Unveil the Nambu- Goldstone Nature of the Higgs boson by precision measurement

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Motivation

- Many lepton colliders have been proposed, ILC (this workshop), CLIC, FCC-ee,CEPC
- The precision for the electroweak sector will be significantly improved
- Higgs property is of the essential goal
- Any hints for new physics pattern from coupling measurement?

Higgs as Goldstone bosons

- The effective Lagrangians possess two expansion h/f, E/Λ

- Non-Linearly realised symmetry allow us to include all the orders of h/f, CCWZ, IR Construction etc.

- Couplings are strongly correlated with each other at fixed order E/Λ

- Possible relationships could be found

Construct the covariant objects

$$\pi^{a\,\prime} = \pi^a + [F_1(\mathcal{T})]_{ab}\,\varepsilon^b \qquad F_1(0) = 1 \qquad \mathcal{T}_{ab} = \frac{1}{f^2}(\delta^{ab}\pi^c\pi^c - \pi^a\pi^b).$$

$$\begin{aligned} d_\mu^a(\pi,\partial) &= \frac{\sqrt{2}}{f}[\mathcal{F}_2(\mathcal{T})]_{ab}\partial_\mu\pi^b\ , \\ E_\mu^i(\pi,\partial) &= \frac{2}{f^2}\partial_\mu\pi^a[\mathcal{F}_4(\mathcal{T})]_{ab}(T^i\pi)^b\ , \end{aligned}$$

$$\begin{aligned} d_\mu &\to U\ d_\mu\ , \\ E_\mu^i T^i &\to U(E_\mu^i T^i)U^{-1} - iU\partial_\mu(U^{-1})\ , \end{aligned}$$

$$F_1(\mathcal{T}) = \sqrt{\mathcal{T}}\cot\sqrt{\mathcal{T}} \qquad F_2(\mathcal{T}) = \frac{\sin\sqrt{\mathcal{T}}}{\sqrt{\mathcal{T}}}\ . \qquad F_4(\mathcal{T}) = -\frac{2i}{\mathcal{T}}\sin^2\frac{\sqrt{\mathcal{T}}}{2}\ .$$

$$\begin{aligned} [F_1(\mathcal{T})]_{ab} &= \delta^{ab} + \left(\delta^{ab} - \frac{\pi^a\pi^b}{|\pi|^2}\right)\left(\frac{|\pi|}{f}\cot\frac{|\pi|}{f} - 1\right), \\ [F_2(\mathcal{T})]_{ab} &= \delta^{ab} + \left(\delta^{ab} - \frac{\pi^a\pi^b}{|\pi|^2}\right)\left(\frac{f}{|\pi|}\sin\frac{|\pi|}{f} - 1\right), \\ [F_4(\mathcal{T})]_{ab} &= -\frac{i}{2}\left[\delta^{ab} + \left(\delta^{ab} - \frac{\pi^a\pi^b}{|\pi|^2}\right)\left(\frac{4f^2}{|\pi|^2}\sin^2\frac{|\pi|}{2f} - 1\right)\right] \end{aligned}$$

$$\partial_\mu \to \partial_\mu + iA_\mu$$

$$(f_{\mu\nu}^-)^a = \frac{\sqrt{2}i}{f}[F_2(\mathcal{T})]_{ab}(T^i\pi)^b\,F_{\mu\nu}^i\ .$$

$$(f_{\mu\nu}^+)^i = F_{\mu\nu}^i + \frac{2}{f^2}F_{\mu\nu}^j(T^j\pi)^a[F_4(\mathcal{T})]_{ab}(T^i\pi)^b\ .$$

Ian Low, arXiv:1412.2145,1412.2146

For detail, see Zhewei’s talk

Non-linearity at $\mathcal{O}(p^2)$

The Lagrangian at $\mathcal{O}(p^2)$

$$\mathcal{T}_{ab} = \frac{1}{f^2}(\delta^{ab}\pi^c\pi^c - \pi^a\pi^b).$$

$$\mathcal{L}^{(2)} = \frac{1}{2}[F_2(\mathcal{T})^2]_{ab}\,\partial_\mu\pi^a\partial^\mu\pi^b\,, \quad F_2(\mathcal{T}) = \frac{\sin\sqrt{\mathcal{T}}}{\sqrt{\mathcal{T}}}$$

$$F_2(0) = 1$$

In the unitary gauge:

$$\xi = \frac{v^2}{f^2}$$

$$= \frac{1}{2}\partial_\mu h\partial^\mu h + \left[1 + 2\sqrt{1-\xi}\,\frac{h}{v} + (1-2\xi)\,\frac{h^2}{v^2} + \cdots\right] \times \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu\right)\,, \qquad (5)$$

One Higgs and two Higgs couplings are correlated

$$b_h = 2\sqrt{1-\xi}\,, \qquad b_{2h} = 1 - 2\xi\,.$$

SMEFT at $\mathcal{O}(p^2)$

Purely dimension-six operator

$$\mathcal{O}_H = \frac{1}{2}\partial_\mu (H^\dagger H)\partial^\mu (H^\dagger H), \qquad \mathcal{L} = \frac{c_H}{f^2}\mathcal{O}_H$$

The Higgs couplings are

$$b_h = 2\left(1 - \frac{1}{2}c_H\xi\right), \qquad b_{2h} = 1 - 2c_H\xi$$

Compared with the non-linear case:

$$b_h = 2\sqrt{1-\xi}\ , \qquad b_{2h} = 1 - 2\xi\ .$$

Apart from the redefinition of ξ, the difference arise at ξ^2

Non-linearity at $\mathcal{O}(p^4)$

In the unitary gauge:

$$
\begin{aligned}
d_\mu^a &= \sqrt{2}\left[\delta^{a4}\,\partial_\mu\left(\frac{h}{f}\right) + \frac{\delta^{ar}}{2}\sin(\theta + h/f) \right.\\
&\qquad \left. \times \left(W_\mu^r - \delta^{r3}B_\mu\right)\right] , \\
(E_\mu^{L/R})^r &= \frac{1 \pm \cos(\theta + h/f)}{2} W_\mu^r \\
&\quad + \frac{1 \mp \cos(\theta + h/f)}{2} B_\mu \delta^{r3} , \\
(f_{\mu\nu}^-)^a &= \frac{1}{\sqrt{2}} \sin(\theta + h/f)(W_{\mu\nu}^r - \delta^{r3} B_{\mu\nu})\delta^{ra}, \\
(f_{\mu\nu}^{+L/R})^r &= \frac{1 \pm \cos(\theta + h/f)}{2} W_{\mu\nu}^r \\
&\quad + \frac{1 \mp \cos(\theta + h/f)}{2} \delta^{r3} B_{\mu\nu} \qquad (13)
\end{aligned}
$$

Canonical bases of the gauge bosons are obtained by:

$$W_\mu \to -gW_\mu, \qquad B_\mu \to -g' B_\mu$$

All the cosine and sines are indications of Higgs non-linearity

Non-linearity at $\mathcal{O}(p^4)$

Complete independent P-even operators:

$$O_1 = (d^a_\mu d^{\mu a})^2 , \quad O_2 = (d^a_\mu d^a_\nu)^2 ,$$
$$O_3 = \left[(E^L_{\mu\nu})^r\right]^2 - \left[(E^R_{\mu\nu})^r\right]^2 ,$$
$$O_4^\pm = -i\, d^a_\mu d^b_\nu \left[(f^{+L}_{\mu\nu})^r T^r_L \pm (f^{+R}_{\mu\nu})^r T^r_R\right]_{ab} ,$$
$$O_5^+ = \left[(f^-_{\mu\nu})^a\right]^2 , \quad O_5^- = \left[(f^{+L}_{\mu\nu})^r\right]^2 - \left[(f^{+R}_{\mu\nu})^r\right]^2 , \quad (14)$$

R. Contino et al,arXiv:1109.1570

The Lagrangian:

$$S^{(4)}_{SILH} = \int d^4x\, m_\rho^2 f^2 \mathcal{L}^{(4)}\left(\frac{\pi}{f}, \frac{D}{m_\rho}\right) = \sum_i \frac{c_i}{g_\rho^2} O_i , \quad (15)$$

g_ρ factor out to make c_i coupling-dimensionless

c_i are unpredictable, need to be measured

Predictions on the couplings

$$\mathcal{L}_{\rm NL} = \sum_i \frac{m_W^2}{m_\rho^2} \left(C_i^h \mathcal{I}_i^h + C_i^{2h} \mathcal{I}_i^{2h} + C_i^{3V} \mathcal{I}_i^{3V} \right) , \qquad V_{1/2} \in \{W, Z, \gamma\}$$

- HVV CMS, 1707.00541

$$\frac{h}{v} V_{1\,\mu} \mathcal{D}^{\mu\nu} V_{2\,\nu} , \qquad \frac{h}{v} V_{1\,\mu\nu} V_2^{\mu\nu} , \qquad \mathcal{D}^{\mu\nu} = \partial^\mu \partial^\nu - \eta^{\mu\nu} \partial^2$$

- HHVV Need to be done!

$$\frac{h^2}{v^2} V_{1\,\mu} \mathcal{D}^{\mu\nu} V_{2\,\nu} , \quad \frac{h^2}{v^2} V_{1\,\mu\nu} V_2^{\mu\nu} , \quad \frac{\partial_\mu h \partial_\nu h}{v^2} V_1^\mu V_2^\nu .$$

- (H)VVV

$$V_{1\,\mu} V_{2\,\nu} V_3^{\mu\nu} , \qquad \frac{h}{v} V_{1\,\mu} V_{2\,\nu} V_3^{\mu\nu} , \qquad \frac{\partial_\mu h}{v} V_{1\,\nu} V_2^\mu V_3^\nu .$$

Predictions on the couplings

$$\mathcal{L}_{\rm NL} = \sum_i \frac{m_W^2}{m_\rho^2} \left(C_i^h \mathcal{I}_i^h + C_i^{2h} \mathcal{I}_i^{2h} + C_i^{3V} \mathcal{I}_i^{3V} \right) ,$$

$\mathcal{I}_i^h$	C_i^h (NL)	C_i^h (D6)
(1) $\frac{h}{v} Z_\mu \mathcal{D}^{\mu\nu} Z_\nu$	$\frac{4c_{2w}}{c_w^2}\left(-2c_3 + c_4^-\right) + \frac{4}{c_w^2} c_4^+ \cos\theta$	$2(c_W + c_{HW}) + 2t_w^2(c_B + c_{HB})$
(2) $\frac{h}{v} Z_{\mu\nu} Z^{\mu\nu}$	$-\frac{2c_{2w}}{c_w^2}\left(c_4^- + 2c_5^-\right) - \frac{2}{c_w^2}\left(c_4^+ - 2c_5^+\right)\cos\theta$	$-(c_{HW} + t_w^2 c_{HB})$
(3) $\frac{h}{v} Z_\mu \mathcal{D}^{\mu\nu} A_\nu$	$8\left(-2c_3 + c_4^-\right) t_w$	$2t_w(c_W + c_{HW}) - 2t_w(c_B + c_{HB})$
(4) $\frac{h}{v} Z_{\mu\nu} A^{\mu\nu}$	$-4\left(c_4^- + 2c_5^-\right) t_w$	$-t_w(c_{HW} - c_{HB})$
(5) $\frac{h}{v} W_\mu^+ \mathcal{D}^{\mu\nu} W_\nu^- + h.c.$	$4(-2c_3 + c_4^-) + 4c_4^+ \cos\theta$	$2(c_W + c_{HW})$
(6) $\frac{h}{v} W_{\mu\nu}^+ W^{-\mu\nu}$	$-4(c_4^- + 2c_5^-) - 4\left(c_4^+ - 2c_5^+\right)\cos\theta$	$-2c_{HW}$

Predictions on the couplings

$\mathcal{I}_i^{2h}$	C_i^{2h} (NL)	C_i^{2h} (D6)
(1) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} Z_\nu$	$\frac{2c_{2w}}{c_w^2}\left(-2c_3+c_4^-\right)\cos\theta$ $+\frac{2}{c_w^2}c_4^+\cos 2\theta$	$\frac{1}{2}C_1^h$
(2) $\frac{h^2}{v^2} Z_{\mu\nu} Z^{\mu\nu}$	$-\frac{c_{2w}}{c_w^2}\left(c_4^-+2c_5^-\right)\cos\theta$ $-\frac{1}{c_w^2}\left(c_4^+-2c_5^+\right)\cos 2\theta$	$\frac{1}{2}C_2^h$
(3) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} A_\nu$	$4t_w\left(-2c_3+c_4^-\right)\cos\theta$	$\frac{1}{2}C_3^h$
(4) $\frac{h^2}{v^2} Z_{\mu\nu} A^{\mu\nu}$	$-2t_w\left(c_4^-+2c_5^-\right)\cos\theta$	$\frac{1}{2}C_4^h$
(5) $\frac{h^2}{v^2} W_\mu^+ \mathcal{D}^{\mu\nu} W_\nu^- + h.c.$	$2(-2c_3+c_4^-)\cos\theta$ $+2c_4^+\cos 2\theta$	$\frac{1}{2}C_5^h$
(6) $\frac{h^2}{v^2} W_{\mu\nu}^+ W^{-\mu\nu}$	$-2\left(c_4^-+2c_5^-\right)\cos\theta$ $-2\left(c_4^+-2c_5^+\right)\cos 2\theta$	$\frac{1}{2}C_6^h$
(7) $\frac{(\partial_\nu h)^2}{v^2} Z_\mu Z^\mu$	$\frac{8}{c_w^2}c_1\sin^2\theta$	$\times$
(8) $\frac{\partial_\mu h \partial_\nu h}{v^2} Z^\mu Z^\nu$	$\frac{8}{c_w^2}c_2\sin^2\theta$	$\times$
(9) $\frac{(\partial_\nu h)^2}{v^2} W_\mu^+ W^{-\mu}$	$16c_1\sin^2\theta$	$\times$
(10) $\frac{\partial^\mu h \partial^\nu h}{v^2} W_\mu^+ W_\nu^-$	$16c_2\sin^2\theta$	$\times$

Predictions on the couplings

$\mathcal{I}_i^{3V}$	C_i^{3V} (NL)	C_i^{3V} (D6)
$(\delta\tilde{g}_1^Z)\ igc_w W^{+\mu\nu}W^-_\mu Z_\nu$ $+h.c.$	$-\frac{2}{c_w^2}\left[(-2c_3+c_4^-)\cos\theta+c_4^+\right]$	$-\frac{c_W+c_{HW}}{c_w^2}$
$(\delta\tilde{\kappa}_\gamma) ieW^+_\mu W^-_\nu A^{\mu\nu}$	$-4\left(c_4^+-2c_5^+\right)$	$-\left(c_{HW}+c_{HB}\right)$
$(\delta\tilde{\kappa}_Z) igc_w W^+_\mu W^-_\nu Z^{\mu\nu}$	$-\frac{2}{c_w^2}\left(-2c_3+c_4^-\right)\cos\theta$ $-\frac{2}{c_w^2}(c_4^+c_{2w}+4c_5^+s_w^2)$	$-\frac{c_W}{c_w^2}-c_{HW}+t_w^2c_{HB}$
(1) $igc_w\frac{h}{v}W^{+\mu\nu}W^-_\mu Z_\nu$ $+h.c.$	$-\frac{4}{c_w^2}\left[(-2c_3+c_4^-)(1-\frac{3}{2}\sin^2\theta)\right.$ $\left.+c_4^+\cos\theta\right]$ $+16(c_3+c_5^- -c_5^+\cos\theta)$	$-\frac{2}{c_w^2}(c_W+c_{HW})-4c_W$
(2) $ie\frac{h}{v}W^{+\mu\nu}W^-_\mu A_\nu$ $+h.c.$	$16(c_3+c_5^- -c_5^+\cos\theta)$	$-4c_W$
(3)$igc_w\frac{h}{v}W^+_\mu W^-_\nu Z^{\mu\nu}$	$-\frac{4}{c_w^2}\left(1-\frac{3}{2}\sin^2\theta\right)\left(-2c_3+c_4^-\right)$ $+16(c_3+c_5^-)$ $-\frac{4}{c_w^2}(c_4^+c_{2w}+4c_5^+)\cos\theta$	$-\frac{2(1+2c_w^2)}{c_w^2}c_W-2c_{HW}$ $+2t_w^2c_{HB}$
(4)$ie\frac{h}{v}W^+_\mu W^-_\nu A^{\mu\nu}$	$16(c_3+c_5^-)-8c_4^+\cos\theta$	$-4c_W-2c_{HW}-2c_{HB}$
(5)$ieW^+_{[\mu}W^-_{\nu]}A^\mu\frac{\partial^\nu h}{v}$ (6) $-ig's_wW^+_{[\mu}W^-_{\nu]}Z^\mu\frac{\partial^\nu h}{v}$	$-8\left[(-2c_3+c_4^-)+\cos\theta c_4^+\right.$ $\left.-\sin^2\theta c_3\right]$	$-4\left(c_W+c_{HW}\right)$

Possible relations testing Higgs non-linearity

Class I

$$\mathrm{UR1}: \frac{C_6^h - C_4^h/t_w}{\delta\tilde{\kappa}_\gamma} = \frac{2c_w^2\, C_2^h - c_{2w}\, C_4^h/t_w}{\delta\tilde{\kappa}_\gamma} = \cos\theta \approx 1 - \frac{1}{2}\xi,$$

$$\mathrm{UR2}: \frac{c_{2w}}{t_w} C_3^h - 2c_w^2\, C_1^h = 4c_w^2\, \delta\tilde{g}_1^Z \cos\theta + \frac{1}{t_w} C_3^h \cos^2\theta\ ,$$

$$\mathrm{UR3}: C_5^h = -2c_w^2\, \delta\tilde{g}_1^Z \cos\theta + \frac{1}{2t_w} C_3^h \sin^2\theta\ ,$$

Class II

$$\mathrm{UR4}: \frac{C_3^{2h}}{C_3^h} = \frac{C_4^{2h}}{C_4^h} = \frac{1}{2}\cos\theta\ ,$$

$$\mathrm{UR5}: \frac{C_5^{2h} - C_3^{2h}/2t_w}{C_5^h - C_3^h/2t_w} = \frac{C_6^{2h} - C_4^{2h}/t_w}{C_6^h - C_4^h/t_w} = \frac{\cos 2\theta}{2\cos\theta} \approx \frac{1}{2}\left(1 - \frac{3}{2}\xi\right),$$

$$\mathrm{UR6}: \frac{s_{2w}\, C_1^{2h} - c_{2w}\, C_3^{2h}}{s_{2w}\, C_1^h - c_{2w}\, C_3^h} = \frac{s_{2w}\, C_2^{2h} - c_{2w}\, C_4^{2h}}{s_{2w}\, C_2^h - c_{2w}\, C_4^h} = \frac{\cos 2\theta}{2\cos\theta} \approx \frac{1}{2}\left(1 - \frac{3}{2}\xi\right).$$

$$\mathrm{UR7}: \frac{(C_3^h - 2t_w C_5^h) - s_{2w}(C_1^{3V} - C_2^{3V})}{C_3^h} = 1 - \frac{3}{2}\sin^2\theta\ .$$

All the ratios fixed by ξ,
can be thought as different ways of measuring ξ

What about the case of fundamental scalar?

$$\mathcal{L}_{\text{SMEFT}} \supset \sum_{i=W,B,HW,HB} \frac{c_i}{m_\rho^2}\mathcal{O}_i + \frac{c_i^8}{f^2 m_\rho^2}(H^\dagger H)\mathcal{O}_i \ ,$$

$$\mathcal{O}_W = \frac{ig}{2}\left(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H\right) D^\nu W^a_{\mu\nu}, \qquad \mathcal{O}_B = \frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right) \partial^\nu B_{\mu\nu}$$

$$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}, \qquad \mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu} \ .$$

Ratios are modified:

$$\frac{C_3^{2h}}{C_3^h} \approx \frac{1}{2}\left(1 + \frac{c_{HW}^8 - c_{HB}^8}{c_{HW} - c_{HB}}\xi\right),$$

$$\frac{C_4^{2h}}{C_4^h} \approx \frac{1}{2}\left(1 + \frac{c_W^8 - c_B^8 + c_{HW}^8 - c_{HB}^8}{c_W - c_B + c_{HW} - c_{HB}}\xi\right),$$

$$\frac{s_{2w}C_1^{2h} - c_{2w}C_3^{2h}}{s_{2w}C_1^h - c_{2w}C_3^h} \approx \frac{1}{2}\left(1 + \frac{c_W^8 + c_B^8 + c_{HW}^8 + c_{HB}^8}{c_W + c_B + c_{HW} + c_{HB}}\xi\right),$$

$$\frac{s_{2w}C_2^{2h} - c_{2w}C_4^{2h}}{s_{2w}C_2^h - c_{2w}C_4^h} \approx \frac{1}{2}\left(1 + \frac{c_{HW}^8 + c_{HB}^8}{c_{HW} + c_{HB}}\xi\right).$$

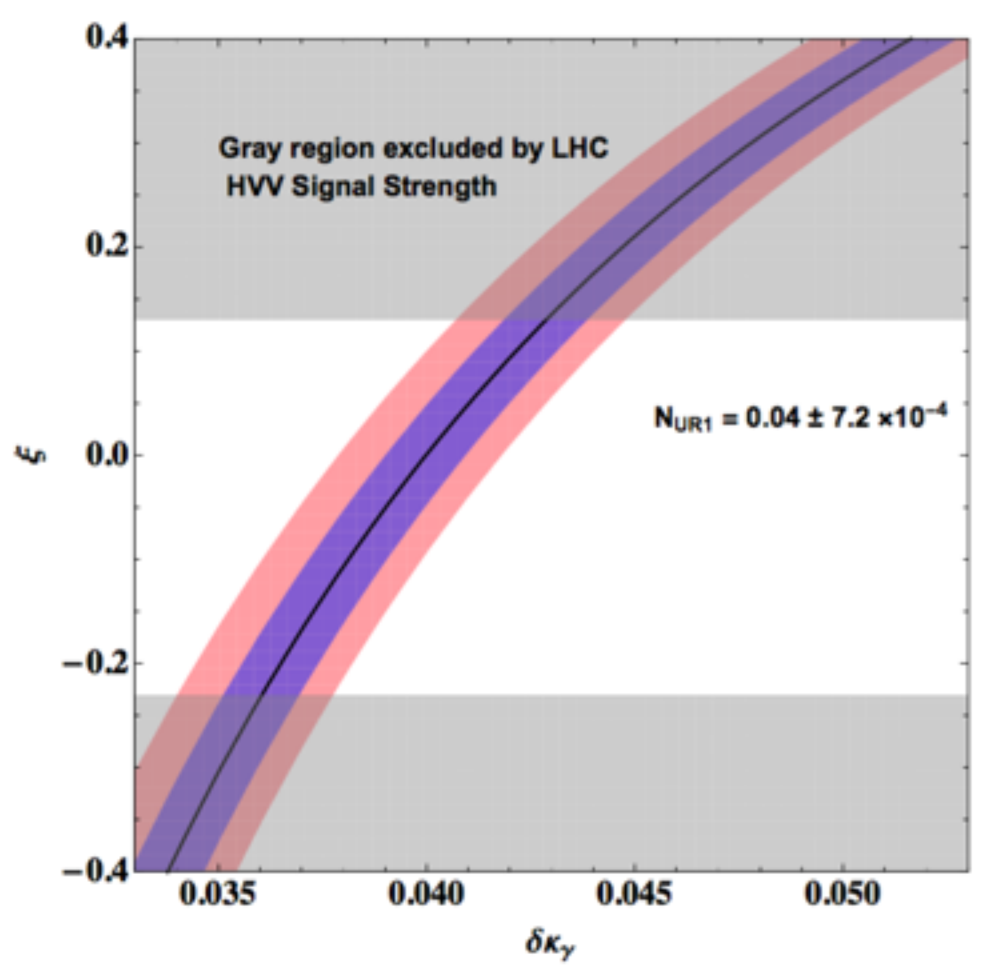
One Example

$$\text{UR1}: \frac{N_{\text{UR1}}}{\delta\kappa_\gamma} = \sqrt{1-\xi}\,.$$

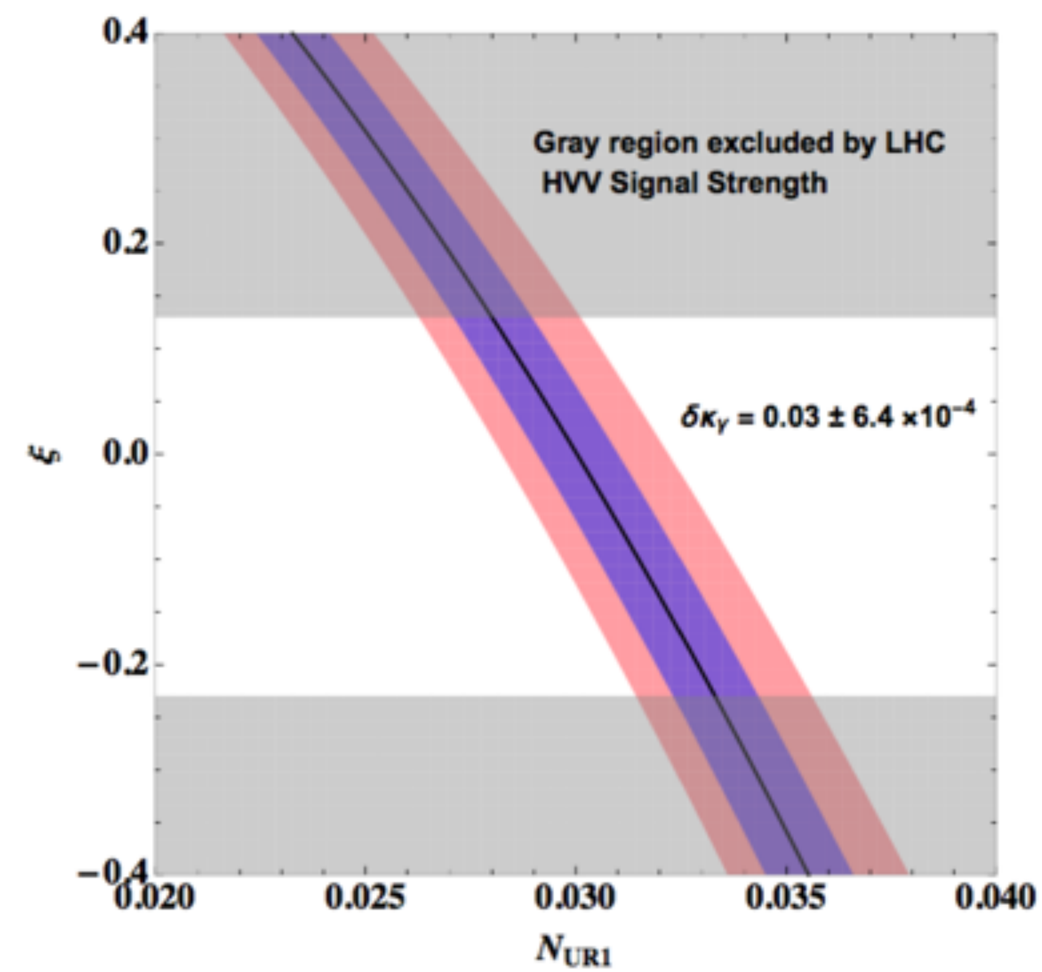
G. Durieux et al,1704.02333 ILC precision

$2\,\text{ab}^{-1}$ at $250\,\text{GeV}$, $200\,\text{fb}^{-1}$ at $350\,\text{GeV}$, and $4\,\text{ab}^{-1}$ at $500\,\text{GeV}$

$$N_{\text{UR1}} \equiv \frac{m_W^2}{m_\rho^2}\left(2c_w^2 C_2^h - c_{2w}C_4^h/t_w\right) : 7.20\times 10^{-4}\,, \quad \delta\kappa_\gamma : 6.4\times 10^{-4}\,,$$

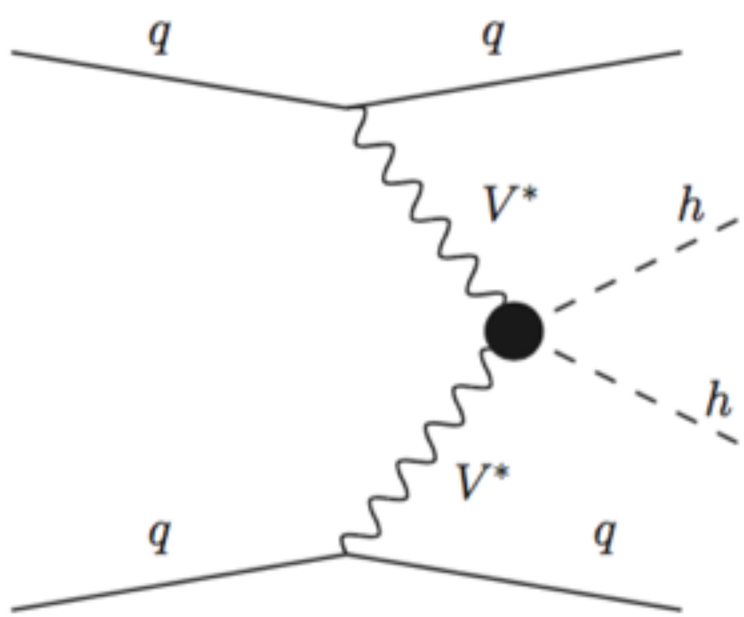


(a) Measured ξ as a function of $\delta\kappa_\gamma$, using N_{UR1} as an input.

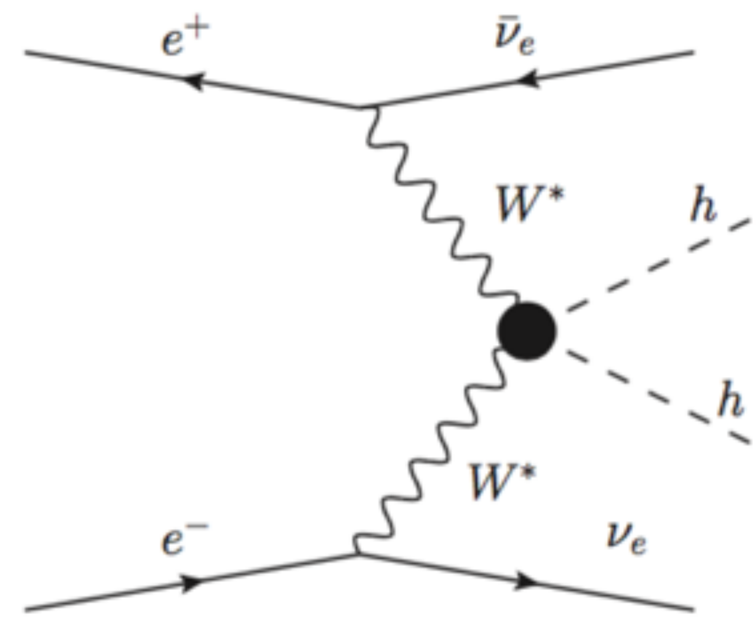


(b) Measured ξ as a function of N_{UR1}, using $\delta\kappa_\gamma$ as an input.

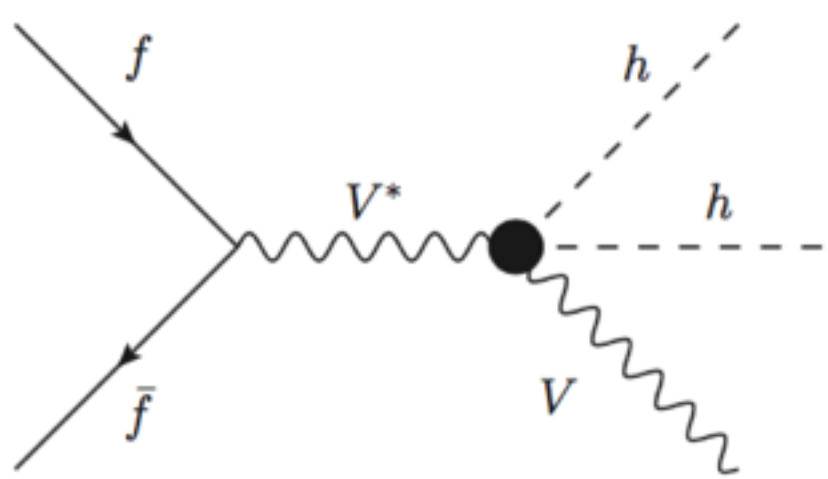
Future prospective HHVV



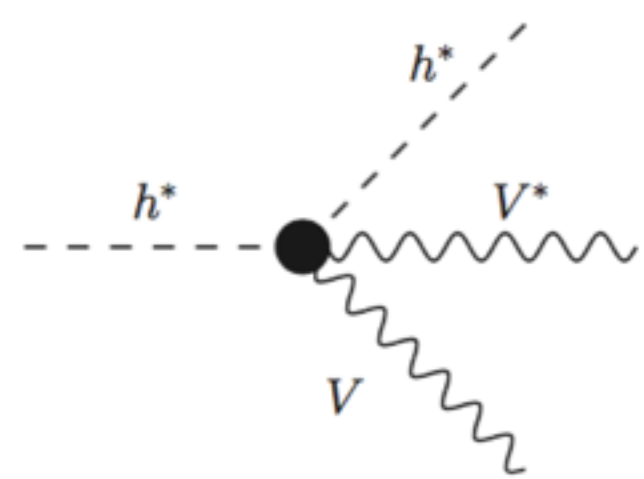
(a) Double Higgs production through vector boson fusion at a hadron collider.



(b) Double Higgs production through vector boson fusion at a lepton collider.



(c) Double Higgs production in association with a vector boson.



(d) Off-shell Single Higgs decay.

Conclusion

- We have proposed some relations that can potentially test the Higgs non-linearity.

- We have performed a phenomenological study on one universal relation by using the projection by ILC