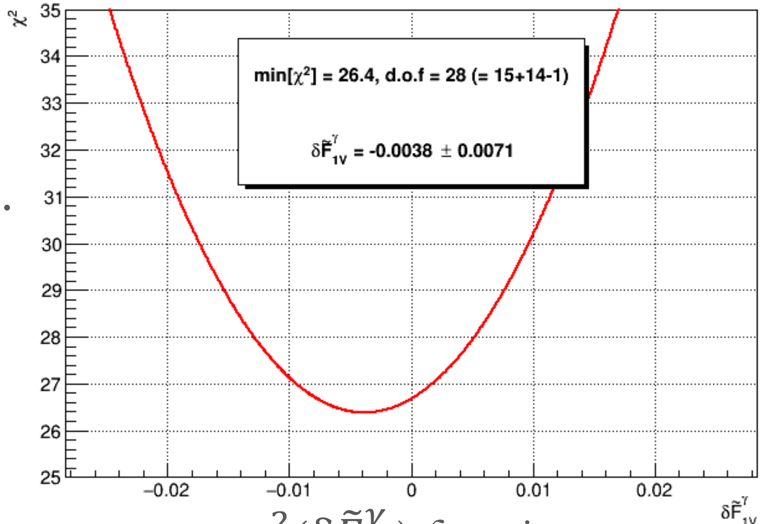
(e.g.) $\delta \tilde{F}_{1V}^{\gamma} = -0.0038 \pm 0.0071$

C.L. = 55.2 %

Consistent with SM (input) value



ω distribution of $\delta \tilde{F}_{1V}^{\gamma}$ (Left pol.)



$\chi^2(\delta \tilde{F}_{1V}^{\gamma})$ function

Comparison with previous study

Form factor	Previous (1) semi-lep	This study $bb\mu\mu\nu\nu$
F_{1V}^{γ}	± 0.002	± 0.0071
F_{1V}^{Z}	± 0.003	± 0.0128
F_{1A}^{γ}	---	± 0.0162
F_{1A}^{Z}	± 0.007	± 0.0262
F_{2V}^{γ}	± 0.001	± 0.0058
F_{2V}^{Z}	± 0.002	± 0.0102

Form factor	Previous (2) semi-lep	This study $bb\mu\mu\nu\nu$
ReF_{2A}^{γ}	± 0.005	± 0.0238
ReF_{2A}^{Z}	± 0.007	± 0.0351
ImF_{2A}^{γ}	± 0.006	± 0.0223
ImF_{2A}^{Z}	± 0.010	± 0.0394

Difference of N_{signal} is

$$\frac{N_{\text{semi-lep}}}{N_{bb\mu\mu\nu\nu}} \simeq \frac{\frac{6}{9} \times \frac{2}{9} \times 2}{\frac{1}{9} \times \frac{1}{9}} = 24$$

→ A factor of 5 can be expected

- Consistent with the previous study(*)
- If this method is applied for the semi-leptonic process, it's possible that the precision will be improved

(*) Although some results of previous study are from multi-fit, the correlation is small.

(1) Eur.Phys.J. C75 (2015) no.10, 512

(2) arXiv:1710.06737 [hep-ex].

Progress from 55th General Meeting (3rd Feb.)

Apply this method to the di-leptonic process, $e^-e^+ \rightarrow b\bar{b}l^-l^+\nu\bar{\nu}$.

Single Parameter fit

- Precision becomes twice better because the number of signal events is about 4 times larger.

Form factor	$bb\mu\mu\nu\nu$	$bbll\nu\nu$
F_{1V}^{γ}	± 0.0071	± 0.0034
F_{1V}^{Z}	± 0.0128	± 0.0061
F_{1A}^{γ}	± 0.0162	± 0.0082
F_{1A}^{Z}	± 0.0262	± 0.0133
F_{2V}^{γ}	± 0.0058	± 0.0028
F_{2V}^{Z}	± 0.0102	± 0.0049

Form factor	$bb\mu\mu\nu\nu$	$bbll\nu\nu$
ReF_{2A}^{γ}	± 0.0238	± 0.0123
ReF_{2A}^{Z}	± 0.0351	± 0.0180
ImF_{2A}^{γ}	± 0.0223	± 0.0110
ImF_{2A}^{Z}	± 0.0394	± 0.0194

Progress from 55th General Meeting (3rd Feb.)

Multi-Parameter fit

- ω's are not correlated → Each parameter can be measured independently.
  (10 Parameters can be separated into top 6, middle 2 and bottom 2)
- ω's are correlated or there is a structure in a 2D histogram
  → The 2D histogram is needed for the multi-parameter fit
- ω's are strongly correlated.
  → The 1D histogram can measure multi parameter (not optimal way)

(eg) Use a 1D histogram of ω of F_{1V}^{γ}

Form factor	Single	Multi
F_{1V}^{γ}	± 0.0034	± 0.0038
F_{1V}^{Z}	± 0.0061	± 0.0068
F_{1A}^{γ}	± 0.0082	± 0.0099
F_{1A}^{Z}	± 0.0133	± 0.0155

Correlation matrix

$$\begin{pmatrix} +1.000 & -0.232 & -0.183 & +0.380 \\ -0.232 & +1.000 & +0.328 & -0.123 \\ -0.183 & +0.328 & +1.000 & -0.280 \\ +0.380 & -0.123 & -0.280 & +1.000 \end{pmatrix}$$

Progress from 55th General Meeting (3rd Feb.)

(eg) Use a 2D histogram of $\omega's$ of $F_{1V}^{\gamma}, F_{2V}^{\gamma}$

Form factor	Single	Multi
F_{1V}^{γ}	± 0.0034	± 0.0264
F_{1V}^{Z}	± 0.0061	± 0.0437
F_{1A}^{γ}	± 0.0082	± 0.0102
F_{1A}^{Z}	± 0.0133	± 0.0157
F_{2V}^{γ}	± 0.0028	± 0.0210
F_{2V}^{Z}	± 0.0049	± 0.0344

Correlation matrix

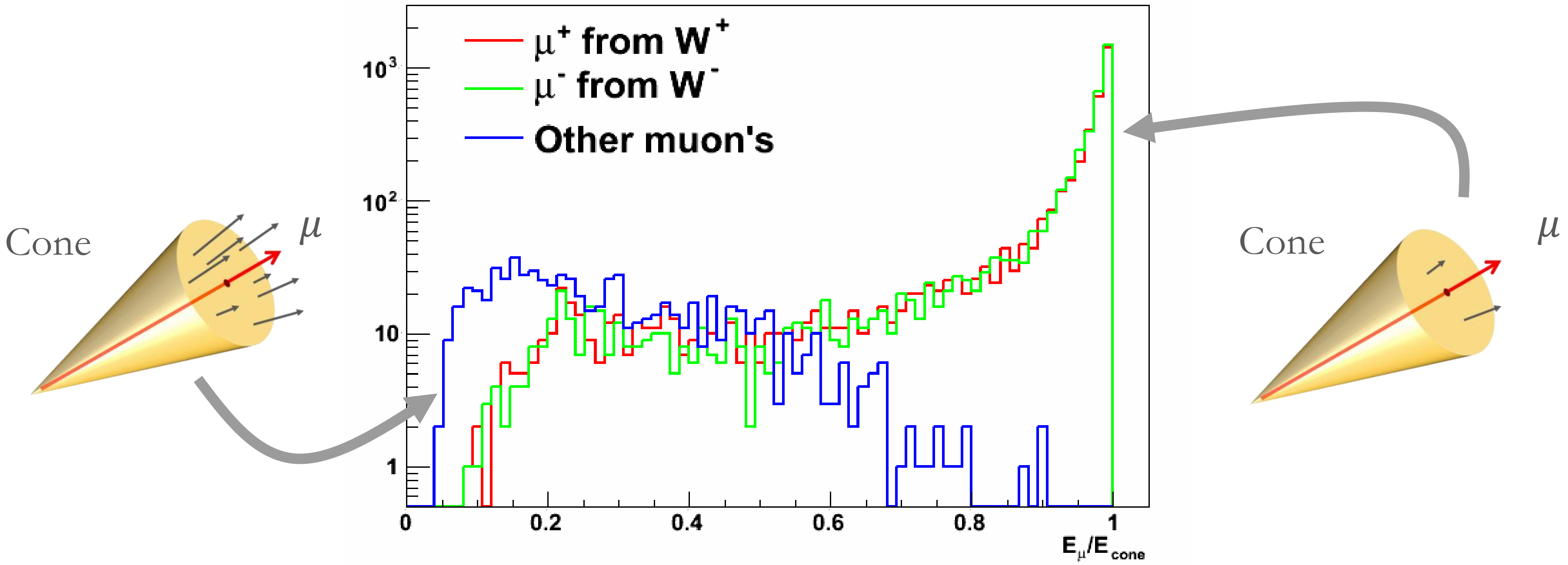
$$\begin{pmatrix} +1.000 & -0.237 & -0.098 & +0.296 & -0.990 & +0.232 \\ -0.237 & +1.000 & +0.322 & -0.058 & +0.231 & -0.989 \\ -0.098 & +0.322 & +1.000 & -0.310 & +0.070 & -0.276 \\ +0.296 & -0.058 & -0.310 & +1.000 & -0.246 & +0.034 \\ -0.990 & +0.231 & +0.070 & -0.246 & +1.000 & -0.231 \\ +0.232 & -0.989 & -0.276 & +0.034 & -0.231 & +1.000 \end{pmatrix}$$

Summary

- Development of the search technique for the anomalous ttZ/γ couplings with full angular analysis based on the ILD full simulation.
- Reconstructed full kinematics of the di-leptonic process (especially μ μ) from the kinematical reconstruction.
- Estimated the statistical errors from the binned likelihood fit for the ω distribution and confirmed the validity of this method.
- The precision is consistent with the previous study and there's a possibility of improvement if this method is applied for the semi-leptonic state.

Backup

Isolated muon finder



Energy ratio between μ and a cone

$R = E_{\mu}/E_{cone}$ is a quantity to evaluate how isolated the muon is.

(E_{cone} : total energy of particles in the cone)

μ from W boson is more isolated than other μ

Isolated muon finder

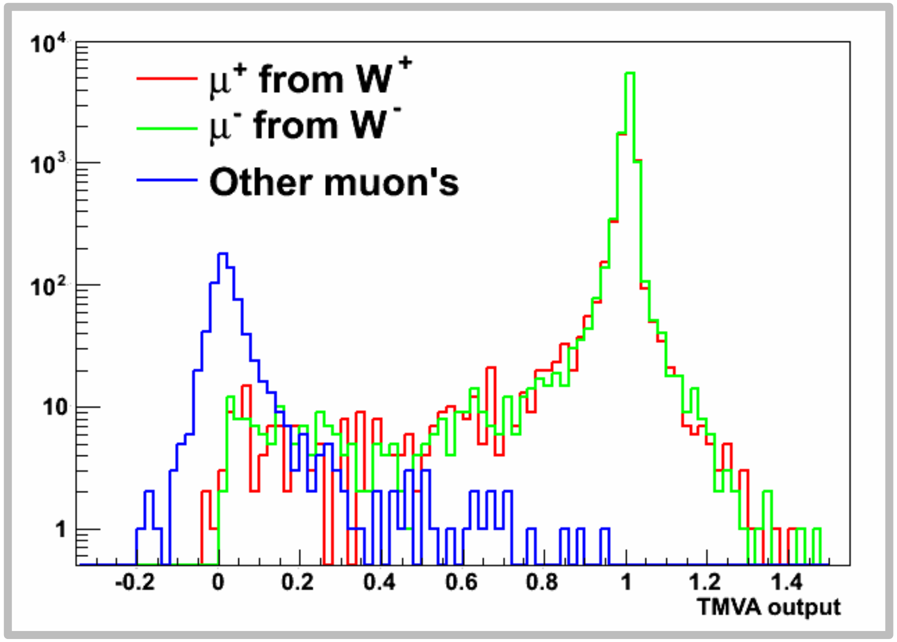
Quantities

$R = E_\mu / E_{cone}$, $E_{cone,neutral}$, $E_{cone,charged}$

$\cos\theta = \frac{P_\mu \cdot P_{cone}}{|P_\mu| \times |P_{cone}|}$, ΔE_{ECAL}, ΔE_{Yoke}, …

TMVA

Multi variable analysis tool



Jet clustering

General strategy

Merge a pair of particles whose "**Distance**" is the smallest until a condition meets "**Criteria**"

"**Distance**"

Durham algorithm : $Y_{ij} = 2\frac{\min\left[E_i^2, E_j^2\right](1-\cos\theta_{ij})}{E_{vis}^2}$, θ_{ij} : angle between P_i and P_j

k_t algorithm : $d_{ij} = \min\left[{p_T}_i^2, {p_t}_j^2\right]\frac{R_{ij}}{R}$ or $d_{iB} = {p_t}_i^2, R_{ij}^2 = \left(\eta_i - \eta_j\right)^2 + \left(\phi_i - \phi_j\right)^2$

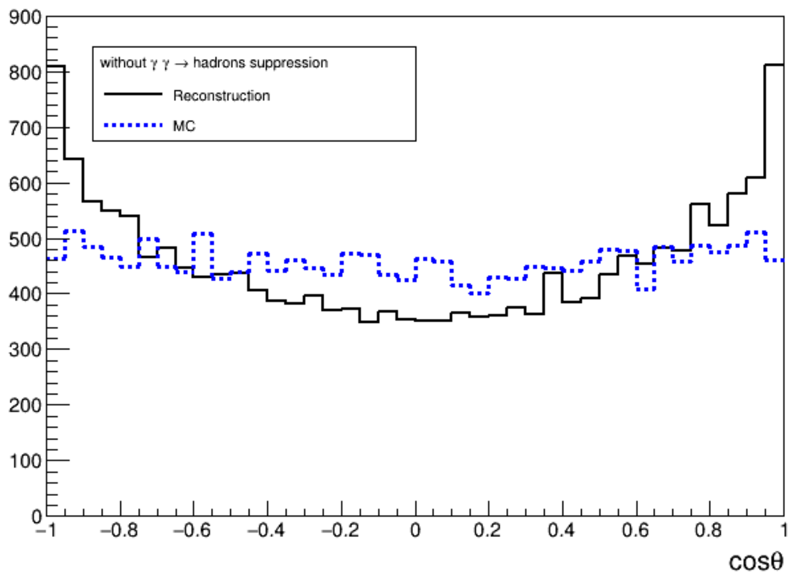
η : pseudo rapidity, ϕ azimuthal angle

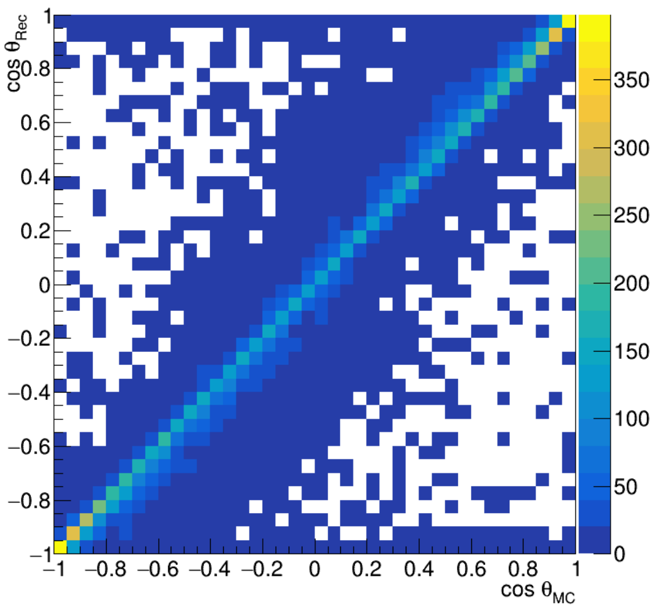
"**Criteria**"

- Number of remaining particles is equal to N_{Req}
- The smallest distance is smaller than D_{Req}

$\gamma\gamma \to$ hadrons rejection

$b, \bar{b}$ are reconstructed from the rest of particles with LCFIPlus





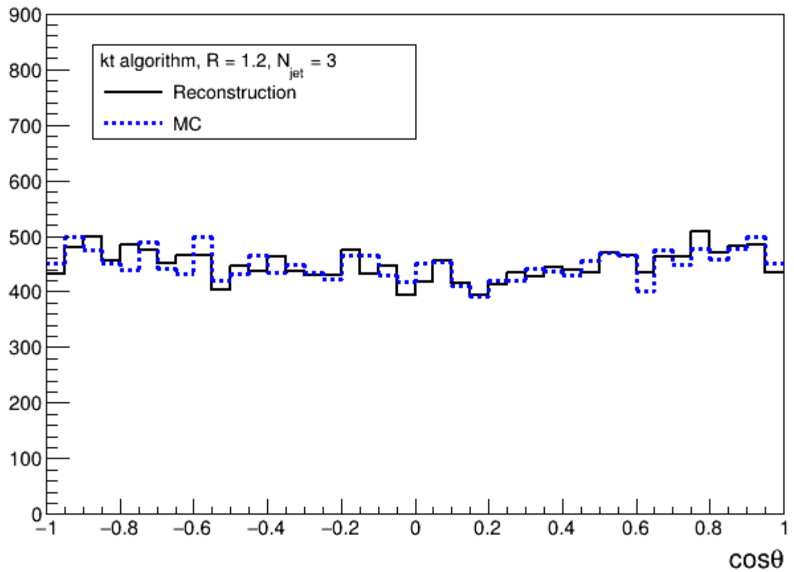
$\cos\theta_{jet}$ distribution

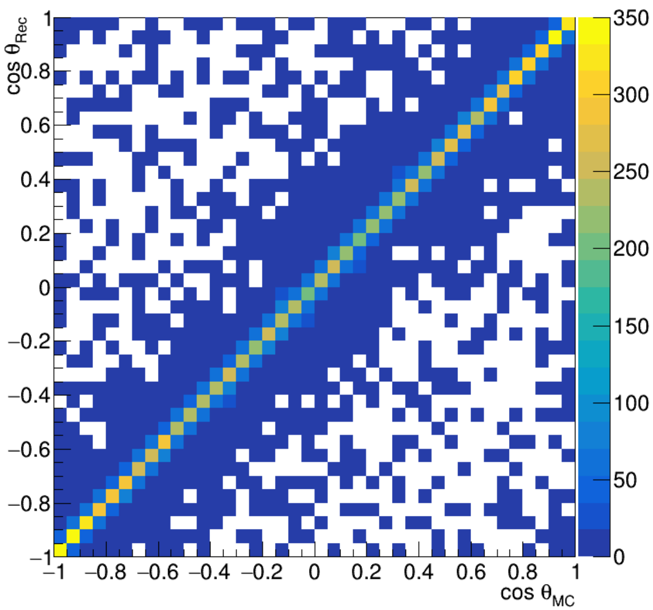
Strongly peaked at very forward region by mistake

$\gamma\gamma \to$ hadrons are emitted along the beam direction

$\gamma\gamma \rightarrow$ hadrons rejection

Eliminate particles close to beam direction rather than other particles with kt algorithm.





$\cos\theta_{jet}$ distribution

Good agreement between Rec and MC

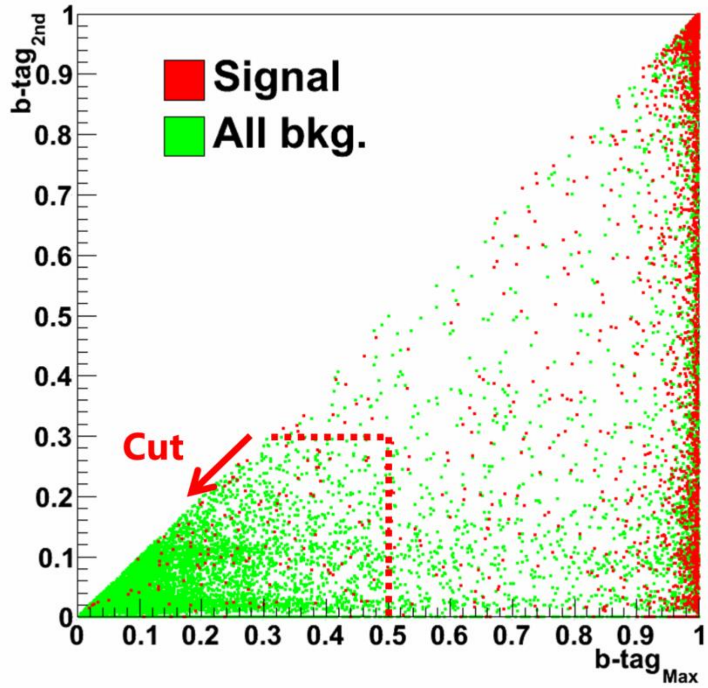
b-tagging with LCFIPlus

b-tag is TMVA output indicating “b-likeness” of a jet obtained by the LCFIPlus(*).

- b-tag_{Max} : the largest b-tag
- b-tag_{2nd} : the 2nd largest b-tag

- Signal has large b-tag_{Max}
- Many of bkg. have small b-tag_{Max} and b-tag_{2nd}

$$b - tag_{Max} > 0.5 \text{ or } b - tag_{2nd} > 0.3$$



b-tag value (Left polarization)

(*) A software package of Marlin for the multi-jet analysis.

Algorithm of the Kinematical Reconstruction

- Introduce 4 free parameters : $\vec{P}_\nu, P_{\gamma,z}$

$\vec{P}_{\bar{\nu}}$ can be computed using the initial momentum constraints

$$\vec{P}_{\bar{\nu}} = -\vec{P}_{\text{vis.}} - \vec{P}_\nu - \vec{P}_\gamma, \qquad \left(\vec{P}_{\text{vis.}} = \vec{P}_b + \vec{P}_{\bar{b}} + \vec{P}_{\mu^+} + \vec{P}_{\mu^-}\right)$$

- Define the likelihood function :

$$L_0\left(\vec{P}_\nu, P_{\gamma,z}\right) = BW(m_t)BW(m_{\bar{t}})BW(m_{W^+})BW(m_{W^-})Gaus(E_{\text{total}})$$

- To correct the energy resolution of b-jets, add 2 parameters, $E_b, E_{\bar{b}}$, with the resolution functions to L_0 :

$$L\left(\vec{P}_\nu, P_{\gamma,z}, E_b, E_{\bar{b}}\right) = L_0 \times Res(E_b, E_b^{\text{meas.}})Res\left(E_{\bar{b}}, E_{\bar{b}}^{\text{meas.}}\right)$$

Define $q\left(\vec{P}_\nu, P_{\gamma,z}, E_b, E_{\bar{b}}\right) = -2\log L + \text{Const.}$

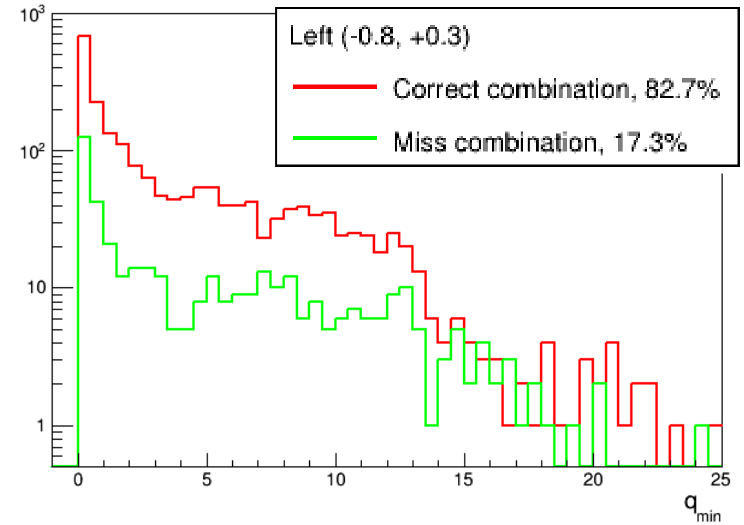
(scaled as the minimum of each component ($BW(m_t)$, etc) is equal to 0)

Combination of μ and b-jet

Choice of a combination of μ and b-jet

There are two candidates for the combination

- Select one having smaller q, defined as $\boldsymbol{q}_{\mathbf{min}}$
- Fraction of correct combination is ~83%

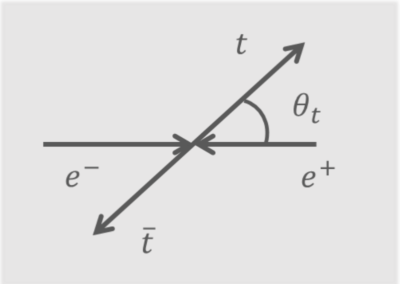


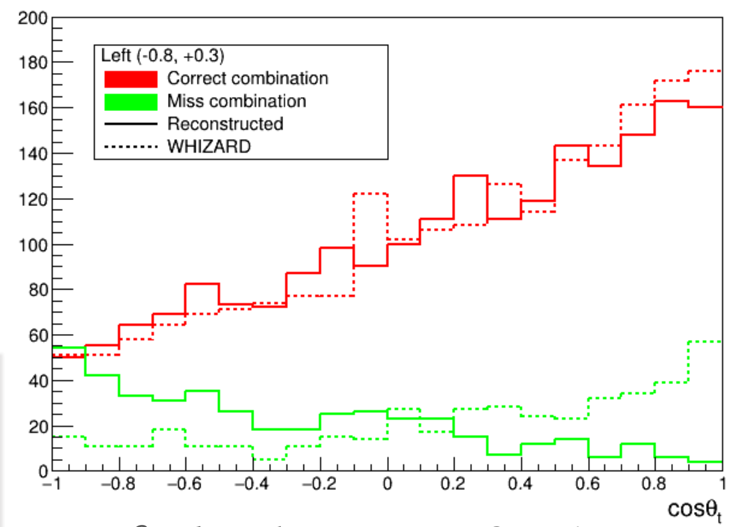
q_{min} distribution (Left polarization)

$\cos\theta_t$ distribution (Rec vs. MC Truth)

- Correct combination: OK !
- Miss combination: Disagree with the MC truth.

Need to estimate an effect of the miss combination for the analysis.





$\cos\theta_t$ distribution (Left polarization)

Event Selection

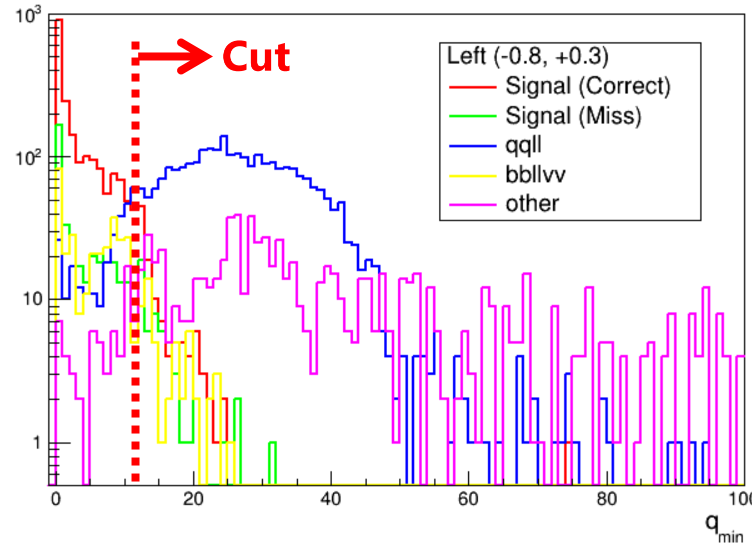
Quality cut :

$q_{\min}$ means the quality of reconstruction.
Useful to suppress the backgrounds.

Criteria are optimized for the significance,

$$S = \frac{N_{\text{signal}}}{\sqrt{N_{\text{signal}} + N_{\text{background}}}}$$

$q_{\min}$ distribution (Left polarization)



Left Polarization **Cut Criteria**	**Signal** $bb\mu\mu\nu\nu$	tt	**except for** tt	**All bkg.**	$qqll$	$bbll\nu\nu$
No cut	2837			8410633	91478	23312
$N_{\mu^-} = 1$ & $N_{\mu^+} = 1$	2618			327488	13827	387
b-tag cut	2489	2215	273	4143	2943	363
Quality cut ($q_{\min} < 11.5$)	2396	2103	195	624	258	313

(*) Separate signals into $t\bar{t}$ and the other process from WHIZARD information

Kinematical Reconstruction

$$BW(x; m, \Gamma) \propto \frac{1}{1 + \left(\frac{x^2 - m^2}{m\Gamma}\right)^2}$$

$$Gaus(x; \mu, \sigma) \propto \exp\left[-\left(\frac{x - \mu}{\sqrt{2}\sigma}\right)^2\right]$$

Detail definition of L_0 is

$$L_0\left(\vec{P}_\nu, P_{\gamma,Z}\right) = BW(m_t; 174{,}5)BW(m_{\bar{t}}; 174{,}5)$$
$$\cdot BW(m_{W^+}; 80.4{,}5)BW(m_{W^-}; 80.4{,}5)Gaus(E_{\mathrm{total}}; 500{,}0.39)$$

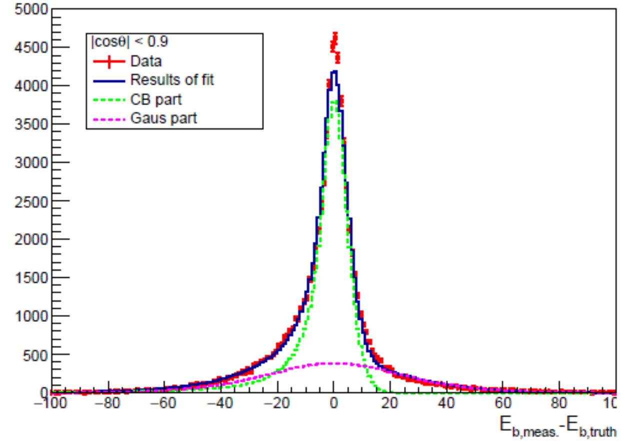
- Larger value for Γ than theoretical value is set because of detector effects
- σ is caused by the Beam energy spread.

Energy resolution of b-jet

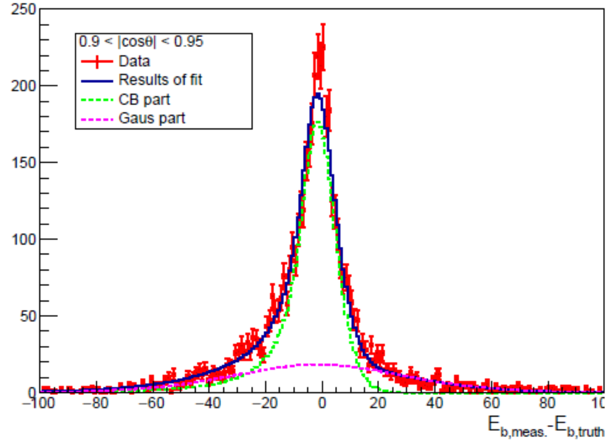
Estimate the energy resolution of b-jet with the following $Res(E_b, E_b^{\text{meas.}})$;

$$Res(E_b, E_b^{\text{meas.}}) = (1-f)CB(\Delta E_b; \alpha, n, \mu_{CB}, \sigma_{CB}) + f * Gaus(\Delta E_b; \mu_{Gaus}, \sigma_{Gaus})$$

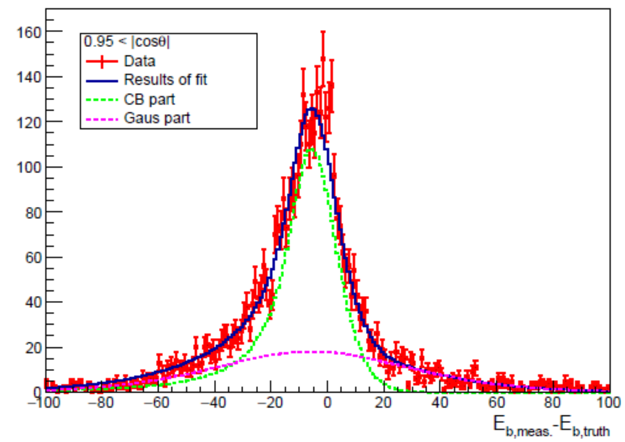
Divide into 3 regions ; $|\cos\theta| = (0, 0.9), (0.9, 0.95), (0.95, 1)$



(a) $|\cos\theta| < 0.9$



(b) $0.9 < |\cos\theta| < 0.95$



(c) $0.95 < |\cos\theta|$

Crystal Ball function

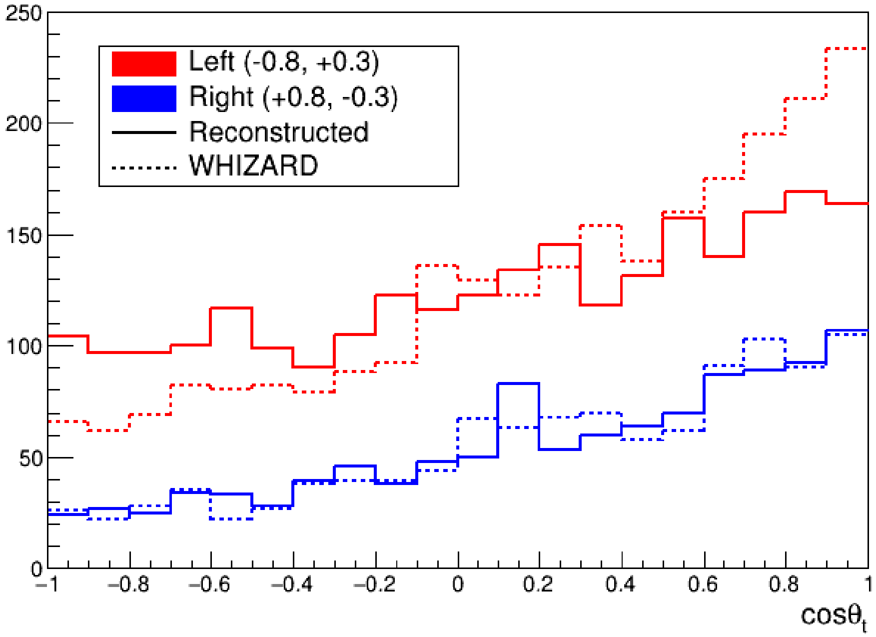
Crystal Ball function consists of a Gaussian core portion and power-law tail.

$$CB(x;\alpha,n,\bar{x},\sigma) = N \cdot \begin{cases} \exp\left(-\frac{(x-\bar{x})^2}{2\sigma^2}\right) & \frac{x-\bar{x}}{\sigma} > -\alpha \\ A \cdot \left(B - \frac{x-\bar{x}}{\sigma}\right)^{-n} & \frac{x-\bar{x}}{\sigma} \leq -\alpha \end{cases}$$

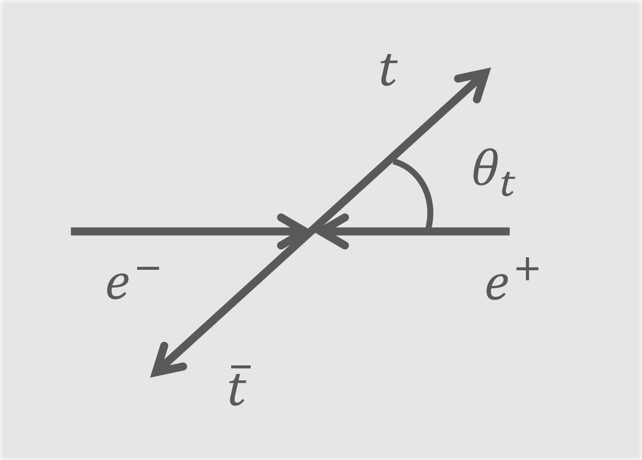
$$\begin{aligned} A &= \left(\frac{n}{|\alpha|}\right)^n \cdot \exp\left(-\frac{|\alpha|^2}{2}\right) \\ B &= \frac{n}{|\alpha|} - |\alpha| \\ N &= \frac{1}{\sigma(C+D)} \\ C &= \frac{n}{|\alpha|} \cdot \frac{1}{n-1} \cdot \exp\left(-\frac{|\alpha|^2}{2}\right) \\ D &= \sqrt{\frac{\pi}{2}}\left(1 + \operatorname{erf}\left(\frac{|\alpha|}{\sqrt{2}}\right)\right) \end{aligned}$$

Results of Reconstruction

Top quark polar angle distribution, $\cos\theta_t$



Definition of θ_t

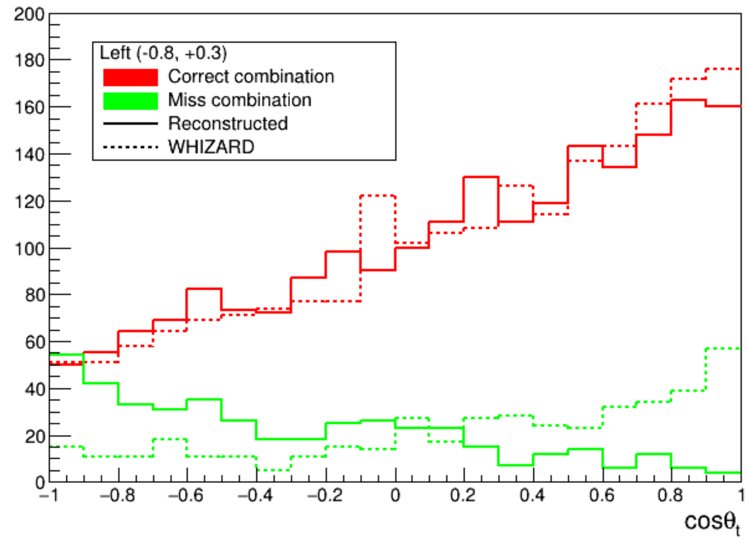


Considerable migration occurs in the Left polarization case

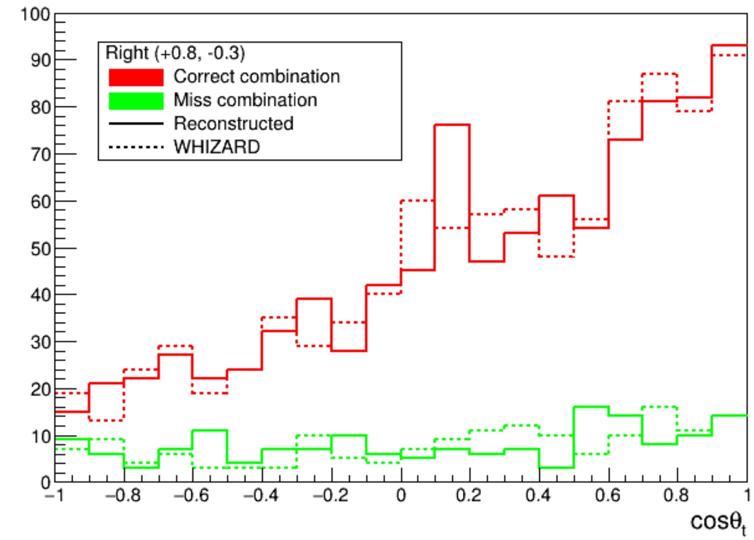
Some events pass from forward to backward because of the miss combination of μ and b-jet.

Dependence from the beam polarization

$\cos\theta_t$ distribution (Left polarization)



$\cos\theta_t$ distribution (Right polarization)



Left polarization

Reconstructed distribution of miss combination is very different from the MC truth.

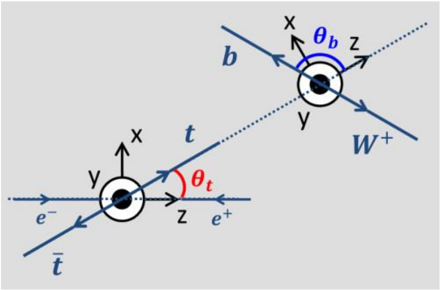
Right polarization

Similar distribution can be reconstructed even when the miss combination is selected.

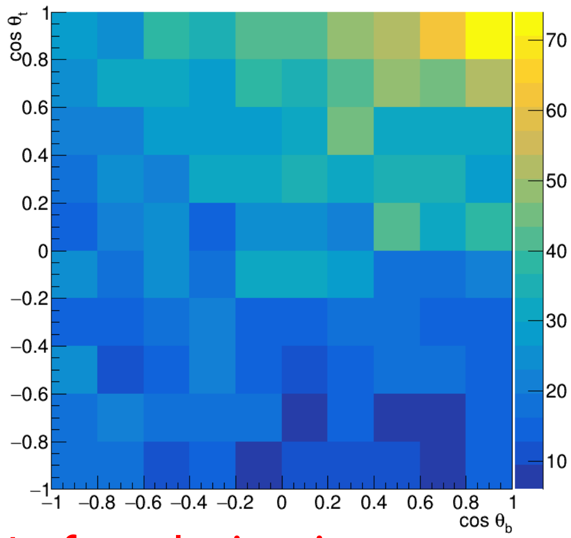
Dependence from the beam polarization

$\cos\theta_b \simeq 1$ → b-jets are energetic

→ **Migration effect is strong**



$\cos\theta_t$ vs. $\cos\theta_b$ (Left polarization)

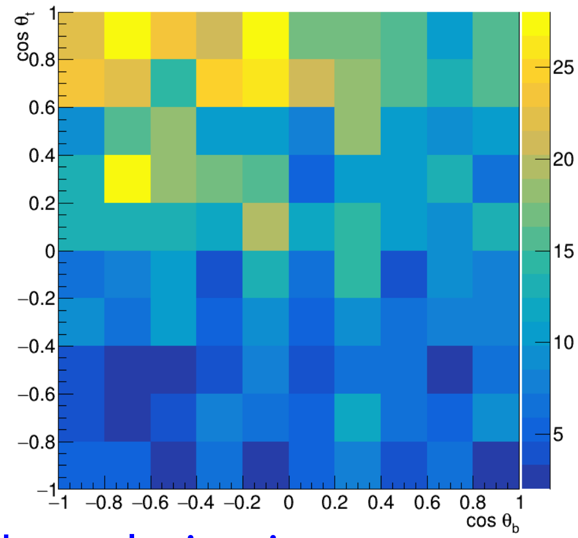


Left polarization

Peak at $\cos\theta_t \simeq 1$ & $\cos\theta_b \simeq 1$

→ Migration is asymmetry

$\cos\theta_t$ vs. $\cos\theta_b$ (Right polarization)



Right polarization

Peak at $\cos\theta_t \simeq 1$ & $\cos\theta_b \simeq -1$

→ Migration is symmetry

Cut table (Right Polarization)

Right Polarization **Cut Criteria**	**Signal** $bb\mu\mu\nu\nu$	tt	**except for** tt	**All bkg.**	$qqll$	$bbll\nu\nu$
No cut	1261			3751175	46344	10117
$N_{\mu^-} = 1$ & $N_{\mu^+} = 1$	1170			230260	6987	189
b-tag cut	1097	1046	79	2118	1468	181
Quality cut ($q_{\min} < 12.5$)	1046	976	70	297	132	151

Criteria of $t\bar{t}$:

$$|M_{b\mu^+\nu} - 174| < 15 \,\&\, |M_{\bar{b}\mu^-\bar{\nu}} - 174| < 15$$

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Matrix element method

- Based on the unbinned likelihood method. The likelihood function is computed from the amplitude.
  → Full kinematics are used = The most sensitive method in principle.
- Fit results are almost consistent with SM values.
  - ~1.5 σ biases are observed for several form factors

$$\begin{pmatrix} \delta\tilde{F}^{\gamma}_{1V,\mathrm{fit}} \\ \delta\tilde{F}^{Z}_{1V,\mathrm{fit}} \\ \delta\tilde{F}^{\gamma}_{1A,\mathrm{fit}} \\ \delta\tilde{F}^{Z}_{1A,\mathrm{fit}} \\ \delta\tilde{F}^{\gamma}_{2V,\mathrm{fit}} \\ \delta\tilde{F}^{Z}_{2V,\mathrm{fit}} \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2A,\mathrm{fit}} \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{2A,\mathrm{fit}} \\ \mathcal{I}m\ \delta\tilde{F}^{\gamma}_{2A,\mathrm{fit}} \\ \mathcal{I}m\ \delta\tilde{F}^{Z}_{2A,\mathrm{fit}} \end{pmatrix} = \begin{pmatrix} +0.0031 \pm 0.0130 \\ -0.0334 \pm 0.0231 \\ -0.0314 \pm 0.0192 \\ +0.0241 \pm 0.0301 \\ -0.0146 \pm 0.0366 \\ -0.0650 \pm 0.0592 \\ +0.0214 \pm 0.0241 \\ -0.0131 \pm 0.0415 \\ -0.0086 \pm 0.0255 \\ +0.0081 \pm 0.0360 \end{pmatrix}$$

Correlation coefficient for $\tilde{F}$

$$V_C = \begin{pmatrix} +1.000 & -0.141 & +0.027 & +0.085 & \underline{+0.598} & -0.067 & -0.026 & -0.018 & -0.006 & -0.012 \\ -0.141 & +1.000 & +0.093 & +0.066 & -0.028 & \underline{+0.606} & -0.033 & -0.065 & +0.002 & +0.012 \\ +0.027 & +0.093 & +1.000 & -0.082 & +0.012 & +0.034 & -0.038 & -0.096 & -0.024 & -0.013 \\ +0.085 & +0.066 & -0.082 & +1.000 & +0.003 & +0.033 & -0.075 & -0.040 & +0.005 & -0.027 \\ +0.598 & -0.028 & +0.012 & +0.003 & +1.000 & -0.107 & +0.037 & -0.038 & -0.021 & +0.019 \\ -0.067 & +0.606 & +0.034 & +0.033 & -0.107 & +1.000 & -0.064 & +0.006 & +0.013 & -0.020 \\ -0.026 & -0.033 & -0.038 & -0.075 & +0.037 & -0.064 & +1.000 & -0.103 & -0.013 & +0.045 \\ -0.018 & -0.065 & -0.096 & -0.040 & -0.038 & +0.006 & -0.103 & +1.000 & +0.047 & +0.004 \\ -0.006 & +0.002 & -0.024 & +0.005 & -0.021 & +0.013 & -0.013 & +0.047 & +1.000 & -0.074 \\ -0.012 & +0.012 & -0.013 & -0.027 & +0.019 & -0.020 & +0.045 & +0.004 & -0.074 & +1.000 \end{pmatrix}$$

- Correlation coefficient between $\tilde{F}_{1V}^{Z/\gamma}$ and $\tilde{F}_{2V}^{Z/\gamma}$ is about 0.6
- The others are less than 0.15

Correlation coefficient for F

$$V_C = \begin{pmatrix}
+1.000 & -0.356 & -0.140 & +0.276 & \underline{-0.970} & +0.313 & -0.049 & +0.065 & -0.089 & +0.097 \\
-0.356 & +1.000 & +0.173 & -0.215 & +0.281 & \underline{-0.971} & +0.053 & -0.038 & +0.113 & -0.066 \\
-0.140 & +0.173 & +1.000 & -0.273 & +0.113 & -0.133 & +0.038 & -0.045 & +0.051 & -0.009 \\
+0.276 & -0.215 & -0.273 & +1.000 & -0.233 & +0.188 & -0.055 & +0.037 & -0.033 & +0.051 \\
-0.970 & +0.281 & +0.113 & -0.233 & +1.000 & -0.254 & +0.046 & -0.063 & +0.099 & -0.104 \\
+0.313 & -0.971 & -0.133 & +0.188 & -0.254 & +1.000 & -0.047 & +0.040 & -0.120 & +0.085 \\
-0.049 & +0.053 & +0.038 & -0.055 & +0.046 & -0.047 & +1.000 & -0.287 & +0.036 & -0.036 \\
+0.065 & -0.038 & -0.045 & +0.037 & -0.063 & +0.040 & -0.287 & +1.000 & -0.059 & +0.024 \\
-0.089 & +0.113 & +0.051 & -0.033 & +0.099 & -0.120 & +0.036 & -0.059 & +1.000 & -0.229 \\
+0.097 & -0.066 & -0.009 & +0.051 & -0.104 & +0.085 & -0.036 & +0.024 & -0.229 & +1.000
\end{pmatrix}$$

- Correlation coefficient between $F_{1V}^{Z/\gamma}$ and $F_{2V}^{Z/\gamma}$ is about 0.97
- The others are less than 0.36

Goodness of fit for the MEM

Expectation value of ω when the fit results are assigned should be equal to mean of reconstructed ω distribution

$$\chi^2_{\mathrm{GoF},k}(\delta F_{\mathrm{fit}}) = \frac{\left(\langle\omega_k\rangle - \Omega_k(\delta F_{\mathrm{fit}})\right)^2}{\left(\langle\omega_k^2\rangle - \langle\omega_k\rangle^2\right)/N_{data}}$$

$$\tilde{\chi}^2_{\mathrm{GoF},kl}(\delta F_{\mathrm{fit}}) = \frac{\left(\langle\tilde{\omega}_{kl}\rangle - \tilde{\Omega}_{kl}(\delta F_{\mathrm{fit}})\right)^2}{\left(\langle\tilde{\omega}_{kl}^2\rangle - \langle\tilde{\omega}_{kl}\rangle^2\right)/N_{data}}$$

Some χ^2_{GoF} have large value (6~10).

→ Goodness of fit for the MEM is bad.

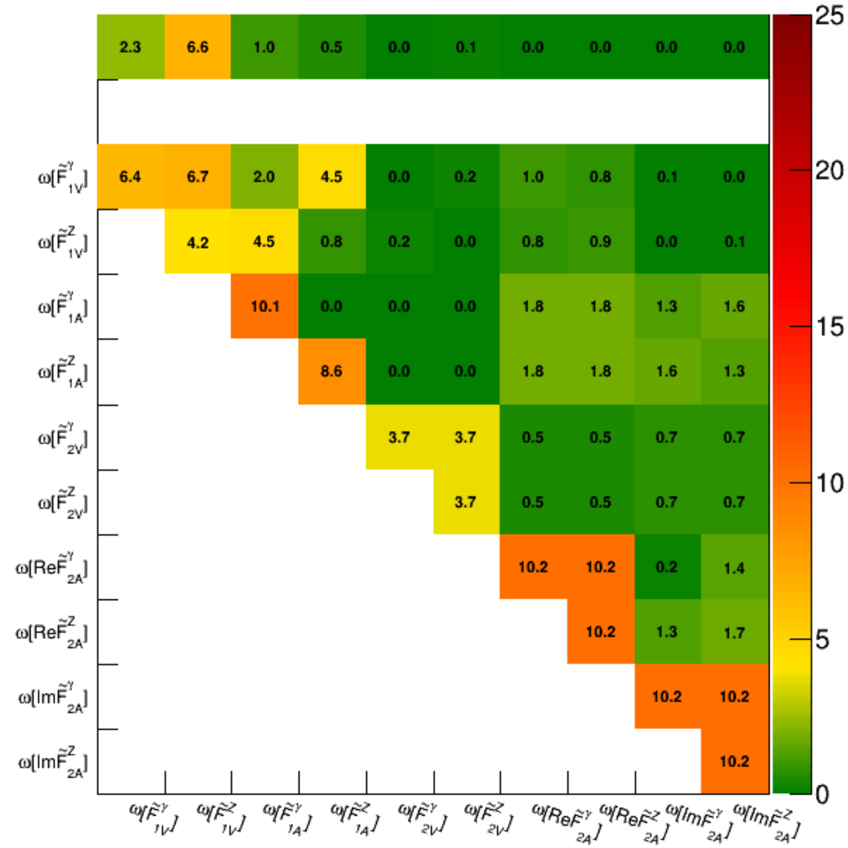

Table of $\chi^2_{\mathrm{GoF}}, \tilde{\chi}^2_{\mathrm{GoF}}$ (Left polarization)

Reweighting (Template-like) Technique

Binned likelihood method : $\chi^2(\delta F) = \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data}} - n_i^{\text{Sim.}}(\delta F)}{\sqrt{n_i^{\text{Data}}}} \right)^2$

$n_i^{\text{Sim.}}(\delta F)$ is obtained from the large full simulation

Reweighting technique :

Produce a sample using SM value, then change the weight of events.

$$\begin{aligned} n_i^{\text{Sim.}}(\delta F) &= n_i^{\text{Sim.,sig}}(\delta F) + n_i^{\text{Sim.,bkg}} \\ &= n_i^{\text{Sim.,sig}}(0)(1\langle\omega\rangle_i \delta F + \langle\widetilde{\omega}\rangle_i \delta F^2) + n_i^{\text{Sim.,bkg}} \\ &\simeq n_i^{\text{Sim.,sig}}(0)(1 + \langle\omega\rangle_i \delta F) + n_i^{\text{Sim.,bkg}} \end{aligned}$$

Template technique : Produce many samples using different parameters

Overestimate of goodness of fit

We don't have enough statistics for the background events for now.

$$\chi^2(\delta F) = \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data}} - n_i^{\text{Sim.}}(\delta F)}{\sqrt{n_i^{\text{Data}}}} \right)^2 \rightarrow \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data,Sig.}} - n_i^{\text{Sim.,Sig.}}(\delta F)}{\sqrt{n_i^{\text{Data}}}} \right)^2$$

When $n_i^{\text{Data,Sig.}} = \alpha n_i^{\text{Data}}$ $(\alpha < 1)$

$$\sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data,Sig.}} - n_i^{\text{Sim.,Sig.}}(\delta F)}{\sqrt{n_i^{\text{Data}}}} \right)^2 = \alpha \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data,Sig.}} - n_i^{\text{Sim.,Sig.}}(\delta F)}{\sqrt{n_i^{\text{Data,Sig}}}} \right)^2$$

$$\equiv \alpha \chi^2_{\text{Sig}}$$

$\min\left[\chi^2_{\text{Sig}}\right]$ obeys chi-square distribution of $n.d.f. = N_{bin} - N_{\text{para}}$

→ $\chi^2(\delta F)$ may be $1/\alpha$ times larger if backgrounds are included in $\chi^2(\delta F)$