기초과학연구원
Institute for Basic Science

Radiative corrections to triple Higgs coupling and electroweak phase transition: 2-loop analysis

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work in progress

Outline

- Introduction
- Triple Higgs coupling (hhh) at 2-loop level
- Implications for electroweak phase transition (EWPT)
- Summary

Introduction

λ_{hhh}-EWPT correlation

- Triple Higgs coupling ($\lambda_{\rm hhh}$) can be modified if EWPT is strong 1st order.

[S.Kanemura, Y.Okada, E.S., PLB606 (2005) 361;
C.Grojean, G.Servant, J.Wells, PRD71 (2005) 036001]

- O(1) quartic couplings in Higgs potential play an essential role.

Question

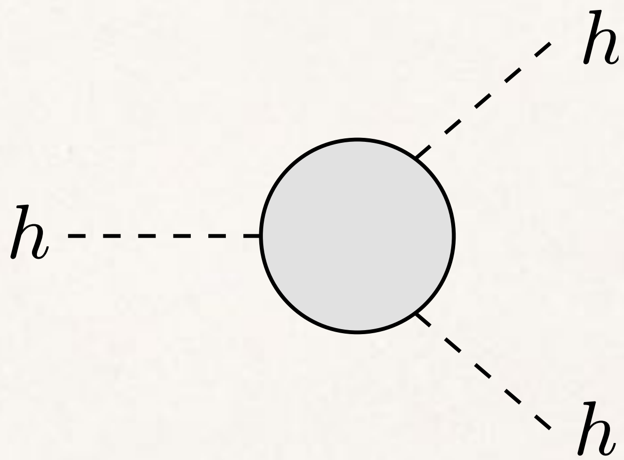
How large is the 2-loop effect on (1) $\lambda_{\rm hhh}$ and (2) 1st-order EWPT?

As an example, we consider Inert Doublet Model (IDM).

hhh coupling

Effective hhh vertex

We will evaluate effective hhh vertex



$$= \hat{\Gamma}_{hhh}(p_1^2, p_2^2, p_3^2)$$

Even though $\hat{\Gamma}_{hhh}(p_1^2, p_2^2, p_3^2) \neq \hat{\Gamma}_{hhh}(0,0,0)$, the ratio can be approximate by

$$\kappa_\lambda \equiv \frac{\hat{\Gamma}^{\rm IDM}_{hhh}(p_1^2, p_2^2, p_3^2)}{\hat{\Gamma}^{\rm SM}_{hhh}(p_1^2, p_2^2, p_3^2)} \simeq \frac{\hat{\Gamma}^{\rm IDM}_{hhh}(0,0,0)}{\hat{\Gamma}^{\rm SM}_{hhh}(0,0,0)}$$

*At 1-loop, this is the good approximation ($\lesssim$ 1% err. in regions of our interest). [S. Kanemura, Y. Okada, E.S. C.-P. Yuan, PRD70,115002(04)]

Effective potential method

$$-\hat{\Gamma}_{hhh}(0,0,0) \equiv \hat{\lambda}_{hhh} = \hat{Z}_h^{3/2} \lambda_{hhh},$$

on-shell MS-bar

where

$$\hat{Z}_h = \frac{Z_h^{\rm OS}}{Z_h^{\overline{\rm MS}}} \qquad \lambda_{hhh} = \left.\frac{\partial^3 V_{\rm eff}}{\partial \varphi^3}\right|_{\varphi = v}$$

V_{eff} is the MS-bar regularized effective potential, which is expanded as

$$V_{\rm eff}(\varphi) = V_0(\varphi) + V_1(\varphi) + V_2(\varphi)$$

We first consider the SM.

λ_{hhh} in the SM

$$V_0(\Phi) = -\mu_\Phi^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger\Phi)^2, \quad \Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix},$$

$$m_h^2 = \left.\frac{\partial^2 V_{\rm eff}}{\partial \varphi^2}\right|_{\varphi=v} = 2\lambda v^2 + \mathcal{D}_m \Delta V_{\rm eff}(\varphi),$$

$$\lambda_{hhh}^{\rm SM} = \left.\frac{\partial^3 V_{\rm eff}}{\partial \varphi^3}\right|_{\varphi=v} = \frac{3m_h^2}{v} + \mathcal{D}_\lambda \Delta V_{\rm eff}(\varphi),$$

where

$$\mathcal{D}_m = \left[\frac{\partial^2}{\partial\varphi^2} - \frac{1}{v}\frac{\partial}{\partial\varphi}\right]_{\varphi=v}, \quad \mathcal{D}_\lambda = \left[\frac{\partial^3}{\partial\varphi^3} - \frac{3}{v}\left(\frac{\partial^2}{\partial\varphi^2} - \frac{1}{v}\frac{\partial}{\partial\varphi}\right)\right]_{\varphi=v},$$

$$\Delta V_{\rm eff}(\varphi) = V_1(\varphi) + V_2(\varphi)$$

NOTE: Higgs pole mass (M_h)

$$M_h^2 = m_h^2 + {\rm Re}\Sigma_h(M_h) - {\rm Re}\Sigma_h(0).$$

At 1-loop

MS-bar regularized 1-loop effective potential

$$V_1(\varphi) = \sum_i c_i \frac{\bar{m}_i^4}{4(16\pi^2)} \left(\ln \frac{\bar{m}_i^2}{\bar{\mu}^2} - c_i \right)$$

$\bar{m}_i$: field-dependent masses of particle i

$\bar{\mu}$: renormalization scale

c=3/2 (scalars, fermions)

c=5/6 (gauge bosons)

Dominant 1-loop correction comes from the top loop.

[W. Hollik and S. Penaranda, EPJC23,163(2002)]

$$\lambda_{hhh}^{\rm SM} = \frac{3m_h^2}{v}\left[1 + \frac{1}{16\pi^2}\left(-\frac{16m_t^4}{m_h^2 v^2}\right)\right]$$

Power correction!! (*log corr. are absorbed into m_h.)

New physics effects can also be power like (see later).

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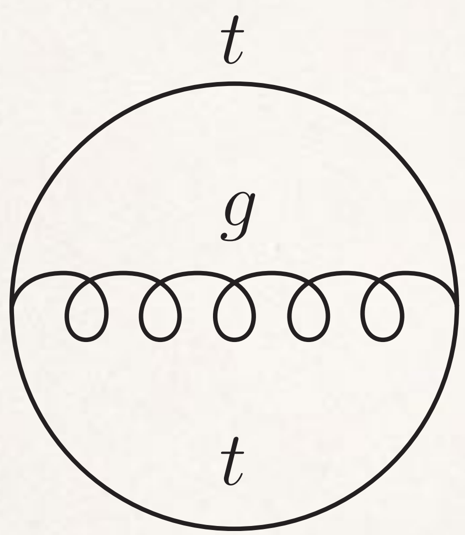
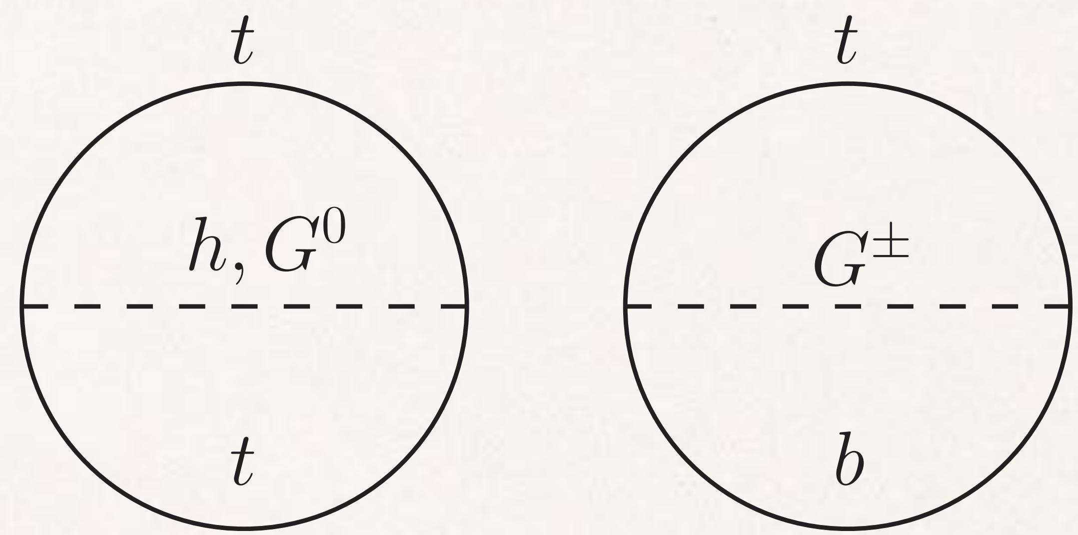
At 2-loop

Dominant diagrams



$\mathcal{O}(g_3^2 y_t^4)$



$\mathcal{O}(y_t^6)$

$$\lambda_{hhh}^{\rm SM} = \frac{3m_h^2}{v}\left[1 + \frac{1}{16\pi^2}\left(-\frac{16m_t^4}{m_h^2 v^2}\right) + \frac{1}{(16\pi^2)^2}\left\{\frac{256 g_3^2 m_t^4}{m_h^2 v^2}\left(\ell_t + \frac{1}{6}\right) - \frac{48 y_t^2 m_t^4}{m_h^2 v^2}\left(\ell_t - \frac{7}{6}\right)\right\}\right],$$

where $\ell_t = \ln \frac{m_t^2}{\bar{\mu}^2}$ (*All parameters are MS-bar variables.)

At 2-loop

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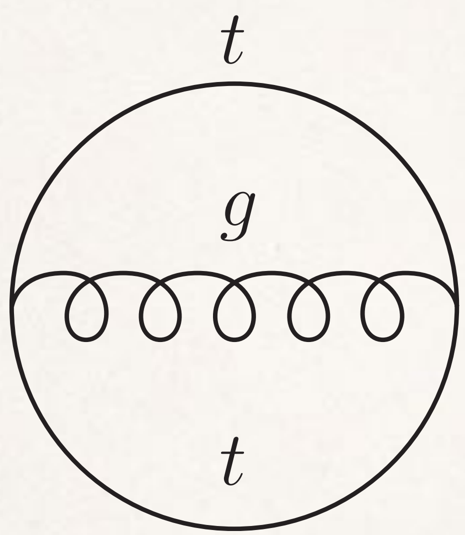

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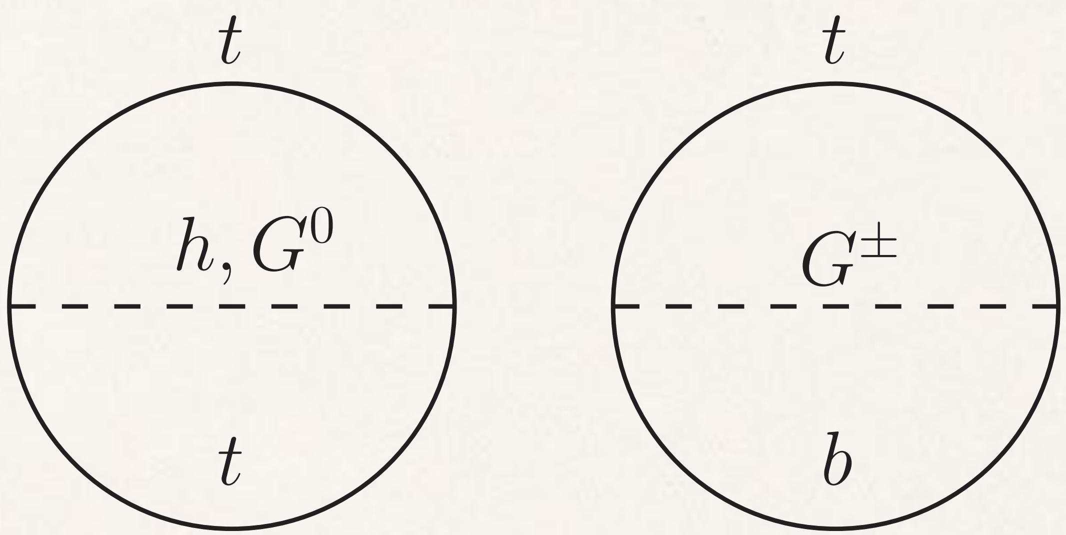

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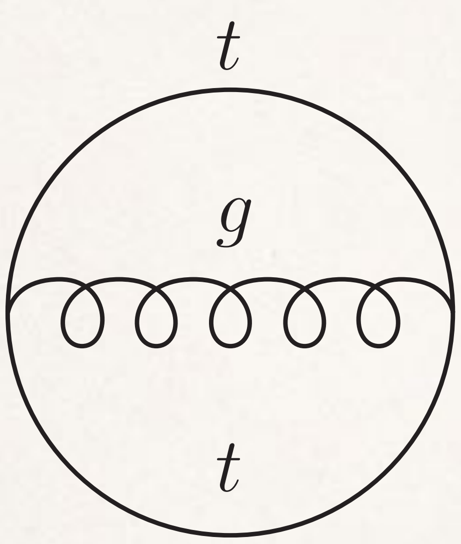

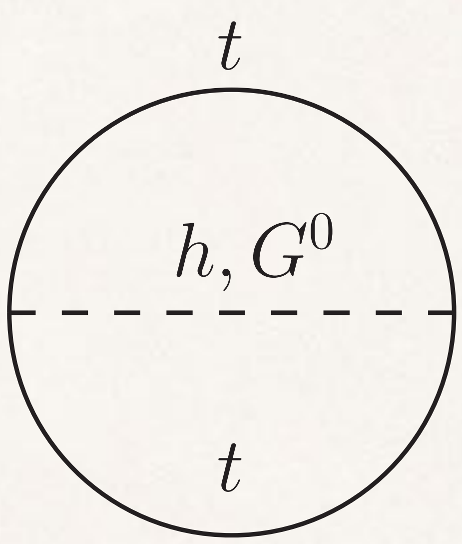

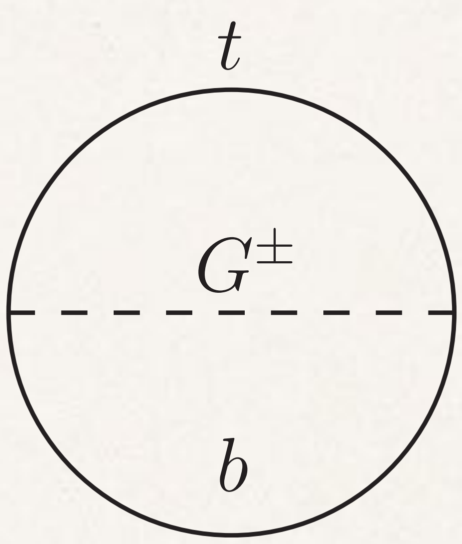

$\mathcal{O}(g_3^2 y_t^4)$ $\mathcal{O}(y_t^6)$

$$\lambda_{hhh}^{\rm SM} = \frac{3m_h^2}{v}\left[1 + \frac{1}{16\pi^2}\left(-\frac{16m_t^4(m_t)}{m_h^2 v^2}\right) + \frac{1}{(16\pi^2)^2}\frac{m_t^4}{m_h^2 v^2}\left(\frac{128 g_3^2}{3} + 56 y_t^2\right)\right],$$

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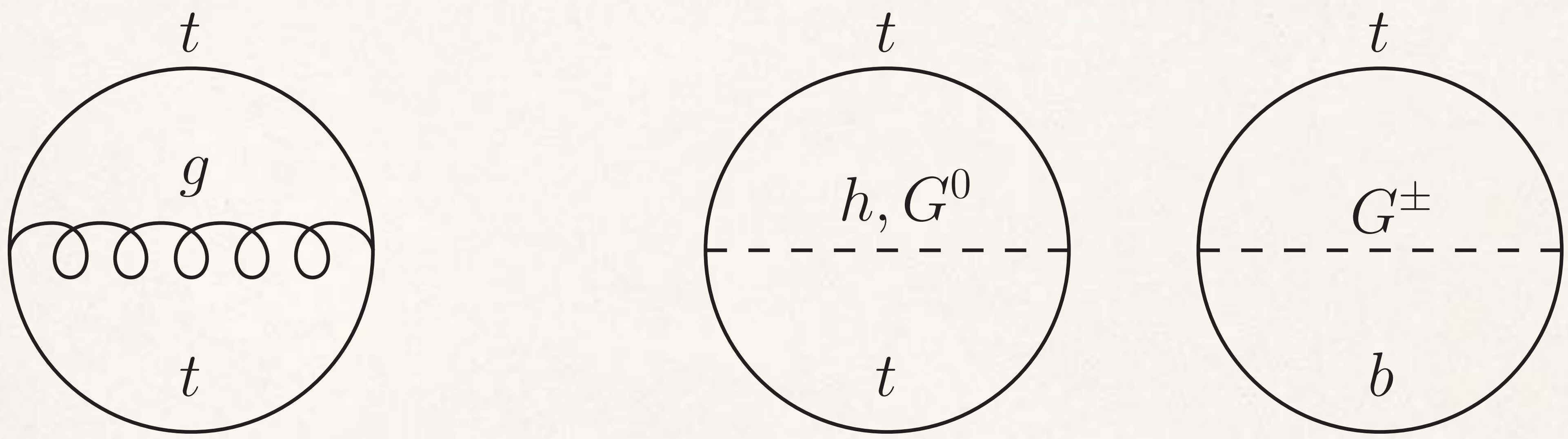

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log terms are absorbed by running top mass.

At 2-loop

After expressing the MS-bar parameters with OS ones, one gets

$$\hat{\lambda}_{hhh}^{\rm SM} \simeq \frac{3M_h^2}{v_{\rm phys}}\left[1 + \frac{1}{16\pi^2}\left(-\frac{16M_t^4}{M_h^2 v_{\rm phys}^2} + \frac{7}{2}\frac{M_t^2}{v_{\rm phys}^2}\right) + \frac{1}{(16\pi^2)^2}\frac{16M_t^4}{M_h^2 v_{\rm phys}^2}\left(24g_3^2 + \frac{7M_t^2}{v_{\rm phys}^2}\right)\right]$$

$$= (190.4~\text{GeV}) \times \left[1 - 8.5\% + 2.2\%\right] = 178.4~\text{GeV},$$

$$v_{\rm phys}^2 = 1/(\sqrt{2}G_F) = (246.22~\text{GeV})^2$$

2-loop contribution is about 1/4 of 1-loop top effect.

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Inert Doublet Model (IDM)

particle content

SM + Z_2-odd doublet η

tree-level potential w/ Z_2 $\Phi \to \Phi$ and $\eta \to -\eta$

$$V_0(\Phi,\eta) = \mu_1^2 \Phi^\dagger \Phi + \mu_2^2 \eta^\dagger \eta + \frac{\lambda_1}{2}(\Phi^\dagger\Phi)^2 + \frac{\lambda_2}{2}(\eta^\dagger\eta)^2 + \lambda_3(\Phi^\dagger\Phi)(\eta^\dagger\eta) + \lambda_4(\Phi^\dagger\eta)(\eta^\dagger\Phi) + \left[\frac{\lambda_5}{2}(\Phi^\dagger\eta)^2 + \mathrm{h.c}\right]$$

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+iG^0) \end{pmatrix}, \quad \eta = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H+iA) \end{pmatrix}.$$

H or A (lightest Z_2 odd particle) can be a dark matter (DM).

Higgs masses

$$m_h^2 = \lambda_1 v^2,$$

$$m_H^2 = \mu_2^2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5) v^2,$$

$$m_A^2 = \mu_2^2 + \frac{1}{2}(\lambda_3 + \lambda_4 - \lambda_5) v^2,$$

$$m_{H^\pm}^2 = \mu_2^2 + \frac{1}{2}\lambda_3 v^2.$$

- Heavy Higgs masses come from both μ_2^2 and symmetry breaking term (λv^2).

- We may know which part is dominant by measuring Higgs couplings precisely.

Model parameters

Original parameters:	$\mu_1^2,\ \mu_2^2,\ \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5.$
Converted parameters:	$v,\ \mu_2^2,\ \lambda_2,\ m_h,\ m_H,\ m_A,\ m_{H^\pm},$

At tree level

$$\lambda_1 = \frac{m_h^2}{v^2}, \quad \lambda_3 = \frac{2}{v^2}(m_{H^\pm}^2 - \mu_2^2),$$
$$\lambda_4 = \frac{1}{v^2}(m_H^2 + m_A^2 - 2m_{H^\pm}^2), \quad \lambda_5 = \frac{1}{v^2}(m_H^2 - m_A^2).$$

In our study, H is DM, and $M_H \simeq M_h/2$ is taken.

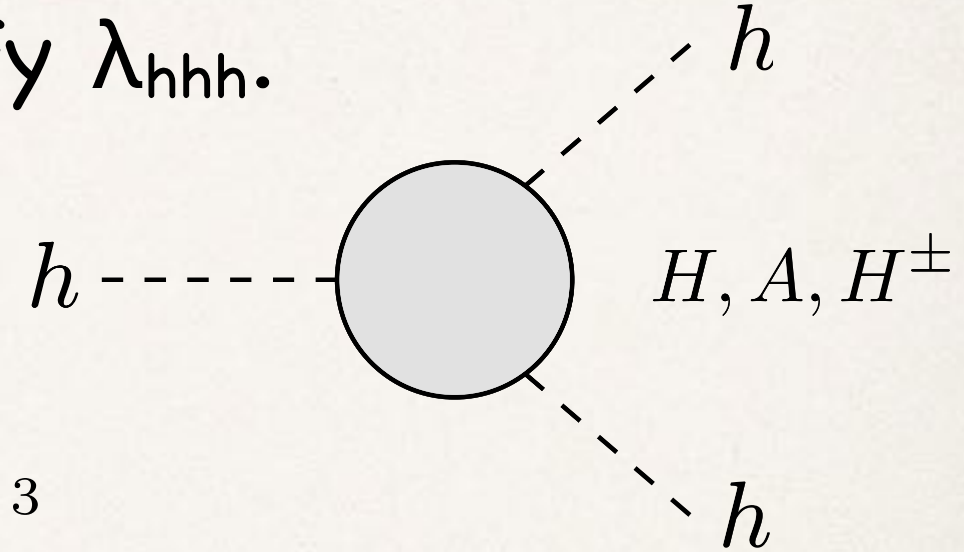
DM physics point of view, $\mu_2^2 \longrightarrow \bar{\lambda}_{hHH} \equiv \lambda_3 + \lambda_4 + \lambda_5$

Also, we take $M_A = M_{H+}$ to avoid ρ-parameter constraint.

At 1-loop

[S.Kanemura, M. Kikuchi, K. Sakurai, PRD94,115011(2016)]

- Extra Higgs boson loops can modify λ_{hhh}.



$$\lambda_{hhh}^{\rm IDM} = \frac{3m_h^2}{v} + \Delta^{(1)}\lambda_{hhh}^{\rm IDM},$$

$$\Delta^{(1)}\lambda_{hhh}^{\rm IDM} = \sum_{\phi=H,A,H^\pm} n_\phi \frac{4m_\phi^4}{16\pi^2 v^3}\left(1-\frac{\mu_2^2}{m_\phi^2}\right)^3$$

$$m_\phi^2 = \mu_2^2 + \frac{1}{2}\bar{\lambda}_{h\phi\phi}v^2$$

$$n_H = n_A = 1, \; n_{H^\pm} = 2$$

$$= \begin{cases} \sum_\phi n_\phi \frac{4m_\phi^4}{16\pi^2 v^3} & \text{for} \quad \mu_2^2 \ll \frac{1}{2}\bar{\lambda}_{h\phi\phi}v^2, \\ \sum_\phi n_\phi \frac{4}{16\pi^2 v^3}\frac{(\bar{\lambda}_{h\phi\phi}v^2/2)^3}{m_\phi^2} & \text{for} \quad \mu_2^2 \gg \frac{1}{2}\bar{\lambda}_{h\phi\phi}v^2. \end{cases}$$

$\mu_2^2 \ll \bar{\lambda}_{h\phi\phi}v^2/2$ is necessary for 1st-order EWPT (see later).

* H is DM, so A and H+ are the dominant contributors.

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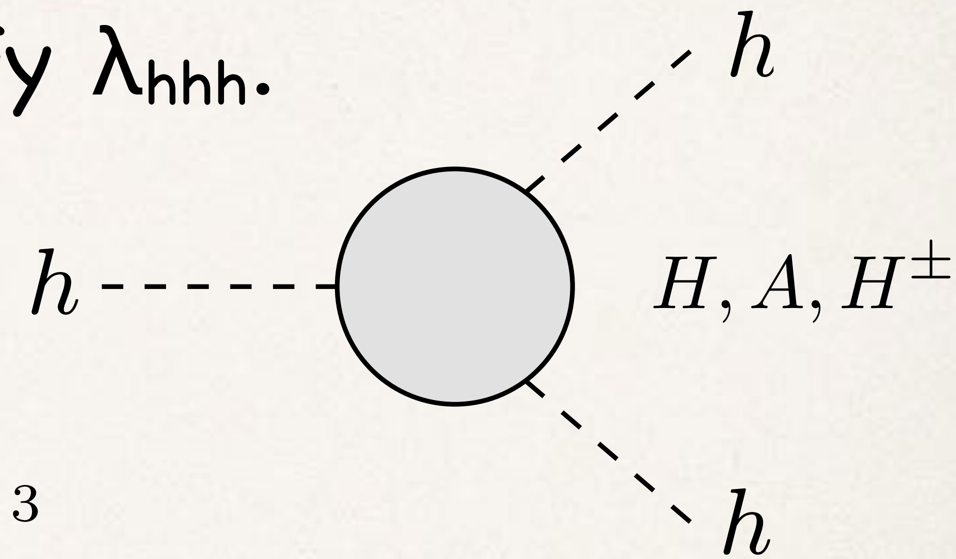

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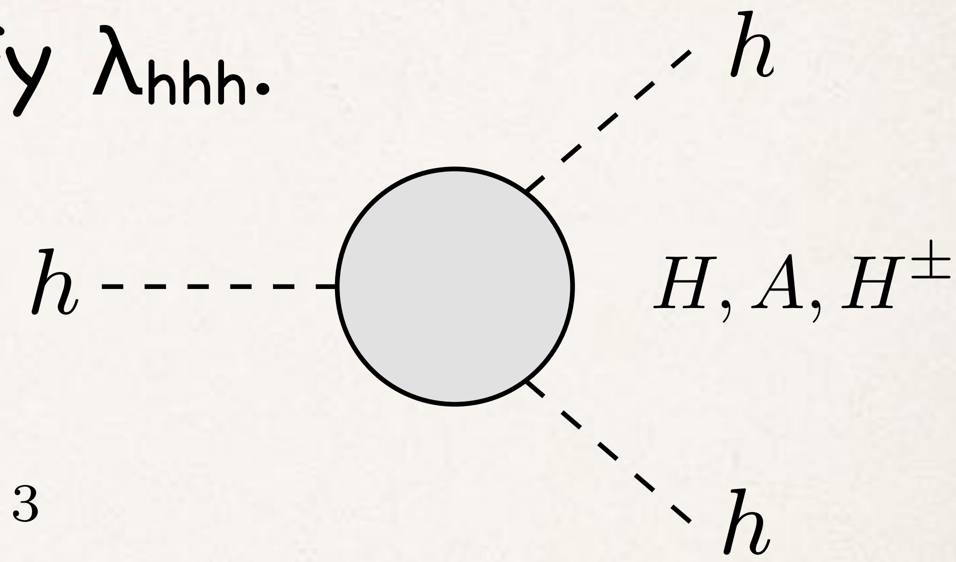

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non-decoupling

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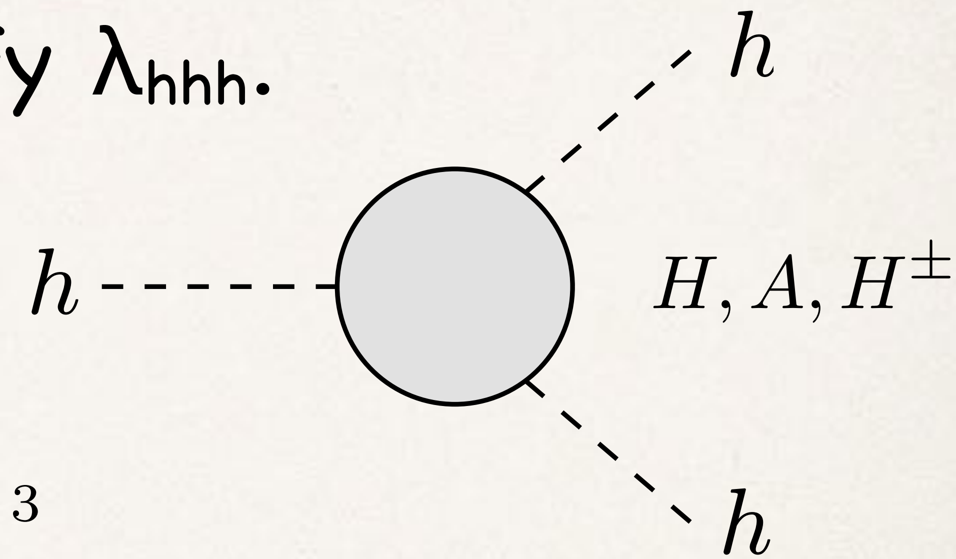

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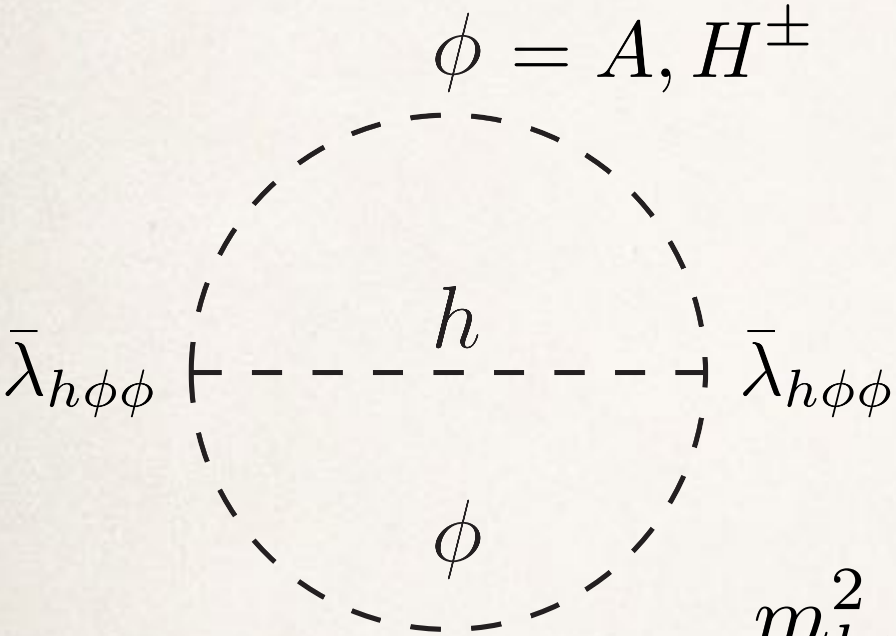
At 2-loop

Dominant 2-loop diagrams



$$\simeq \sum_{\phi=A,H^\pm} \frac{8n_\phi \bar{\lambda}^2_{h\phi\phi}}{(16\pi^2)^2} \frac{m_\phi^2}{v} \left(\ln \frac{m_\phi^2}{\bar{\mu}^2} - \frac{1}{2} \right) + \cdots$$

$$m_h^2, \mu_2^2 \ll m_A^2, m_{H^\pm}^2$$

Combining with 1-loop contributions,

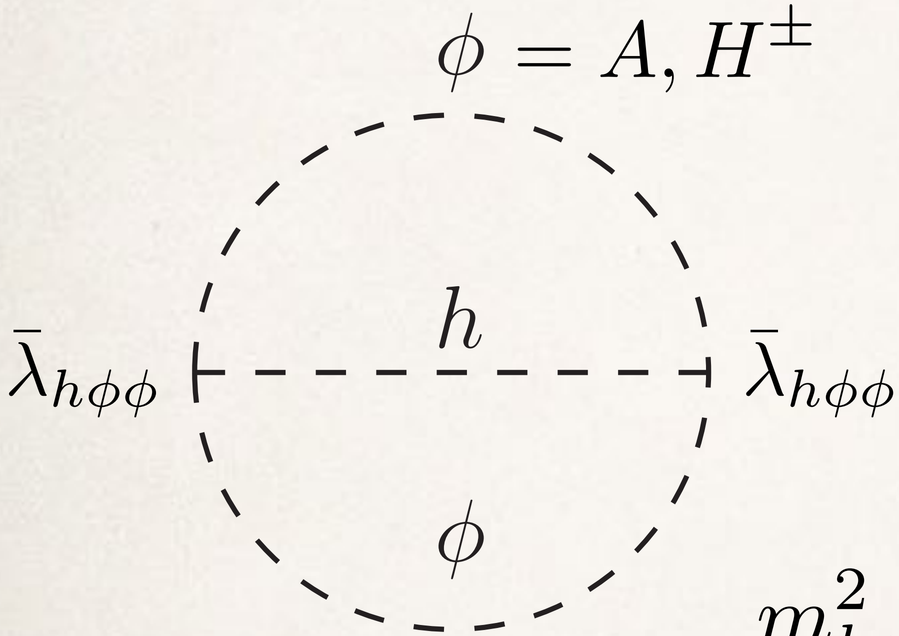
$$\Delta^{(1)} \lambda_{hhh}^{\rm IDM} + \Delta^{(2)} \lambda_{hhh}^{\rm IDM}$$

$$\simeq \sum_{\phi=A,H^\pm} \frac{4n_\phi}{16\pi^2 v^3} \left[m_\phi^4(m_\phi) - \frac{4m_\phi^6}{16\pi^2 v^2} \right] + \cdots .$$

Leading 2-loop effect comes through RG running of m_ϕ.

At 2-loop

Dominant 2-loop diagrams



$$\simeq \sum_{\phi=A,H^{\pm}} \overset{\displaystyle 4m_\phi^4/v^4 \atop \approx}{\frac{8n_\phi \bar{\lambda}_{h\phi\phi}^2}{(16\pi^2)^2}} \frac{m_\phi^2}{v} \left(\ln \frac{m_\phi^2}{\bar{\mu}^2} - \frac{1}{2} \right) + \cdots$$

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Leading 2-loop effect comes through RG running of m_ϕ.

Numerical results

$$M_H = 62.7\ \text{GeV},\ \lambda_2 = 0.02\ \text{and}\ \bar{\lambda}_{hHH} = 4.6 \times 10^{-3}\ \text{at}\ M_Z.$$

$$M_A = M_{H^\pm}$$

- Both 1 and 2-loops corrs. grow w/ increasing M_A.

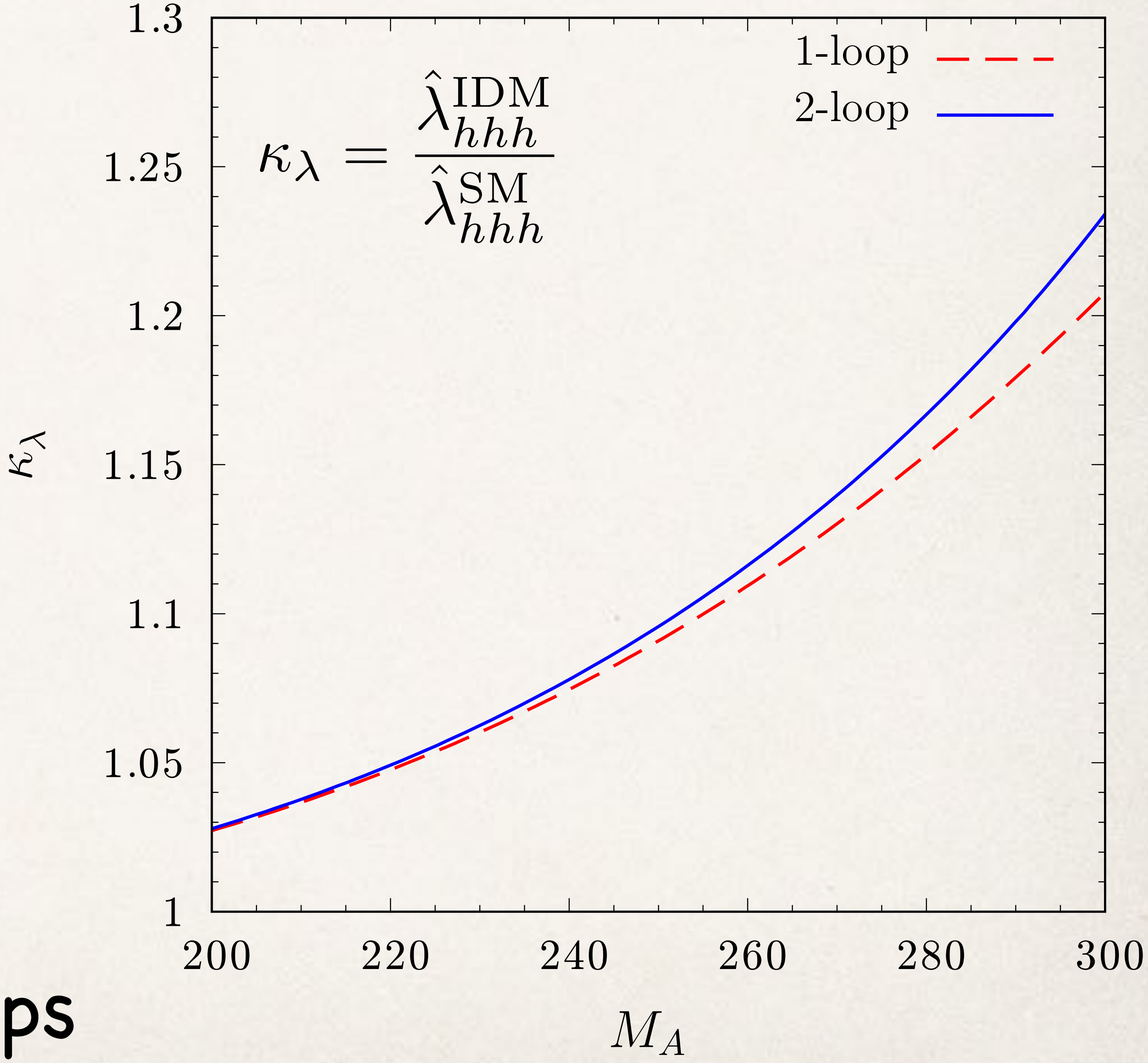
  1 loop corr. is consistent with HCOUP [Kanemura, Kikuchi, Sakurai, Yagyu, 1710.04603].

- M_A has the upper bound.

$$M_H = \mu_2^2 + \frac{\bar{\lambda}_{hHH}}{2} v^2 + \Sigma_H(M_H)$$

$\Sigma_H(M_H) > 0$

- Difference btw 1 and 2-loops is less than about 2%.



EW Phase transition

EW baryogenesis (EWBG)

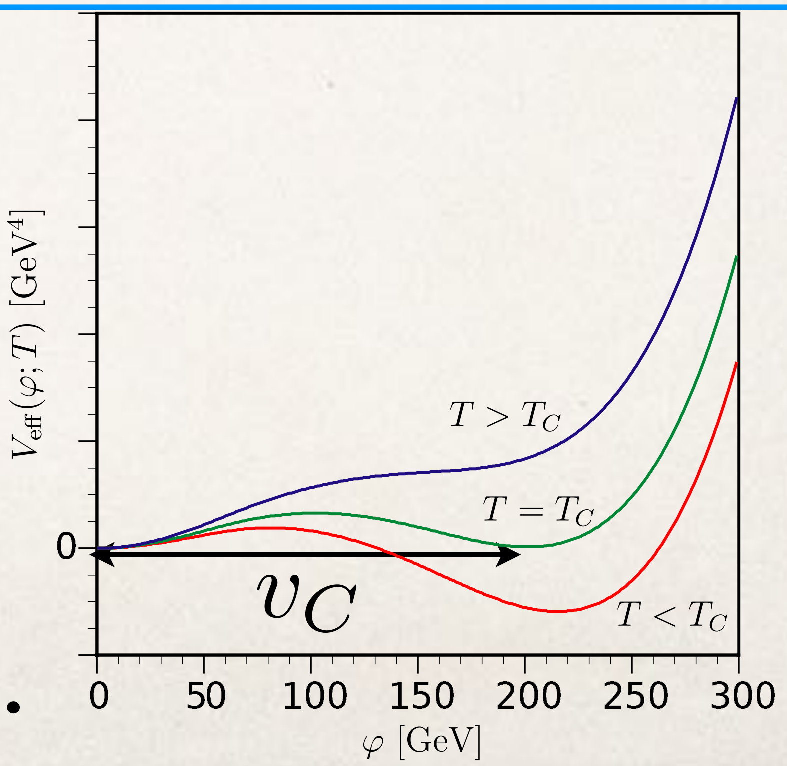
Sakharov's conditions [Kuzmin, Rubakov, Shaposhnikov, PLB155,36 ('85)]

- B violation: anomalous (sphaleron) process
- C violation: chiral gauge interaction
- CP violation: KM phase and/or other sources in beyond the SM
- Out of equilibrium: 1st-order EW phase transition (EWPT) with expanding bubble walls

Baryon number preservation criterion

$$\frac{v_C}{T_C} \gtrsim 1$$

T_C: critical temperature; V_C: VEV at T_C.



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Related to Higgs physics

- Out of equilibrium: 1st-order EW phase transition (EWPT) with expanding bubble walls

Baryon number preservation criterion

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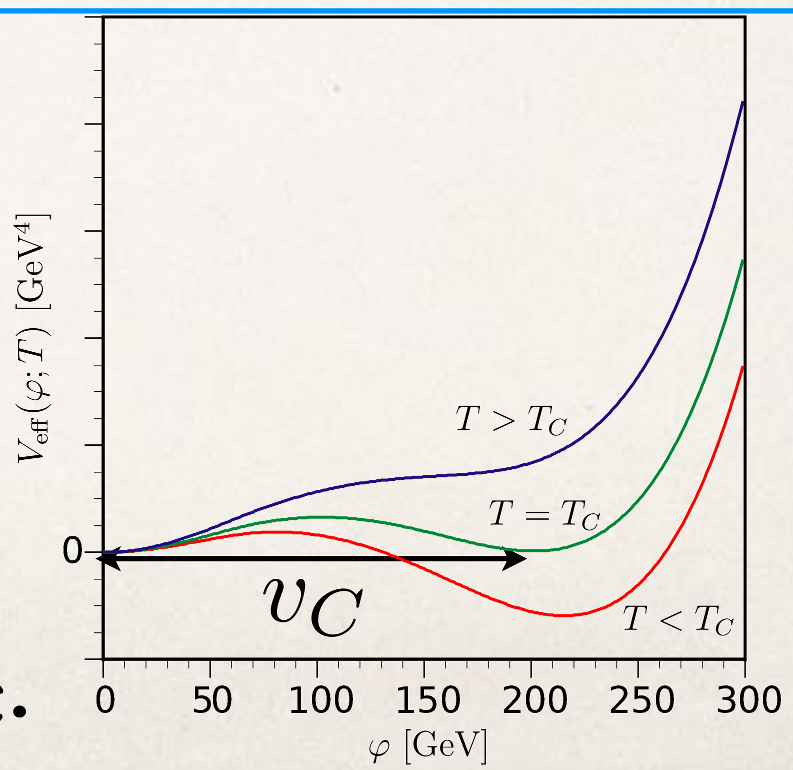
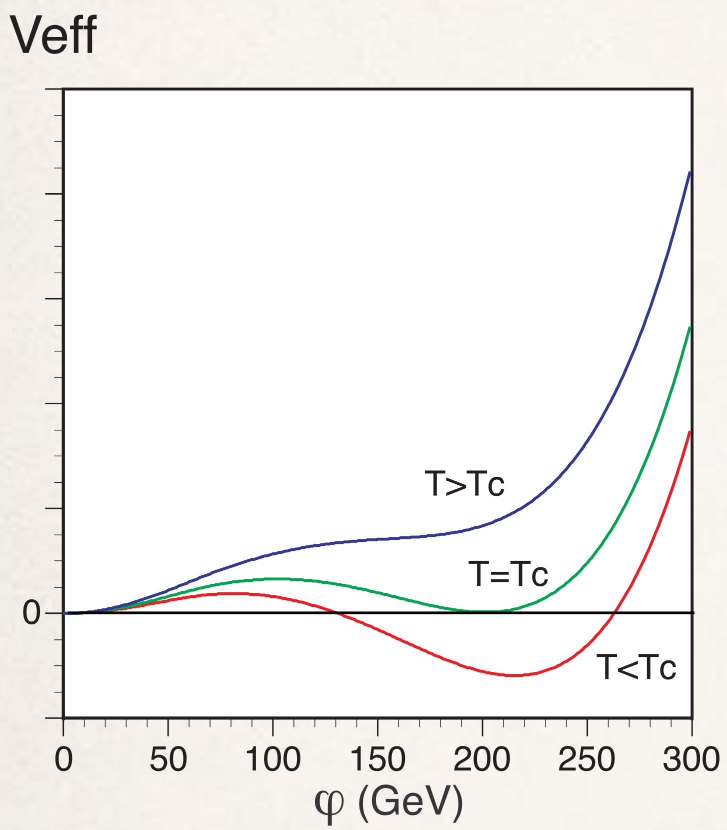


T_C: critical temperature; V_C: VEV at T_C.

1^{st}-order phase transition

$$V_{\rm eff} \simeq D(T^2 - T_0^2)\varphi^2 - ET\varphi^3 + \frac{\lambda_T}{4}\varphi^4 \underset{T=T_C}{\rightarrow} \frac{\lambda_{T_C}}{4}\varphi^2(\varphi - v_C)^2$$



$$v_C = \frac{2ET_C}{\lambda_{T_C}} \quad \Rightarrow \quad \frac{v_C}{T_C} = \frac{2E}{\lambda_{T_C}}$$

Extra Higgs loops can enhance E.

$$\bar{m}_\phi^2 = \mu_2^2 + \frac{1}{2}\bar{\lambda}_{h\phi\phi}\varphi^2$$

At 1-loop

$$V_{\rm eff} \ni \begin{cases} -(\bar{\lambda}_{h\phi\phi}/2)^{3/2} T \left(1 + \frac{\mu_2^2}{\bar{\lambda}_{h\phi\phi}\varphi^2/2}\right)^{3/2} \varphi^3 & \text{for} \quad \mu_2^2 \ll \bar{\lambda}_{h\phi\phi}\varphi^2/2, \\ -(\mu_2^2)^{3/2} T \left(1 + \frac{\bar{\lambda}_{h\phi\phi}\varphi^2}{\mu_2^2}\right)^{3/2} & \text{for} \quad \mu_2^2 \gg \bar{\lambda}_{h\phi\phi}\varphi^2/2. \end{cases}$$

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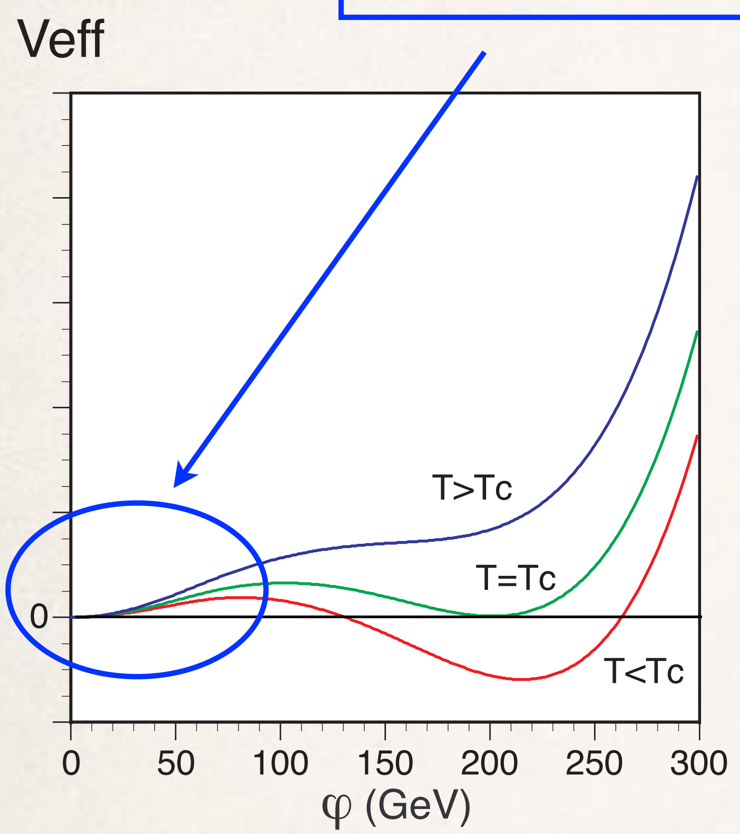

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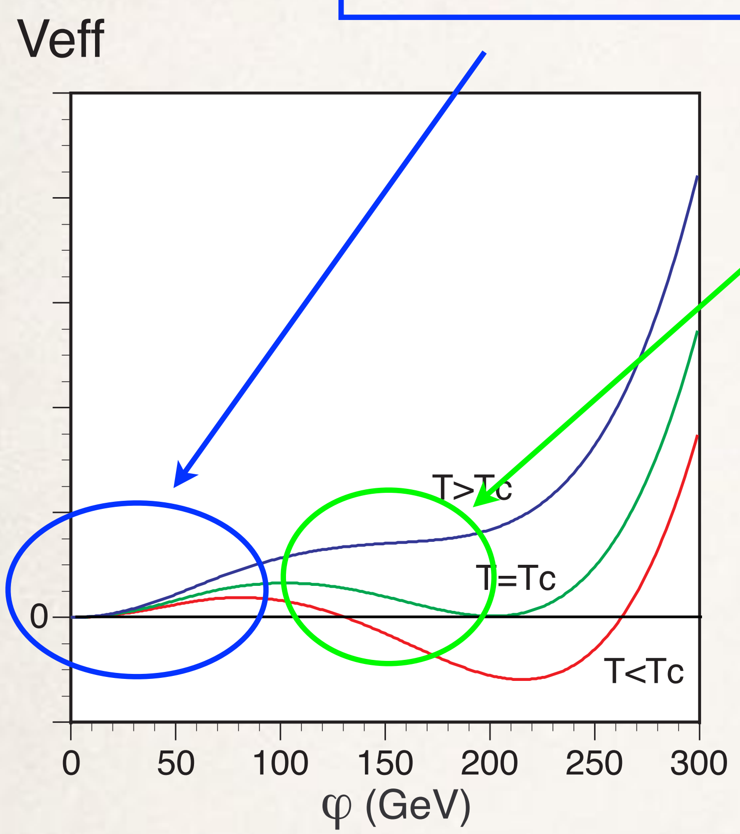

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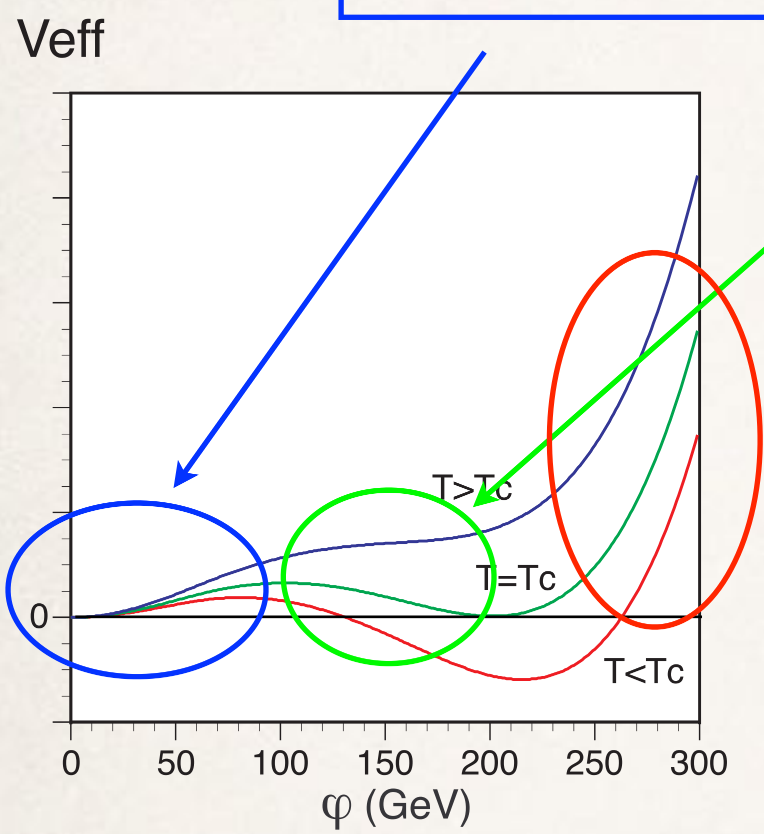

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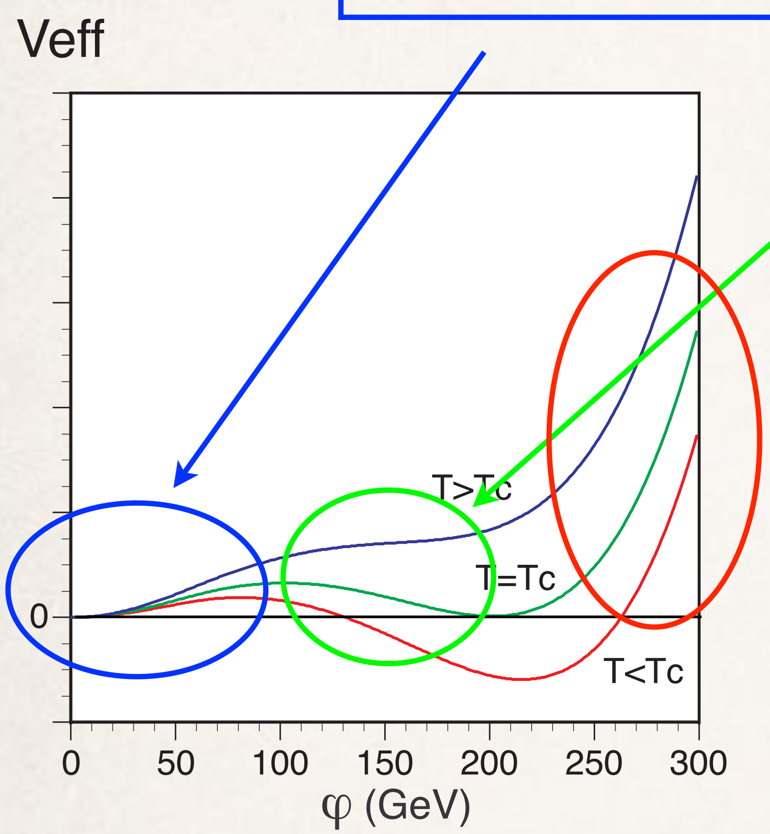

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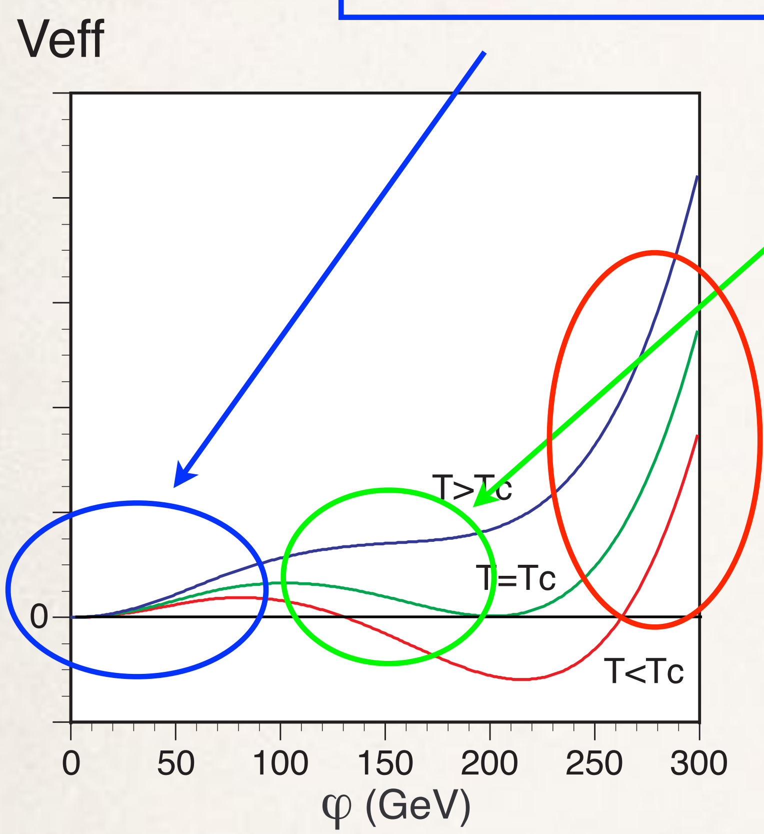

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At 1-loop

non-decoupling

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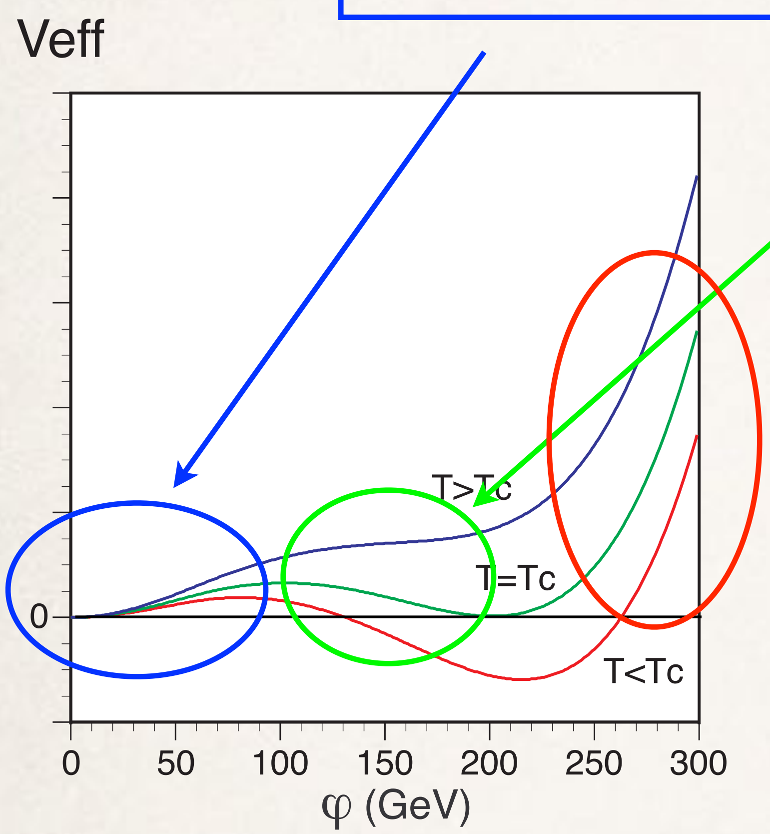

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decoupling

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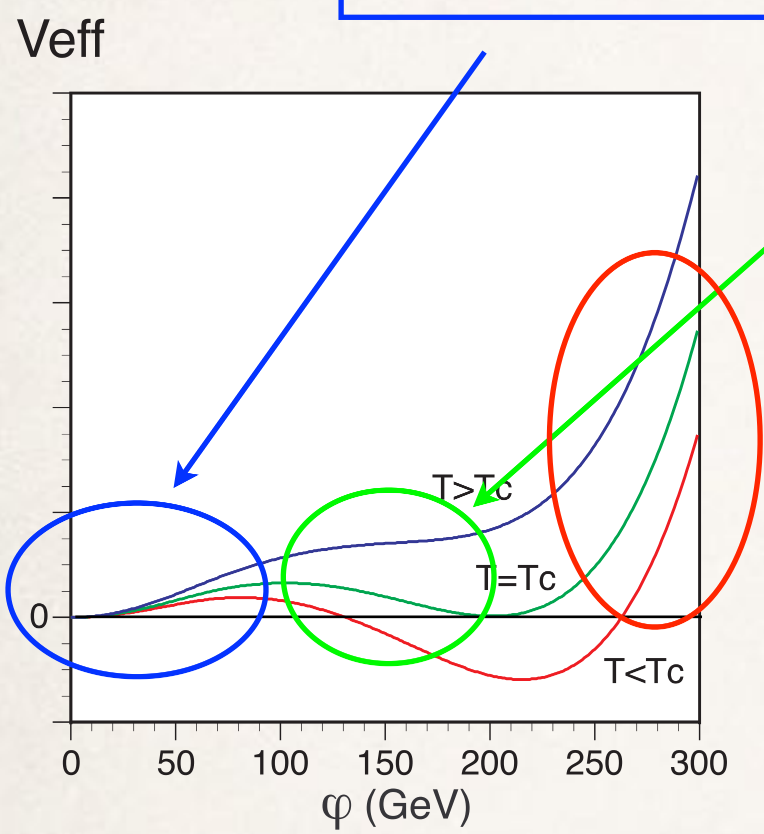

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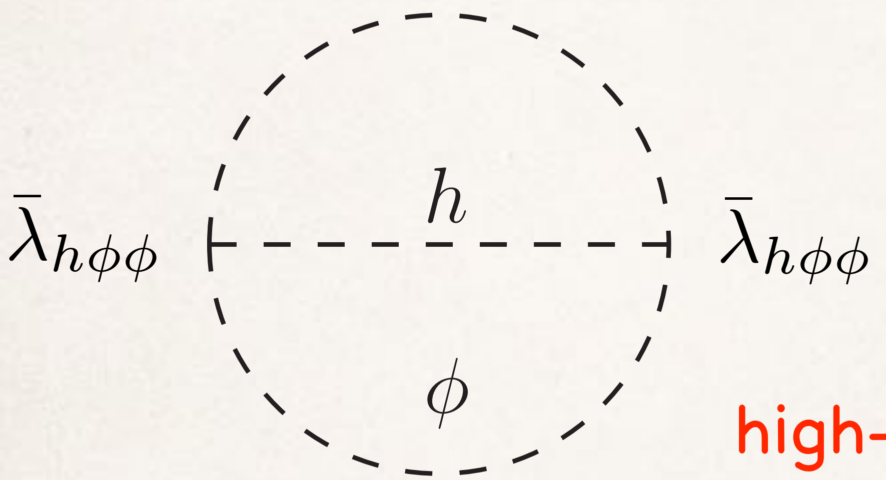
decoupling

Non-decoupling heavy Higgs bosons play a central role in enhancing E.

At 2-loop

Dominant 2-loop diagrams

$\phi = A, H^{\pm}$



$$\simeq \sum_{\phi} n_{\phi} \frac{T^2 \bar{\lambda}^2_{h\phi\phi} \varphi^2}{128\pi^2} \ln \frac{\bar{m}^2_{\phi}}{T^2},$$

high-T expansion

$$m_h^2 \ll m_A^2, m_{H^\pm}^2 \lesssim T^2.$$

Known fact: [J.E.Bagnasco and M.Dine, PLB303,308(1993)]

$$+\varphi^2 \ln\left(\frac{\bar{m}^2}{T^2}\right) \longrightarrow$$ 1st-order EWPT is weakened.

$$-\varphi^2 \ln\left(\frac{\bar{m}^2}{T^2}\right) \longrightarrow$$ 1st-order EWPT is strengthened.

At 2-loop, v_C/T_C would be weakened by extra Higgs bosons.

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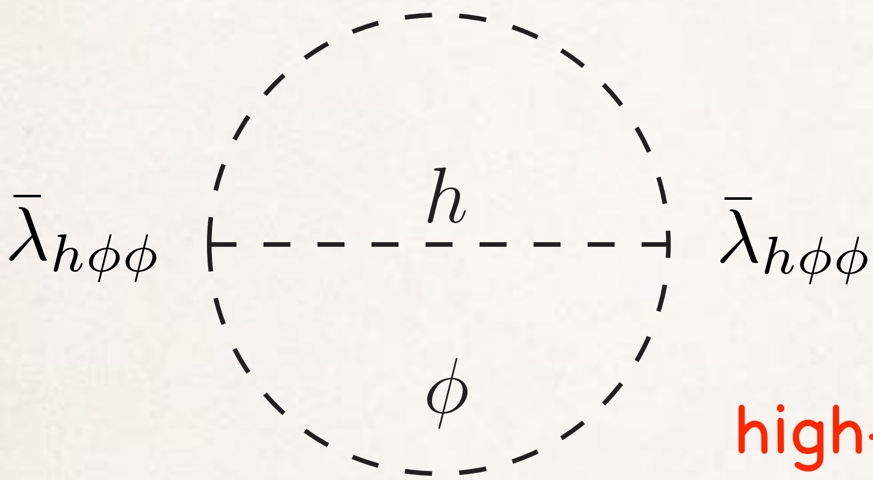

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$\bar{m}^2_{\phi} \simeq \bar{\lambda}_{h\phi\phi}\varphi^2/2$

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At 2-loop, v_C/T_C would be weakened by extra Higgs bosons.

v_C/T_C

$$M_H = 62.7 \text{ GeV}, \ \lambda_2 = 0.02 \text{ and } \bar{\lambda}_{hHH} = 4.6 \times 10^{-3} \text{ at } M_Z.$$

$$M_A = M_{H^\pm} \qquad \bar{\mu} = M_A$$

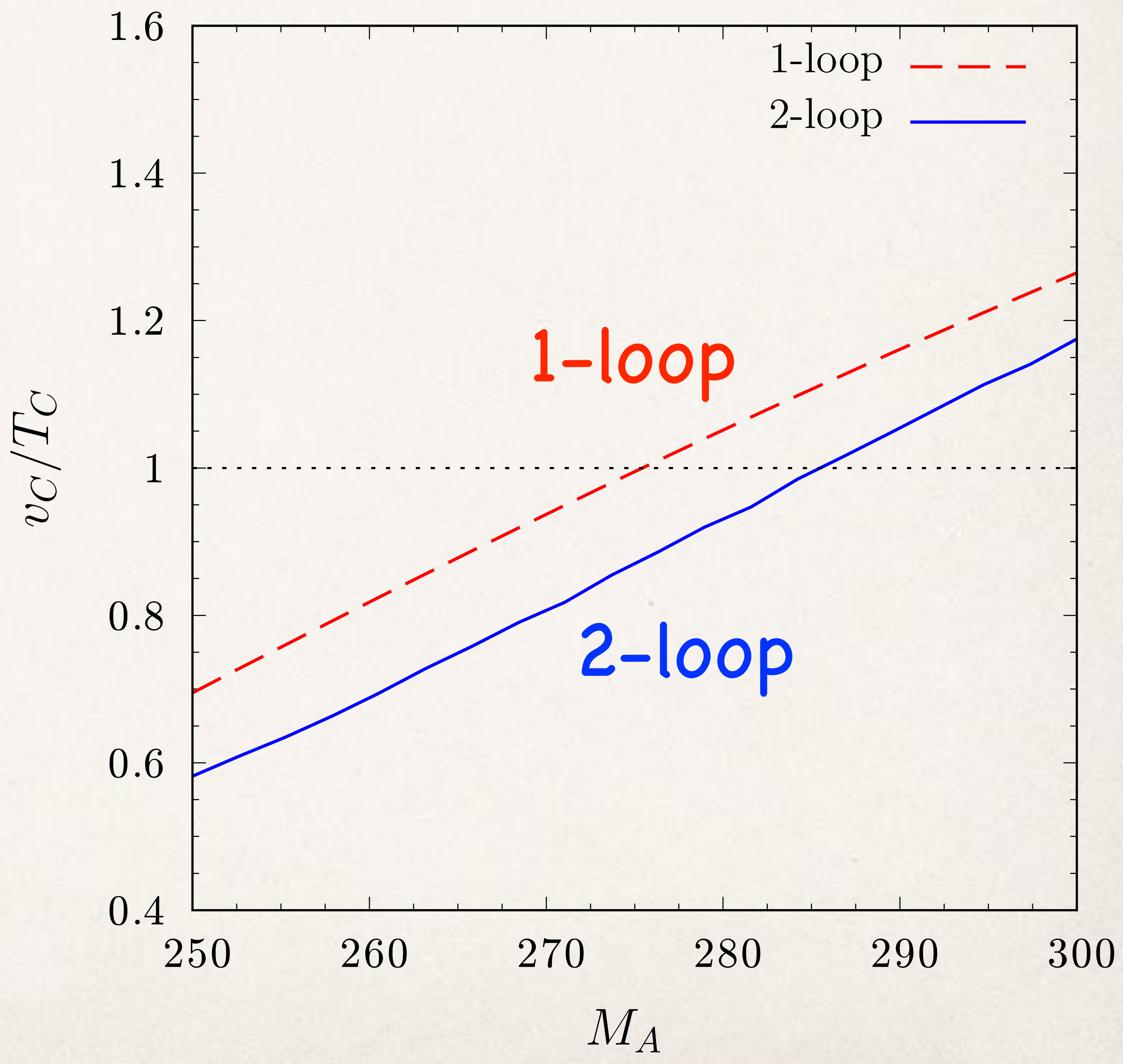
@ 2-loop

- v_C/T_C is weakened by about (7–16)%.

consistent with Ref. [M.Laine, M.Meyer, G. Nardini, NPB920,565(2017).]

- Larger M_A is needed to realize $v_C/T_C>1$.

$$\longrightarrow \ \kappa_\lambda \nearrow$$



vc/Tc

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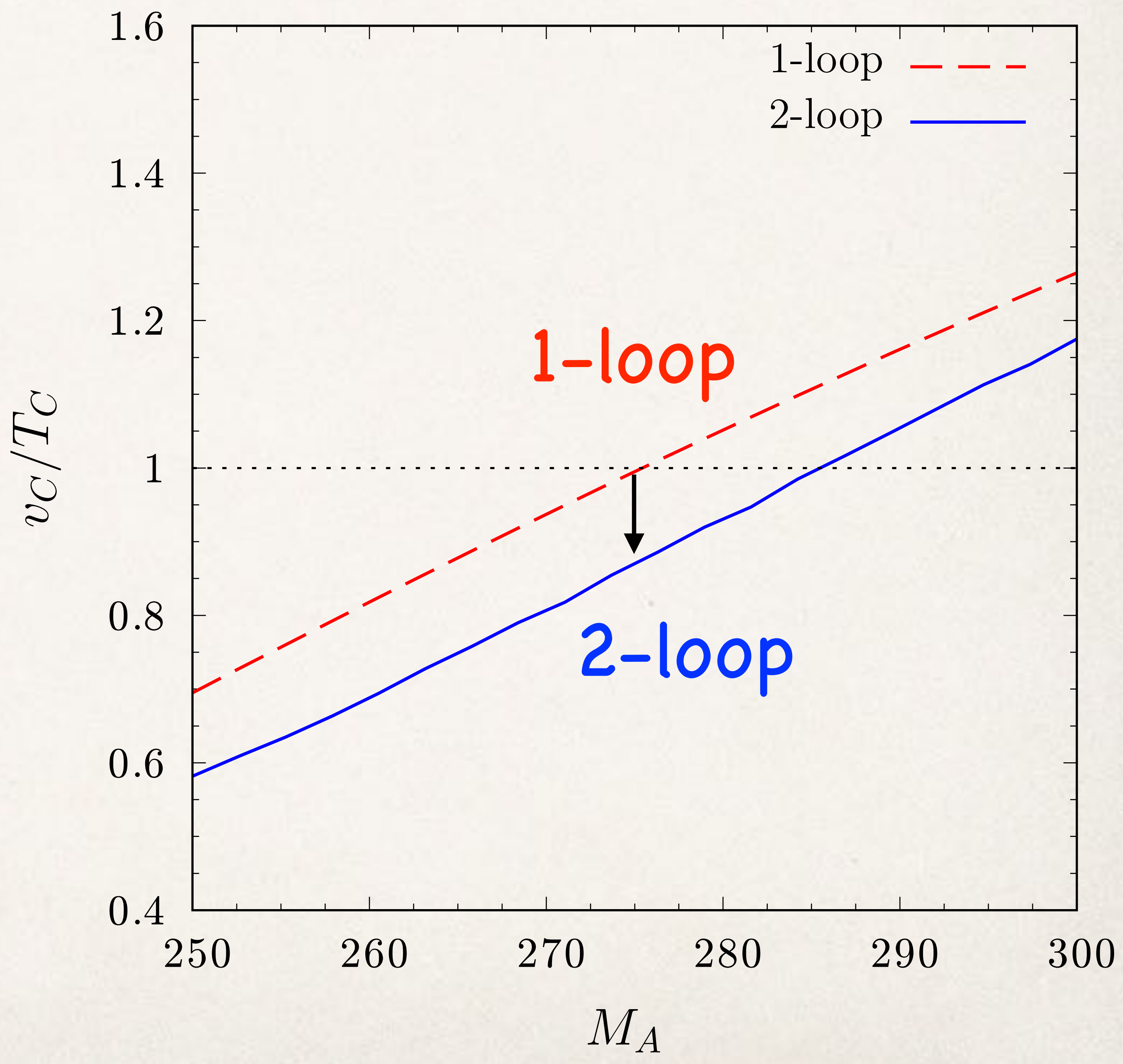

κ_λ-v_C/T_C correlations

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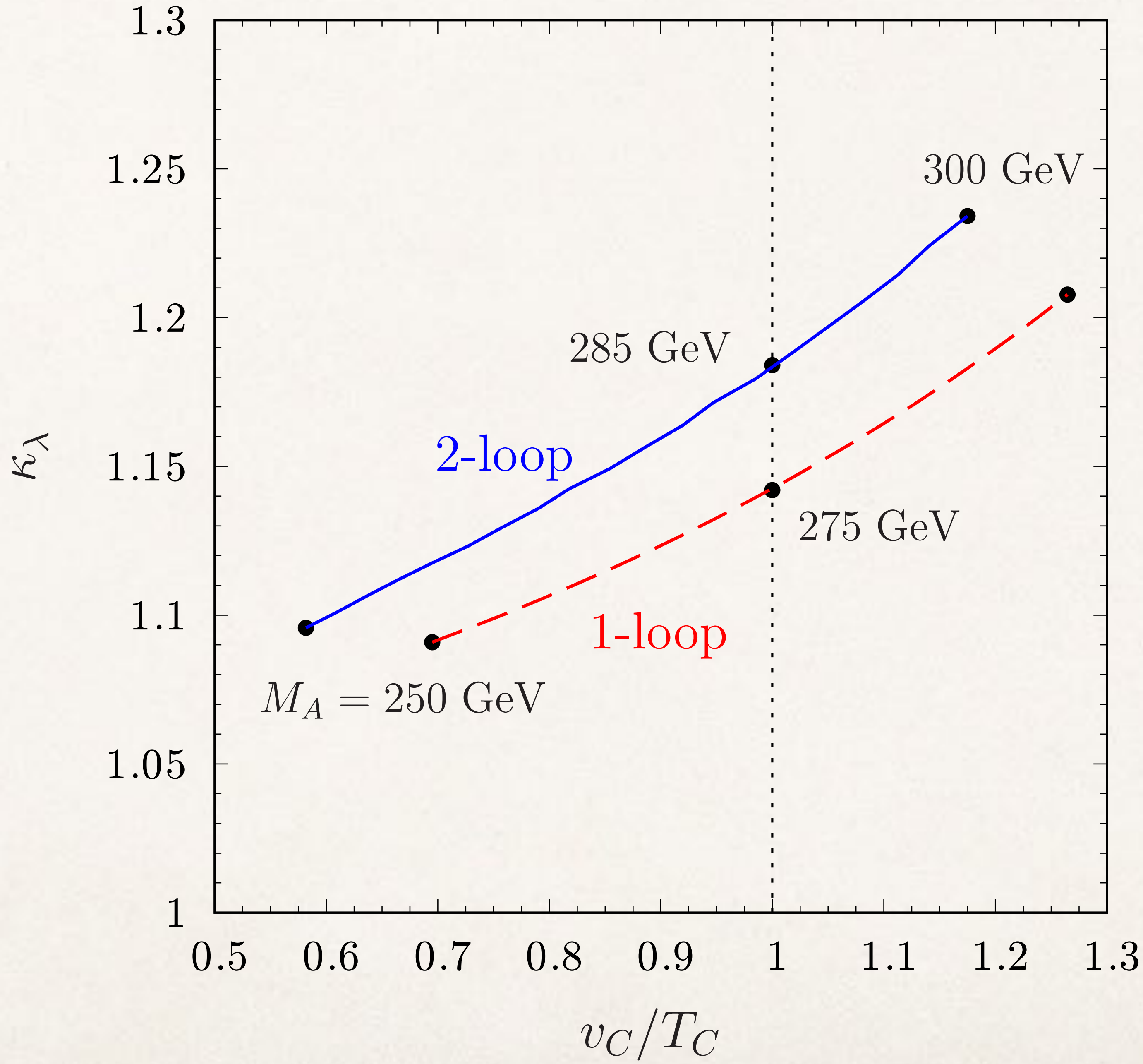

At 2-loop κ_λ is enhanced by about 4% for v_C/T_C>1.

Summary

- We have discussed the hhh coupling and 1st-order EWPT at 2-loop level in the IDM.

@2-loop level

- κ_λ gets larger by at most 2%.
- 1st-order EWPT gets weaken by (7-16)%

- κ_λ-v_C/T_C correlation at 2-loop level is modified by about 4%.

$\kappa_\lambda \gtrsim 1.18$ for $v_C/T_C \gtrsim 1$ @2-loop

This can be tested at future colliders!