The Infrared Construction of Composite Higgs Models

Based on 1805.00489, 1809.09126 with Da Liu and Ian Low

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Composite Higgs Models

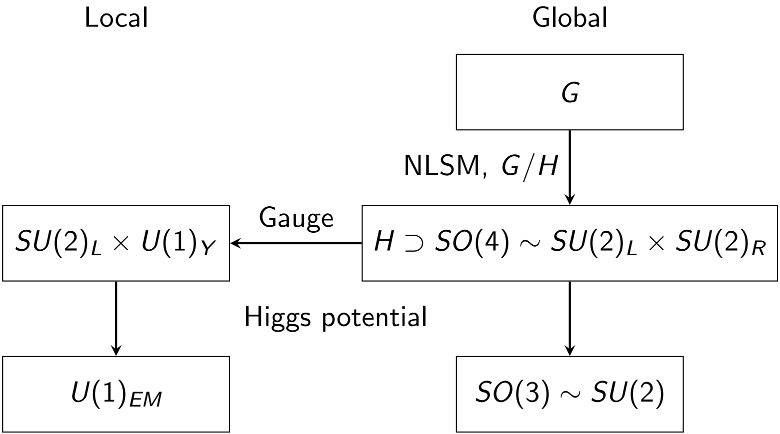
The Standard Model: a Higgs potential

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

Composite Higgs models: additional sponaneous symmetry breaking above electroweak scale

- Higgs as a Nambu-Goldstone boson
- The Higgs potential generated radiactively
- A naturally light Higgs

Symmetry breaking pattern



CCWZ formalism of NLSM

In general, G broken to H,

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Callan, Coleman, Wess & Zumino:

$$-i\xi^{-1}\partial_\mu\xi = d_\mu + E_\mu = \mathcal{D}_\mu\pi^a X^a + E_\mu^i T^i, \qquad \xi = \exp[i\pi^a X^a/f],$$

$$d_\mu \to h d_\mu h^\dagger, \qquad E_\mu \to h E_\mu h^\dagger - ih\partial_\mu h^\dagger$$

$$\mathcal{L} = \frac{f^2}{2}\mathcal{D}_\mu\pi^a\mathcal{D}^\mu\pi^a + \mathcal{O}(\partial^4).$$

Phys. Rev. **177**, 2239 (1969); **177**, 2247 (1969).

The shift symmetry and Adler's zero

The transformation of the Goldstones:

$$\pi^a \to \pi^a + \varepsilon^a + iT^i_{ab}\alpha^i\pi^b + \cdots$$

Current under the shift symmetry contains a one-particle pole:

$$\langle\Omega|\mathcal{J}^\mu(x)|\pi(p)\rangle = ifp^\mu e^{-ip\cdot x},$$

Current conservation leads to Adler's zero condition:

$$\partial_\mu\langle f|\mathcal{J}^\mu|i\rangle = 0, \qquad \langle f+\pi(p)|i\rangle = p_\mu R^\mu(p).$$

Phys. Rev. **137**, B1022 (1965).

Nonlinearity from IR

Trivial case: $G = U(1)$

$$\pi \to \pi + \varepsilon, \qquad \mathcal{L} = \frac{1}{2}(\partial\pi)^2 + \mathcal{O}(\partial^4).$$

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General case: several assumptions

- Nonlinear transformation of π is a field-dependent H rotation:

$$|\pi\rangle \to |\pi\rangle + |\varepsilon\rangle + i\alpha^i(\varepsilon,\pi)|T^i\pi\rangle$$

- Goldstone covariant derivative $|\mathcal{D}\pi\rangle$ transform as a field-dependent H rotation:

$$|\mathcal{D}\pi\rangle \to e^{iu^i(\varepsilon,\pi)T^i/f}|\mathcal{D}\pi\rangle.$$

- All of the above can be reduced to the trivial case.

Universal nonlinearity

Solve α, u and $\mathcal{D}$ simultaneously.

$$\mathcal{D}_\mu \pi^a = \left(\frac{\sin\sqrt{\mathcal{T}}}{\sqrt{\mathcal{T}}} \right)_{ab} \partial_\mu \pi^b, \qquad \mathcal{T} \equiv \frac{1}{f^2} | T^i \pi \rangle \langle \pi T^i |.$$

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Low, 1412.2145.

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- Structure of nonlinear interaction independent of G at UV
- G dependence all included in the value of f
- Can be used to derive interesting soft theorems

Low, Yin, 1709.08639, 1804.08629.

IR construction of Higgs nonlinearity: ingradients

CCWZ: $d_\mu \to h d_\mu h^\dagger$, $E_\mu \to h E_\mu h^\dagger - i h \partial_\mu h^\dagger$.
Can be solved by IR construction:

$$d_\mu^a = \left(\frac{\sin\sqrt{\mathcal{T}}}{\sqrt{\mathcal{T}}} \right)_{ab} \partial_\mu \pi^b, \quad E_\mu^i = -\frac{4i}{f^2} \partial_\mu \pi^a \left[\frac{1}{\mathcal{T}} \sin^2 \frac{\sqrt{\mathcal{T}}}{2} \right]_{ab} (T^i \pi)^b$$

Low, 1412.2145; 1412.2146.

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$$d^a_\mu = \left(\frac{\sin\sqrt{\mathcal{T}}}{\sqrt{\mathcal{T}}}\right)_{ab} D_\mu \pi^b, \quad E^i_\mu = A^i - \frac{4i}{f^2} D_\mu \pi^a \left[\frac{1}{\mathcal{T}} \sin^2 \frac{\sqrt{\mathcal{T}}}{2}\right]_{ab} (T^i \pi)^b$$

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Replace A^i_μ in d^a_μ and E^i_μ with $F^i_{\mu\nu}$:

$$(f^-_{\mu\nu})^a = \left(\frac{\sin\sqrt{\mathcal{T}}}{\sqrt{\mathcal{T}}}\right)_{ab} (T^i \pi)^b F^i_{\mu\nu},$$

$$(f^+_{\mu\nu})^i = F^i_{\mu\nu} - \frac{4i}{f^2} (T^j \pi)^a \left[\frac{1}{\mathcal{T}} \sin^2 \frac{\sqrt{\mathcal{T}}}{2}\right]_{ab} (T^i \pi)^b F^j_{\mu\nu}$$

Liu, Low, Yin, 1805.00489.

The Lagrangian

At $\mathcal{O}(p^2)$:

$$\mathcal{L}^{(2)} = \frac{f^2}{4} d_\mu^a d^{\mu a}.$$

At $\mathcal{O}(p^4)$:

$$\begin{aligned}
O_1 &= \left(d_\mu^a d^{\mu a}\right)^2 , \\
O_2 &= (d_\mu^a d_\nu^a)^2 \ , \\
O_3 &= \left[\left(E_{\mu\nu}^L\right)^r\right]^2 - \left[\left(E_{\mu\nu}^R\right)^r\right]^2 \ , \\
O_4^{\pm} &= -i\, d_\mu^a d_\nu^b \left[(f_{\mu\nu}^{+L})^r\, T_L^r \pm (f_{\mu\nu}^{+R})^r\, T_R^r\right]_{ab} \ , \\
O_5^{+} &= \left[(f_{\mu\nu}^{-})^a\right]^2 , \\
O_5^{-} &= \left[(f_{\mu\nu}^{+L})^r\right]^2 - \left[(f_{\mu\nu}^{+R})^r\right]^2 ,
\end{aligned}$$

Contino et al., 1109.1570.

Conclusion

- NLSM can be constructed from IR constraints
- The nonlinearity of Higgs as pNGB is universal

Backup Slides

Pions as pNGB

The QCD Lagrangian with two light quarks:

$$\mathcal{L} \supset i\bar{u}^L \not{D} u^L + i\bar{u}^R \not{D} u^R + i\bar{d}^L \not{D} d^L + i\bar{d}^R \not{D} d^R$$

$SU(2)_L \times SU(2)_R$ chiral symmetry:

$$\begin{pmatrix} u^L \\ d^L \end{pmatrix} \to g_L \begin{pmatrix} u^L \\ d^L \end{pmatrix}, \qquad \begin{pmatrix} u^R \\ d^R \end{pmatrix} \to g_R \begin{pmatrix} u^R \\ d^R \end{pmatrix}$$

The chiral symmetry is spontaneously broken:

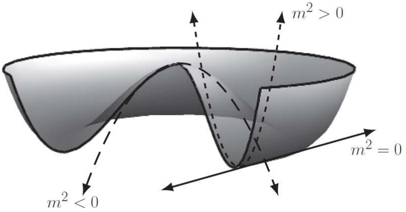
$$\begin{aligned} \langle \bar{u}u \rangle = \langle \bar{d}d \rangle &= V^3, \\ SU(2)_L \times SU(2)_R &\to SU(2)_V. \end{aligned}$$

Given the symmetry breaking pattern, we can describe the Nambu-Goldstone Bosons without QCD

The Sigma model

Introducing the scalar Σ_{ij} that transfrom under $SU(2)_L \times SU(2)_R$:

$$\mathcal{L} = |\partial_\mu \Sigma|^2 + m^2|\Sigma|^2 - \frac{\lambda}{4}|\Sigma|^4.$$



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Decouple the radial part and only keep the NGBs:

$$\Sigma \to U = \exp\left[2i \frac{\pi^a \tau^a}{F_\pi}\right]$$

NLSM:

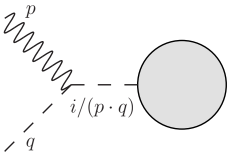
$$\mathcal{L} = \frac{F_\pi^2}{4} \mathrm{Tr}\left[D_\mu U (D_\mu U)^\dagger\right] + \mathcal{O}(D^4).$$

Nonlinear transformation

$$U \to g_L U g_R^\dagger, \qquad \pi^a \to \pi^a + \varepsilon^a + i T^i_{ab} \alpha^i \pi^b + \cdots$$

The regularity condition

For there to be Adler's zero, the remainder R needs to be regular: no "pole digrams" should appear.



The closure condition

The "closure condition" is necessary for solutions to exist in the IR construction of NLSM:

$$(T^i)_{ab}(T^i)_{cd} + (T^i)_{ac}(T^i)_{db} + (T^i)_{ad}(T^i)_{bc} = 0.$$