

The ILC as a natural SUSY discovery machine and precision microscope: from light higgsinos to tests of unification

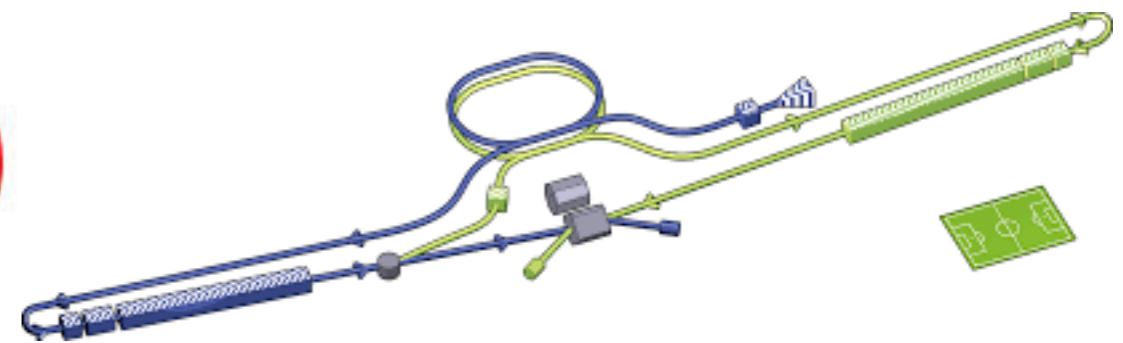
Howard Baer and collaborators
University of Oklahoma

LCWS2018, UT-Arlington, Oct. 23, 2018

or

Why

SUSY



**The hypothesis of weak scale SUSY
(that nature is supersymmetric with SUSY breaking at or
around the weak scale)
is remarkably simple and solves a host of problems**

- SUSY- extension of Poincare group to its most general structure: super-Poincare
- scalar field quadratic divergences cancel thus stabilizing the weak scale: potentially solves SM naturalness problem
- local SUSY: supergravity
- the vague prediction: superpartners around the weak scale

In spirit of Karl Popper,
any scientific hypothesis must be falsifiable

SUSY has already met 3 tests:

- measured gauge coupling strengths consistent with SUSY unification
- $m(t) \sim 173$ GeV consistent with SUSY requirement for (radiative) breakdown of EW symmetry
- $m(h) \sim 125$ GeV in accord with narrow MSSM requirement that $m(h) < 135$ GeV
- BUT where are the sparticles? **And where ought they to be?**

The main *raison d'être* for SUSY is to address the naturalness question:
works admirably by eliminating quadratic divergences to $m(h)$:
BUT if sparticles too heavy, then re-introduce hierarchy problem in form of
Little Hierarchy: why is $m(h) \sim 125$ GeV and not $m(\text{sparticle}) \sim 1-10$ TeV?

The notion of **practical naturalness**:

An observable \mathcal{O} is natural if all *independent* contributions to $\mathcal{O} = a_1 + \dots + a_n$ are comparable to or less than \mathcal{O}

Or else, if one contribution, say $a_1 \gg \mathcal{O}$, then some other (independent) contribution would have to be *fine-tuned* to a large opposite-sign value to compensate and maintain \mathcal{O} at its measured value



“The appearance of fine-tuning in a scientific theory is like a cry of distress from nature complaining that something needs to be better explained”

Pie baking analogy:



1 kg pie = .2 kg(sugar) + .3 kg(flour) +
.1 kg(water) + .5 kg (apples) +
-.1 kg(evaporation)

Voila! It is very natural!

An unnatural recipe:



$$1\text{kg}(\text{pie}) = .2 \text{ kg}(\text{sugar}) + .3 \text{ kg}(\text{flour}) + .5 \text{ kg}(\text{apples}) + 10^4 \text{ kg}(\text{water}) - 10^4 \text{ kg}(\text{evaporation})$$

mathematically, it is possible-
but success seems highly implausible:
it is fine-tuned and hence

unnatural

“...settling the ultimate fate of naturalness is perhaps the most profound theoretical question of our time”



Arkani-Hamed et al.,
arXiv:1511.06495

“Given the magnitude of the stakes involved,
it is vital to get a clear verdict
on naturalness from experiment”

This should be matched by theoretical scrutiny
of what we mean by naturalness

EW naturalness: why are $m(W,Z,h) \sim 100$ GeV
while $m(\text{sparticles}) \sim > 1$ TeV?

$$\text{Let } \mathcal{O} \equiv m_Z^2$$

EW minimization conditions relate $m(Z)$ to SUSY Lagrangian parameters

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u - \mu^2$$

For naturalness:

- $m_{H_u}^2$ driven to $\sim -(100 - 200)^2$ GeV² at weak scale
- superpotential Higgs/higgsino mass contribution $\mu \sim 100 - 200$ GeV
- TeV scale highly mixed top squarks minimize Σ_u^u
(and raise $m_h \sim 125$ GeV)

Chan, Chattopadhyaya, Nath

HB, Barger, Huang

Perelstein, Shakya

Low value of $\Delta_{EW} \equiv |\text{max of each term on RHS}| / (m_Z^2/2)$
is most conservative, unavoidable naturalness condition

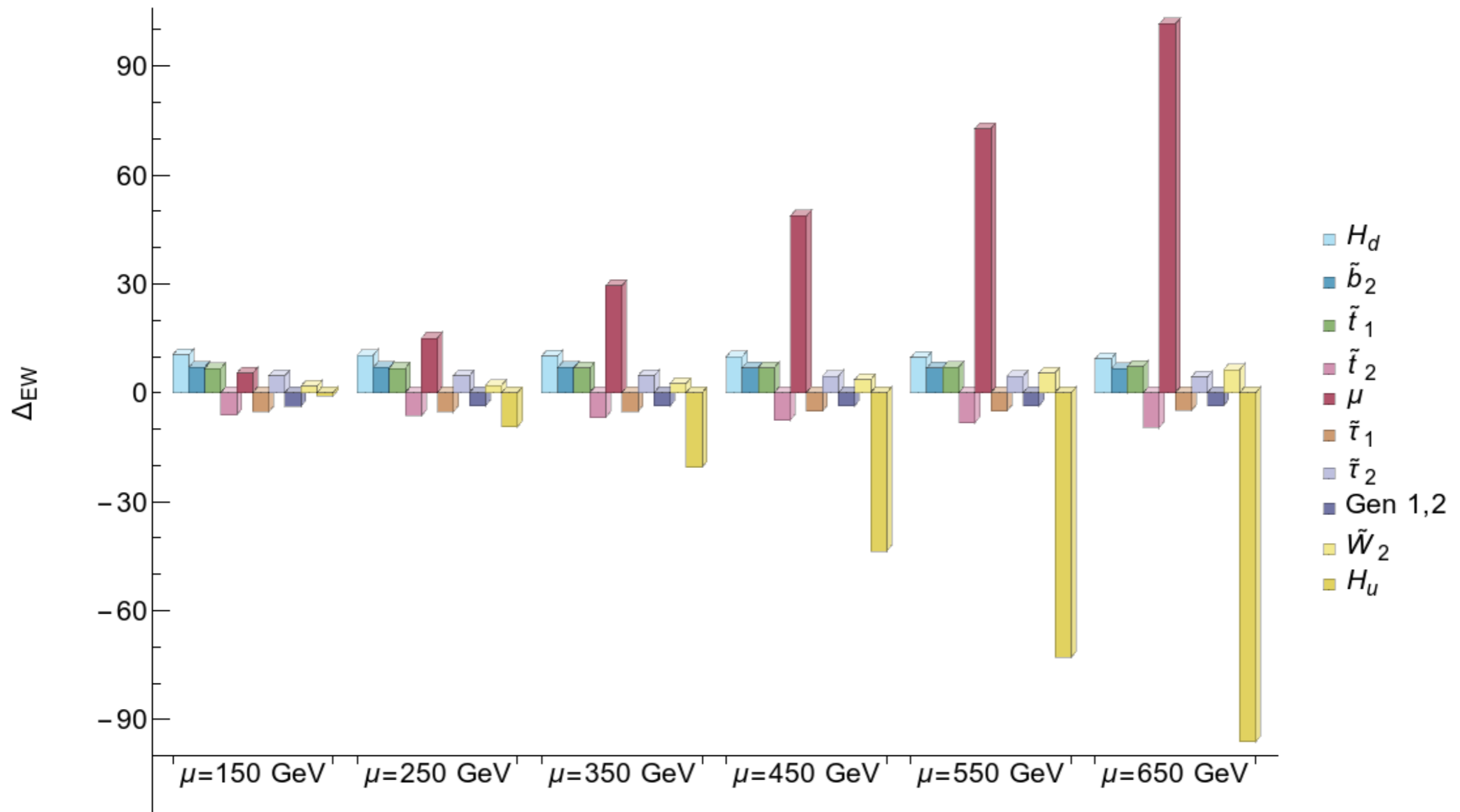
HB, Barger, Huang, Mustafayev, Tata

Most important inference:

light higgsinos of mass $\mu \sim 100 - 200$ GeV hard to see at LHC

but easily discovered at ILC with $\sqrt{s} > 2m(\text{higgsino}) \sim 200 - 500$ GeV!

How much is too much fine-tuning?



HB, Barger, Savoy

Visually, large fine-tuning has already developed by $\mu \sim 350$ or $\Delta_{EW} \sim 30$

Nature is natural $\Rightarrow \Delta_{EW} < 20 - 30$ (take 30 as conservative)

#3. What about EENZ/BG measure?

$$\Delta_{BG} = \max_i \left| \frac{\partial \log m_Z^2}{\partial \log p_i} \right| = \max_i \left| \frac{p_i}{m_Z^2} \frac{\partial m_Z^2}{\partial p_i} \right|$$

p_i are the theory parameters

applied to pMSSM, then $\Delta_{BG} \simeq \Delta_{EW}$

apply to high (e.g. GUT) scale parameters

$$\begin{aligned} m_Z^2 \simeq & -2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\ & + 0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\ & - 0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b \\ & - 1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\ & + 0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\ & + 0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\ & + 0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2, \end{aligned}$$

applied to most parameters,

Δ_{BG} large, looks fine-tuned for e.g. $m_{\tilde{t}_1} \sim 1$ TeV

$$\Delta_{BG}(Q_3) \simeq 0.73 \frac{1000^2}{91.2^2} \sim 100$$

#3. What about EENZ/BG measure?

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applied to pMSSM, then $\Delta_{BG} \simeq \Delta_{EW}$

What if we apply to high (e.g. GUT) scale parameters ?

$$\begin{aligned} m_Z^2 \simeq & -2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\ & + 0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\ & - 0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b \\ & - 1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\ & \hline & + 0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\ & \hline & + 0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\ & \hline & + 0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2, \end{aligned}$$

For correlated scalar masses $\equiv m_0$,

scalar contribution collapses:

what looks fine-tuned isn't: *focus point SUSY*

multi-TeV scalars are *natural*

Feng, Matchev, Moroi

Even with FP, still
fine-tuned on $m(\text{gluino})$:(

But wait! in more complete models,
soft terms **not independent**

violates prime directive!

e.g. in SUGRA, for well-specified hidden sector,
each soft term calculated as multiple of $m_{3/2}$;
soft terms must be combined!

e.g. dilaton-dominated SUSY breaking: $m_0^2 = m_{3/2}^2$ with $m_{1/2} = -A_0 = \sqrt{3}m_{3/2}$

in general:

$$\begin{aligned} m_{H_u}^2 &= a_{H_u} \cdot m_{3/2}^2, \\ m_{Q_3}^2 &= a_{Q_3} \cdot m_{3/2}^2, \\ A_t &= a_{A_t} \cdot m_{3/2}, \\ M_i &= a_i \cdot m_{3/2}, \\ &\dots \end{aligned}$$

since μ hardly runs, then

$$\begin{aligned} m_Z^2 &\simeq -2\mu^2 + a \cdot m_{3/2}^2 \\ &\simeq -2\mu^2 - 2m_{H_u}^2 (weak) \end{aligned}$$

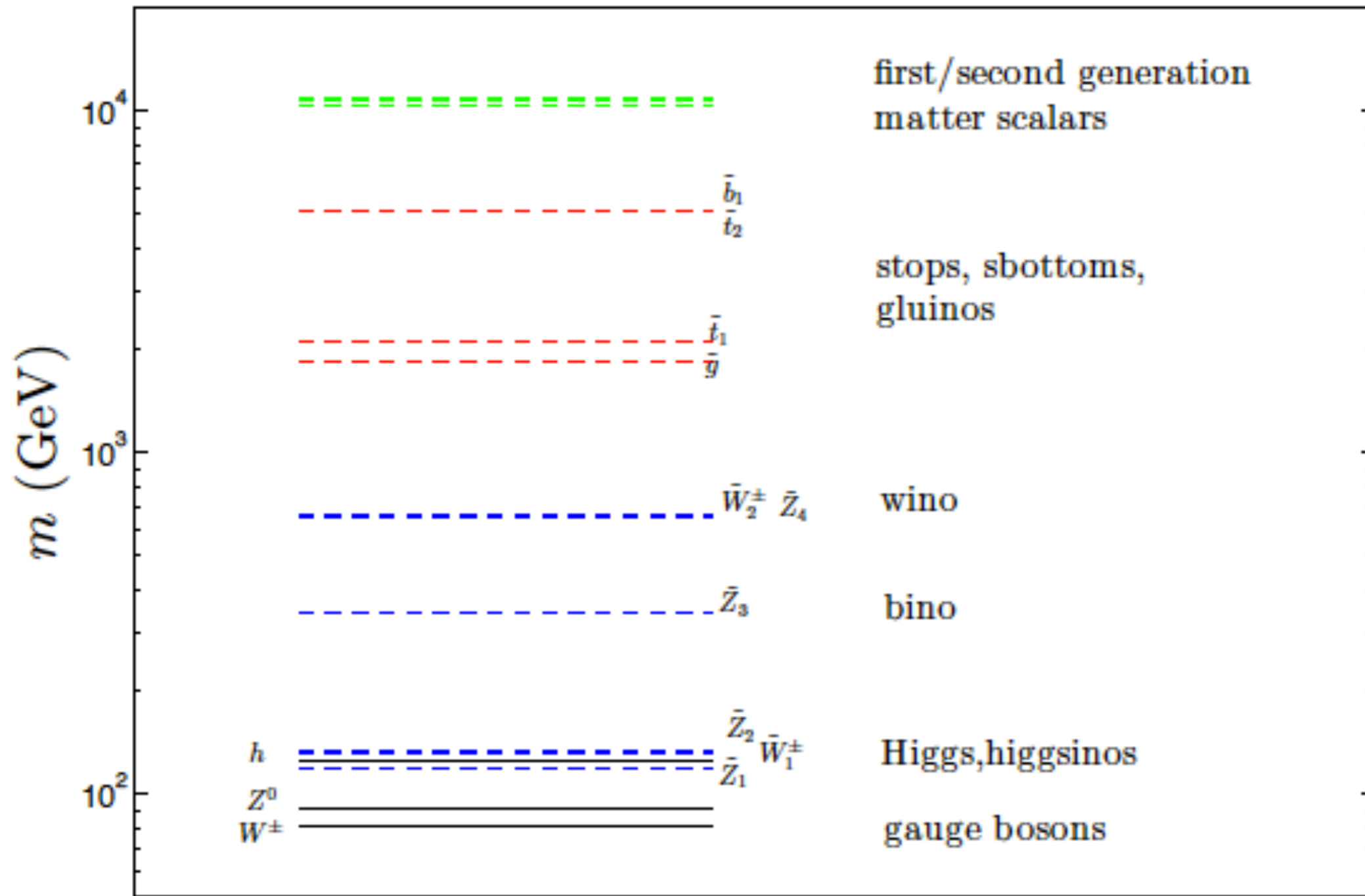
$$m_{H_u}^2 (weak) \sim -(100 - 200)^2 \text{ GeV}^2 \sim -a \cdot m_{3/2}^2/2$$

using μ^2 and $m_{3/2}^2$ as fundamental,
then $\Delta_{BG} \simeq \Delta_{EW}$ even using high scale parameters!

bounds from naturalness (3%)	BG/DG	Delta_EW
mu	350 GeV	0.35 TeV
gluino	400-600 GeV	~6 TeV
t1	450 GeV	3 TeV
sq/sl	550-700 GeV	10-30 TeV

h(125) and LHC limits are perfectly compatible with 3-10% naturalness: **no crisis for SUSY!**

Typical spectrum for low Δ_{EW} models

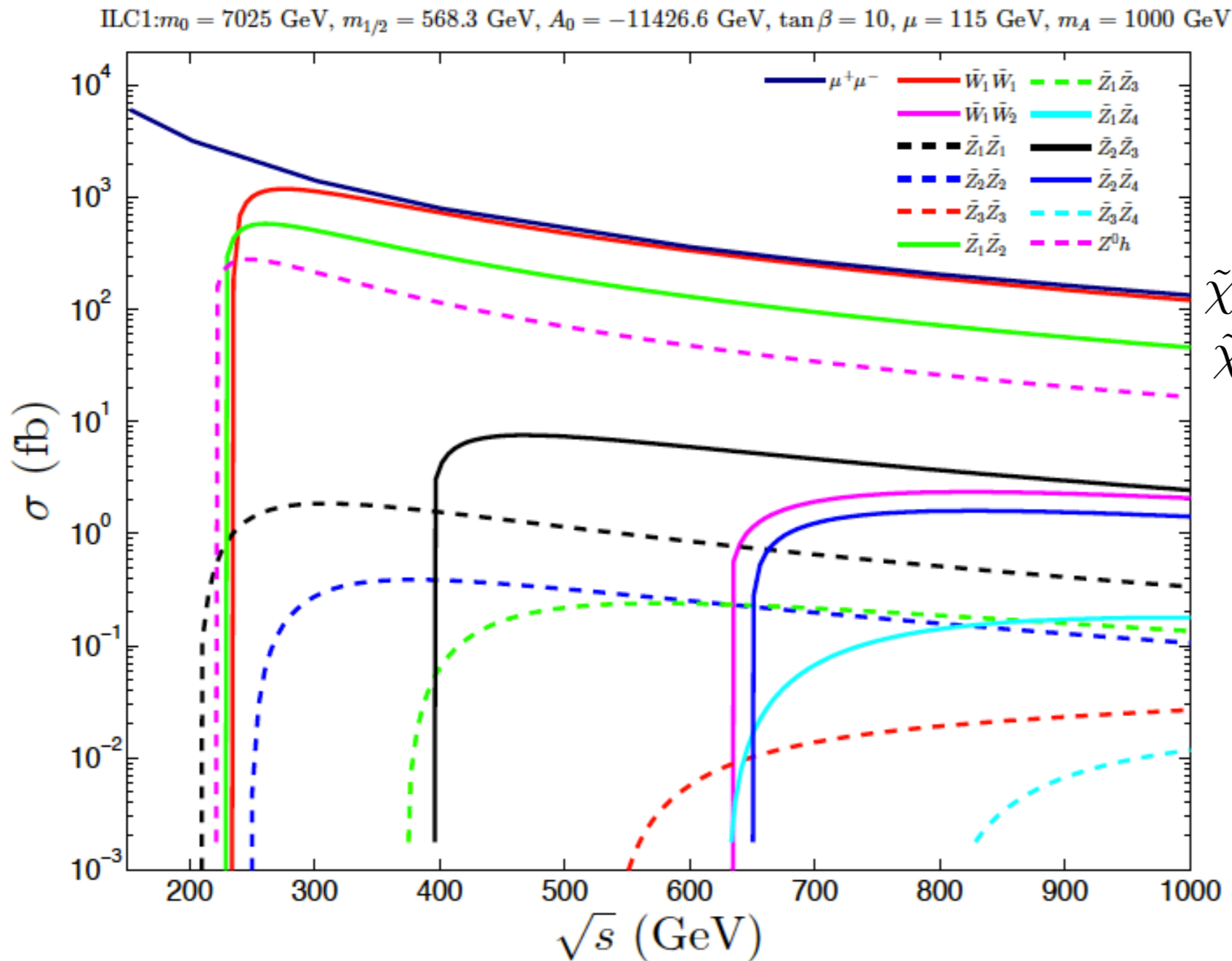


There is a Little Hierarchy, but it is **no problem**

$$\mu \ll m(\text{soft}) \text{ is OK}$$

Smoking gun signature: light higgsinos at ILC:

ILC is Higgs/higgsino factory!



$$\sigma(\text{higgsino}) \gg \sigma(Zh)$$

$\tilde{\chi}_1^+ \tilde{\chi}_1^-$
 $\tilde{\chi}_1^0 \tilde{\chi}_2^0$

3-15 GeV higgsino mass
gaps no problem
in clean ILC environment

HB, Barger, Mickelson, Mustafayev, Tata
arXiv:1404.7510

Why might $\mu \ll m(\text{soft})$?

SUSY μ problem: μ term is SUSY, not SUSY breaking:
expect $\mu \sim M(\text{Pl})$ but phenomenology requires $\mu \sim m(\text{Z})$

- NMSSM: $\mu \sim m(\text{soft})$; but beware singlets!
- Giudice–Masiero: μ forbidden by some symmetry: generate via Higgs coupling to hidden sector: $\mu \sim m(\text{soft})$
- **Kim–Nilles**: invoke SUSY version of DFSZ axion solution to strong CP:

KN: PQ symmetry forbids μ term,
but then it is generated via PQ breaking

$$\mu \sim \lambda_\mu f_a^2 / m_P$$

Little Hierarchy due to mismatch between
PQ breaking and SUSY breaking scales?

$$m(\text{soft}) \sim m_{3/2} \sim m_{\text{hidden}}^2 / m_P$$

$$f_a < m_{\text{hidden}} \Rightarrow \mu \ll m(\text{soft})$$

**Higgs mass $m(h) \sim \mu$
tells us where to look for axion!**

$$m_a \sim 6.2 \mu\text{eV} \left(\frac{10^{12} \text{ GeV}}{f_a} \right)$$

Gravity safe, electroweak natural axionic solution to strong CP and SUSY μ problems

HB, Barger, Sengupta, arXiv:1810.03713

1. Global symmetries fundamentally incompatible with gravity completion
2. Expect global symmetry to emerge as accidental (approximate) symmetry from some more fundamental gravity-safe (e.g. gauge or R-) symmetry
3. Krauss-Wilczek: gauge symmetry with charge N object condensing leaves charge e fields with Z_N discrete gauge symmetry
4. Babu et al.: Z_{22} symmetry works but charge 22 object in swampland?
5. **Better choice: discrete R-symmetries which arise from compactification of extra dimensions in string theory**

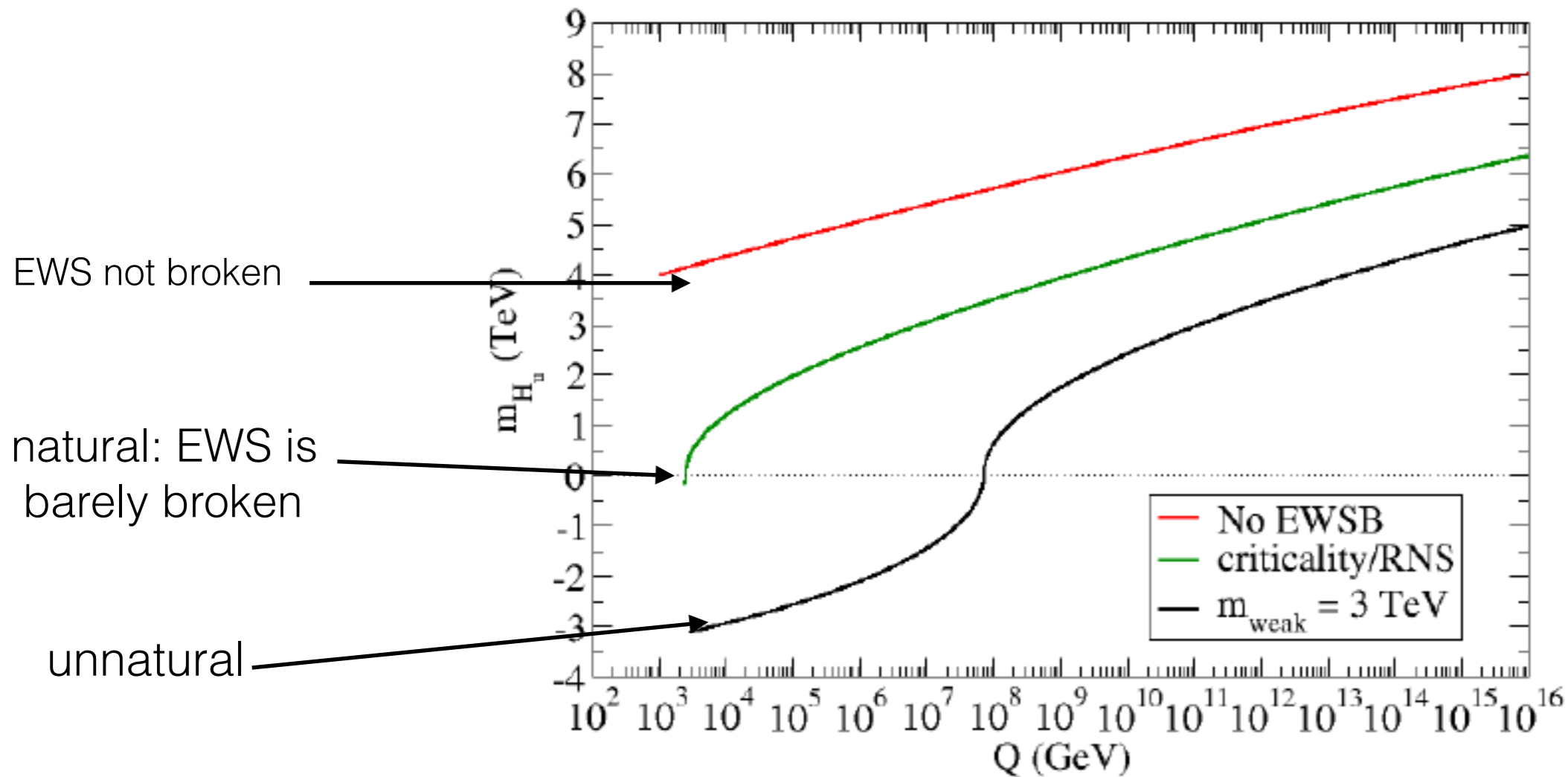
A model which works: $Z(24)$ R symmetry (see also Lee et al.)

$$W \ni f_u Q H_u U^c + f_d Q H_d D^c + f_\ell L H_d E^c + f_\nu L H_u N^c + M_N N^c N^c / 2 + \lambda_\mu X^2 H_u H_d / m_P + f X^3 Y / m_P + \lambda_3 X^p Y^q / m_P^{p+q-3}$$

- Lowest dimension PQ breaking operator contributing to scalar PQ potential $\sim 1/m_P^8$: enough suppression so that PQ is gravity-safe
- Also forbids/suppresses RPV/p-decay operators
- $\mu \sim \lambda_\mu f_a^2 / m_P$

What about $m(H_u)^2$?

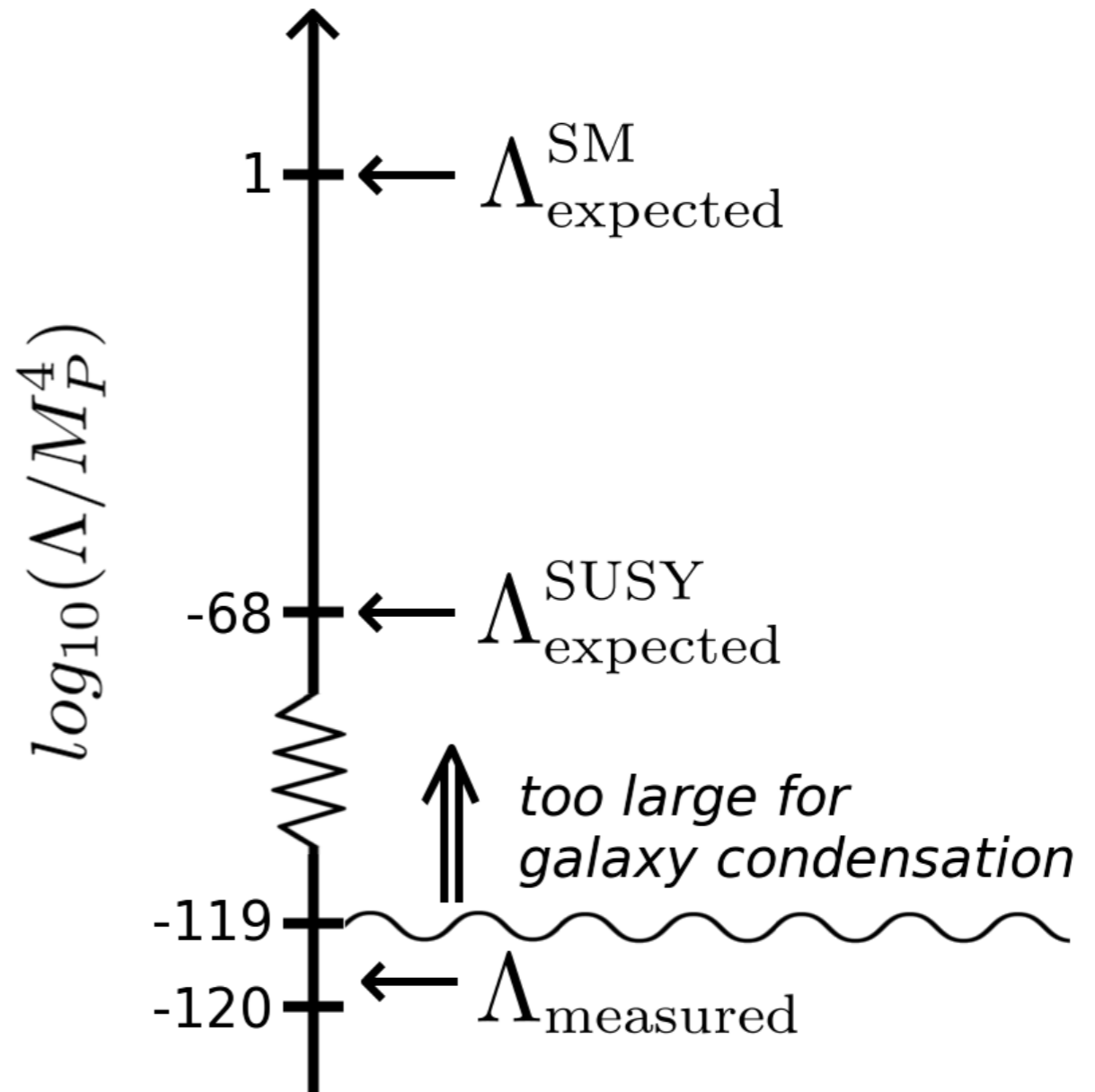
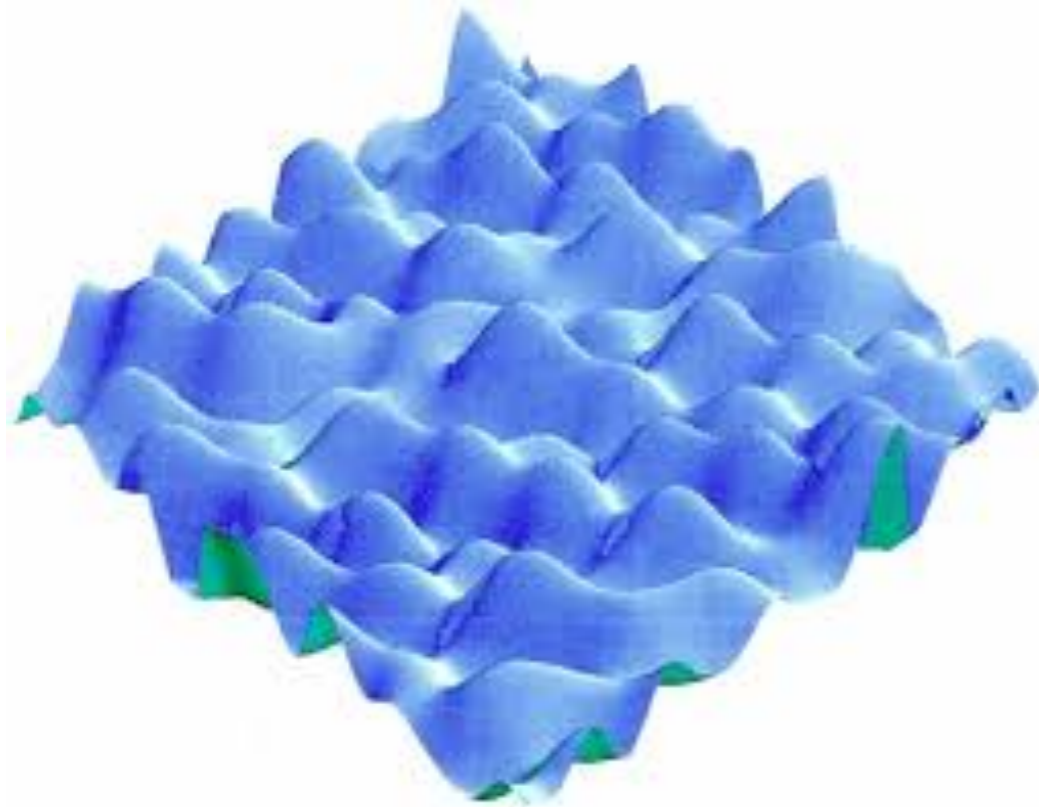
radiative corrections drive $m_{H_u}^2$ from unnatural GUT scale values to naturalness at weak scale:
radiatively-driven naturalness



Evolution of the soft SUSY breaking mass squared term $sign(m_{H_u}^2)\sqrt{|m_{H_u}^2|}$ vs. Q

Landscape of string theory vacua provides solution to cosmological constant

$$n_{vac} \sim 10^{500}?$$



Weinberg; Bousso Polchinski; Denef Douglas;...

Can similar reasoning explain scale of soft SUSY breaking?

Statistical analysis of SUSY breaking scale in IIB theory: M. Douglas, hep-th/0405279

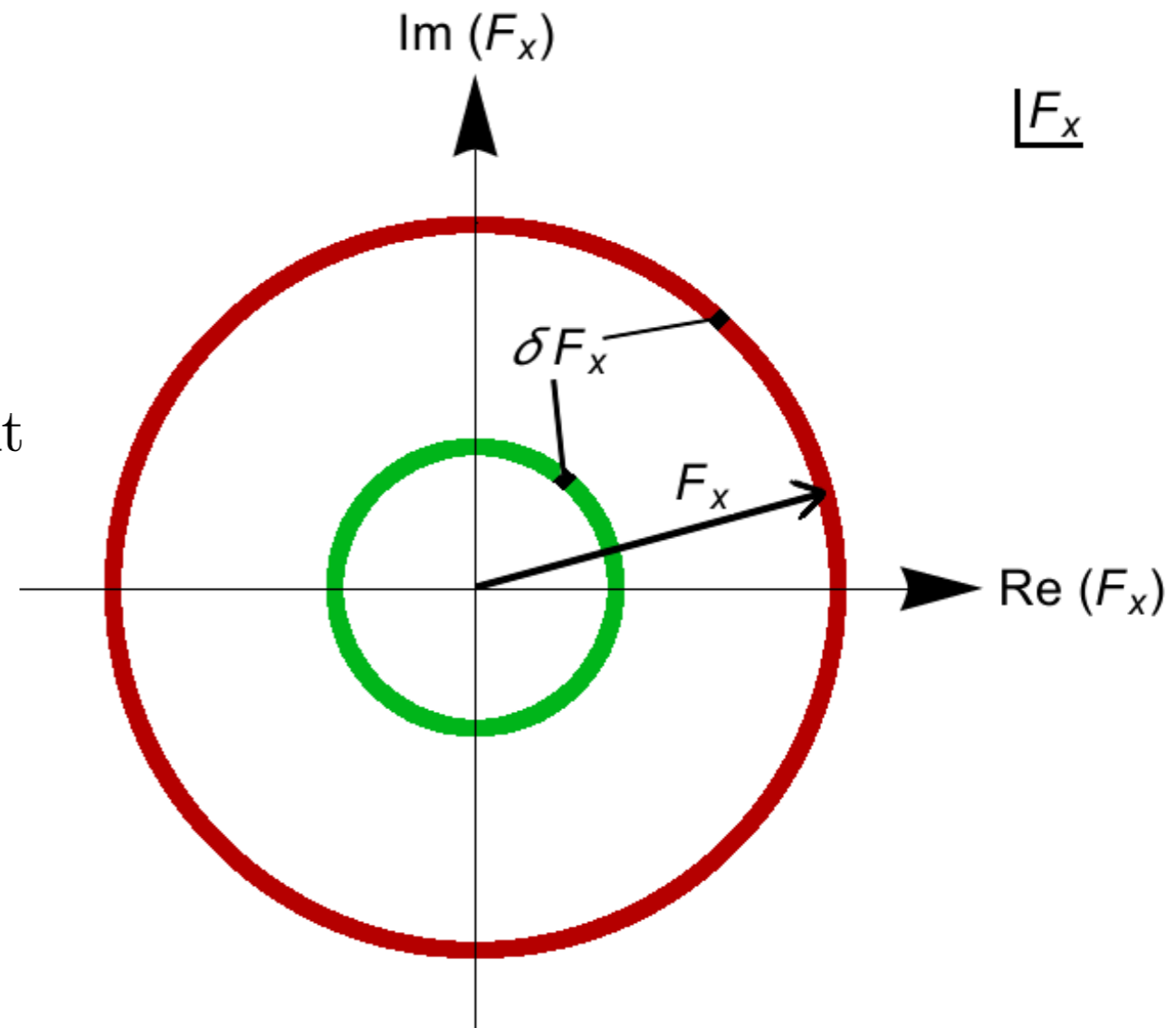
start with 10^{500} string vacua states

- string theory landscape contains vast ensemble of $N=1, d=4$ SUGRA EFTs at high scales
- the EFTs contain the SM as weak scale EFT
- the EFTs contain visible sector +potentially large hidden sector
- visible sector contains MSSM plus extra gauge singlets (e.g. a PQ sector, RN neutrinos,...)
- SUGRA is broken spontaneously via superHiggs mechanism via either F- or D- terms or in general a combination

Why do soft terms take on values needed for natural (barely-broken) EWSB? string theory landscape?

- assume model like MSY/CCK where $\mu \sim 100$ GeV
- then $m(\text{weak})^2 \sim |m_{H_u}^2|$
- If all values of SUSY breaking field $\langle F_X \rangle$ equally likely, then mild (linear) statistical draw towards large soft terms
- This is balanced by anthropic requirement of weak scale $m_{\text{weak}} \sim 100$ GeV

Anthropic selection of $m_{\text{weak}} \sim 100$ GeV:
 If m_W too large, then weak interactions $\sim (1/m_W^4)$ too weak
 weak decays, fusion reactions suppressed
 elements not as we know them



Denef&Douglas: statistics of SUSY breaking in landscape

DD observation: W_0 distributed uniformly as complex variable allows dynamical neutralization of Λ while not influencing SUSY breaking

Then, number of flux vacua containing spontaneously broken SUGRA with SUSY breaking scale m_{hidden}^2 is:

$$dN_{vac}[m_{hidden}^2, m_{weak}, \Lambda] = f_{SUSY}(m_{hidden}^2) \cdot f_{EWFT} \cdot f_{cc} dm_{hidden}^2$$

- $f_{cc} \sim \Lambda/m^4$ where DD maintain $m \sim m_{string}$ and not m_{hidden}
- $f_{SUSY}(m_{hidden}^2) \sim (m_{hidden}^2)^{2n_F+n_D-1}$ for uniformly distributed values of F and D breaking fields
- $f_{EWFT} \sim m_{weak}^2/m_{soft}^2$ (?) where $m_{soft} \sim m_{3/2} \sim m_{hidden}^2/m_P$

$$n = 2n_F + n_D - 1$$

$$f_{SUSY} \sim m_{soft}^n$$

n_F	n_D	n
0	1	0
1	0	1
0	2	1
1	1	2
0	3	2
2	0	3
2	1	4

landscape favors high scale SUSY breaking
tempered by $f(EWFT)$ anthropic penalty!

What about DD/AD anthropic penalty $f_{EWFT} \sim m_{weak}^2/m_{soft}^2$?

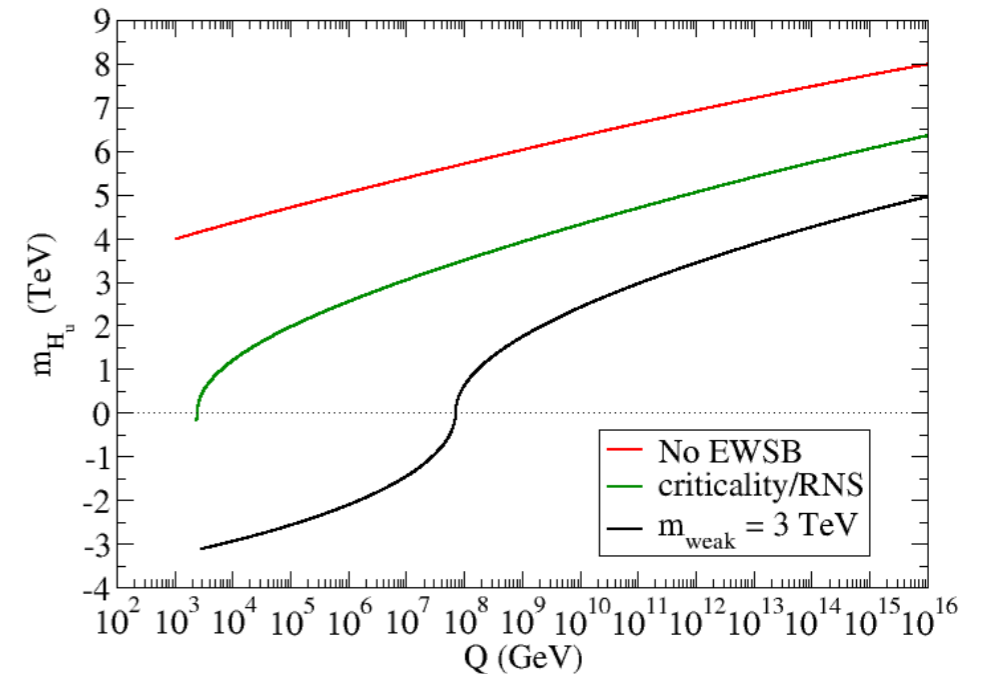
This fails in a variety of *practical* cases:

- A -terms get large: \Rightarrow CCB minima
- $m_{H_u}^2$ too large: fail to break EW symmetry

Must require proper EWSB!

Even if EWS properly broken, then

- large A_t reduces EWFT in the $\Sigma_u^u(\tilde{t}_{1,2})$
- large $m_{H_u}^2(m_{GUT})$ needed to radiatively drive $m_{H_u}^2$ to natural value at weak scale



Better proposal: $f_{EWFT} \Rightarrow \Theta(30 - \Delta_{EW})$

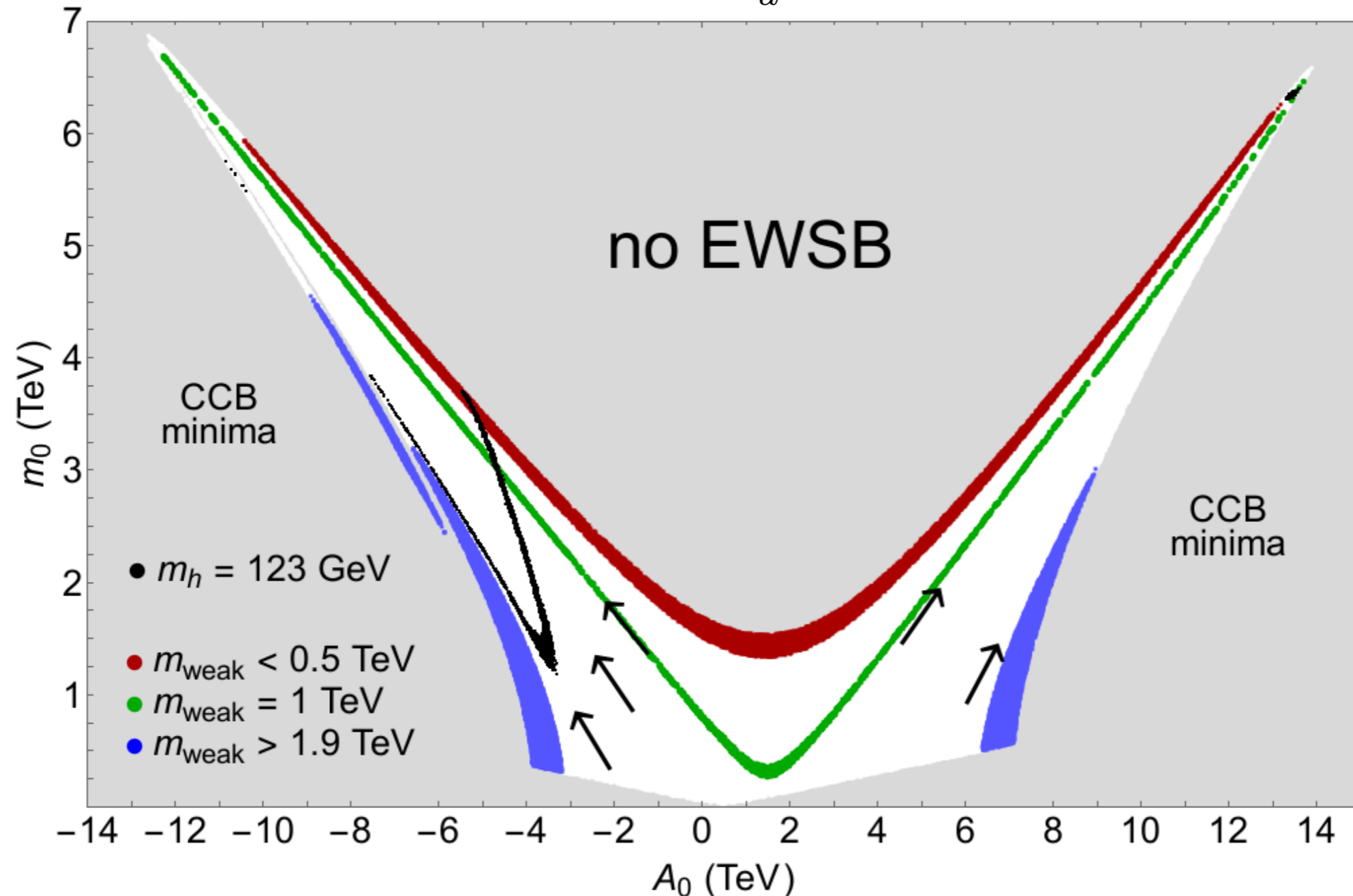
keeps calculated weak scale within factor ~ 4 of measured weak scale

$$m_{weak} \equiv m_{W,Z,h} \sim 100 \text{ GeV}$$

Assume $\mu \sim 100 - 200 \text{ GeV}$ via *e.g.* rad PW breaking: then m_Z variable and may be large depending on soft terms $m_{H_{u,d}}^2$ and $\Sigma_{u,d}^{u,d}(i)$

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2$$

$$m_{H_u} = 1.3m_0$$



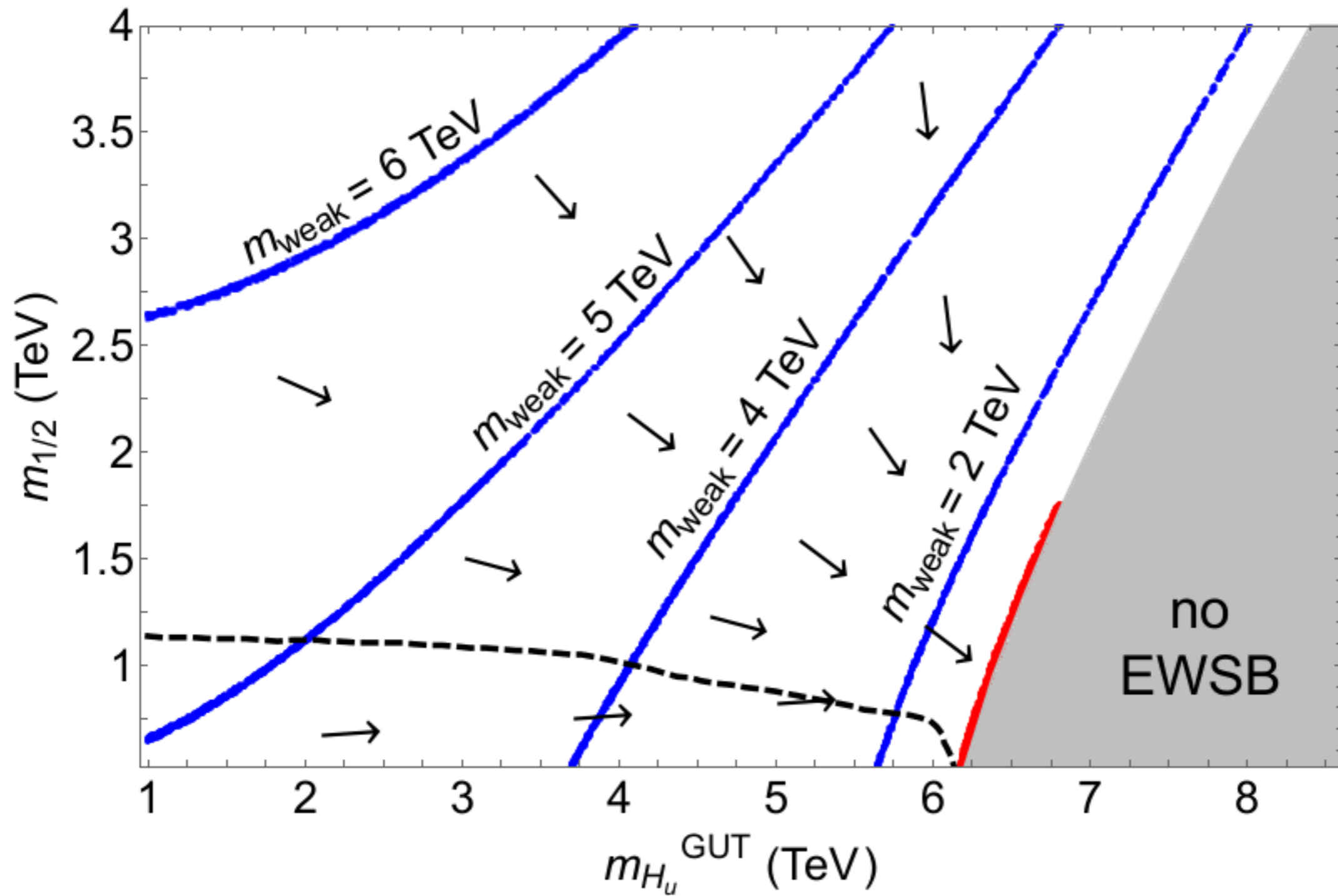
statistical draw to large soft terms balanced by anthropic draw toward red ($m(\text{weak}) \sim 100 \text{ GeV}$): then $m(\text{Higgs}) \sim 125 \text{ GeV}$ and natural SUSY spectrum!

Denef, Douglas, JHEP0405 (2004) 072

Giudice, Rattazzi, NPB757 (2006) 19;

HB, Barger, Savoy, Serce, PLB758 (2016) 113

$$m_0 = 5 \text{ TeV}$$



statistical/anthropic draw toward FP-like region

Recent work: place on more quantitative footing:
scan soft SUSY breaking parameters in NUHM3 model
as $m(\text{soft})^n$ along with $f(\text{EWFT})$ penalty

We scan according to m_{soft}^n over:

- $m_0(1, 2) : 0.1 - 40 \text{ TeV},$

- $m_0(3) : 0.1 - 20 \text{ TeV},$

- $m_{1/2} : 0.5 - 10 \text{ TeV},$

- $A_0 : 0 - -60 \text{ TeV},$

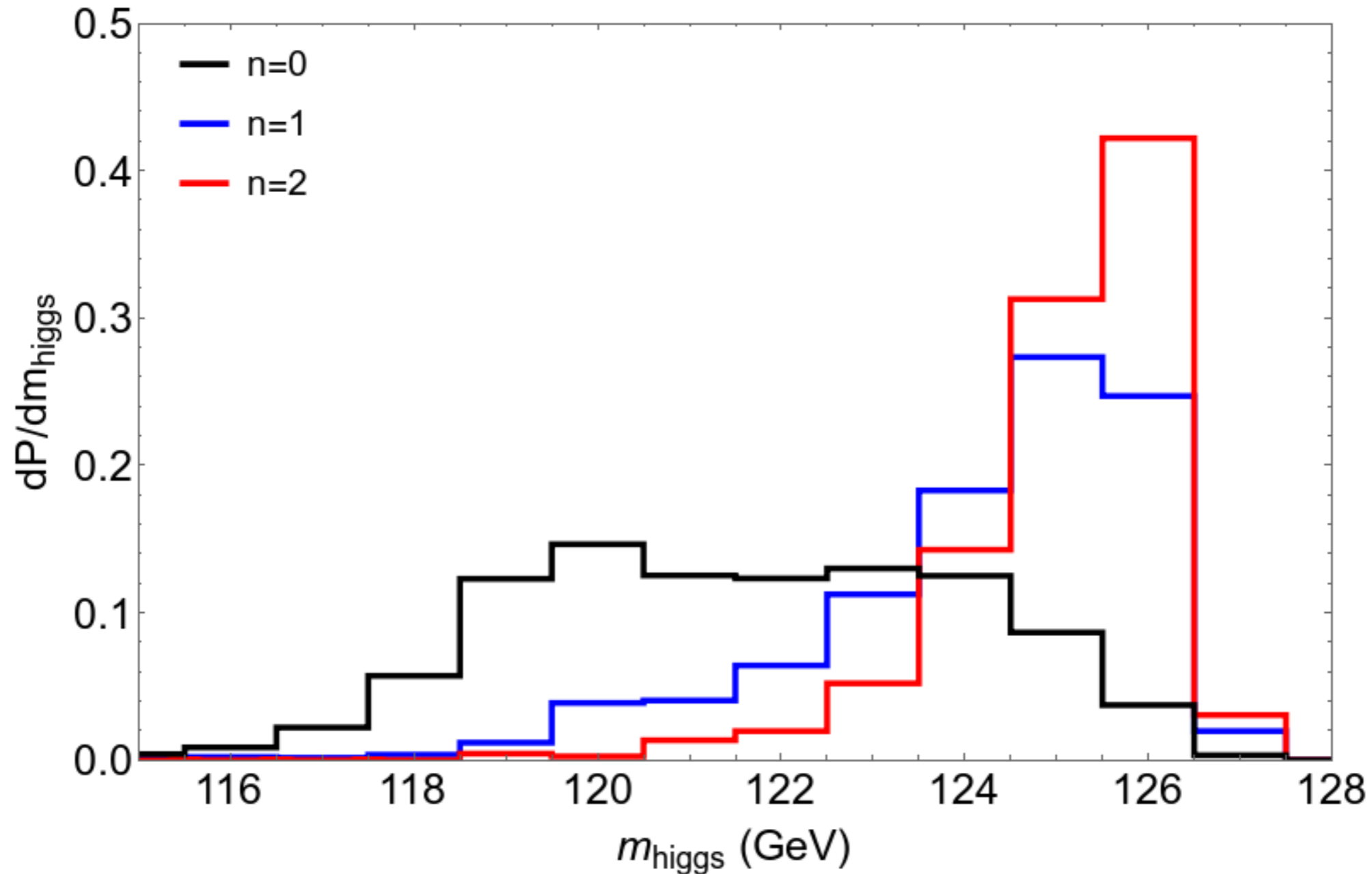
- $m_A : 0.3 - 10 \text{ TeV},$

$\tan \beta : 3 - 60 \quad (\text{flat})$

$\mu = 150 \text{ GeV (fixed)}$

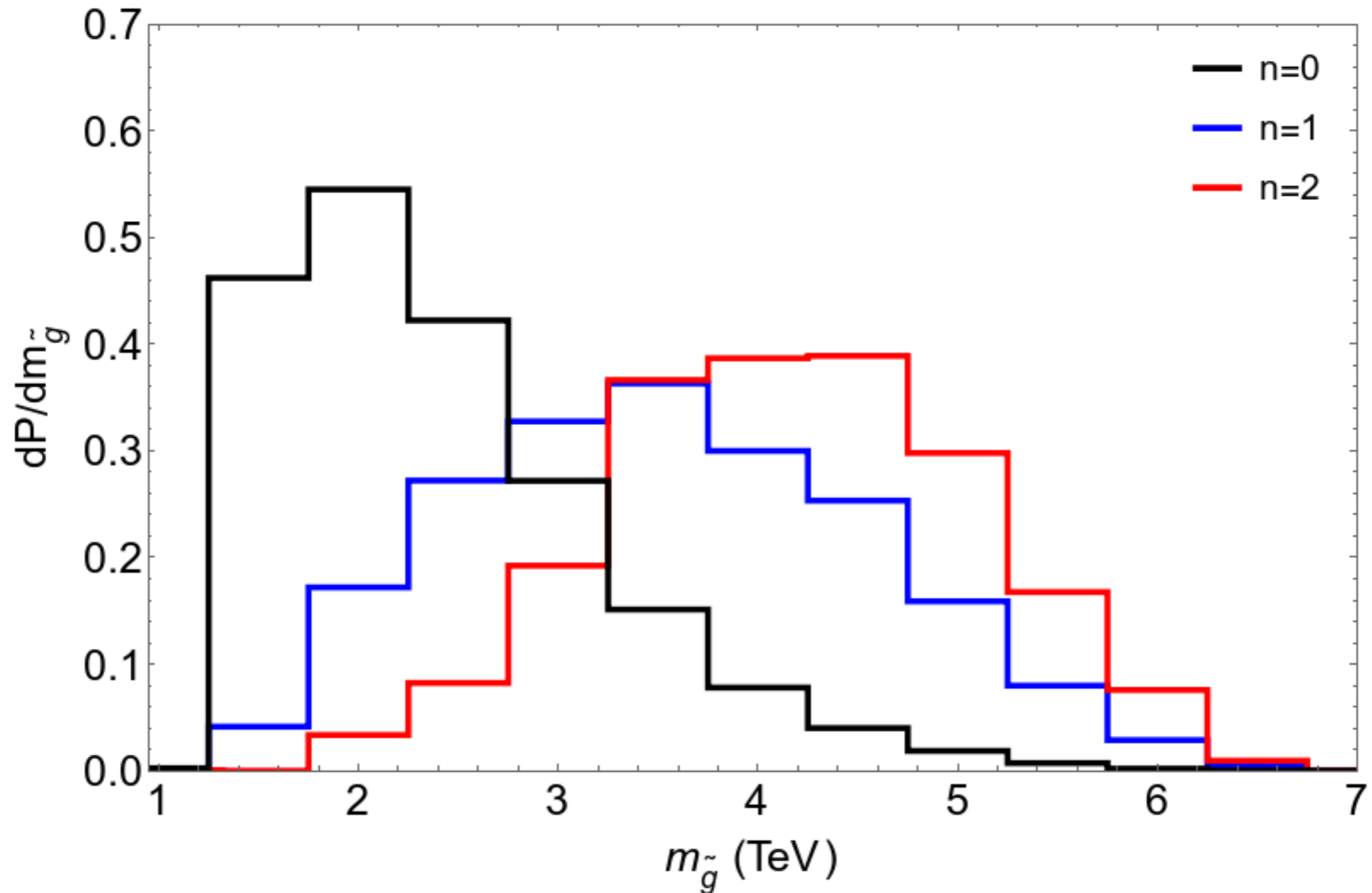
Making the picture more quantitative:

$$dN_{vac}[m_{hidden}^2, m_{weak}, \Lambda] = f_{SUSY}(m_{hidden}^2) \cdot f_{EFT} \cdot f_{cc} dm_{hidden}^2$$



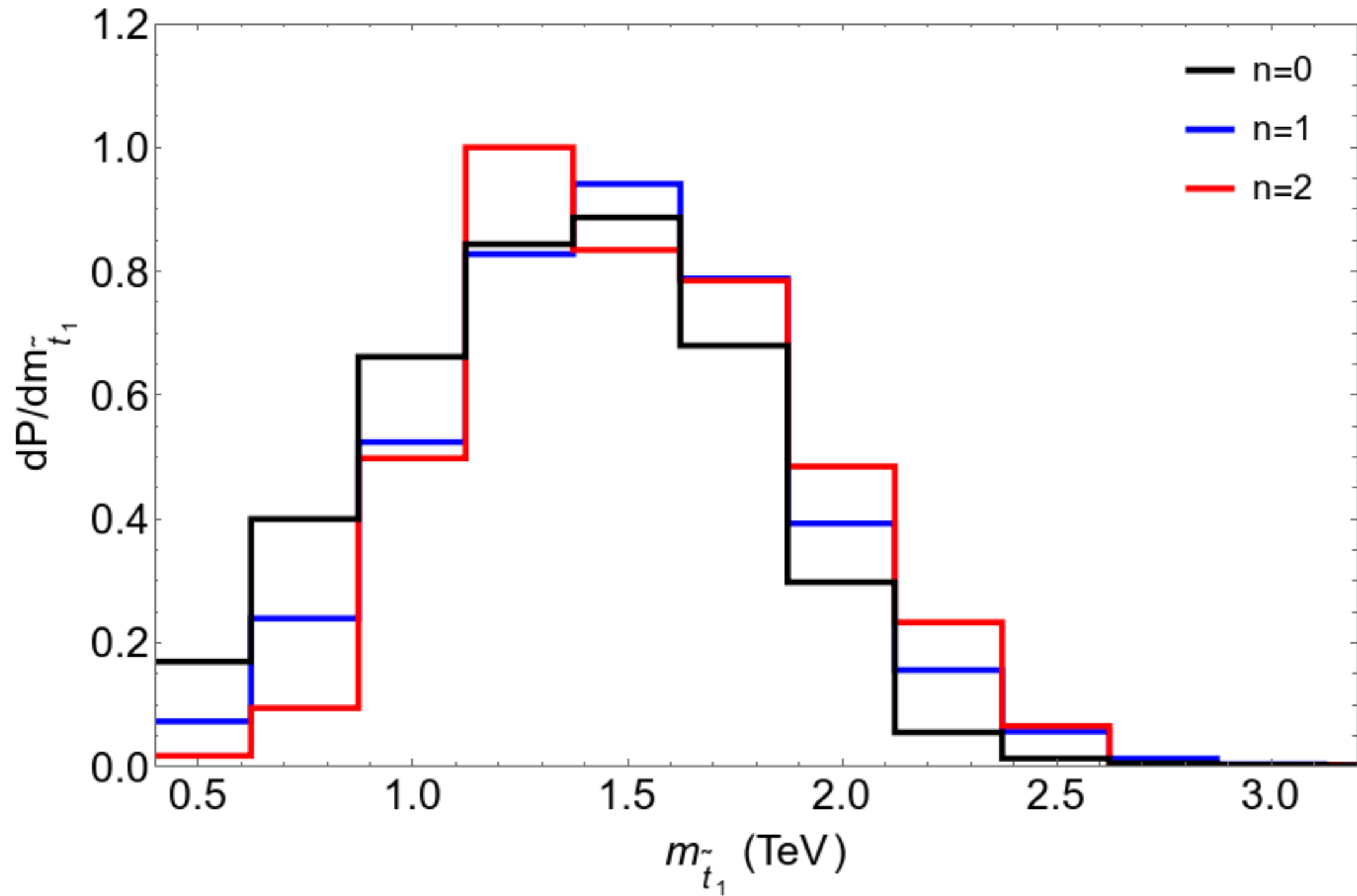
$m(h) \sim 125$ most favored for $n=1,2$

What is corresponding distribution for gluino mass?

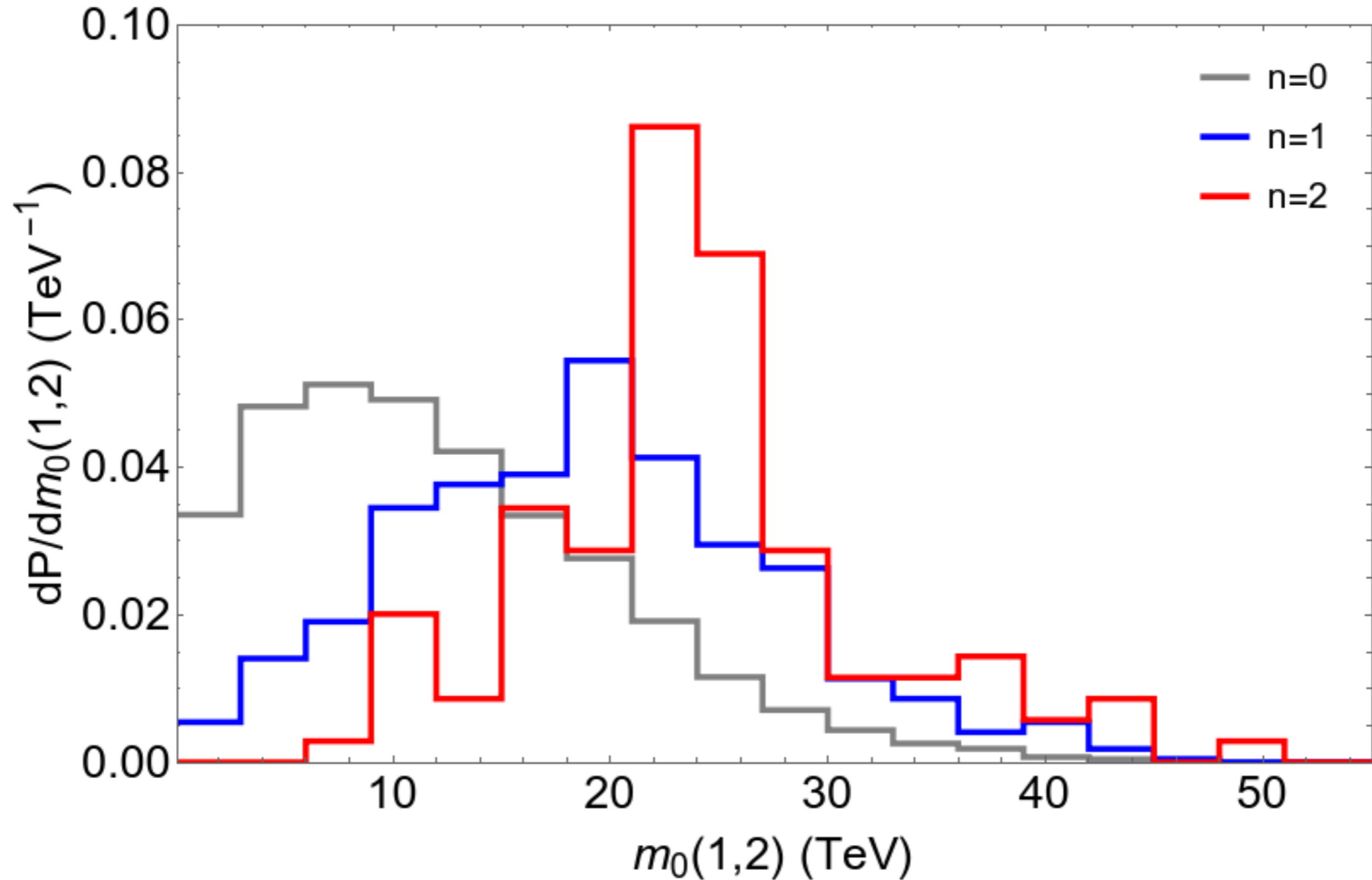


typically beyond LHC 14 reach (may need HE-LHC)

and m_{t_1} ?



first/second generation sfermions pulled
to 10–30 TeV thus softening any SUSY flavor/CP problems



What role would ILC play with predicted light higgsinos?

The ILC as a natural SUSY discovery machine and precision microscope: from light higgsinos to tests of unification

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$$e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow (\ell\nu_\ell \tilde{\chi}_1^0) + (q\bar{q}' \tilde{\chi}_1^0)$$

measure $m(jj) < m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$ and $E(jj)$

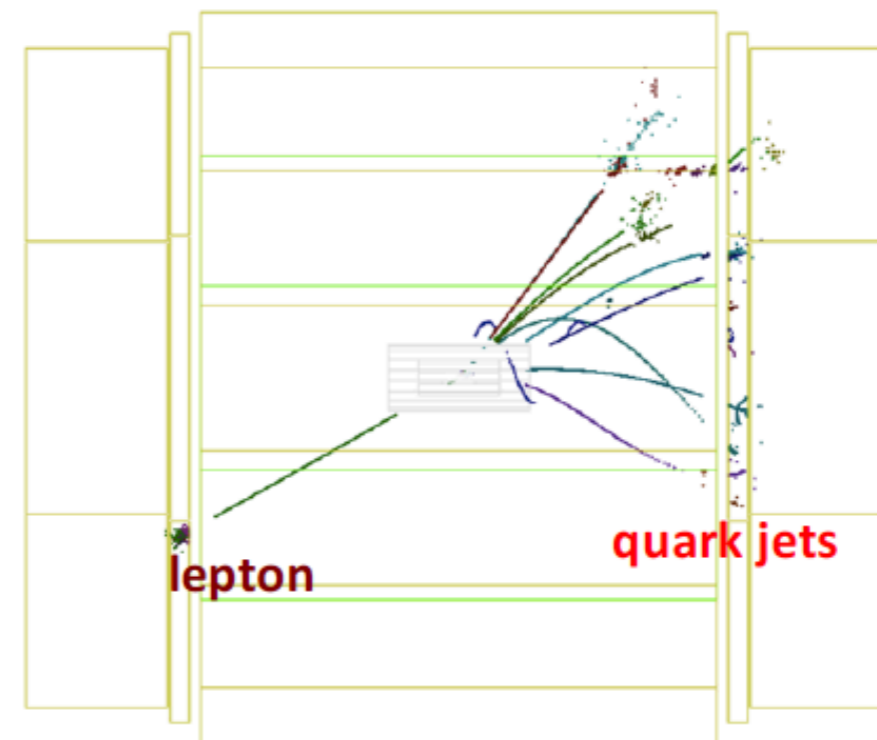
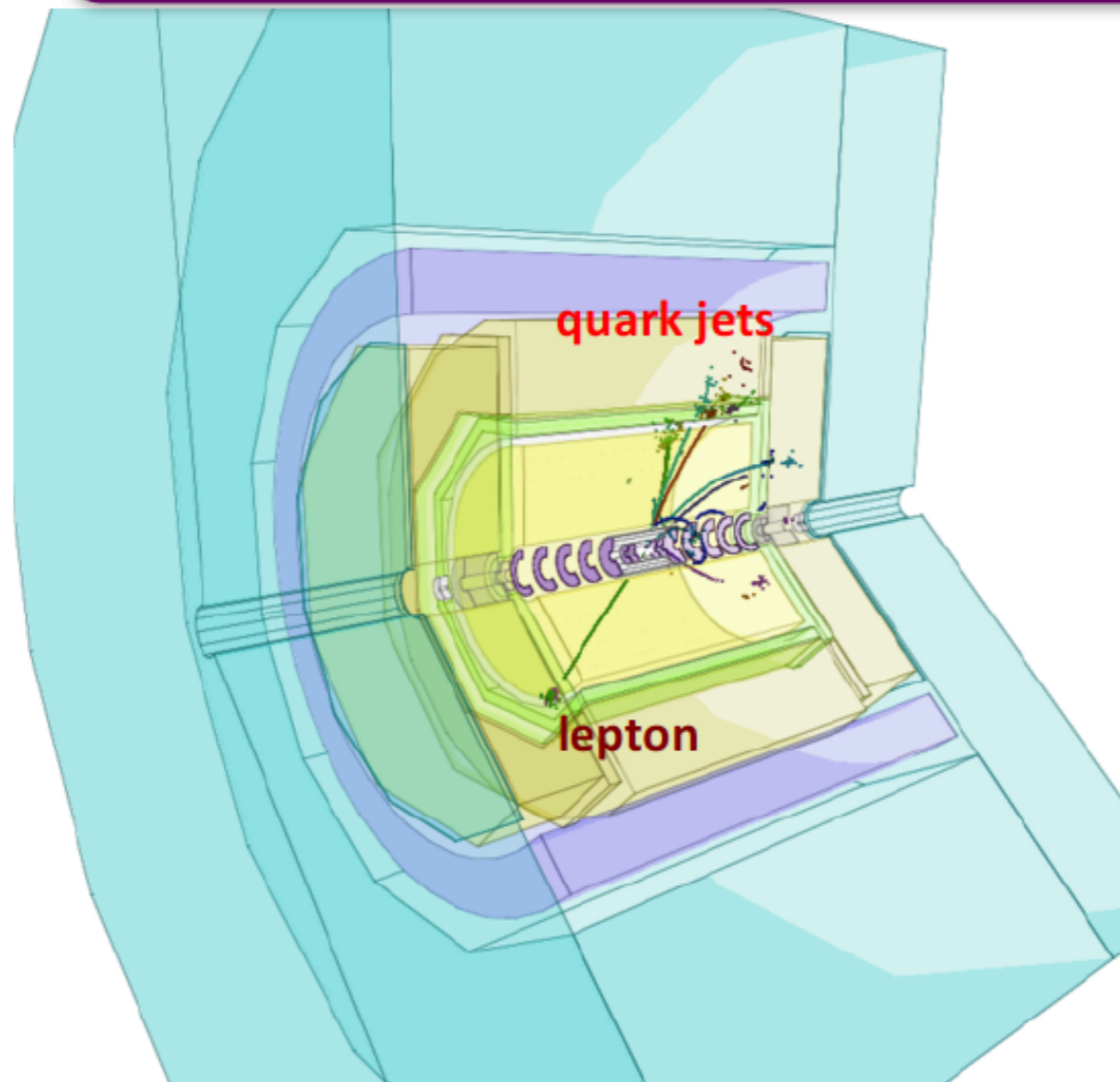
soft visible particles since small higgsino mass gaps

How do these signals look in the detector? (2)

$\sqrt{s} = 500 \text{ GeV}$

Chargino pair production with semileptonic decay

$$e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 qq' \ell \nu$$



$$e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 + (\ell^+\ell^-\tilde{\chi}_1^0)$$

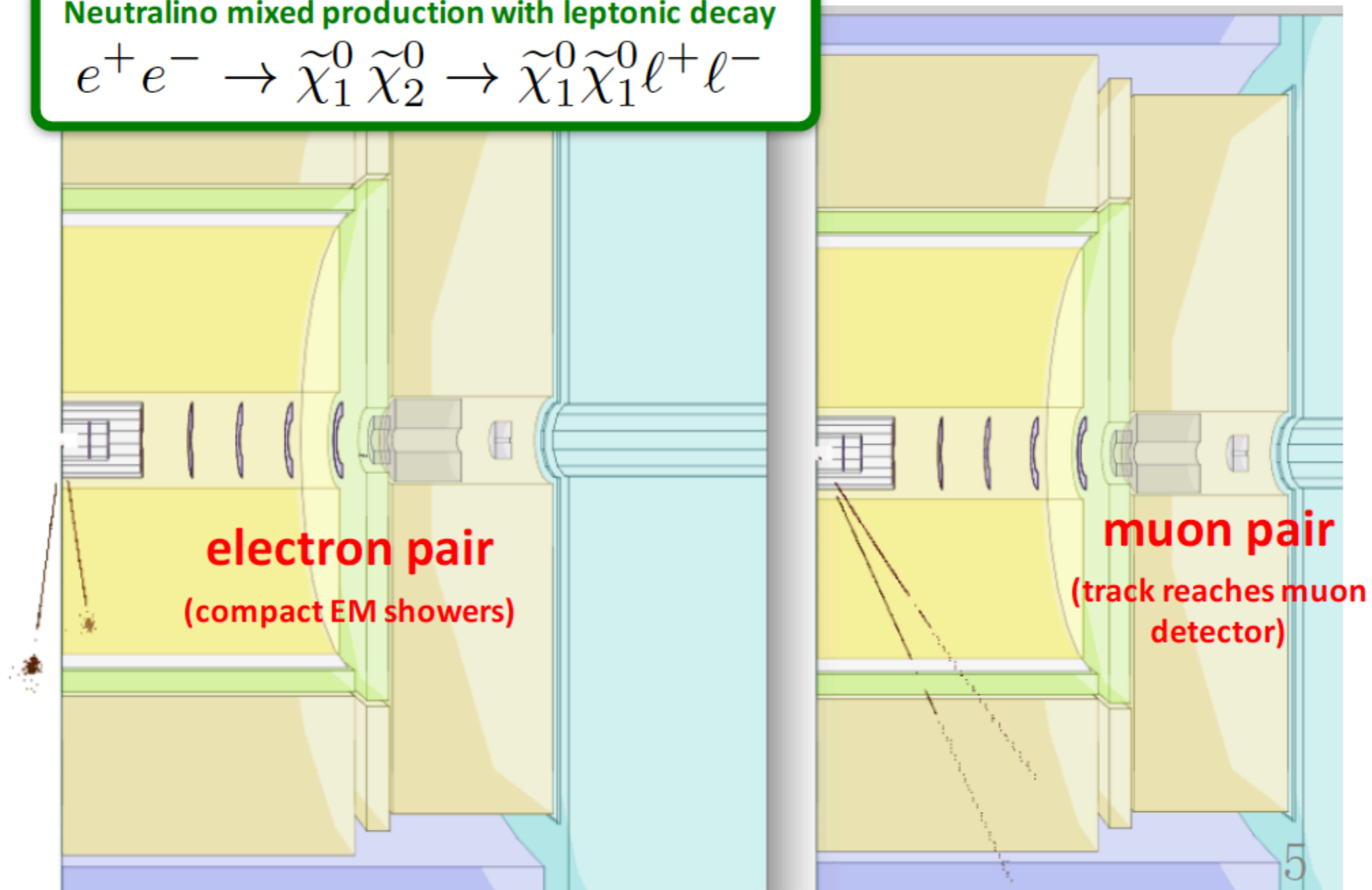
measure $m(\ell^+\ell^-) < m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ and $E(\ell^+\ell^-)$

How do these signals look in the detector? (1)

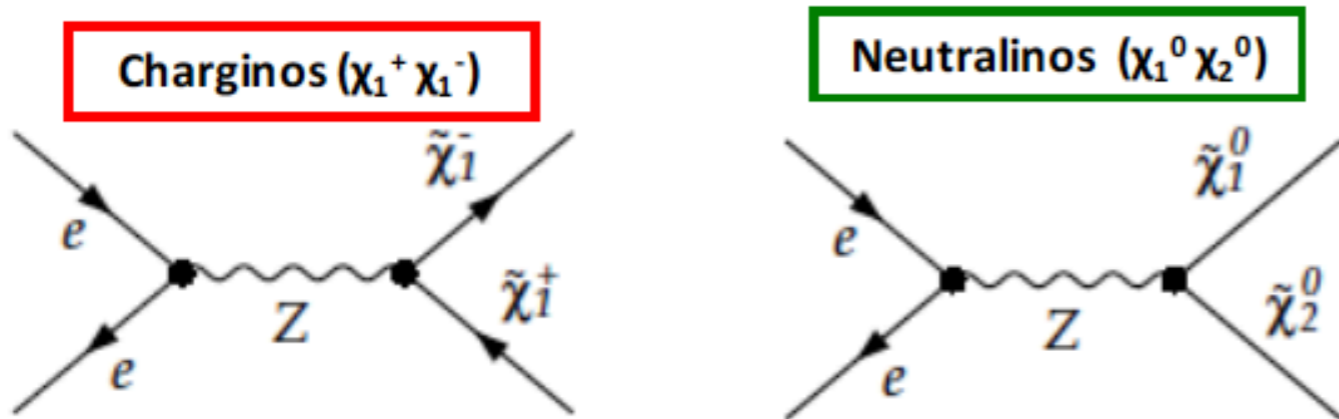
$\sqrt{s} = 500$ GeV

Neutralino mixed production with leptonic decay

$$e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0\ell^+\ell^-$$



Benchmarks in this Study



ΔM complies with naturalness (no use of ISR tag)

Unit: GeV	ILC1	ILC2	nGMM1
M(N1)	102.7	148.1	151.4
M(N2)	124.0	157.8	155.8
$\Delta M(N2,N1)$	21.3	9.7	4.4
M(C1)	117.3	158.3	158.7
$\Delta M(C1,N1)$	14.6	10.2	7.3

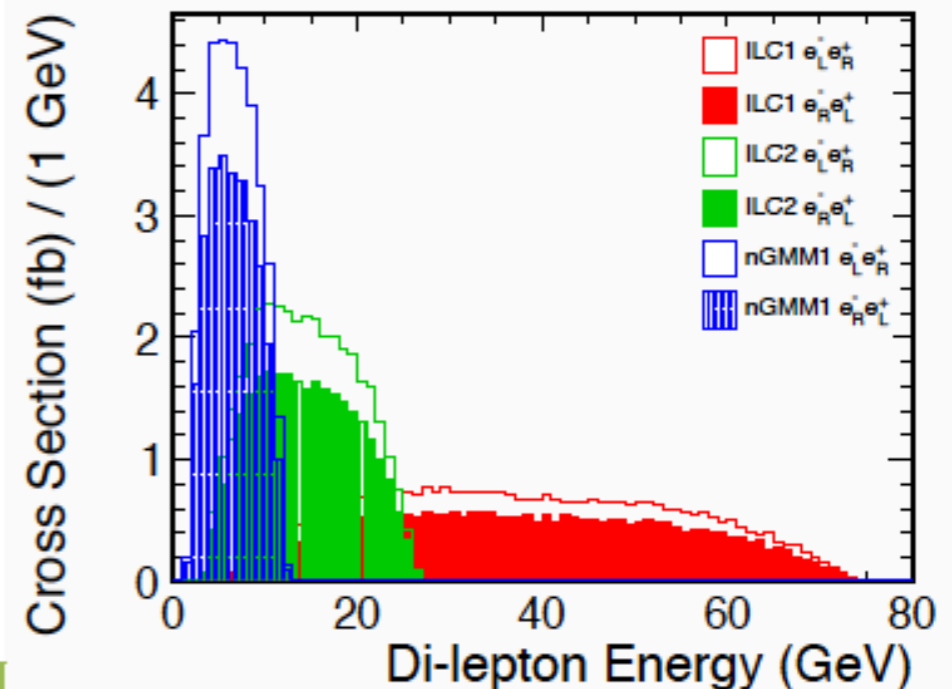
Process (Pe-,Pe+)	ILC1	ILC2	nGMM1
C1C1 (-1,+1)	1799.9	1530.5	1520.6
C1C1 (+1,-1)	334.5	307.2	309.5
N1N2 (-1,+1)	490.9	458.9	463.5
N1N2 (+1,-1)	378.5	353.8	357.3

Event Generator: WHIZARD v1.95, DBD setup, TDR beam parameters

4 light Higgsinos

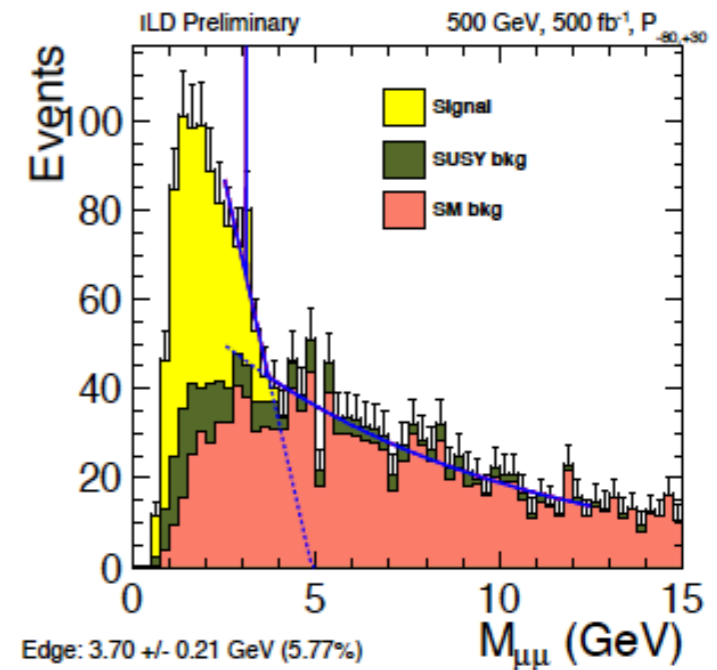
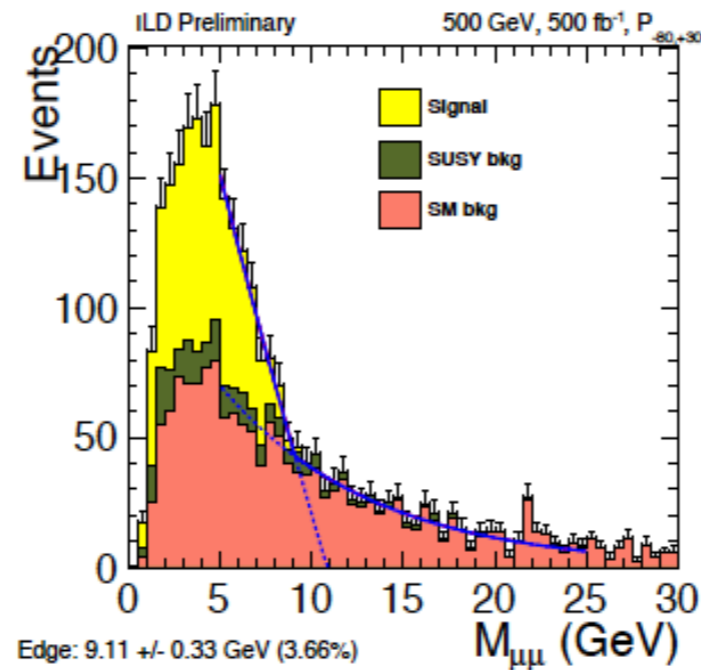
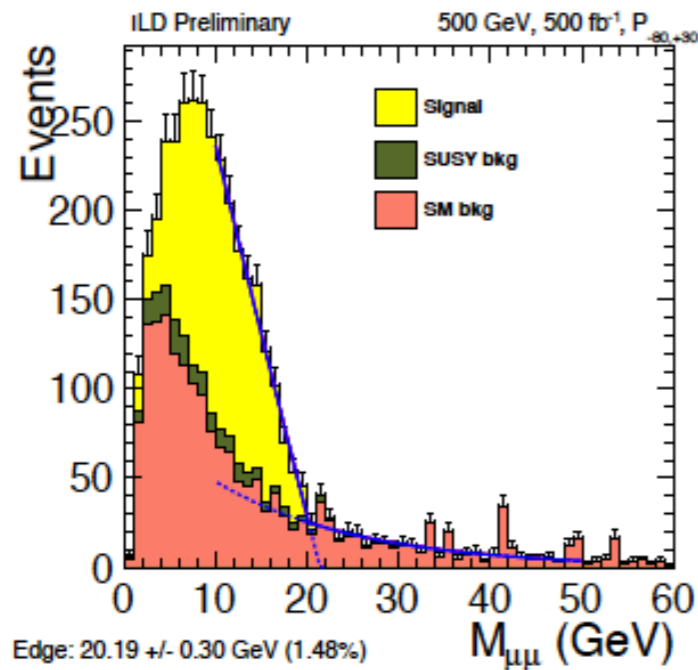
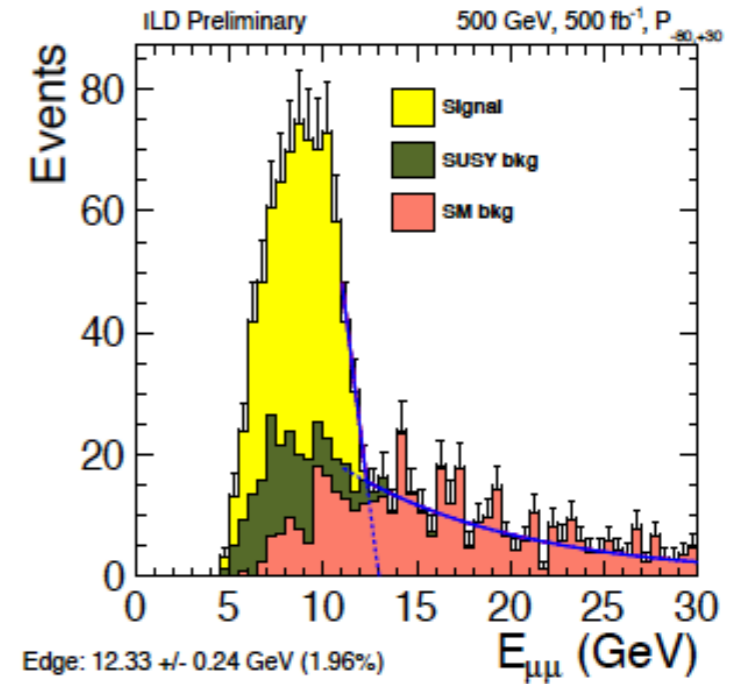
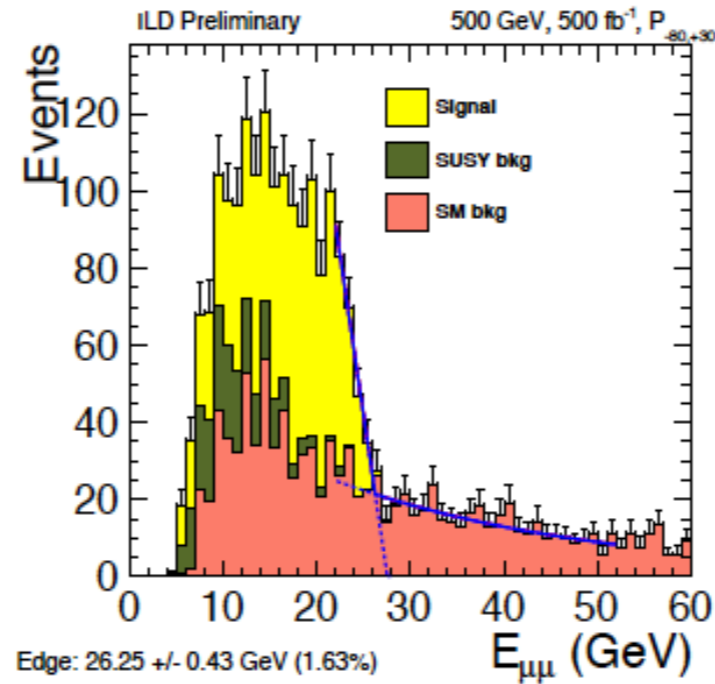
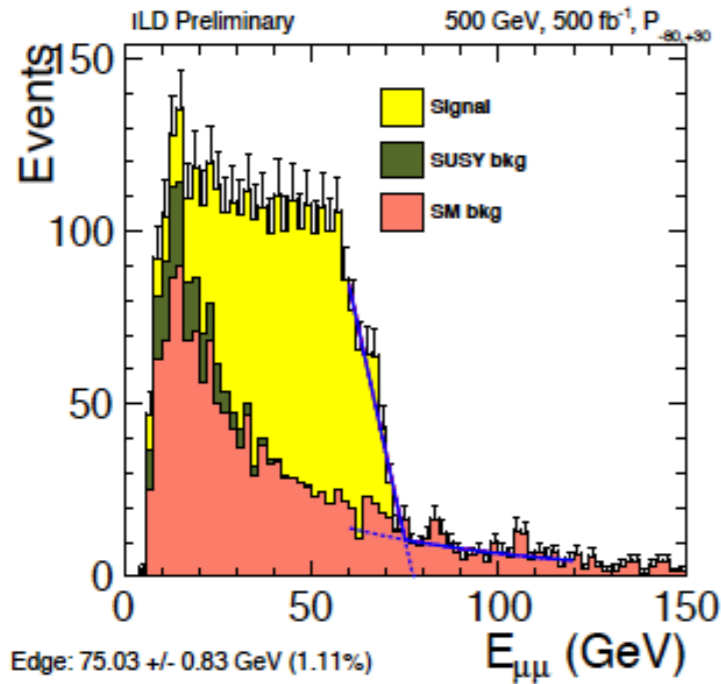
- $\sqrt{s} = 500$ GeV
- full ILD detector simulation

Good precision achievable even for challenging ΔM with soft leptons/jets

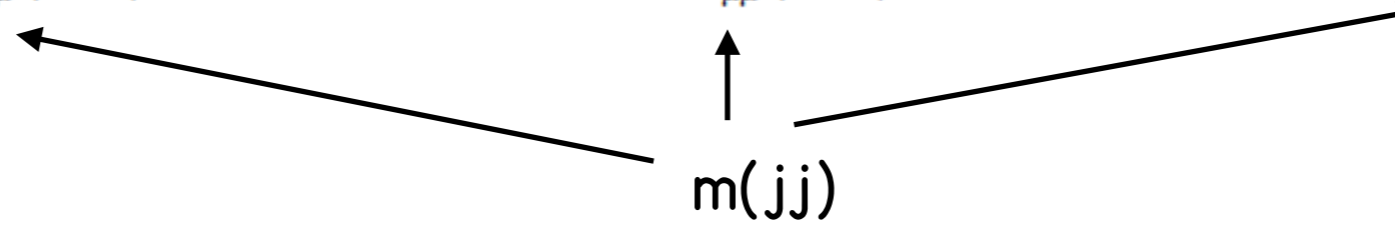
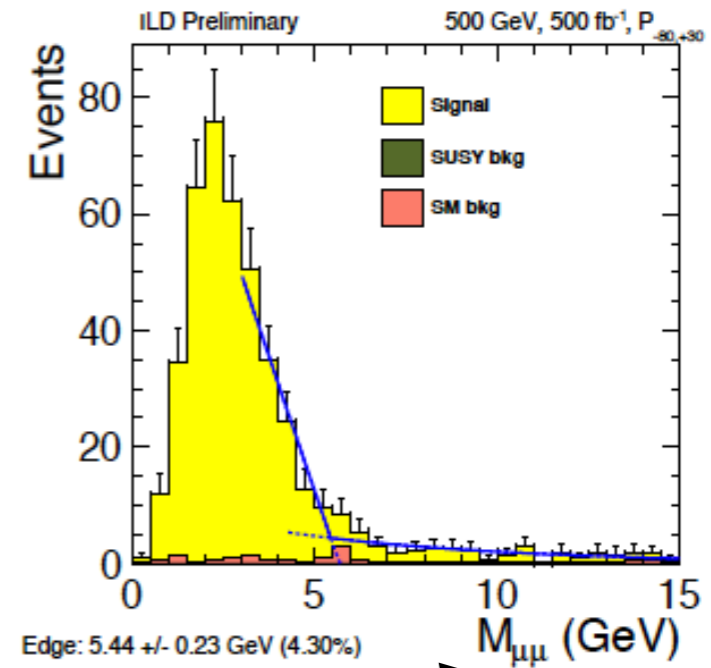
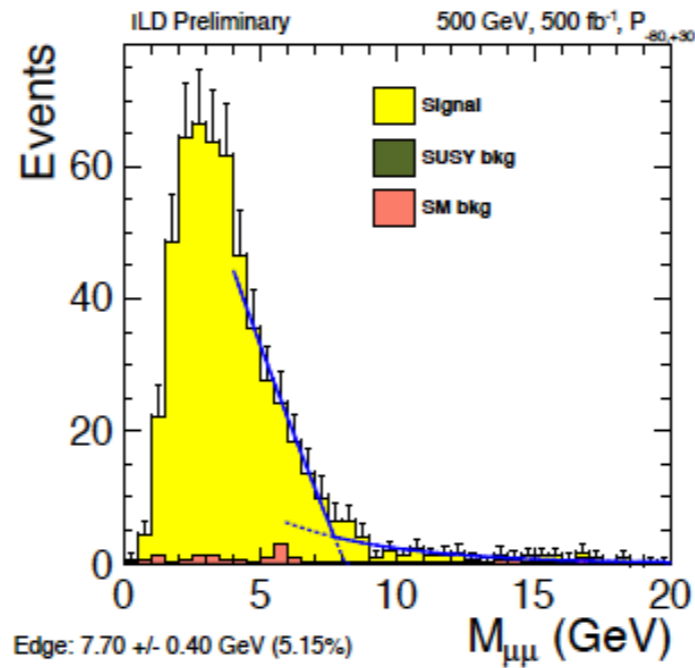
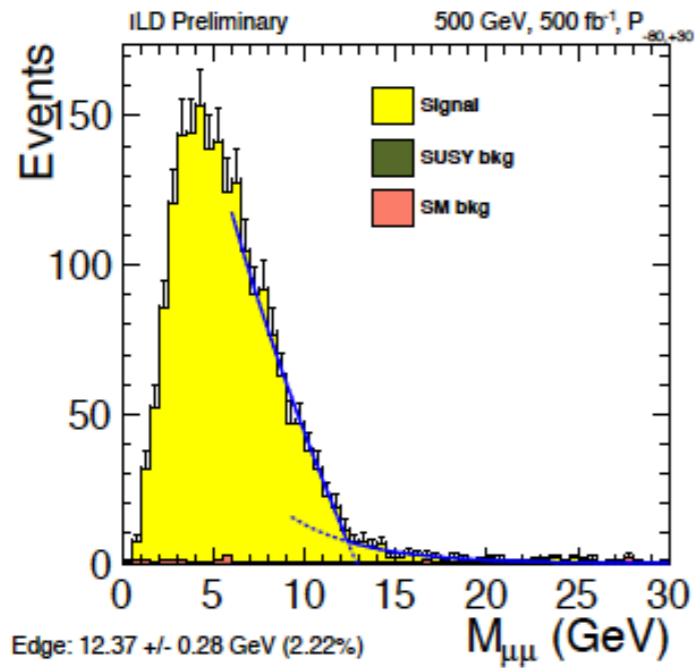
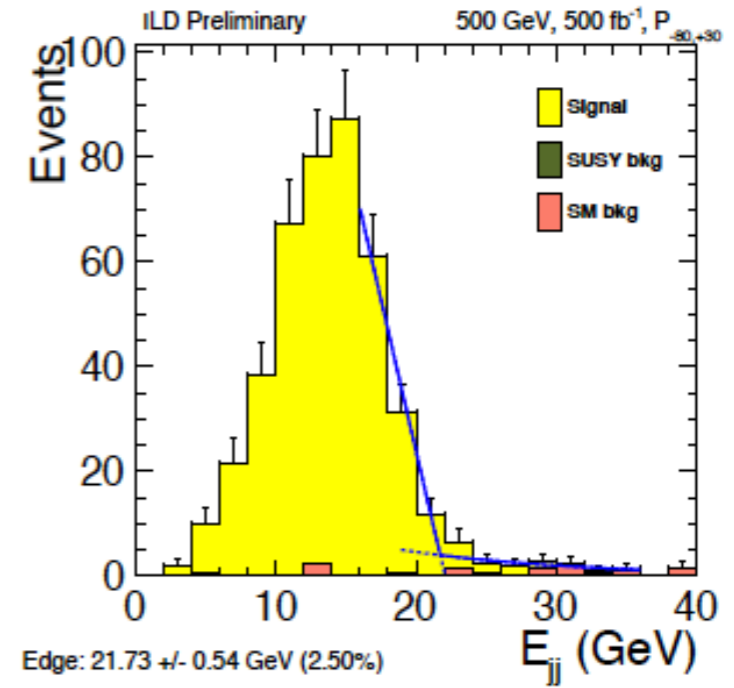
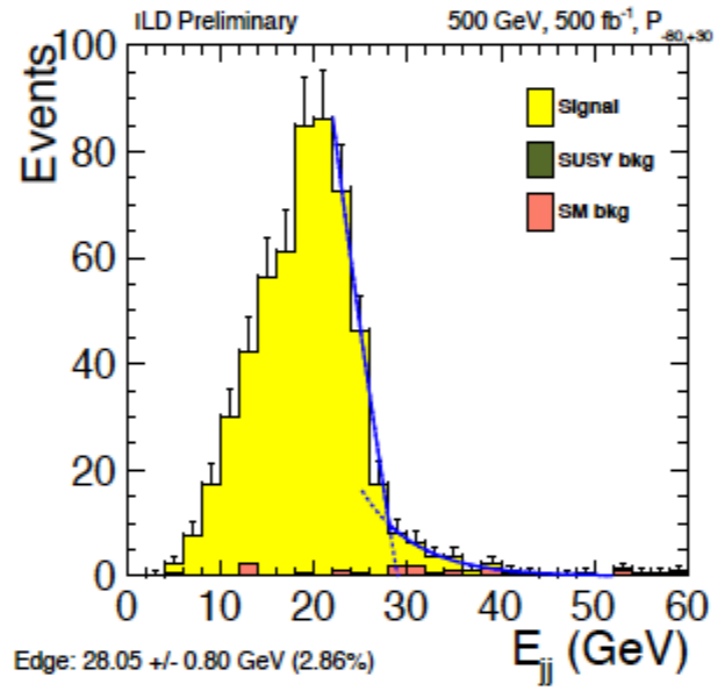
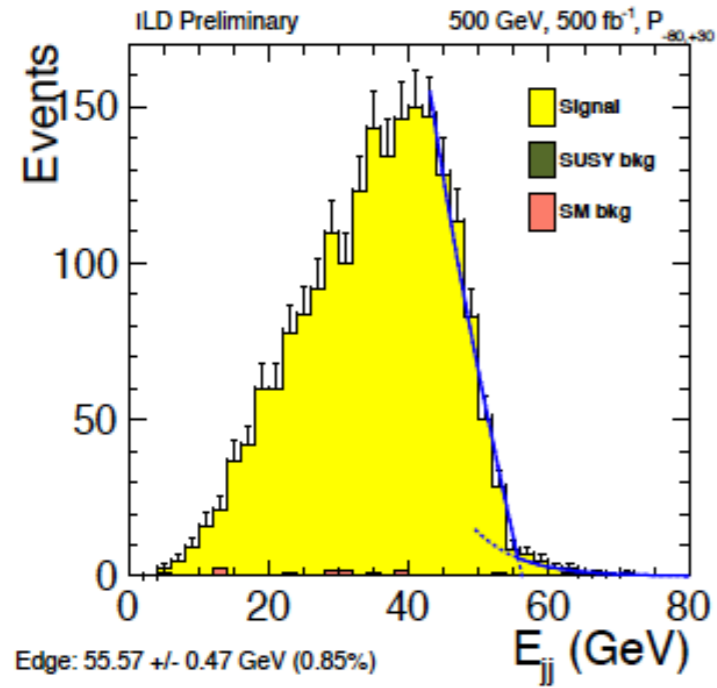


Cross sections for $\sqrt{s} = 500$ GeV
Similar for all benchmarks

$E(\ell^+\ell^-)$ and $m(\ell^+\ell^-)$ measurements $\Rightarrow m_{\tilde{\chi}_2^0}$ and $m_{\tilde{\chi}_1^0}$ to $\sim 1\%$ typically



$E(jj)$ and $m(jj)$ measurements $\Rightarrow m_{\tilde{\chi}_1^\pm}$ and $m_{\tilde{\chi}_1^0}$ to $\sim 1\%$ typically

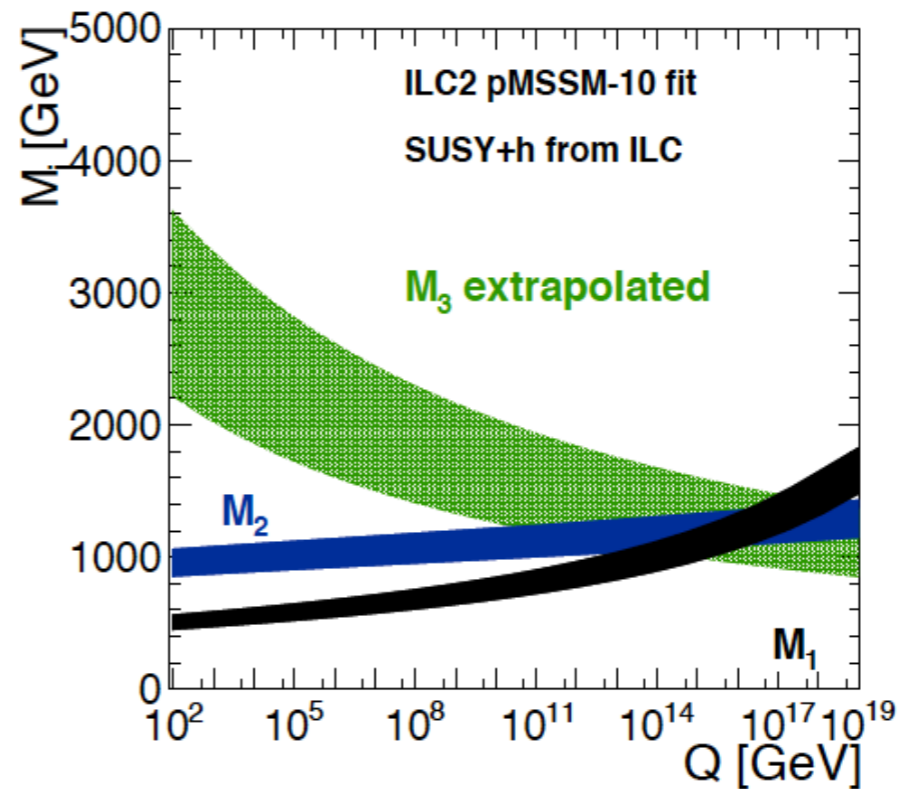
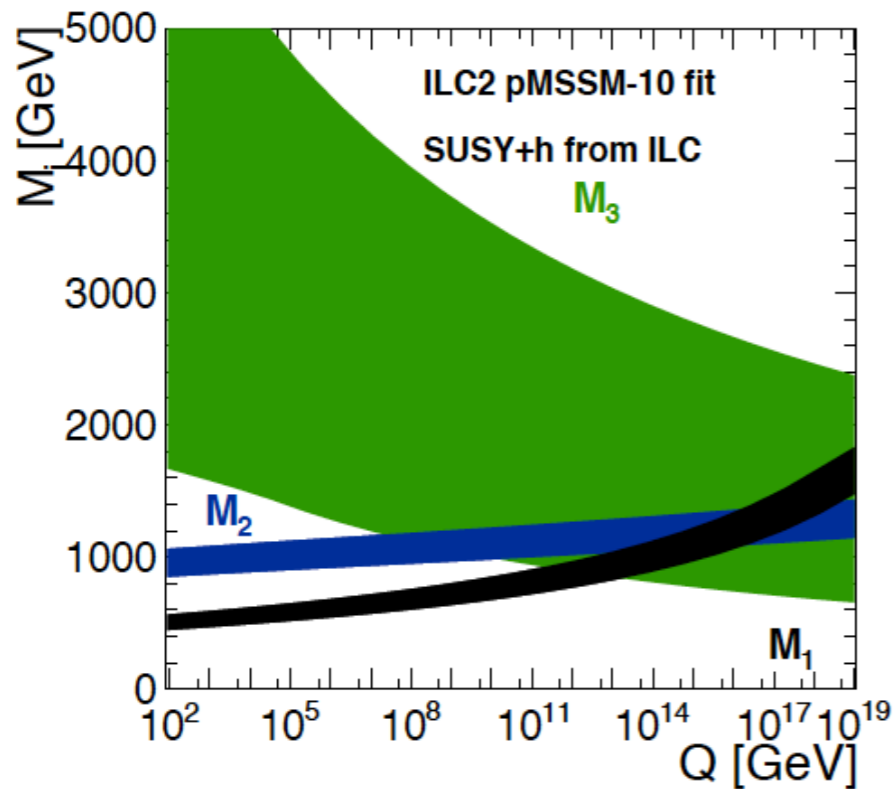
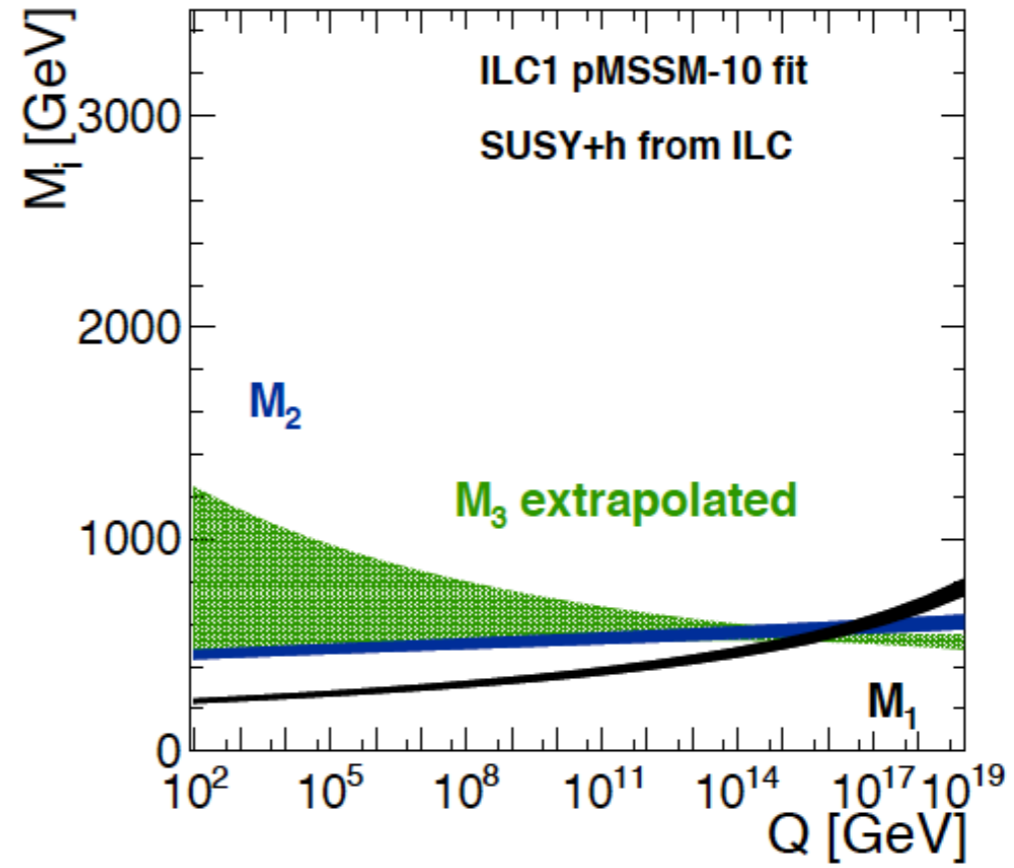
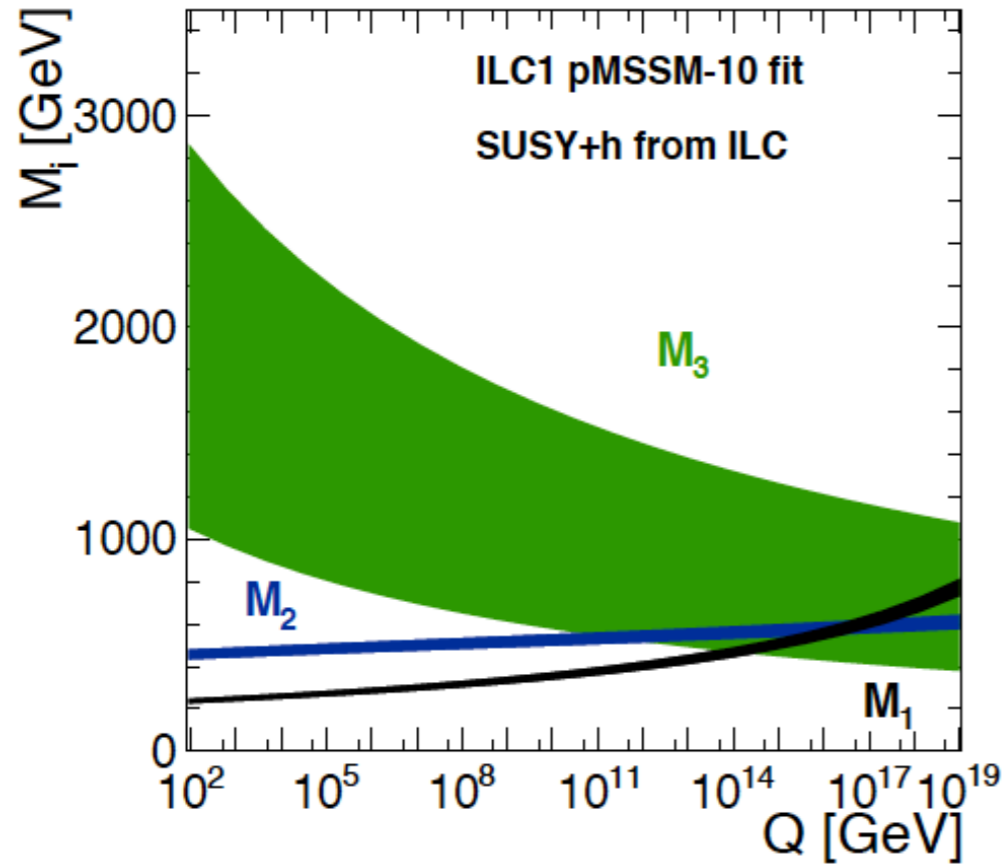


- weak scale fits \Rightarrow absolute higgsino masses and mass gaps
- masses and gaps allow sensitivity to gaugino masses $M_1(\text{bino})$ and $M_2(\text{wino})$
- combine with polarized beam σ s
- combine with fits to h properties
- extract gaugino masses and other SUSY parameters

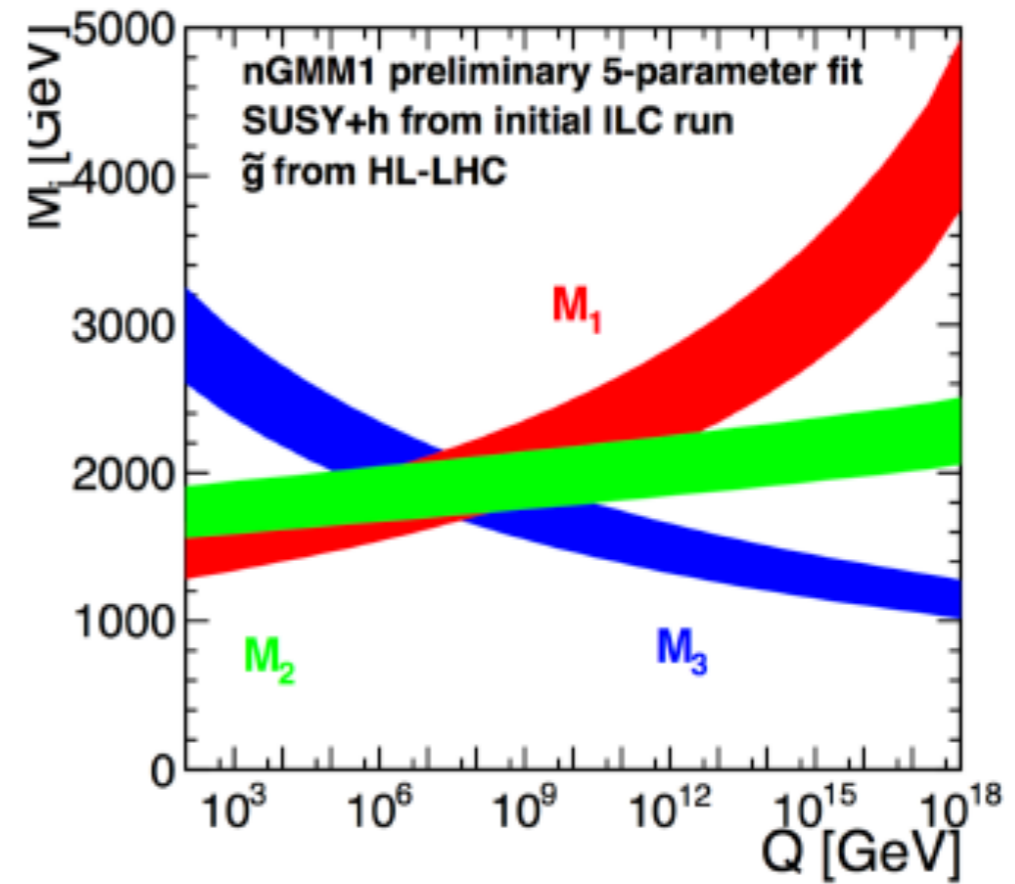
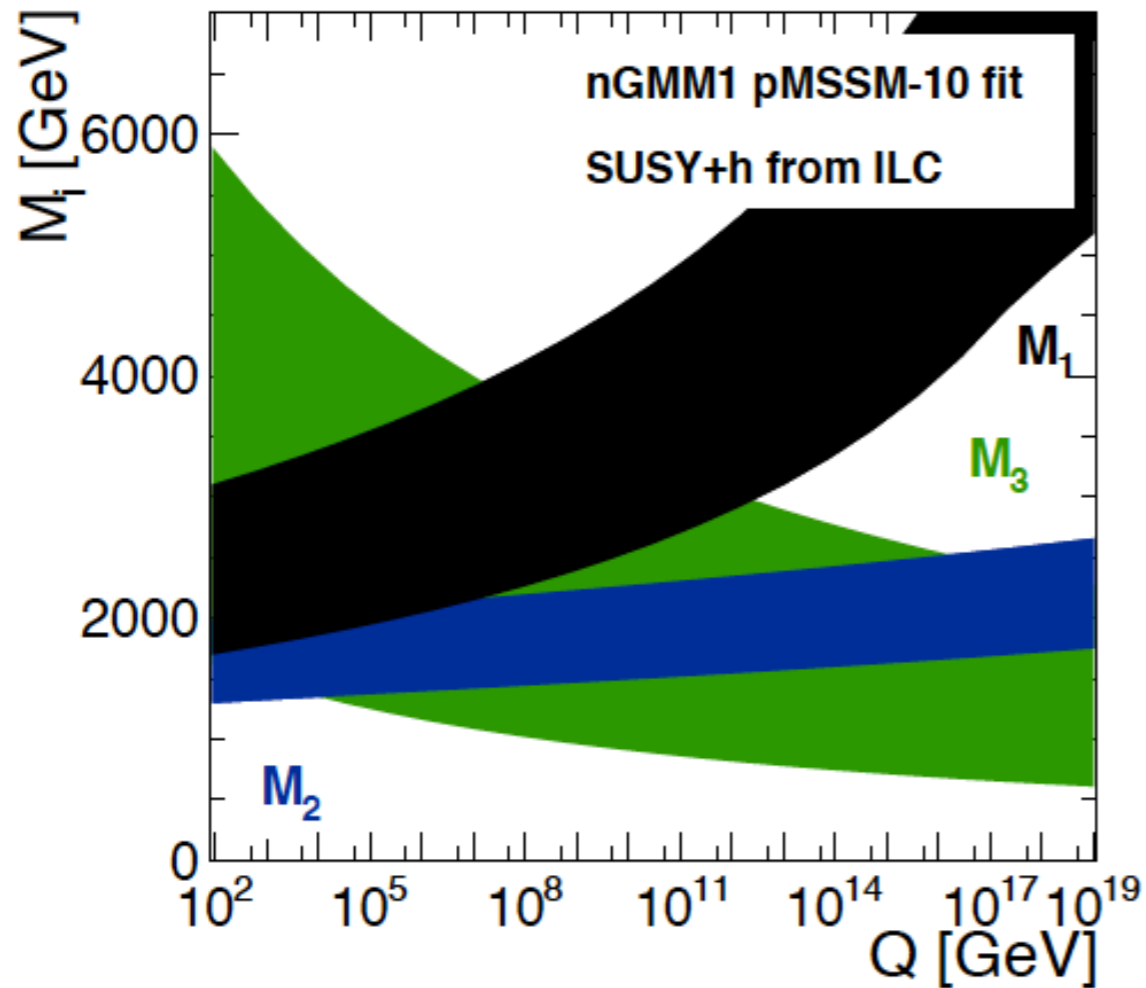
parameter	ILC1 NUHM2 true	best fit point	1σ CL	2σ CL
$M_{1/2}$	568.3	556.7	+24.3 -20.3	+37.7 -43.1
μ	115.0	105.3	+12.8 -8.2	+14.0 -14.5
$\tan\beta$	10.0	11.4	+5.6 -1.6	+11.4 -1.6
m_A	1000	968	+167 -65	+288 -130
M_0	7025	7685	+1243 -1917	+2311 -2095
A_0	-10427	-11064	+2695 -1422	+2927 -2698
χ^2	0.0013	0.0011		

parameter	ILC1 pMSSM true	pMSSM-4			pMSSM-10		
		best fit point	1σ CL	2σ CL	best fit point	1σ CL	2σ CL
M_1	250	250.2	+8.2 -7.7	+17.1 -15.1	251.3	+8.6 -15.7	+17.2 -23.7
M_2	463	463.3	+8.0 -8.1	+16.2 -14.9	465.8	+24.2 -23.0	+31.4 -49.8
μ	115.0	115.0	+0.2 -0.2	+0.3 -0.3	115.7	+10.9 -4.7	+20.3 -6.1
$\tan\beta$	10.0	10.0	+0.1 -0.1	+0.2 -0.2	9.7	+8.8 -3.0	+45.3 -3.5
m_A	1000				1050	+310 -180	+607 -296
M_3	1270				1412	+1791 -1104	+1411 -2843
$M_{L(3)}$	7150				7063	+2029 -4311	+2645 -5632
$M_{U(3)}$	1670				1751	+2414 -628	+4498 -740
$M_{Q(3)}$	4820				4951	+2324 -3226	+3858 -3226
$A_{t=b=\tau}$	-4400				-4591	+1371 -973	+1647 -2949
χ^2		0.0011			0.1360		

Check scale for $M_1=M_2$ in mass unification?
Combine with LHC gluino mass measurement?

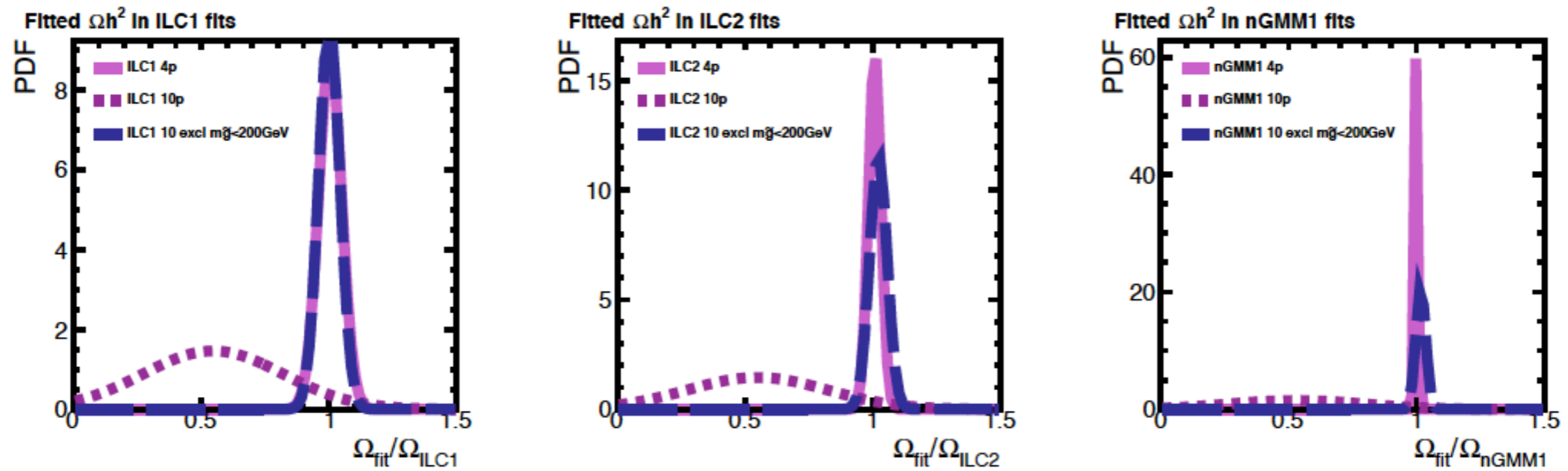


Compare with mirage unification scenario where gaugino masses unify at intermediate scale:



nGMM1 (ILC initial stage)

Can also test WIMP dark matter properties:



Higgsino-like DM underproduced-
cross check with direct detection rates:
confirm need 2nd DM particle: axion?

Conclusions:

- Naturalness: light higgsinos $\sim 100\text{-}300\text{ GeV}$
- Stops, gluinos OK in multi-TeV range
- μ emerges from gravity-safe SUSY axion model: $Z(24)^R$
- DM= axion+higgsino admixture
- natural soft terms, $m(h)=125\text{ GeV}$ from statistics of string landscape
- ILC500: SUSY discovery machine and higgsino factory
- precision higgsino measurements allow tests of gaugino unification

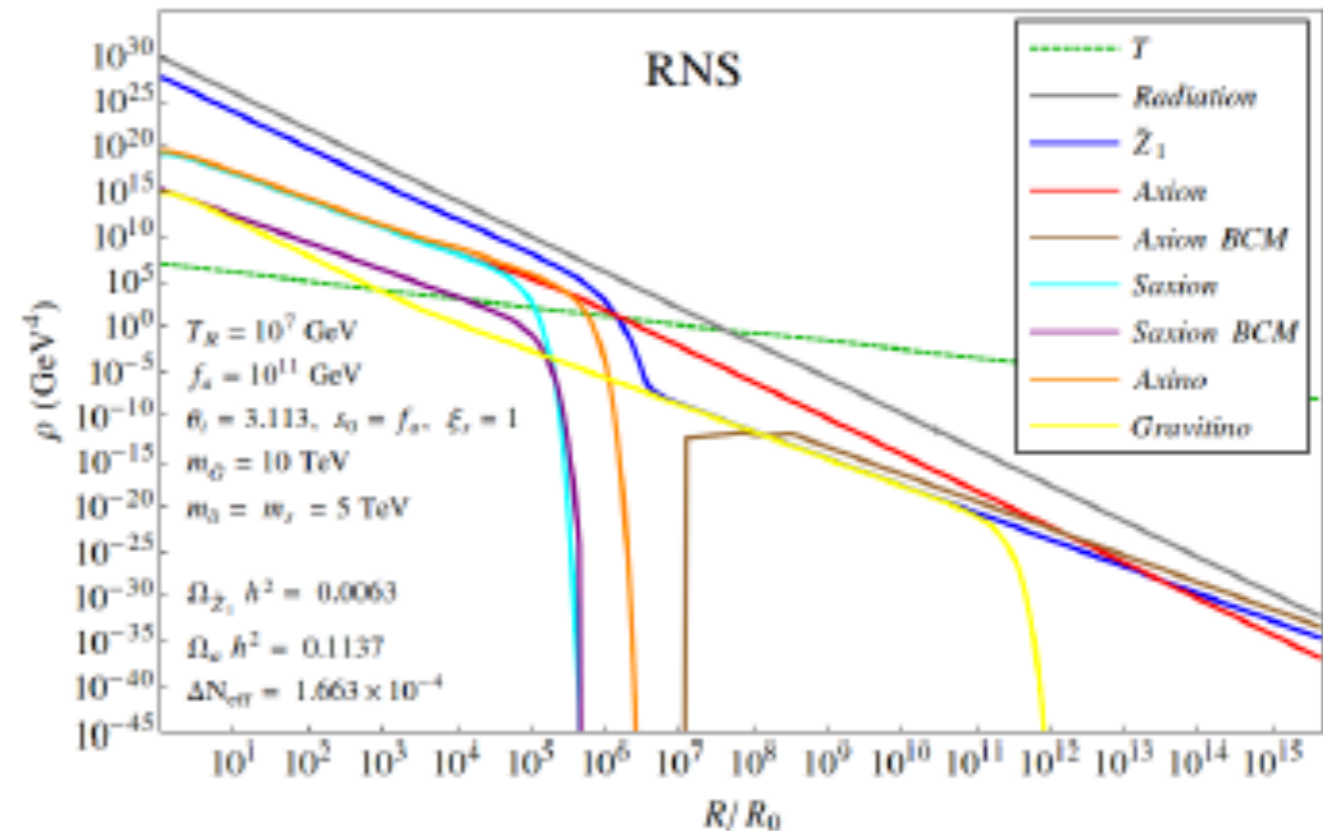
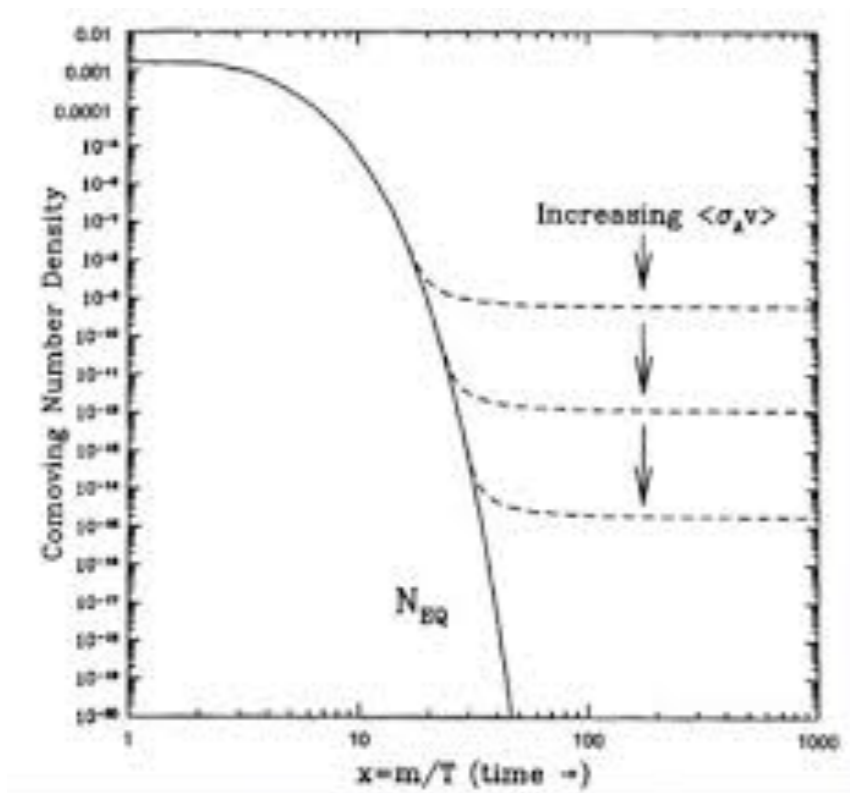
What happens to SUSY WIMP dark matter?

- higgsino-like WIMPs thermally underproduced
- 3 not four light pions \Rightarrow QCD theta vacuum
- EDM(neutron) \Rightarrow axions: no fine-tuning in QCD sector
- SUSY context: axion superfield, axinos and saxions
- DM= axion+higgsino-like WIMP admixture
- DFSZ SUSY axion: solves mu problem with $\mu \ll m_{3/2}$!
- ultimately detect both WIMP and axion?

usual picture

=>

mixed axion/WIMP



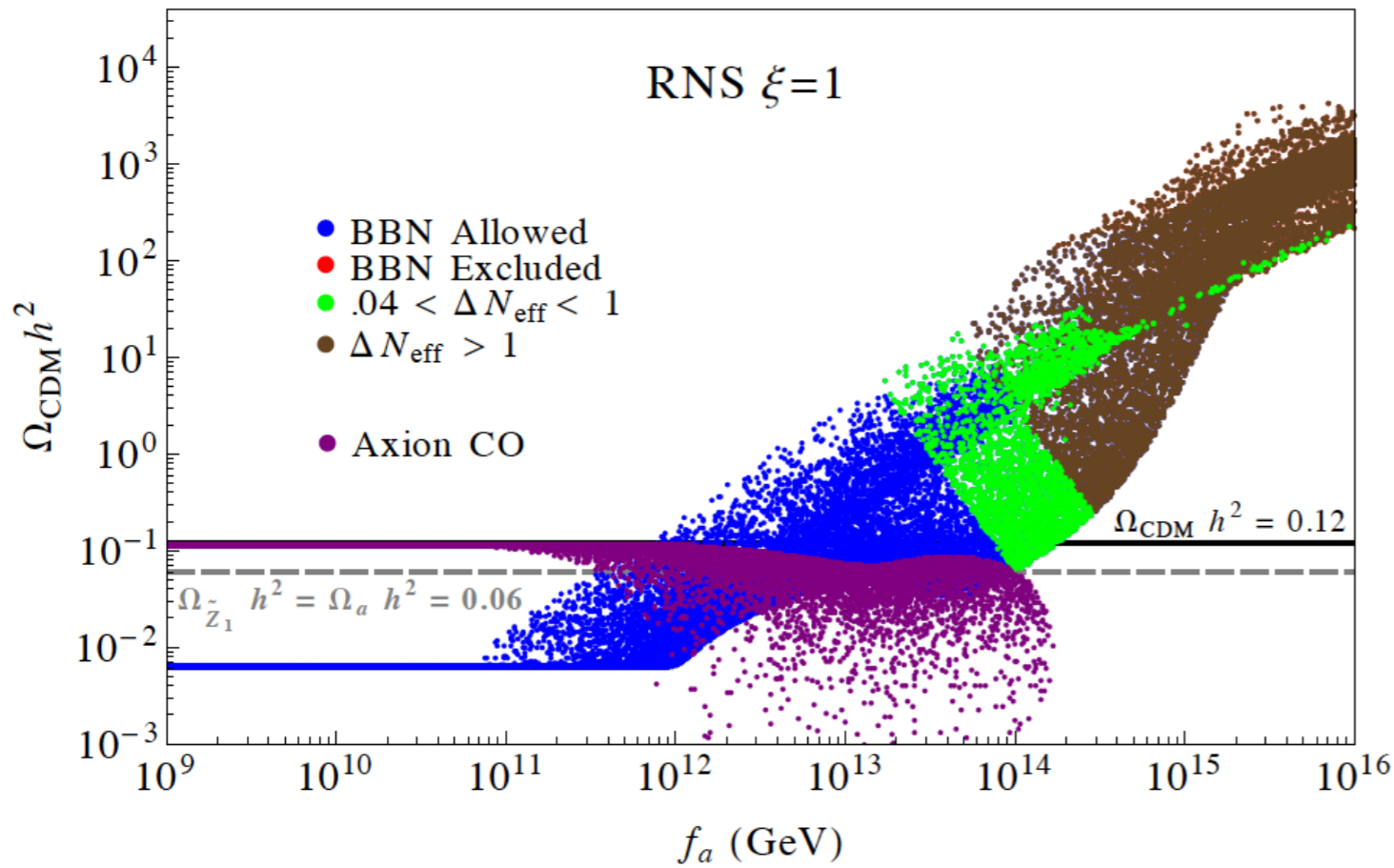
KJ Bae, HB, Lessa, Serce

much of parameter space is axion-dominated
with 10-15% WIMPs



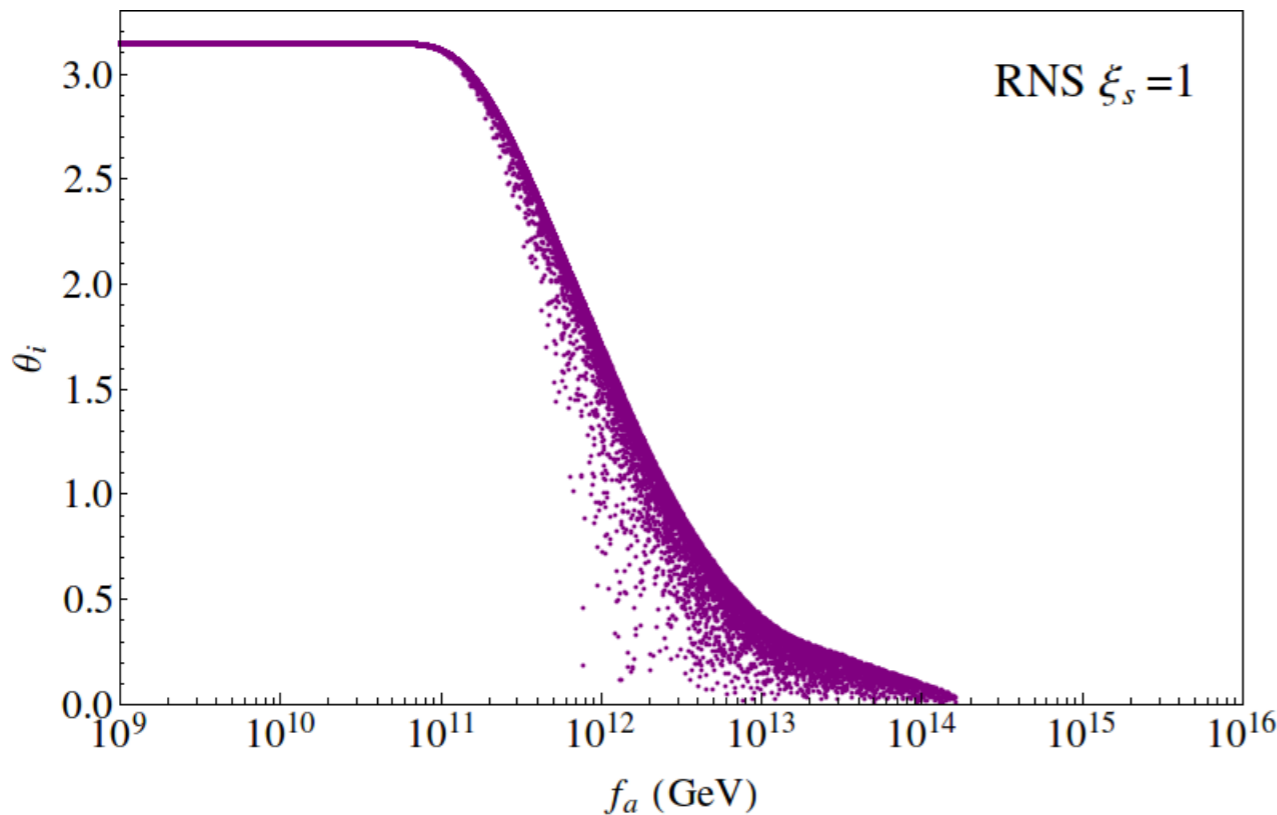
\Rightarrow





higgsino abundance

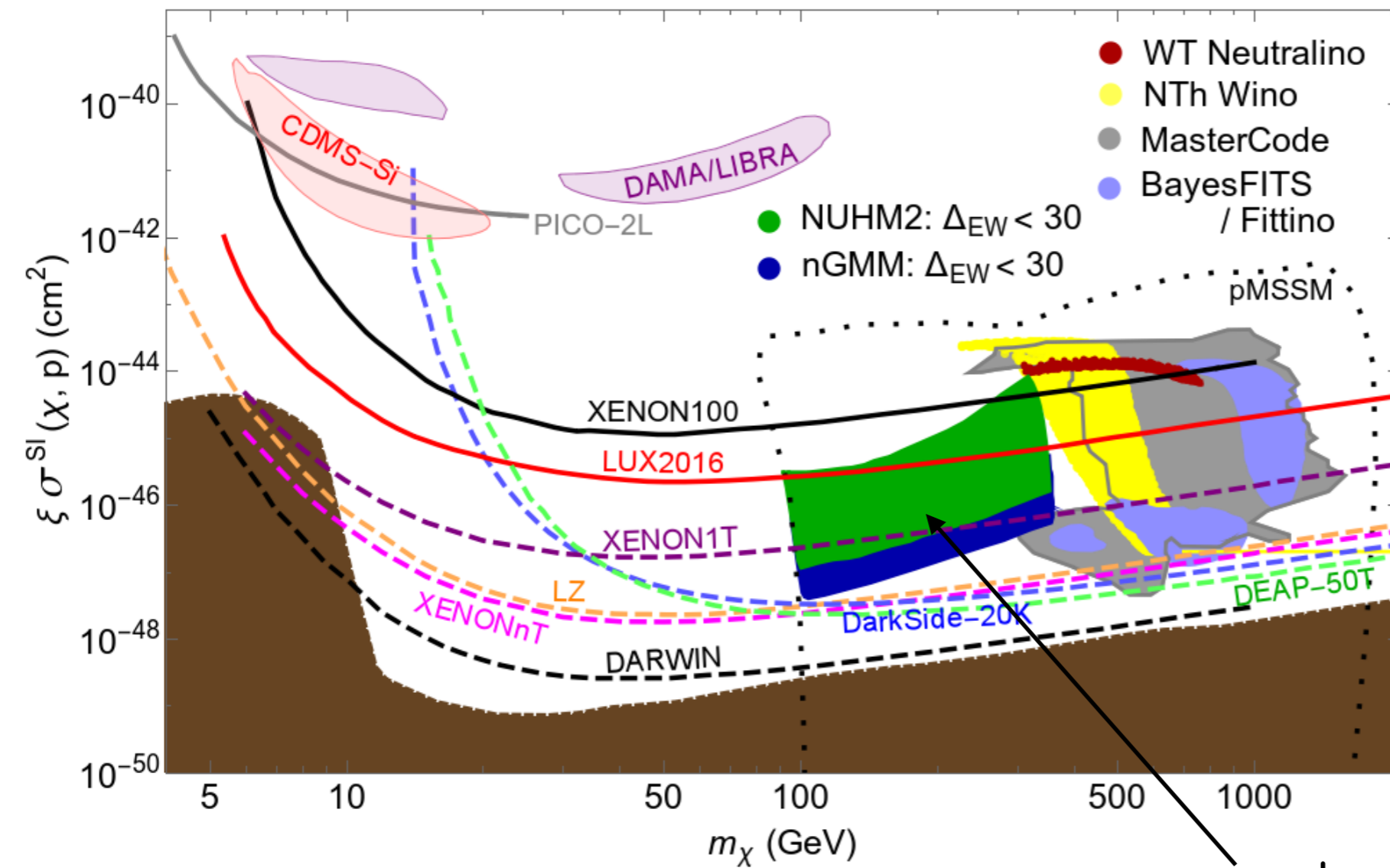
axion abundance



mainly axion CDM
 for $f_a < \sim 10^{12}$ GeV;
 for higher f_a , then
 get increasing wimp
 abundance

Direct higgsino detection rescaled

for minimal local abundance $\xi \equiv \Omega_{\chi}^{TP} h^2 / 0.12$



Bae, HB, Barger, Savoy, Serce

$$\mathcal{L} \ni -X_{11}^h \bar{\tilde{Z}}_1 \tilde{Z}_1 h$$

$$X_{11}^h = -\frac{1}{2} (v_2^{(1)} \sin \alpha - v_1^{(1)} \cos \alpha) (g v_3^{(1)} - g' v_4^{(1)})$$

Xe-1-ton
now operating!

natural SUSY

Can test completely with ton scale detector
or equivalent (subject to minor caveats)

Conclusion: SUSY is alive and well!

- old calculations of naturalness over-estimate fine-tuning
- naturalness: Little Hierarchy $\mu \ll m(\text{SUSY})$ allowed
- radiatively-driven naturalness: $\mu \sim 100\text{--}200$ GeV, $m(t_1) < 3$ TeV, $m(\text{gluino}) < 5\text{--}6$ TeV
- SUSY DFSZ axion: solve strong CP, solve SUSY μ problem; generate $\mu \ll m(\text{SUSY})$
- landscape pull on soft terms towards RNS, $m(h) \sim 125$ GeV
- natural mirage-mediation/mini-landscape
- natural NUHM2: HL-LHC can cover via $SSdB+Z1Z2j$ channels
- natural mirage/mini-landscape may escape detection at HL-LHC; need LHC33!
- expect ILC as higgsino factory
- DM= axion+higgsino-like WIMP admixture: detect both?
- higgsino-like WIMP detection likely; axion more difficult

#2: Higgs mass or large-log fine-tuning Δ_{HS}

It is tempting to pick out one-by-one quantum fluctuations **but** must combine log divergences before taking any limit

$$m_h^2 \simeq \mu^2 + m_{H_u}^2(\text{weak}) \simeq \mu^2 + m_{H_u}^2(\Lambda) + \delta m_{H_u}^2$$

$$\frac{dm_{H_u}^2}{dt} = \frac{1}{8\pi^2} \left(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10}g_1^2 S + 3f_t^2 X_t \right) \quad X_t = m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + A_t^2$$

neglect gauge pieces, S, m_{H_u} and running;
then we can integrate from $m(\text{SUSY})$ to Λ

$$\delta m_{H_u}^2 \sim -\frac{3f_t^2}{8\pi^2} (m_{Q_3}^2 + m_{U_3}^2 + A_t^2) \ln(\Lambda/m_{\text{SUSY}})$$

$$\Delta_{HS} \sim \delta m_h^2 / (m_h^2/2) < 10$$

$$m_{\tilde{t}_{1,2}, \tilde{b}_1} < 500 \text{ GeV}$$

$$m_{\tilde{g}} < 1.5 \text{ TeV}$$

old natural SUSY

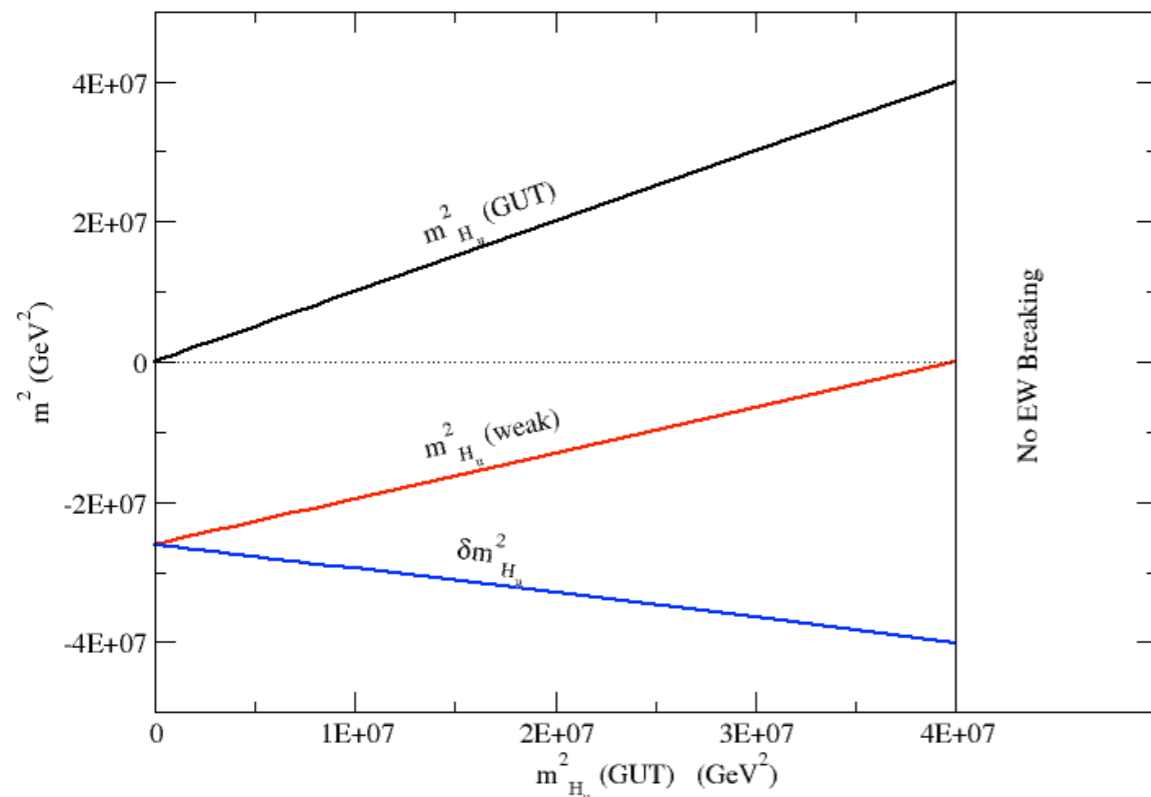
then

A_t can't be too big

What's wrong with this argument?
 In zeal for simplicity, have made several simplifications: most **egregious** is that one sets $m(H_u)^2=0$ at beginning to simplify

$m_{H_u}^2(\Lambda)$ and $\delta m_{H_u}^2$ are *not* independent!

violates prime directive!



The larger $m_{H_u}^2(\Lambda)$ becomes, then the larger becomes the cancelling correction!

HB, Barger, Savoy

To fix: combine dependent terms:

$$m_h^2 \simeq \mu^2 + (m_{H_u}^2(\Lambda) + \delta m_{H_u}^2) \text{ where now both } \mu^2 \text{ and } (m_{H_u}^2(\Lambda) + \delta m_{H_u}^2) \text{ are } \sim m_Z^2$$

After re-grouping: $\Delta_{HS} \simeq \Delta_{EW}$

Instead of: the radiative correction $\delta m_{H_u}^2 \sim m_Z^2$
we now have: the radiatively-corrected $m_{H_u}^2 \sim m_Z^2$

Recommendation: put this horse out to pasture

$$\delta m_{H_u}^2 \sim -\frac{3f_t^2}{8\pi^2} (m_{Q_3}^2 + m_{U_3}^2 + A_t^2) \ln(\Lambda/m_{SUSY})$$

R.I.P.

sub-TeV 3rd generation squarks **not** required for naturalness

If one has the right parameter correlations, can always get generalized focus point behavior for m_{H_u} :

$$m_0^2 = m_{3/2}^2$$

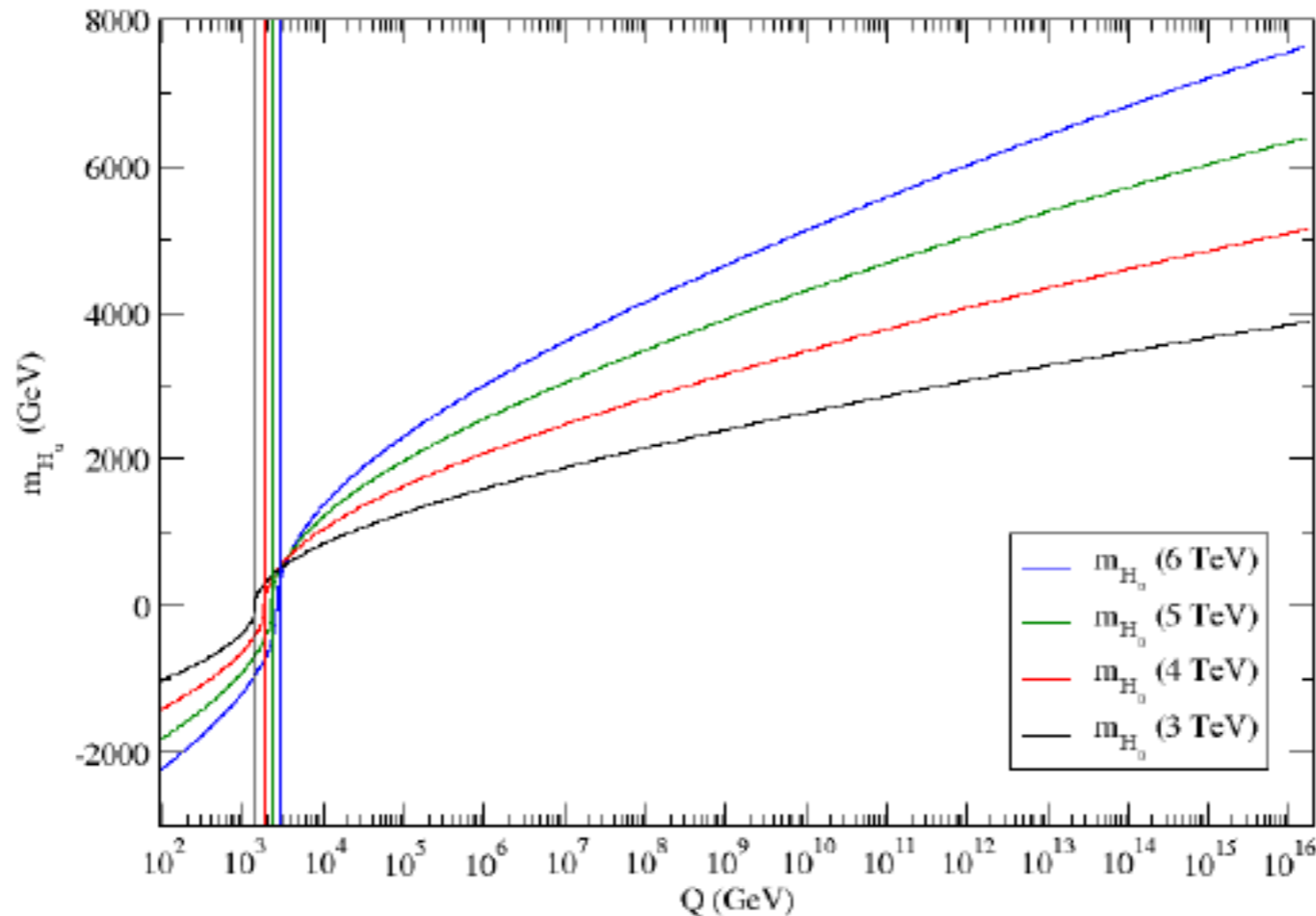
$$A_0 = -1.6m_{3/2}$$

$$m_{1/2} = m_{3/2}/5$$

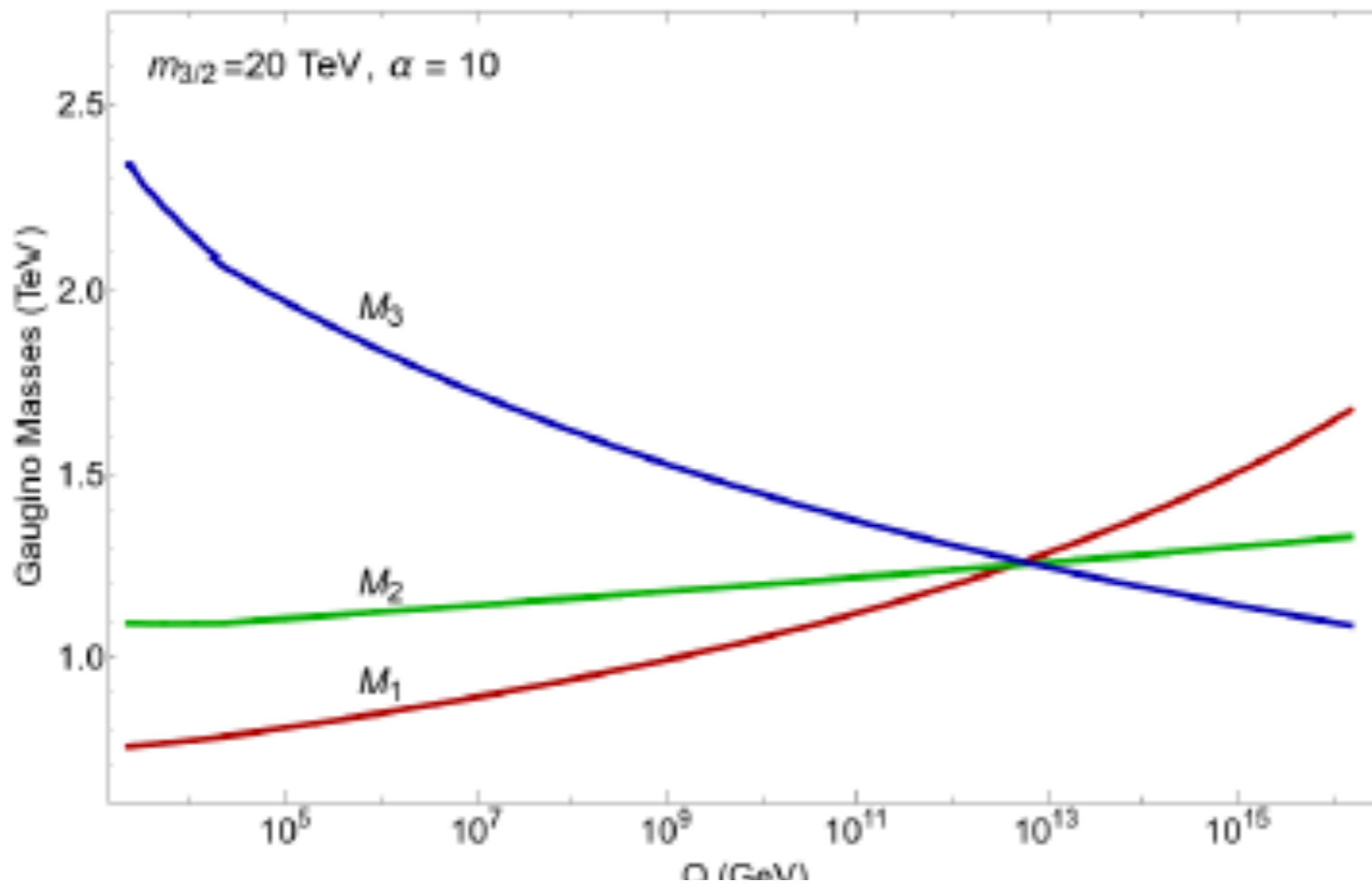
$$m_{H_d}^2 = m_{3/2}^2/2.$$

$$\mu \simeq 150 \text{ GeV}$$

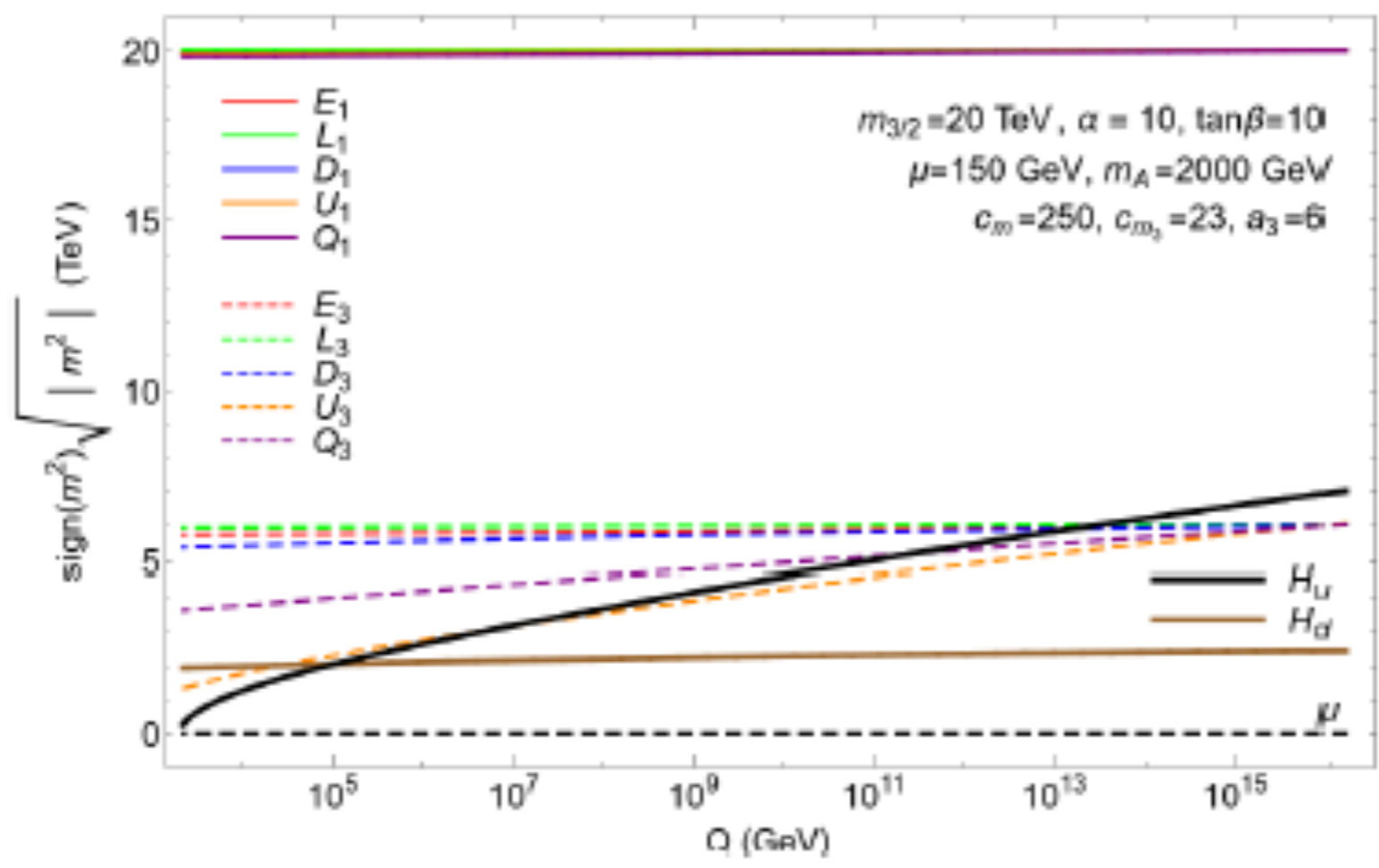
$$m_{H_u}^2(GUT) = 1.8m_{3/2}^2 - (212.52 \text{ GeV})^2.$$



HB, Barger, Savoy

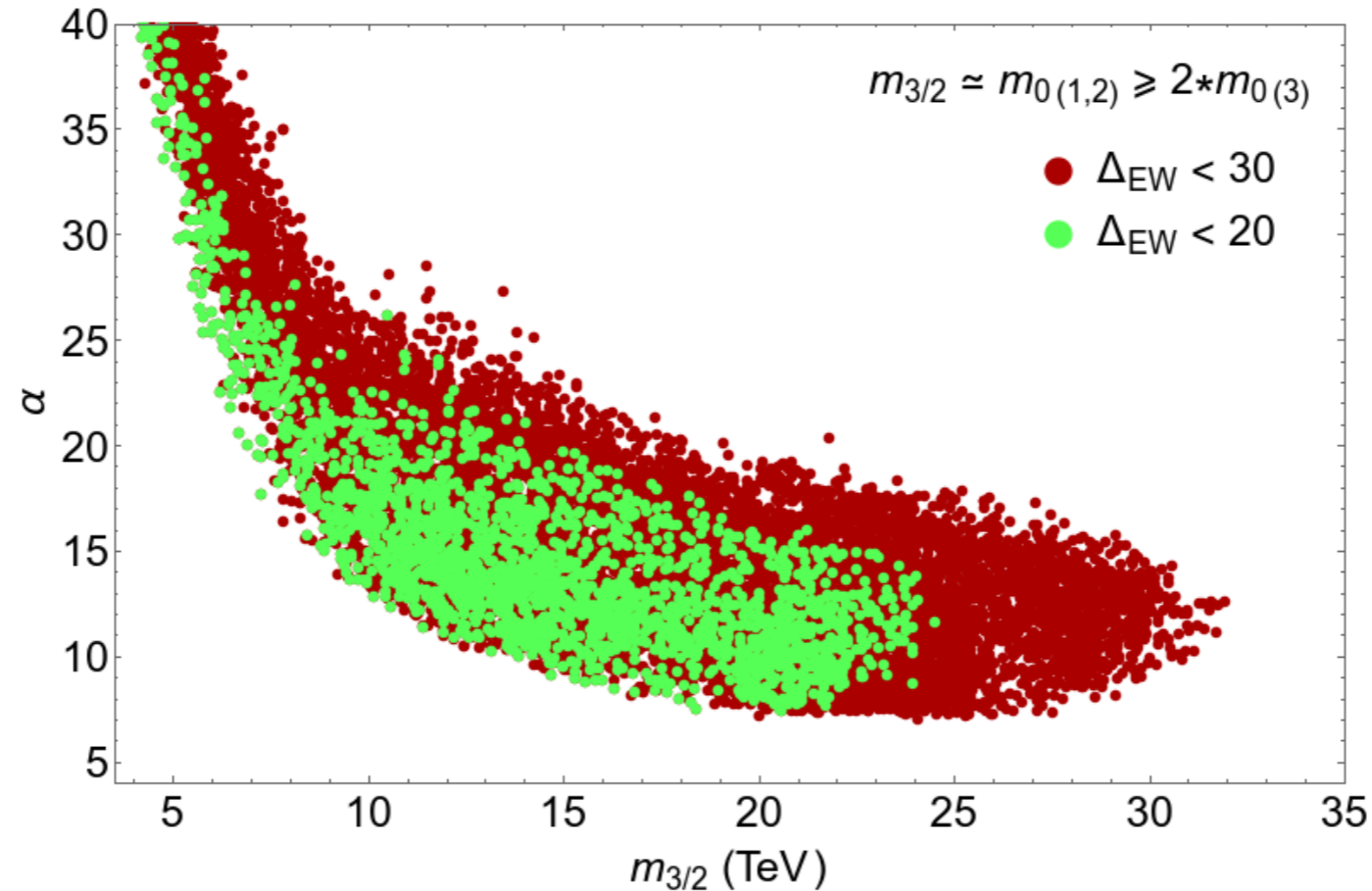


$$\Delta_{EW} = 17.6$$



To generate minilandscape, take:

$$c_m = (16\pi^2/\alpha)^2 \text{ so that } m_0(1,2) \simeq m_{3/2}$$



Then get upper bound $m_{3/2} < 25 - 30$ TeV and $\alpha > 7$
else too large $m_0(1,2)$ drives 3rd generation tachyonic

Martin, Vaughn, 2-loop RGEs

Increased upper bound on $m(\text{gluino}) < 6$ TeV

Alpha bound \Rightarrow mirage unif scale $> 10^{11}$ GeV

(not too much compression of inos)

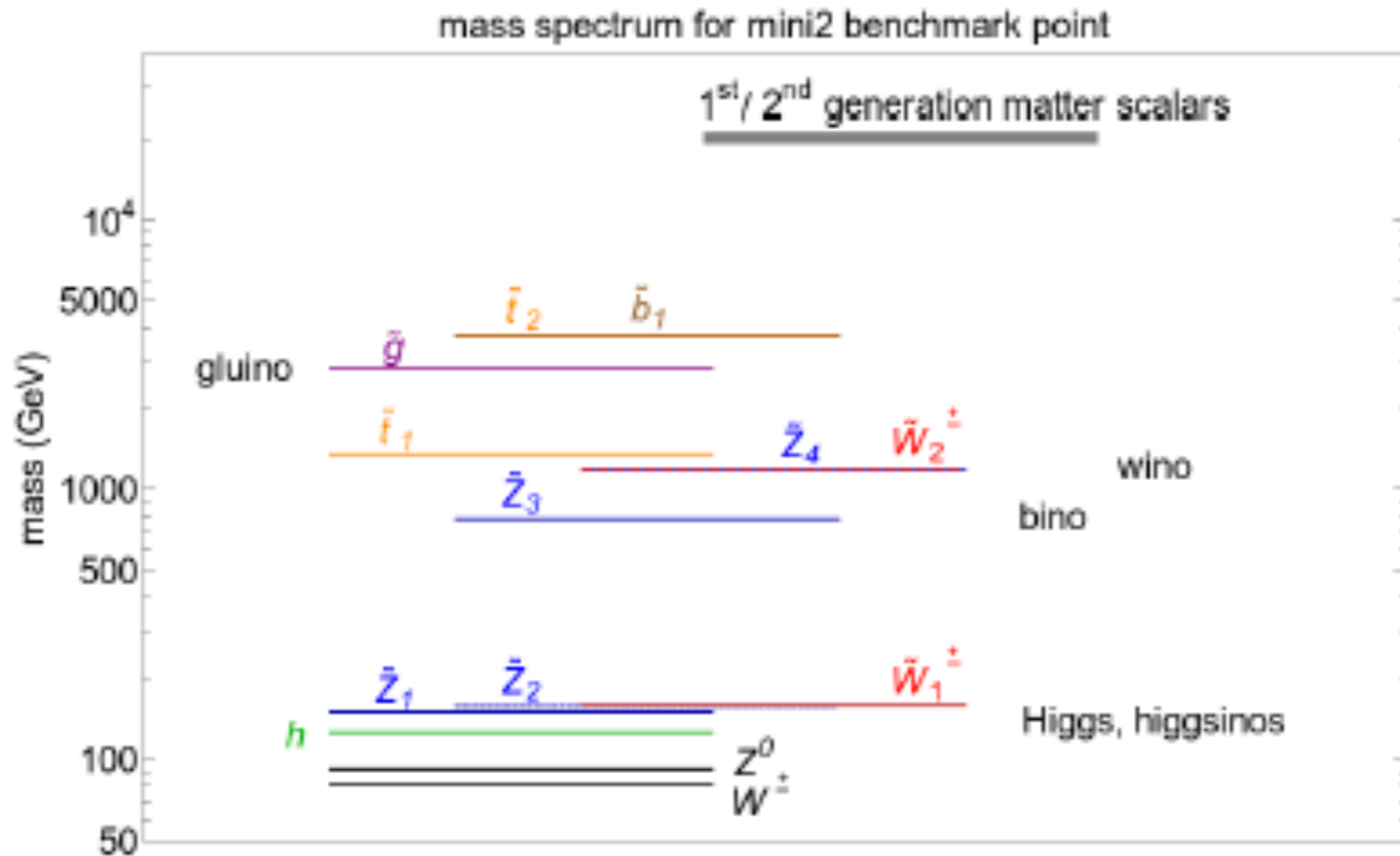
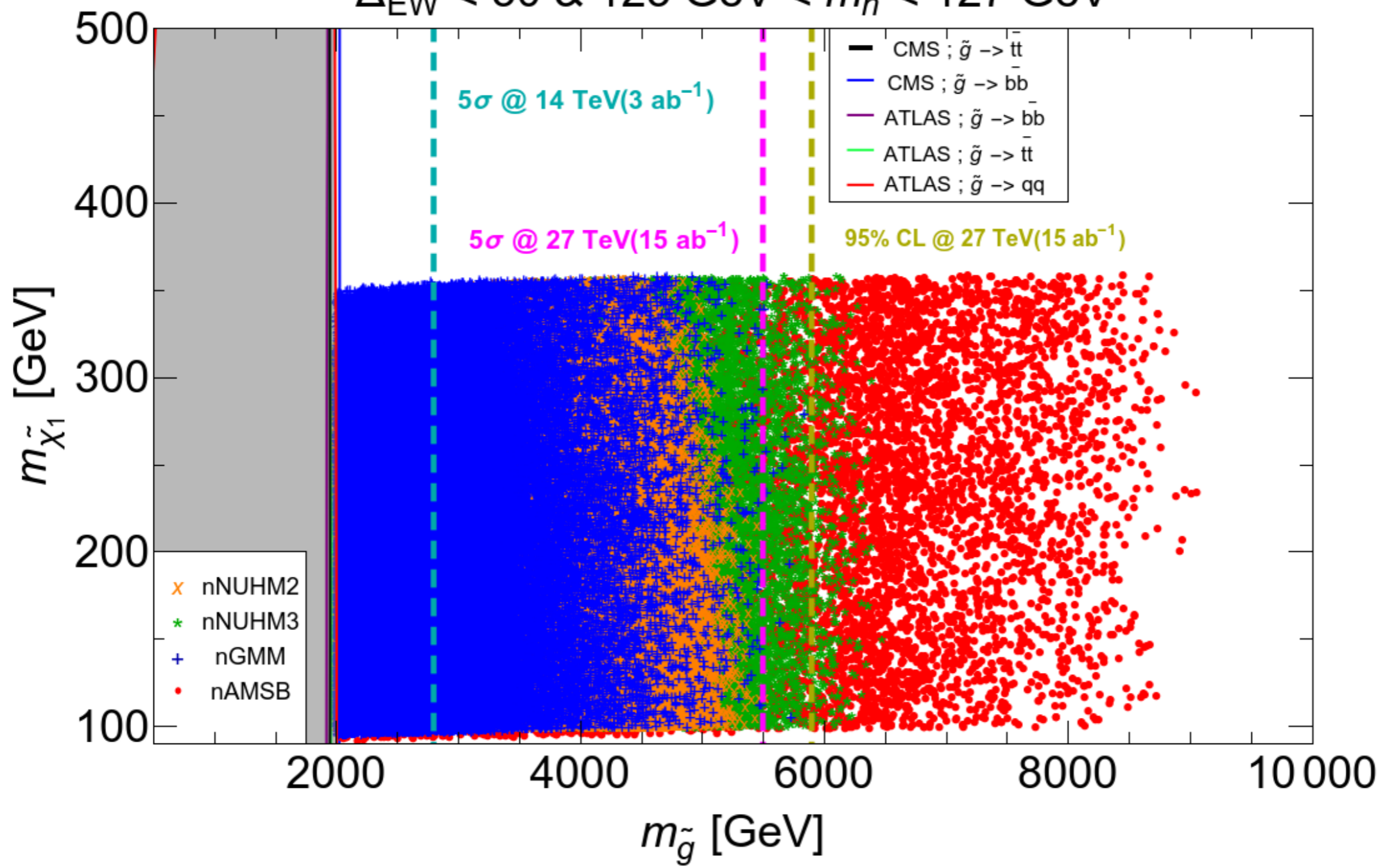


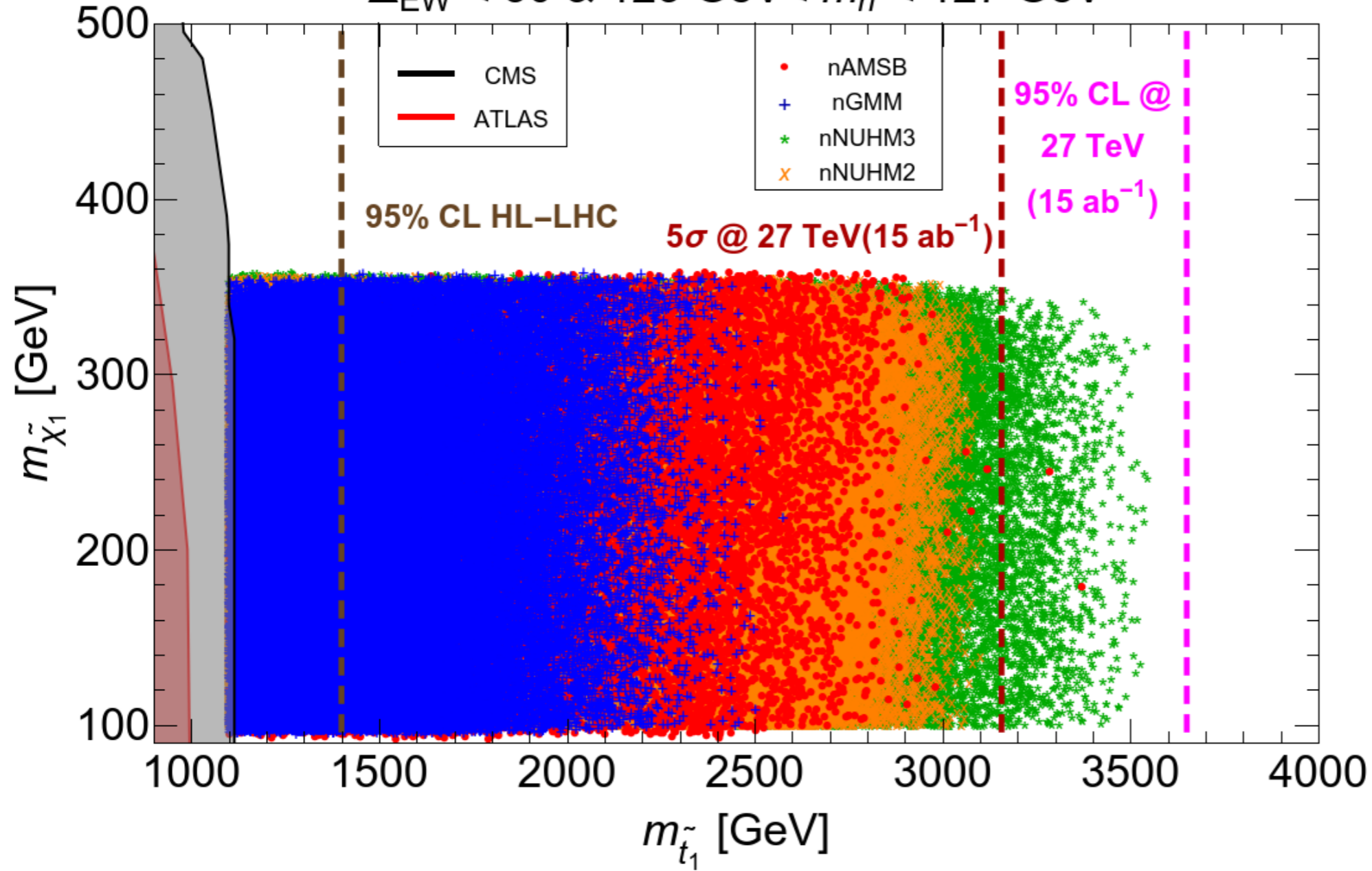
Figure 7: The superparticle mass spectra from the natural mini-landscape point mini2 of Table 1.

Due to compressed gaugino spectra, minilandscape can probably hide from HL-LHC while maintaining naturalness

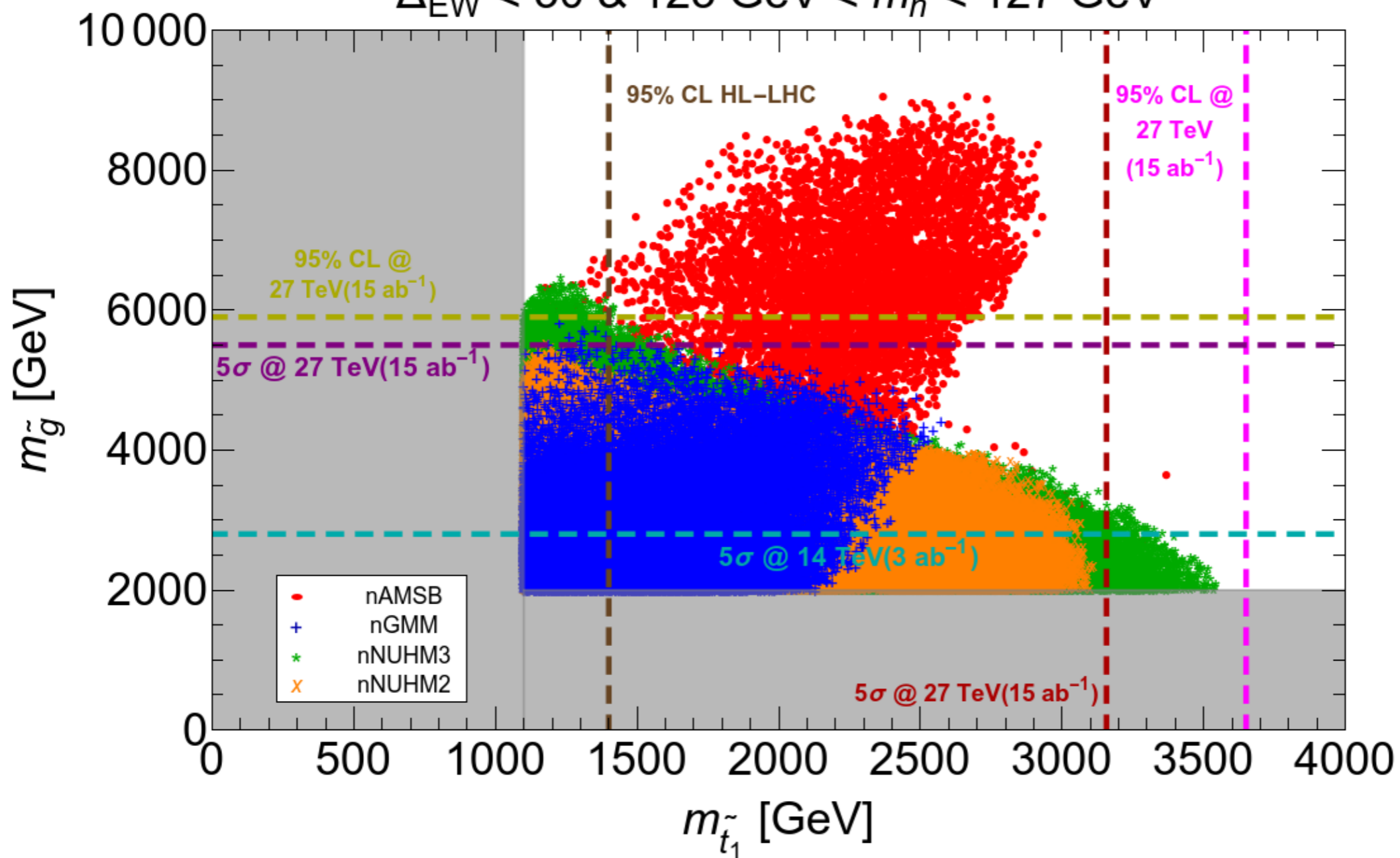
$\Delta_{EW} < 30$ & $123 \text{ GeV} < m_h < 127 \text{ GeV}$



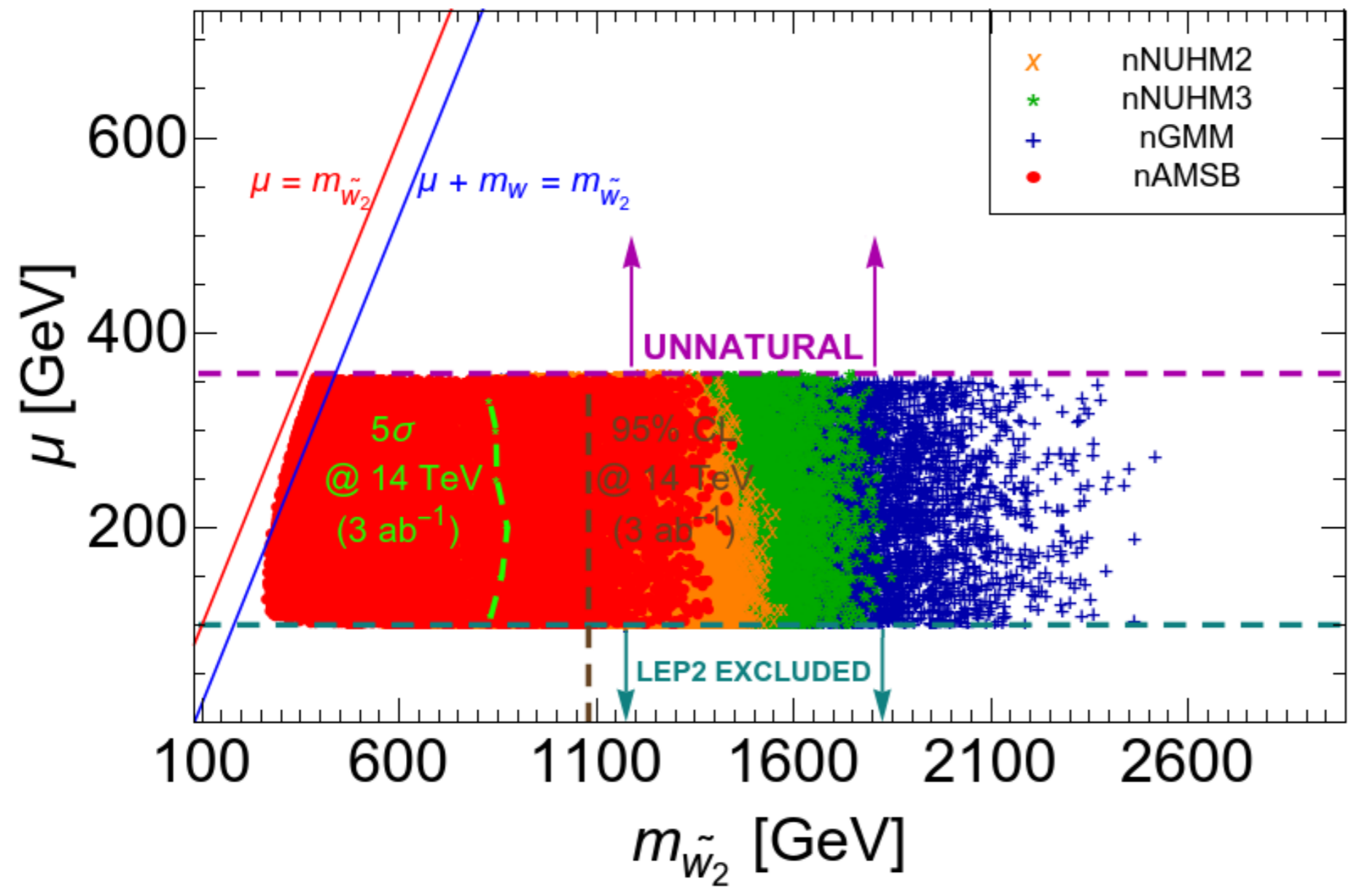
$\Delta_{EW} < 30$ & $123 \text{ GeV} < m_h < 127 \text{ GeV}$



$\Delta_{EW} < 30$ & $123 \text{ GeV} < m_h < 127 \text{ GeV}$



$\Delta_{EW} < 30$ & $123 \text{ GeV} < m_h < 127 \text{ GeV}$



$\Delta_{EW} < 30$ & $123 \text{ GeV} < m_h < 127 \text{ GeV}$

