リニアコライダーのビーム力学とATF

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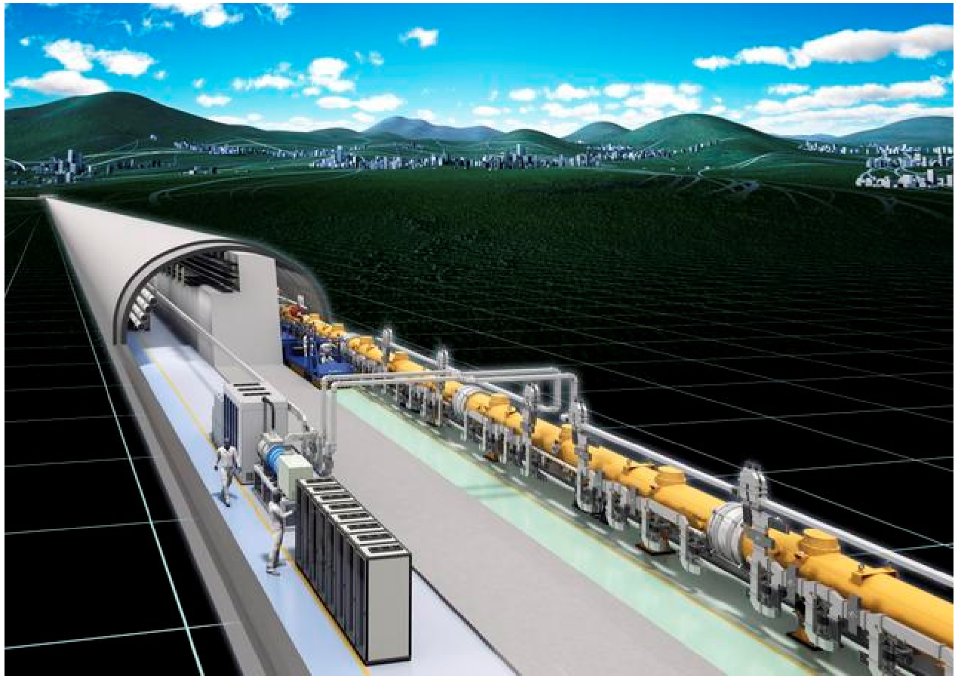
Contents

Introduction

ILC (International Linear Collider)



Superconducting RF cavities

(higher beam current acceleration)

Damping ring

(low emittance beam generation)

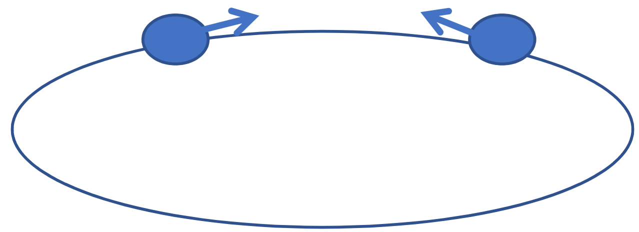
Final Focus Beamline

(small beam focusing without aberrations)

Technical R&D items for ILC

Circular Collider

Beams collide every turns.



Linear Collider

Beams can collide only once !

In order to have enough luminosity,

- *High current beam generation*
  => Superconducting acceleration.
- ***Small beam generation at interaction point.***

What is important to generate small beam at IP ?

1. To generate high quality beam

2. To use high quality final lens system

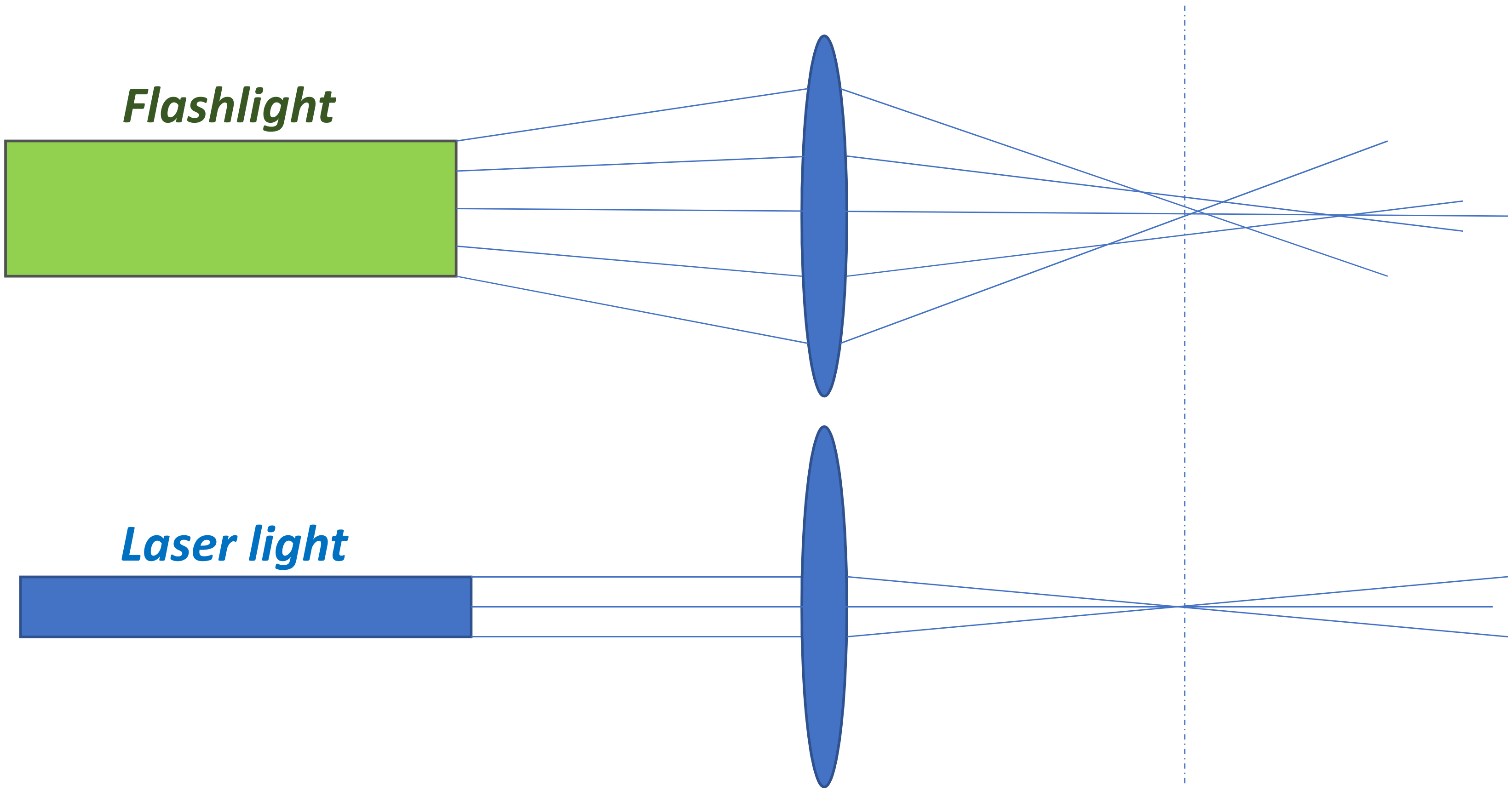
What is high quality beam ?

When we focus the light with same lens system,

The laser light can be focused smaller than flashlight,
because the quality (parallelism) of laser light is better than that for flashlight.

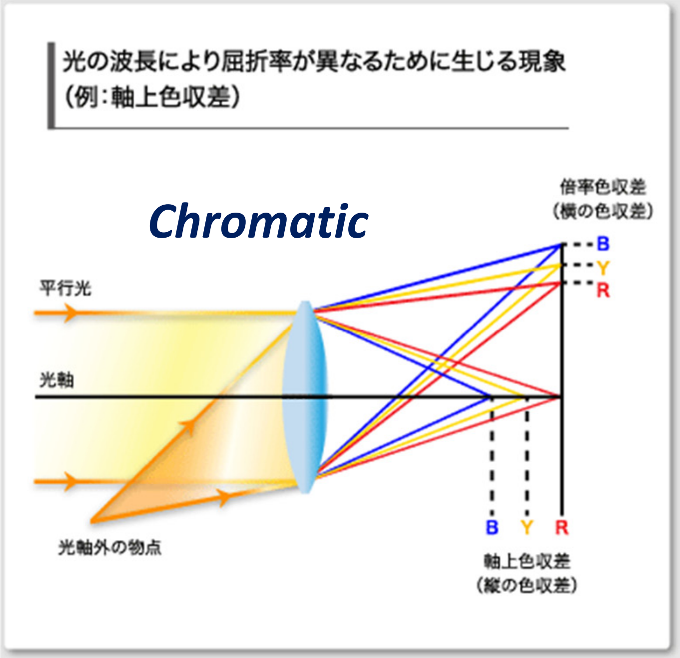
*The electron beam also have the properties of parallelism, which is called to "**emittance**".*

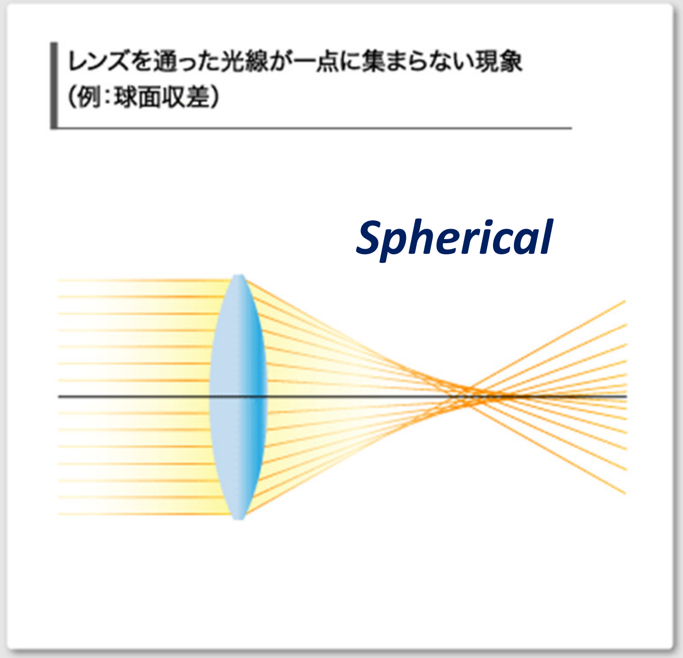
It is very important to use the small emittance beam to make small beam at IP.



What is high quality focal lens system ?

Focal lens has many aberrations.



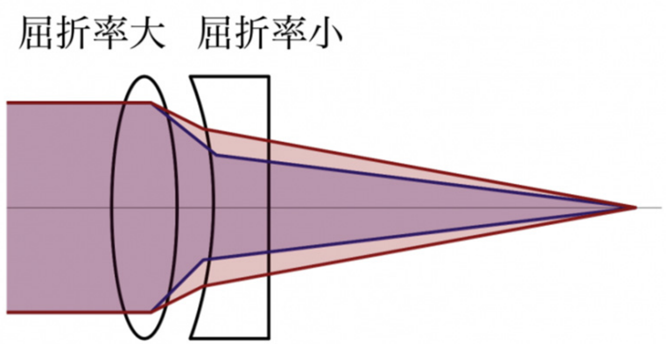


In order to reduce the aberration, we use
- *Aspherical lens*
- *Combination lens systems.*

As well as light,
electron beam must also design the good lens system.

(Main theme of this lecture)

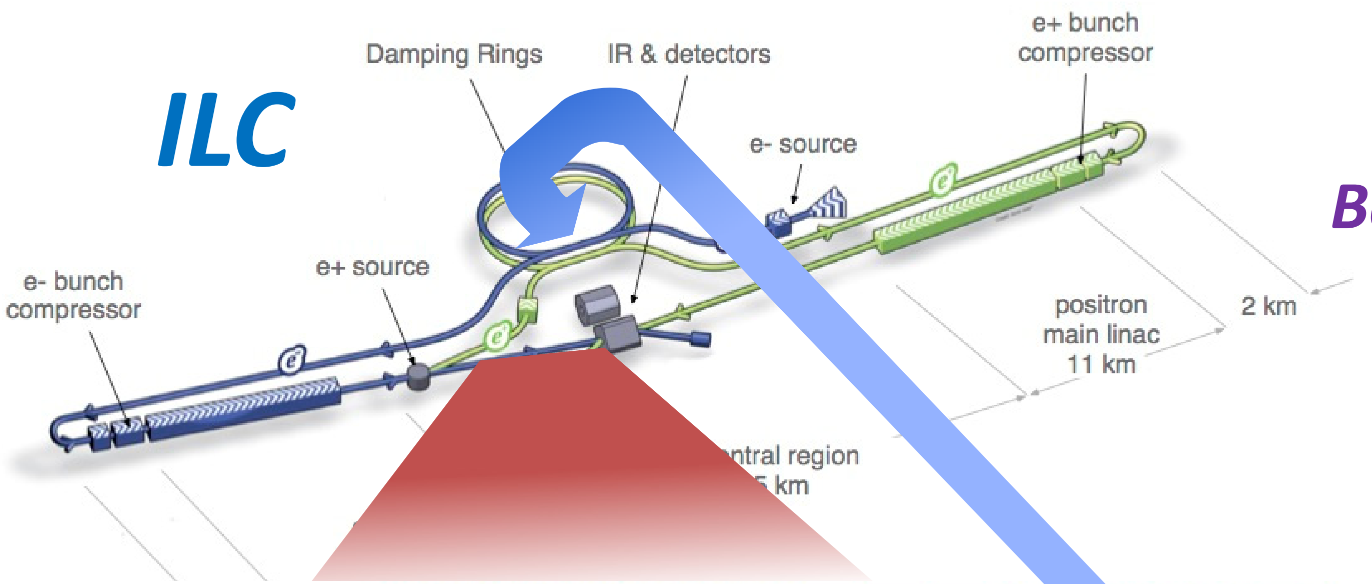
Example of achromatic lens
(lens system to avoid chromatic aberration)



ATF (Accelerator Test Facility)

ATF consists of the damping ring and final focus beamline.

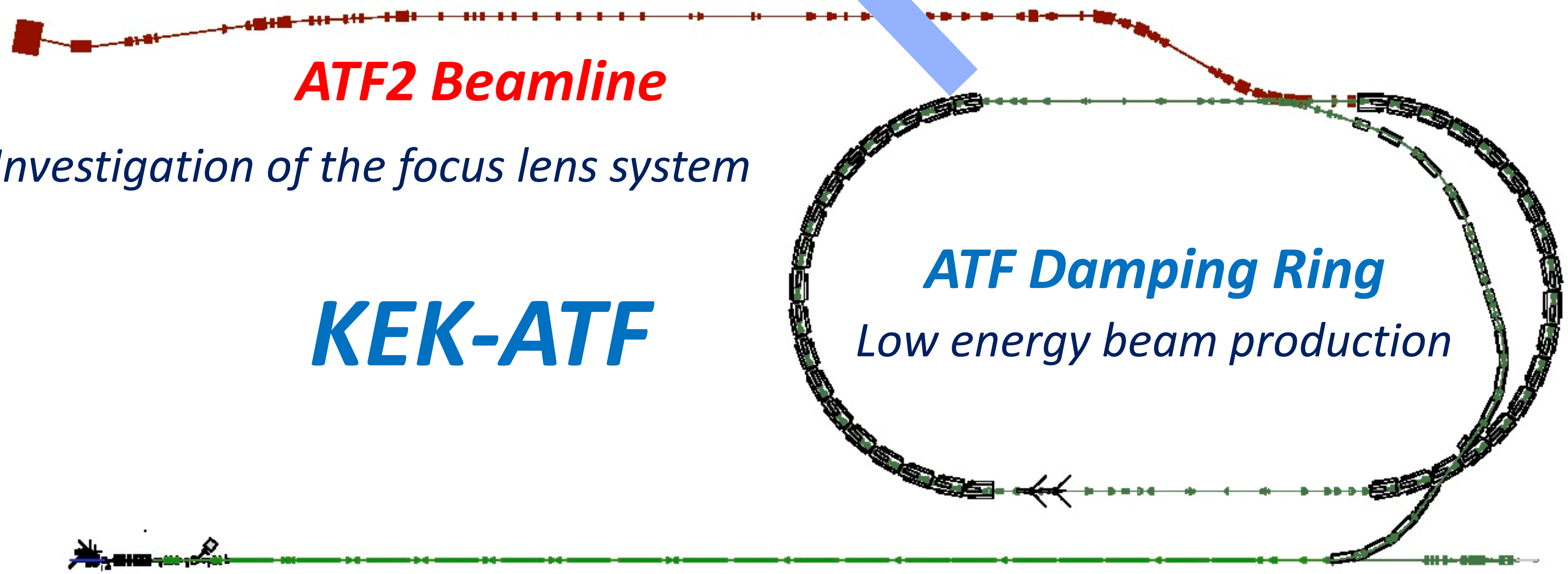
ILC



Beam focusing of ILC

Low emittance beam
⇒ *Damping Ring*

Beam Focusing
⇒ *Final Focus System*



ATF2 Beamline

Investigation of the focus lens system

KEK-ATF

ATF Damping Ring

Low energy beam production

Fundamental Beam Dynamics

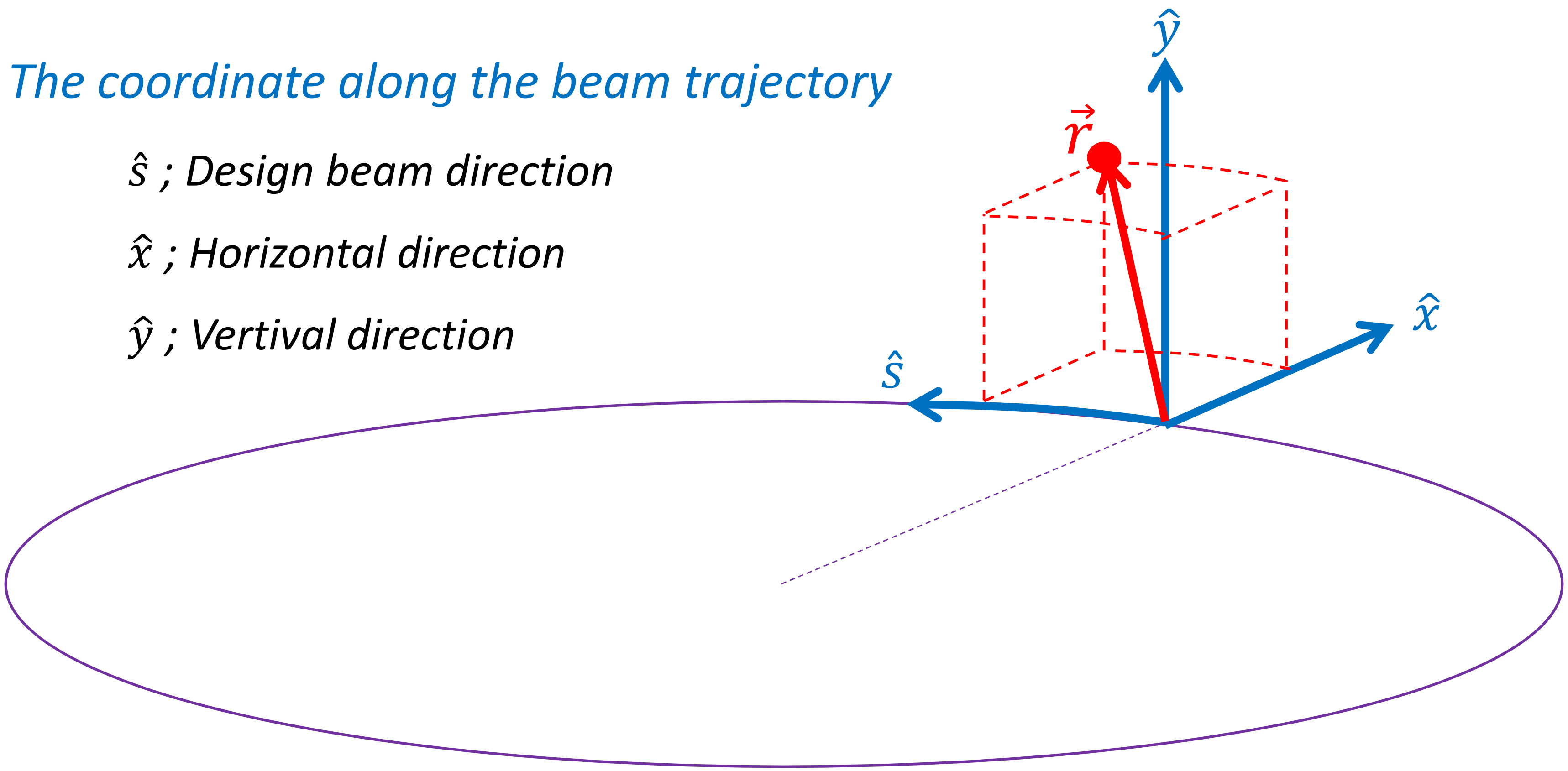
Frenet-Serret Coordinate

The coordinate along the beam trajectory

$\hat{s}$; *Design beam direction*

$\hat{x}$; *Horizontal direction*

$\hat{y}$; *Vertival direction*



Hamiltonian in Canonical Coordinate

$$H(x,y,z) = c\sqrt{m^2c^2 + \left(\vec{p} - q\vec{A}\right)^2} + q\Phi$$

Frenet-Serret Coordinate

$$H(x,y,s) = -\left(1 + \frac{x}{\rho_x} + \frac{y}{\rho_y}\right)\sqrt{(1+\delta)^2 - p_x^2 - p_y^2} - \frac{q}{p_0}A_s$$

Vector Potential in Frenet-Serret Coordinate

Magnetic Field

$$B_x = \frac{1}{h}\left(\frac{\partial A_s}{\partial y} - \frac{\partial A_y}{\partial s}\right)$$

$$B_y = \frac{1}{h}\left(\frac{\partial A_x}{\partial s} - \frac{\partial A_s}{\partial x}\right)$$

$$B_s = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

Vector Potential

$$h = 1 + \frac{x}{\rho_x} + \frac{y}{\rho_y}$$

Vector potential for bending magnet

- the coordinate is changing along the beam line

$$-\frac{q}{p_0}A_{s,0} = \frac{1}{2}\left(1 + \frac{x}{\rho_x} + \frac{y}{\rho_y}\right)^2$$

Vector potential for normal n-th multipole magnets

$$-\frac{q}{p_0}A_{s,nN} = \frac{k_{nN}r^{n+1}}{(n+1)!}\cos[(n+1)\theta]$$

Normal quadrupole

$$\Rightarrow \quad -\frac{q}{p_0}A_{s,1N} = \frac{k_{1N}}{2}(x^2 - y^2)$$

Vector potential for skew n-th multipole magnets

$$-\frac{q}{p_0}A_{s,nS} = \frac{k_{nS}r^{n+1}}{(n+1)!}\sin[(n+1)\theta]$$

Skew quadrupole

$$\Rightarrow \quad -\frac{q}{p_0}A_{s,1S} = k_{1S}xy$$

Equation of motion in Frenet-Serret Coordinate

Hamiltonian with dipole and normal quadrupole field

$$H = -\left(1+\frac{x}{\rho_x}+\frac{y}{\rho_y}\right)\sqrt{(1+\delta)^2 - p_x^2 - p_y^2} + \frac{1}{2}\left(1+\frac{x}{\rho_x}+\frac{y}{\rho_y}\right)^2 + \frac{k_{1N}}{2}(x^2-y^2)$$

$$\approx -\frac{1}{2} - \delta\left(1+\frac{x}{\rho_x}+\frac{y}{\rho_y}\right) + \frac{1}{2}(p_x^2+p_y^2) + \frac{x^2}{2}\left(\frac{1}{\rho_x^2}+k_{1N}\right) + \frac{y^2}{2}\left(\frac{1}{\rho_y^2}-k_{1N}\right)$$

Equation of motion

$$\frac{d}{ds}\vec{q} = S\frac{\partial}{\partial\vec{q}}H(\vec{q}) \qquad \vec{q} = \begin{pmatrix} x \\ p_x \\ y \\ p_y \end{pmatrix} \qquad S = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\Downarrow$$

$$\frac{d^2x}{ds^2} + \left(\frac{1}{\rho_x^2}+k_{1N}\right)x = \frac{\delta}{\rho_x}$$

$$\frac{d^2y}{dy^2} + \left(\frac{1}{\rho_y^2}-k_{1N}\right)y = \frac{\delta}{\rho_y}$$

We can calculate the particle motion in Frenet-Serret Coordinate .

Equation of motion (continued)

Equation of motion

$$\frac{d^2 z}{ds^2} + k_z z = \frac{\delta}{\rho_z} \qquad (z = x, y)$$

The particle motion (z) is defined with on-momentum motion (z_0) and the motion difference, generated to on-momentum particle with momentum offset δ.

$$z = z_0 + \eta_z \delta$$

$$\left(\frac{d^2 z_0}{ds^2} + k_z z_0\right) + \delta\left(\frac{d^2 \eta_z}{ds^2} + k_z \eta_z - \frac{1}{\rho_z}\right) = 0$$

(1st term) ⇒ the equation of motion for on-momentum particle

(2nd term) ⇒ the motion difference to on-momentum particle with momentum offset δ

η is called to “dispersion function”.

Particle Motion for On-momentum Particle

Equation of motion for on-momentum particle ($\delta = 0$)

$$\frac{d^2 z}{ds^2} + k_z z = 0 \quad (z = x, y)$$

$$k_x = \frac{1}{\rho_x^2} + k_{1N}$$

$$k_y = \frac{1}{\rho_y^2} - k_{1N}$$

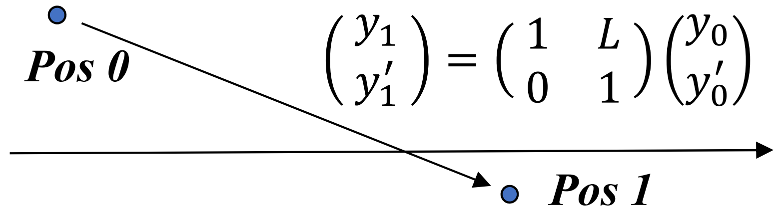
We can express the particle transportation ($s_0 \to s_1$) in the uniform field as

$$\overrightarrow{q_1} = M\,\overrightarrow{q_0} \qquad M = \begin{pmatrix} M_x & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & M_y \end{pmatrix}$$

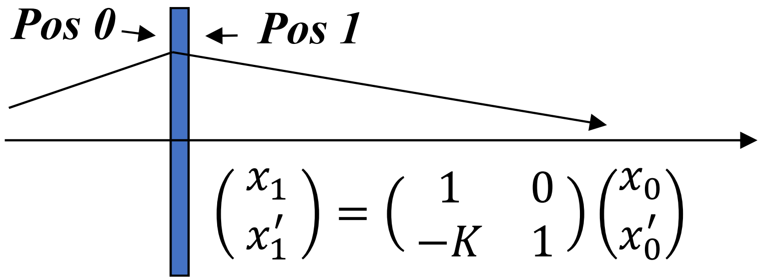
Free Space

$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_1' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_0 \\ y_0' \end{pmatrix}$$



Quadrupole Magnet



$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -K & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_1' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ +K & 1 \end{pmatrix} \begin{pmatrix} y_0 \\ y_0' \end{pmatrix}$$

Transfer Matrix

- $k_z > 0 \quad M_z = \begin{pmatrix} \cos\sqrt{k_z}L & \sin\sqrt{k_z}L/\sqrt{k_z} \\ -\sqrt{k_z}\sin\sqrt{k_z}L & \cos\sqrt{k_z}L \end{pmatrix} \xrightarrow{L\to 0} \begin{pmatrix} 1 & 0 \\ -K_z & 1 \end{pmatrix}$

- $k_z = 0 \quad M_z = \begin{pmatrix} 1 & L \\ 0 & 0 \end{pmatrix}$

- $k_z < 0 \quad M_z = \begin{pmatrix} \cosh\sqrt{-k_z}L & \sinh\sqrt{-k_z}L/\sqrt{-k_z} \\ \sqrt{-k_z}\sinh\sqrt{-k_z}L & \cosh\sqrt{-k_z}L \end{pmatrix} \xrightarrow{L\to 0} \begin{pmatrix} 1 & 0 \\ +K_z & 1 \end{pmatrix}$

Unitary Matrix

Integrated Field Strength

$$K_z = |k_z|L$$

Dispersion Function

The dispersion function of D_z, D'_z (Dispersion function for $\eta_z = \eta'_z = 0$ at $s = 0$)

- $k_z > 0$ $\quad D_z(s) = \dfrac{1 - \cos\sqrt{k_z}\, s}{\rho_z k_z}$ $\quad D'_z(s) = \dfrac{\sin\sqrt{k_z}\, s}{\rho_z \sqrt{k_z}}$
- $k_z = 0$ $\quad D_z(s) = \dfrac{s^2}{2\rho_z}$ $\quad D'_z(s) = \dfrac{s}{\rho_z}$
- $k_z < 0$ $\quad D_z(s) = \dfrac{\cosh\sqrt{-k_z}\, s - 1}{\rho_z k_z}$ $\quad D'_z(s) = \dfrac{\sinh\sqrt{-k_z}\, s}{\rho_z \sqrt{-k_z}}$

The dispersion motion with finite values of η_z, η'_z at $s = 0$

$$\begin{pmatrix} \eta_z(s_1) \\ \eta'_z(s_1) \\ 1 \end{pmatrix} = \begin{pmatrix} M_z(s_0, s_1) & \begin{matrix} D_z(s_1 - s_0) \\ D'_z(s_1 - s_0) \end{matrix} \\ 0 \quad 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_z(s_0) \\ \eta'_z(s_0) \\ 1 \end{pmatrix}$$

We can follow the dispersion function as linear motion.

Collective beam motion

When we define any parameter set of (α, β) at the entrance of the beamline , the parameters are propagated in the beamline with Transfer Matrix components as

$$\begin{pmatrix}\beta_1\\ \alpha_1\\ \gamma_1\end{pmatrix}=\begin{pmatrix}M_{11}^2 & -2M_{11}M_{12} & M_{12}^2\\ -M_{11}M_{21} & 1+2M_{12}M_{21} & -M_{12}M_{22}\\ M_{21}^2 & -2M_{21}M_{22} & M_{22}^2\end{pmatrix}\begin{pmatrix}\beta_0\\ \alpha_0\\ \gamma_0\end{pmatrix}\qquad \gamma=\frac{1+\alpha^2}{\beta}$$

We can select any set of (α_0, β_0) in mathematically.

On the other hand, we can express "Transfer Matrix" itself with the parameters α, β and Δφ at the entrance and exit of the beamline as

$$M(s_1,s_0)=\begin{pmatrix}\sqrt{\beta_1} & 0\\ -\alpha_1/\sqrt{\beta_1} & 1/\sqrt{\beta_1}\end{pmatrix}\begin{pmatrix}\cos\Delta\varphi & \sin\Delta\varphi\\ -\sin\Delta\varphi & \cos\Delta\varphi\end{pmatrix}\begin{pmatrix}1/\sqrt{\beta_0} & 0\\ \alpha_0/\sqrt{\beta_0} & \sqrt{\beta_0}\end{pmatrix}$$

$$=T^{-1}(s_1)\begin{pmatrix}\cos\Delta\varphi & \sin\Delta\varphi\\ -\sin\Delta\varphi & \cos\Delta\varphi\end{pmatrix}T(s_0)$$

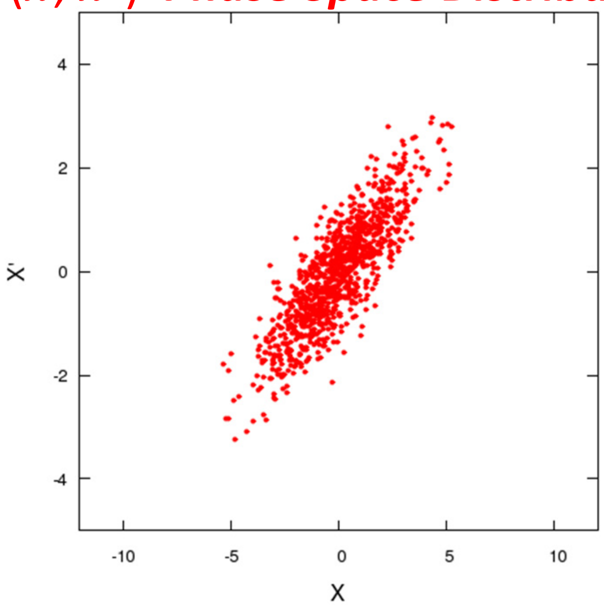
,where $T(s)\equiv\begin{pmatrix}1/\sqrt{\beta} & 0\\ \alpha/\sqrt{\beta} & \sqrt{\beta}\end{pmatrix}$, $\Delta\varphi=\int_{s_0}^{s_1}\frac{1}{\beta(s)}ds$

When we translate the phase space motion $\begin{pmatrix}x\\ x'\end{pmatrix}$ *to the phase space of* $V=\begin{pmatrix}u\\ v\end{pmatrix}\equiv T\begin{pmatrix}x\\ x'\end{pmatrix}$ *, the particle motion along the beamline is expressed only by the rotation in V space.*

Phase Space Distribution

(α, β) are selected **arbitrary** in mathematically.

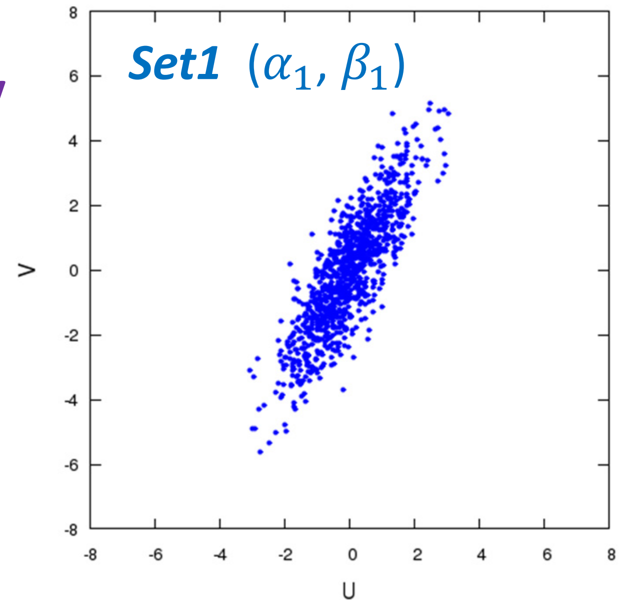
(x, x') Phase Space Distribution

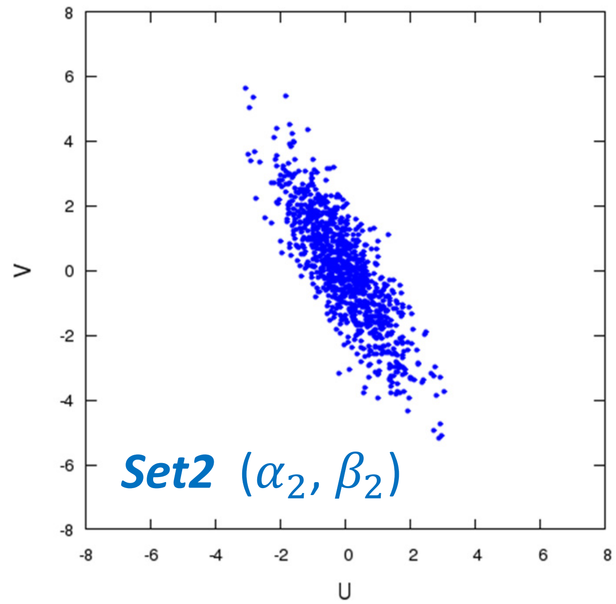


The phase space is transformed to the phase space V as

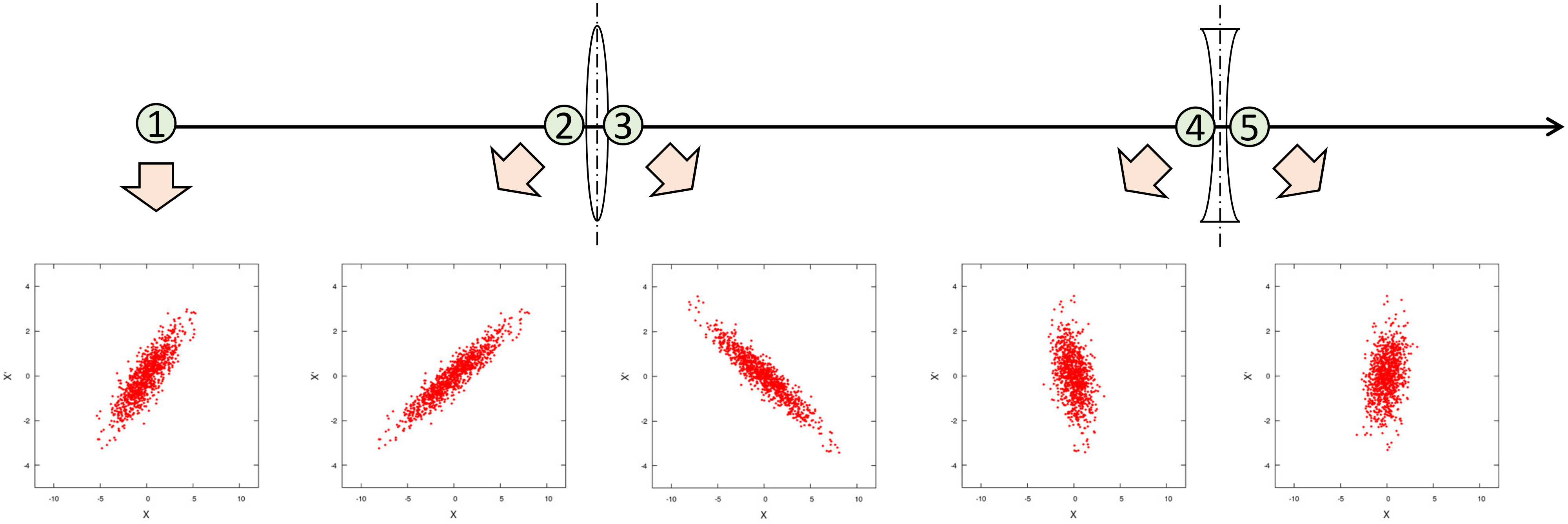
$$V = \begin{pmatrix} u \\ v \end{pmatrix} \equiv T \begin{pmatrix} x \\ x' \end{pmatrix}$$

(u, v) Phase Space Distribution





Phase space propagation along the beamline



(x, x') ***Phase Space Distribution***

The shape of the distribution is changed in the beamline.

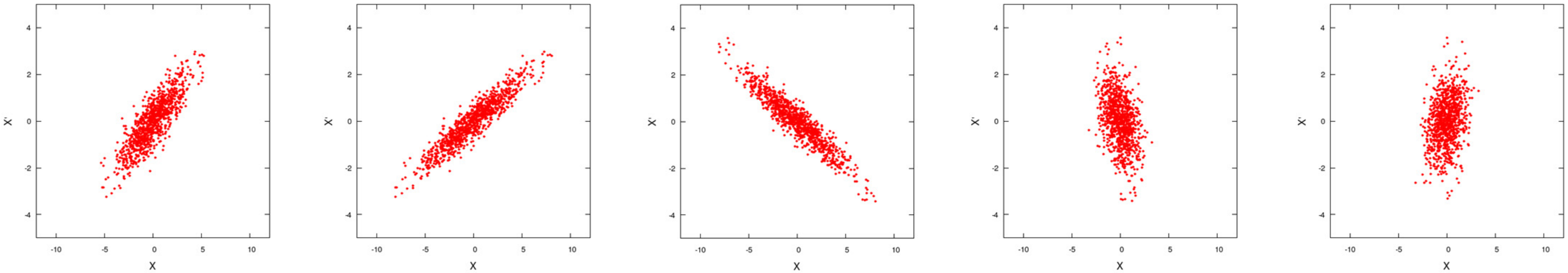
Phase space propagation along the beamline

$$\begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} \cos\Delta\varphi & \sin\Delta\varphi \\ -\sin\Delta\varphi & \cos\Delta\varphi \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}$$

All particles rotate same angle.

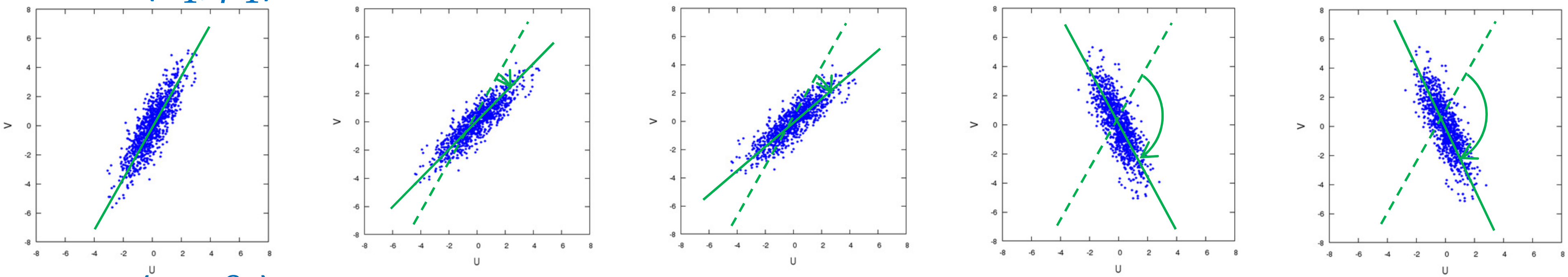
- *The shape is different for the initial setting of (α, β).*
- *Rotation angle is different for the initial setting.*
- *But, the shape will be kept through beamline.*

(x, x') ***Phase Space Distribution***

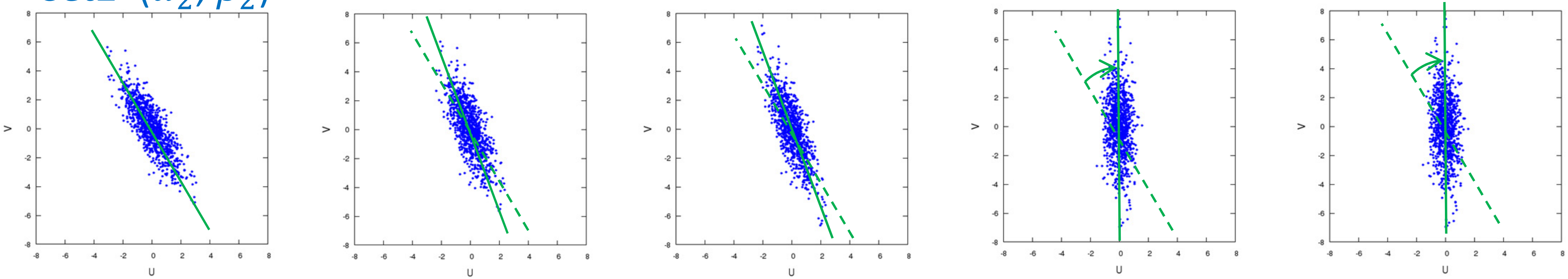


Set1 (α_1, β_1)

(u, v) ***Phase Space Distribution***

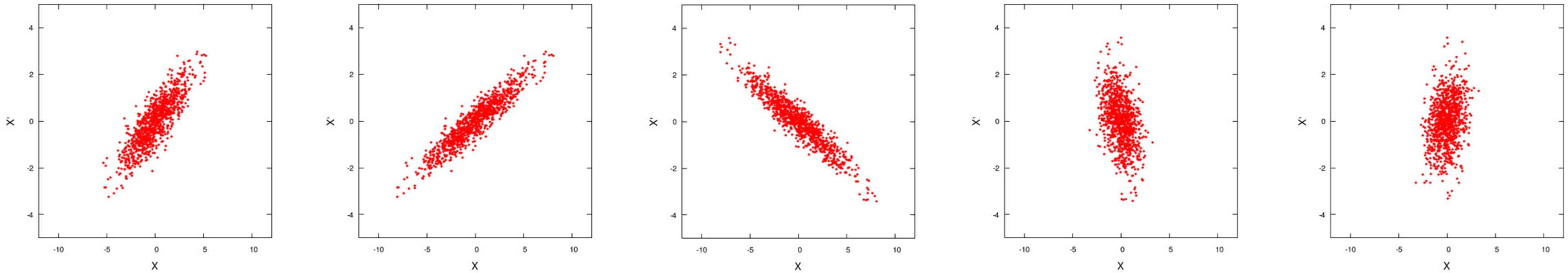


Set2 (α_2, β_2)



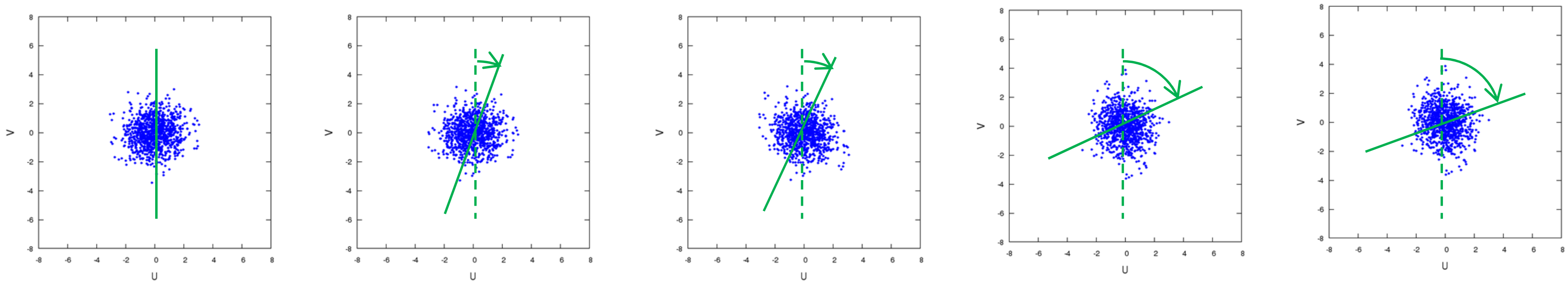
Phase space propagation along the beamline

(x, x') ***Phase Space Distribution***



When we select the parameter (α, β) to make the distribution round shape, the distribution in the phase space V is not changed through the beamline.

(u, v) ***Phase Space Distribution*** ***Phase Advance***



The area of the V phase space distribution is called to "emittance".

$$\langle u^2 \rangle = \langle v^2 \rangle = \varepsilon$$ ***Beam quality parameter***

Phase space propagation along the beamline

Twiss parameters (Beam envelope parameters)

Beam size

$$\langle x^2 \rangle = \beta \langle u^2 \rangle = \beta\varepsilon$$

$x - x'$ correlation

$$\langle xx' \rangle = \langle uv \rangle - \alpha \langle u^2 \rangle = -\alpha\varepsilon$$

When we selected the (α, β) to make V space distribution round shape,

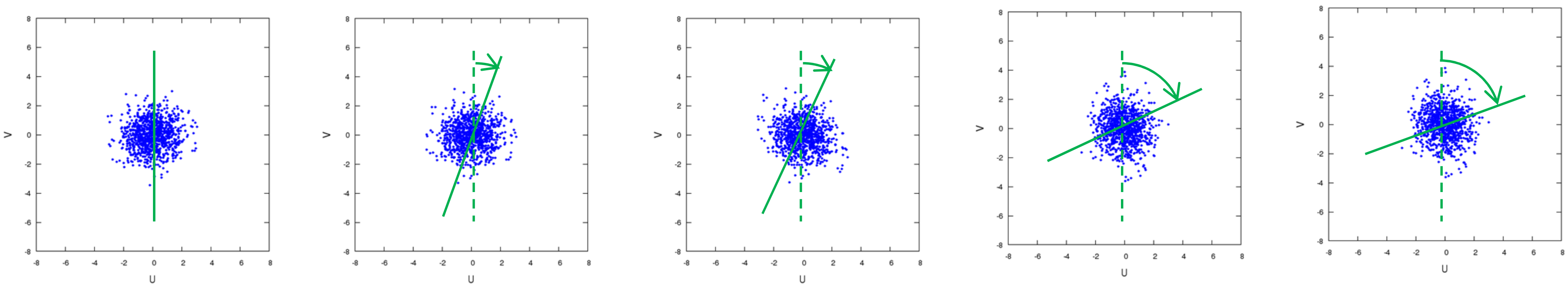
α, β and γ will be the physics parameters.

Beam divergence

$$\langle x'^2 \rangle = \frac{\langle v^2 \rangle - 2\alpha\langle uv \rangle + \alpha^2 \langle u^2 \rangle}{\beta} = \frac{1+\alpha^2}{\beta}\varepsilon = \gamma\varepsilon$$

(u, v) **Phase Space Distribution**

Phase Advance

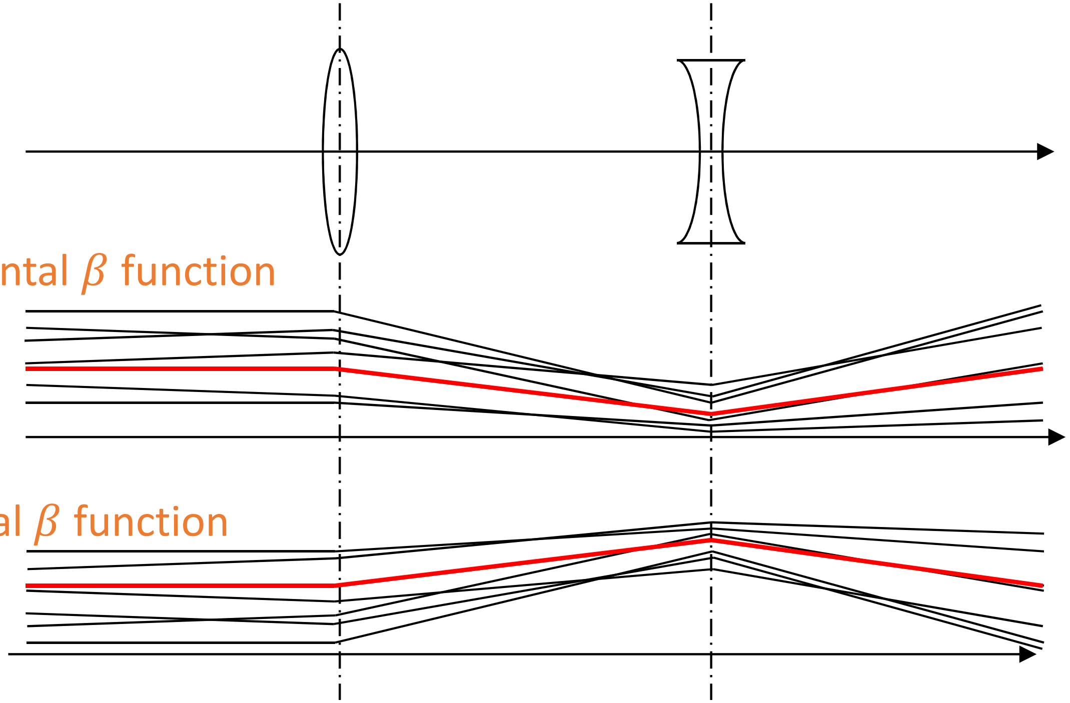


The area of the V phase space distribution is called to "emittance".

$$\langle u^2 \rangle = \langle v^2 \rangle = \varepsilon$$ **Beam quality parameter**

Collective particle motion as a "BEAM"

By selecting the parameter (α, β) to make V space distribution round shape, *we can calculate the beam size through the beamline as a "BEAM".*



Beam Matrix

$$\Sigma = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$\Sigma_1 = M\, \Sigma_0 M^{-1}$$

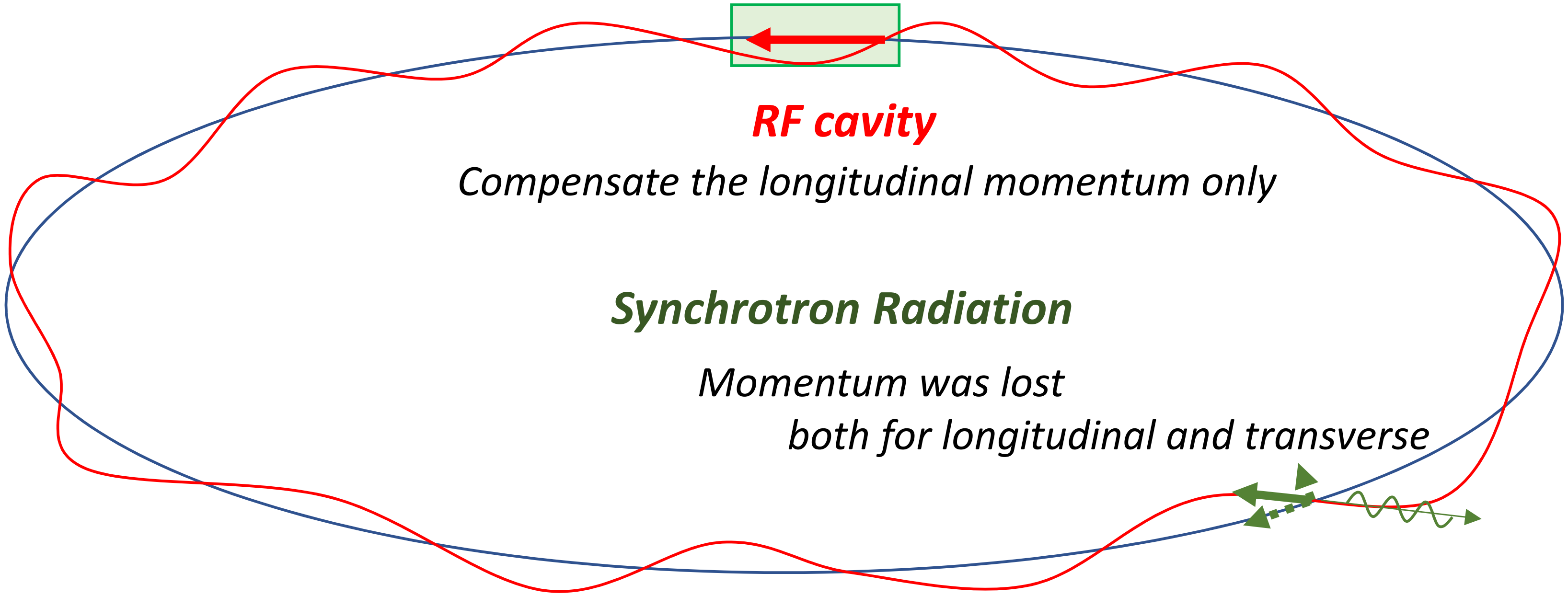
equivalent

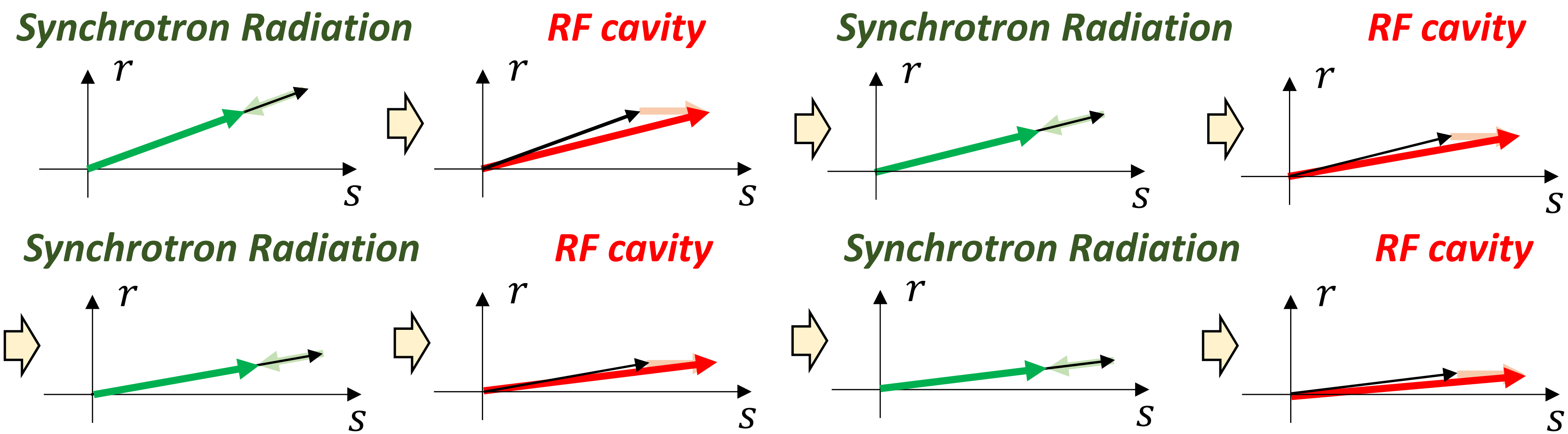
$$\begin{pmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{pmatrix} = \begin{pmatrix} M_{11}^2 & -2M_{11}M_{12} & M_{12}^2 \\ -M_{11}M_{21} & 1+2M_{12}M_{21} & -M_{12}M_{22} \\ M_{21}^2 & -2M_{21}M_{22} & M_{22}^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}$$

"Twiss parameters" expressed the envelopes of beam along the beamline .

Damping Ring

Radiation Damping



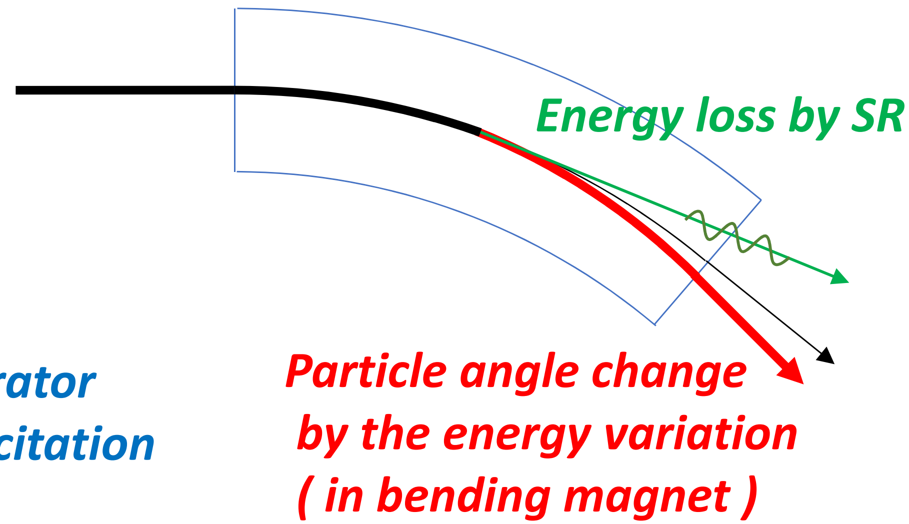


Particle directions in the beam changed to be parallel by radiation damping.

Radiation Excitation

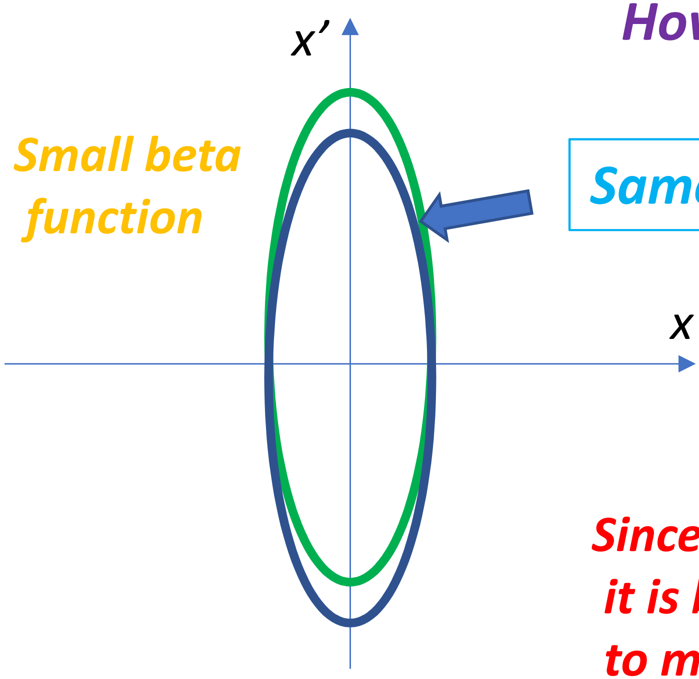
The direction of Synchrotron radiation
is not perfectly parallel to the beam direction.

It has an angle change by synchrotron radiation.

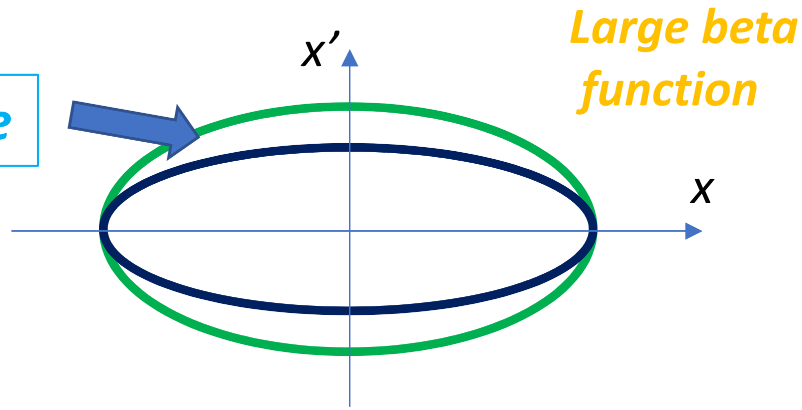


Horizontal beam emittance in the circular accelerator is determined by the balance of the radiation excitation and the radiation damping.

How to make small emittance beam



Same emittance



Since the radiation excitation is larger for larger beta function, it is better to put bending magnet at smaller beta function. to make a smaller emittance beam.

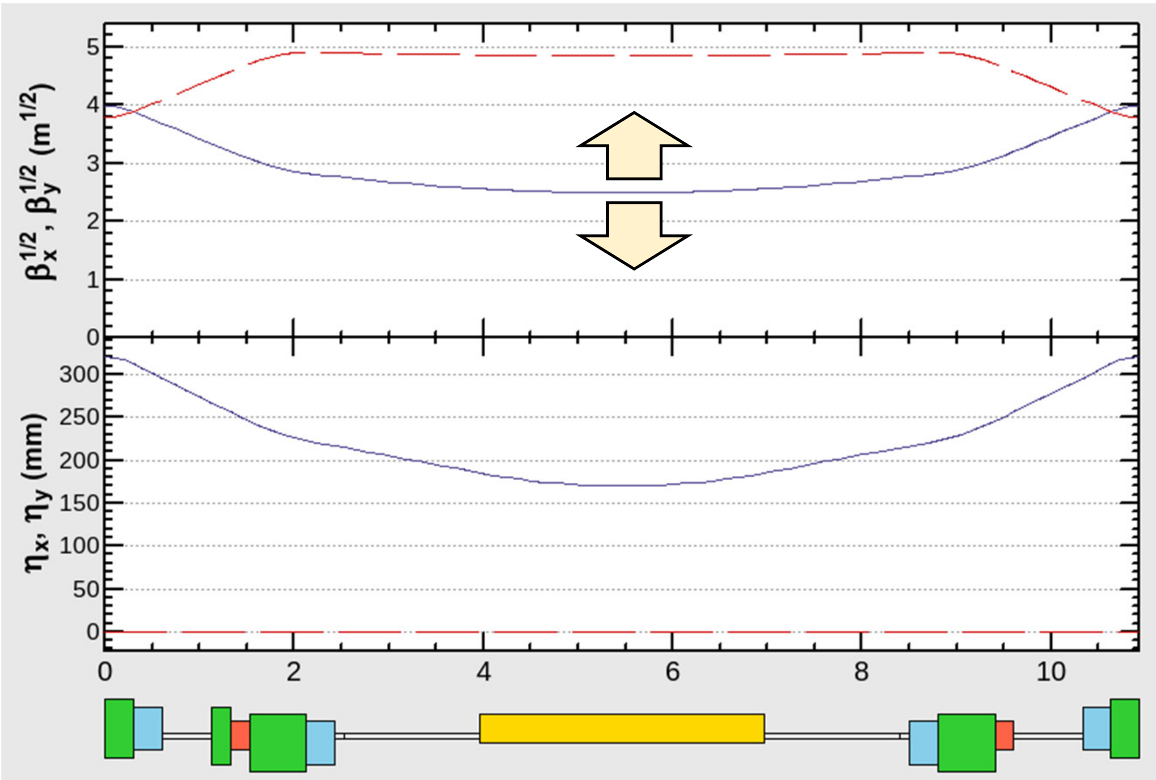
TME lattice for ILC damping ring

The valance of the beta function and dispersion though bending magnet is important to reduce the radiation excitation.

$$\Delta W_x = \langle \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2 \rangle \left(\frac{\Delta p_\gamma}{p} \right)^2$$

(Optimum distribution of beta function in the bending magnet)

→ ***TME (Theoretical Minimum Emittance) lattice***



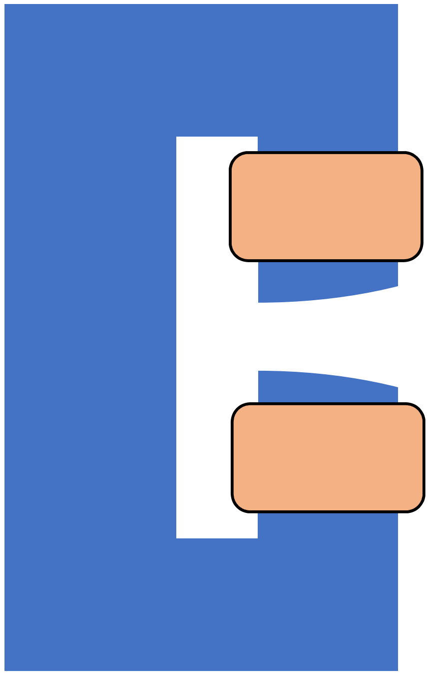
Beam optics of ATF damping ring arc cell

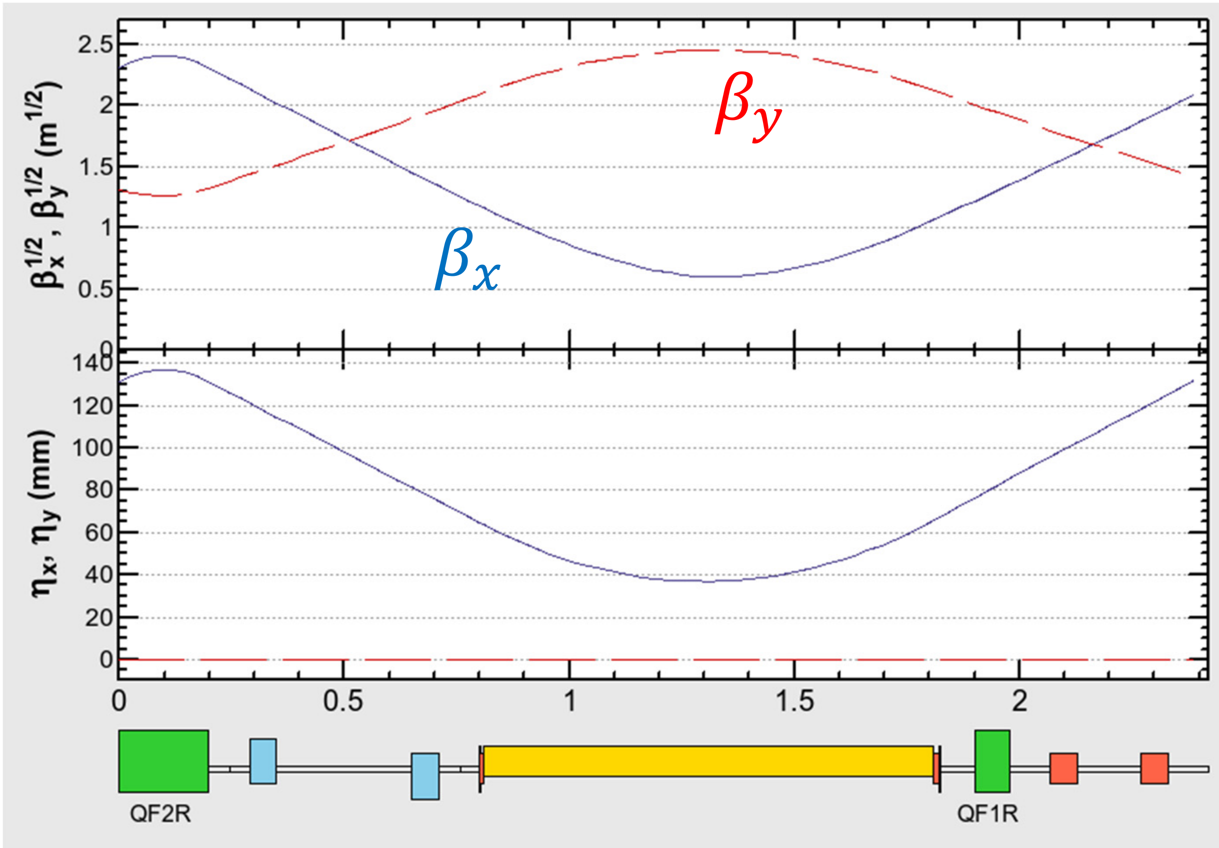
ATF damping ring was operated for the investigation of the DR for linear colliders (JLC/GLC/ILC) from 1997.

The space to build the damping was limited, FOBO lattice was adopted.

In order to make the minimum β_x at the center of bending magnet, the combined function (defocusing) was used in ATF damping ring.

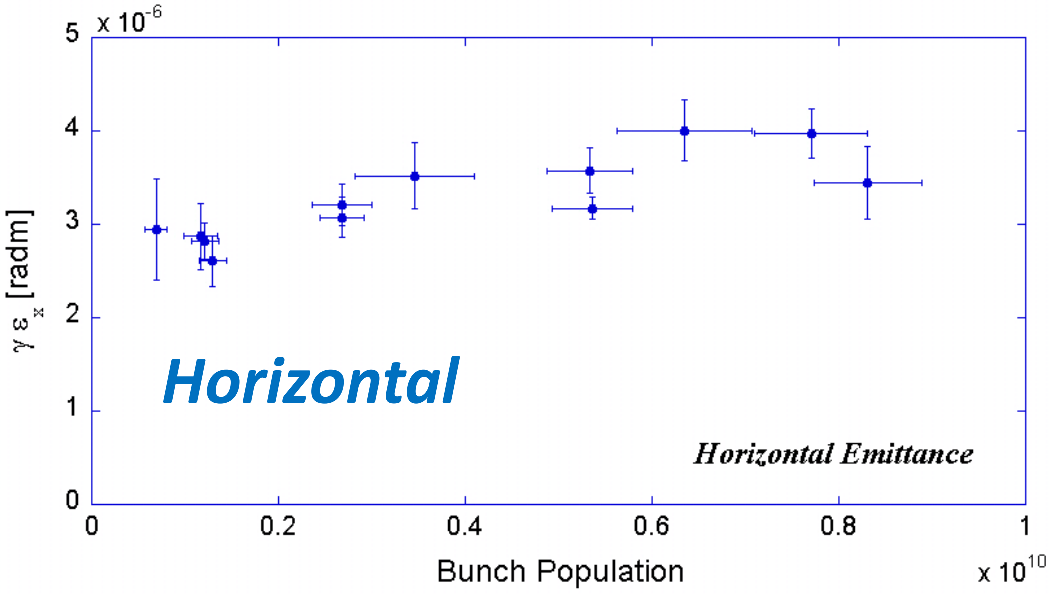
Combined function bending magnet

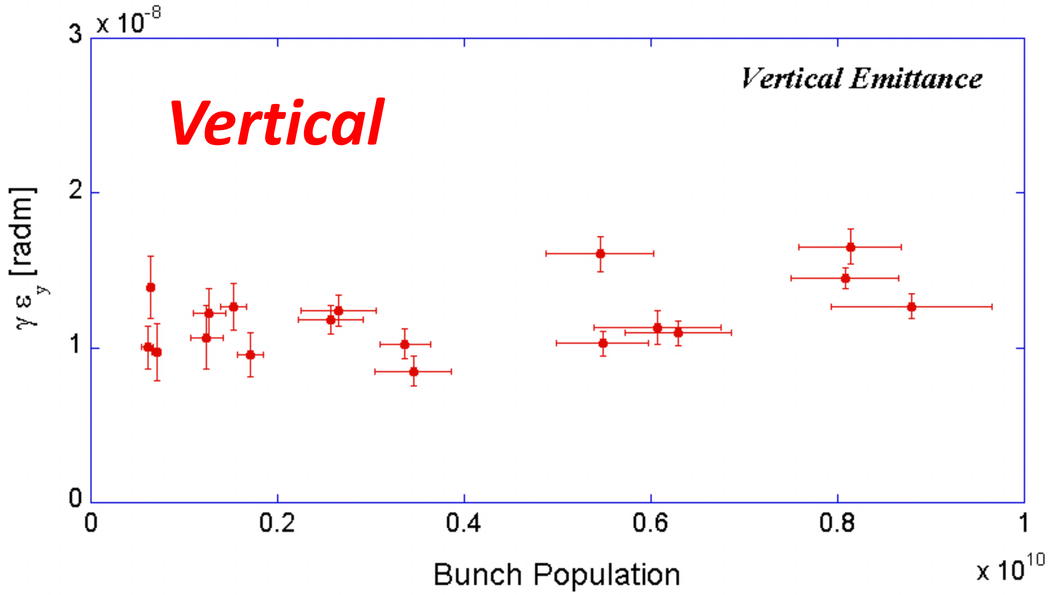




The radiation excitation for ATF damping ring is 1.2 times larger than TME lattice

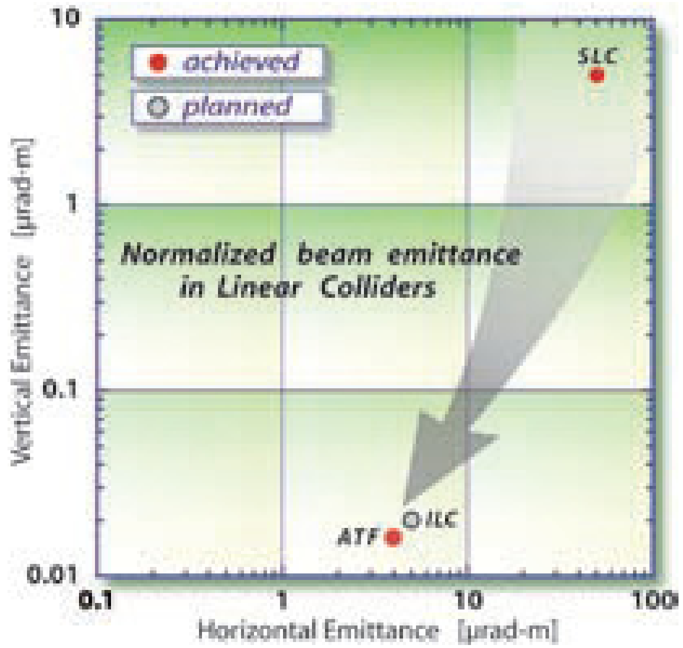
Beam emittance in ATF damping ring





Requirement of ILC emittance at DR

	Normalized emittance
Horizontal ($\gamma\varepsilon_x$)	5.5×10^{-6} rad · m
Vertical ($\gamma\varepsilon_y$)	2.0×10^{-8} rad · m



ATF damping ring was achieved the smaller emittance than ILC requirement.

Vertical emittance was defined by misalignment and x-y coupling.

Chromaticity Correction

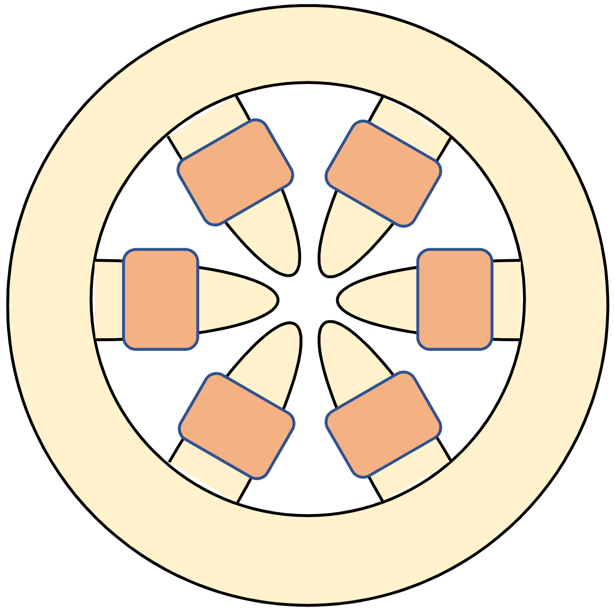
The focusing of quadrupole is different by the particle energy. → ***"chromaticity"***

The sextupole magnets were put to the dispersive location, the chromaticity was generated.
→ *correct the chromaticity of quadrupoles.*

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ -2\eta_x K_2 \sqrt{\beta_x}\, u_x \delta \\ 0 \\ +2\eta_x K_2 \sqrt{\beta_y}\, u_y \delta \end{pmatrix}}_{\text{Chromaticity}} + K_2 \underbrace{\begin{pmatrix} 0 \\ \beta_y u_y^2 - \beta_x u_x^2 - \eta_x^2 \delta^2 \\ 0 \\ 2\sqrt{\beta_x \beta_y}\, u_x u_y \end{pmatrix}}_{\text{2nd order aberrations}}$$

$$\xi_x = -2\eta_x K_2 \beta_x$$
$$\xi_y = +2\eta_x K_2 \beta_y$$

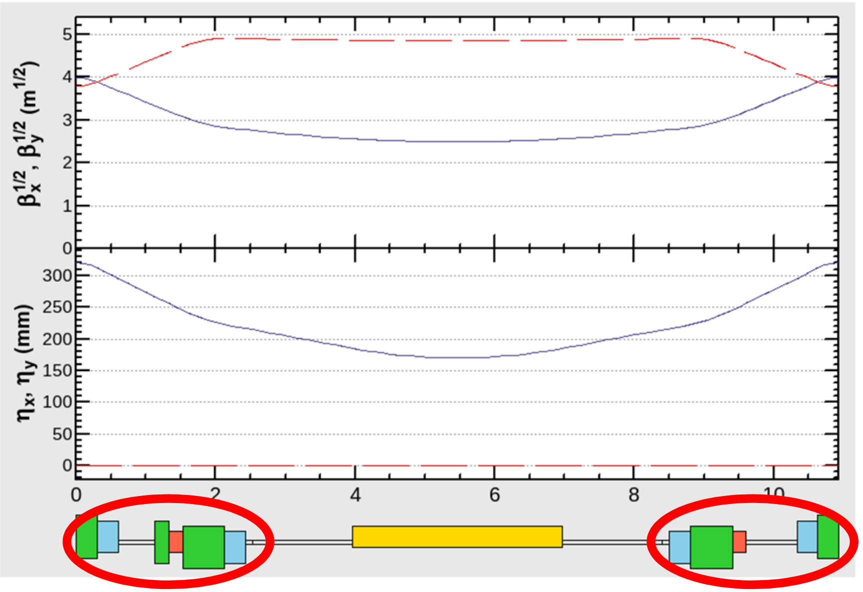
Sextupole magnet



Not only chromaticity, but also geometrical aberration was generated.
→ *The dynamic aperture was reduced.*
→ ***It is better to use the weaker strength of sextupoles in order to increase the dynamic aperture.***

Difference of ILC and ATF damping ring

ILC damping ring (TME lattice)



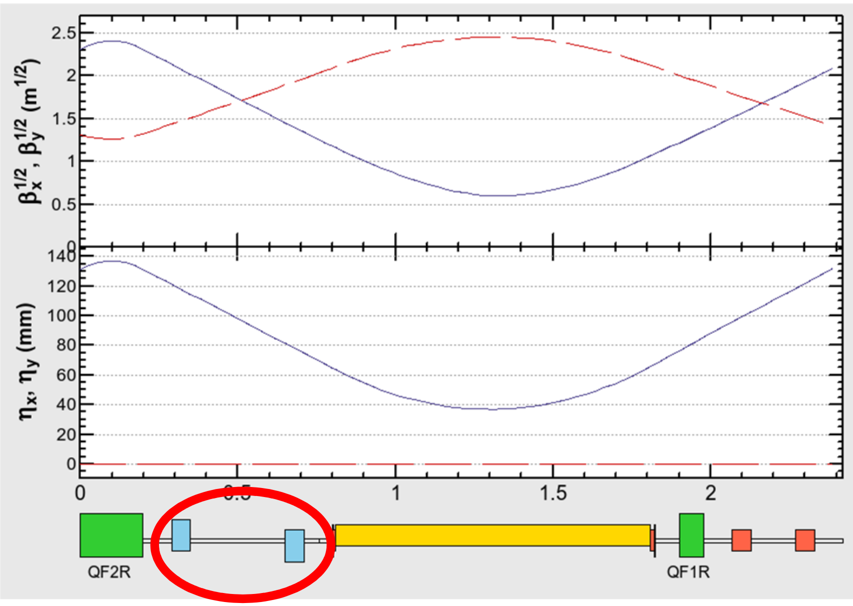
Minimum radiation excitation

Appropriate location of sextupole magnets, 4 sextupoles in single cell.

→ *weaker sextupole magnets*
→ *large dynamic aperture*

→ *no pre-damping ring in design*

ATF damping ring (FOBO lattice)



Smaller radiation excitation (1.2 times larger than TME)

JLC/GLC design

Compact lattice is necessary in order to make a small emittance

→ *matchd to ATF geometry*

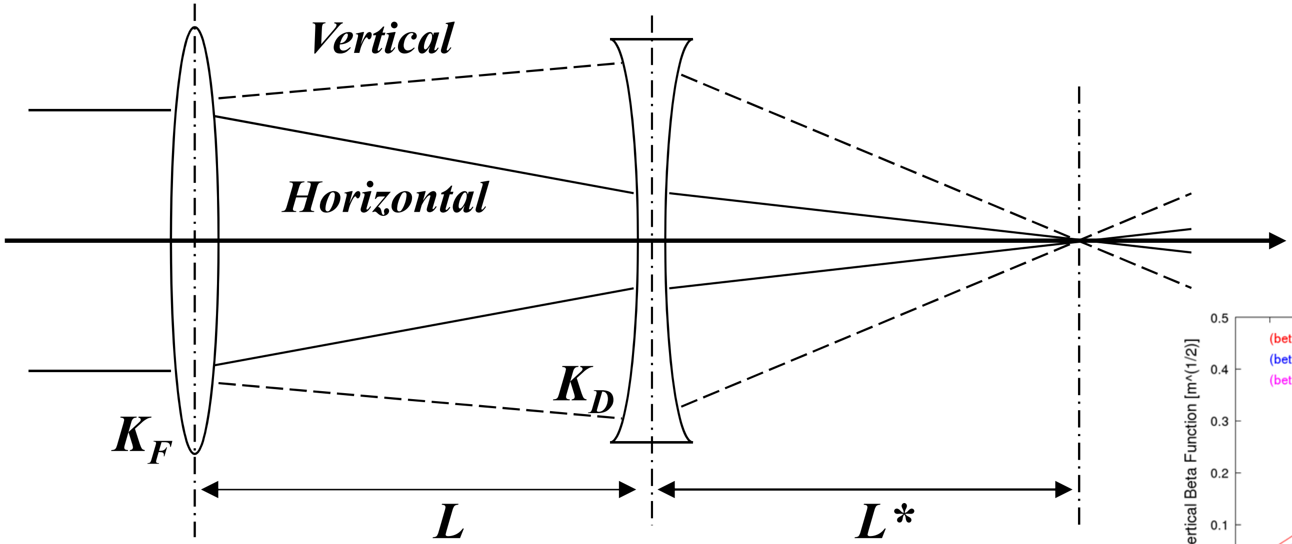
→ *2 sextupoles at small beta and dispersion*
→ *strong sextupole magnets*
→ *small dynamic aperture*

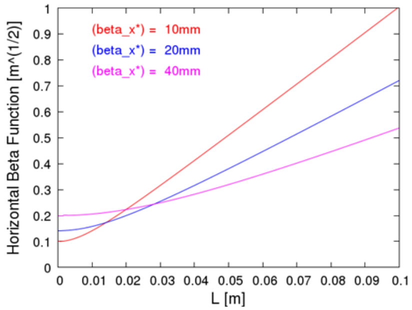
→ *introduce pre-damping ring for positron beam with large dynamic aperture in design*

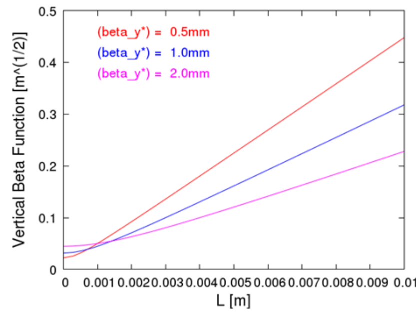
Final Focus Beamline

Final Doublet System

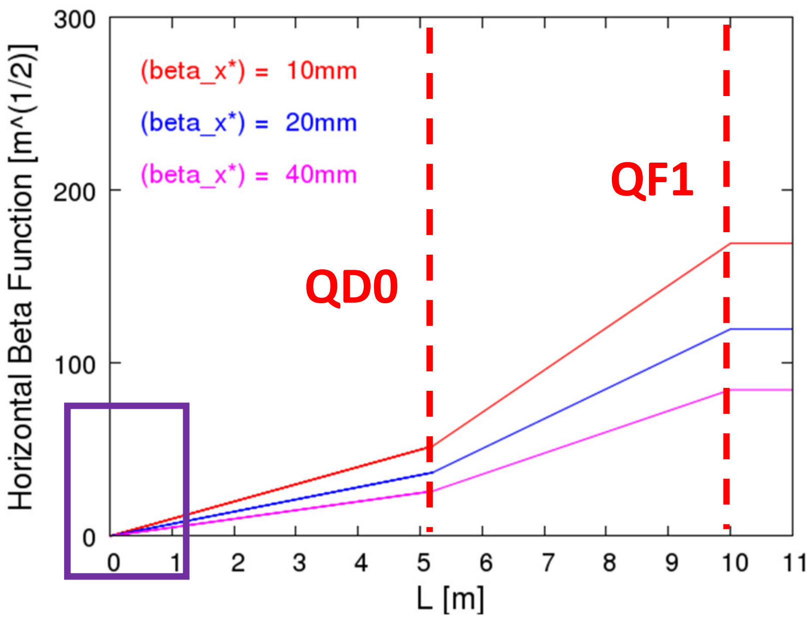
In order to focus the beam both for horizontal and vertical direction, it is necessary to use the combination lens system for final focus lens.



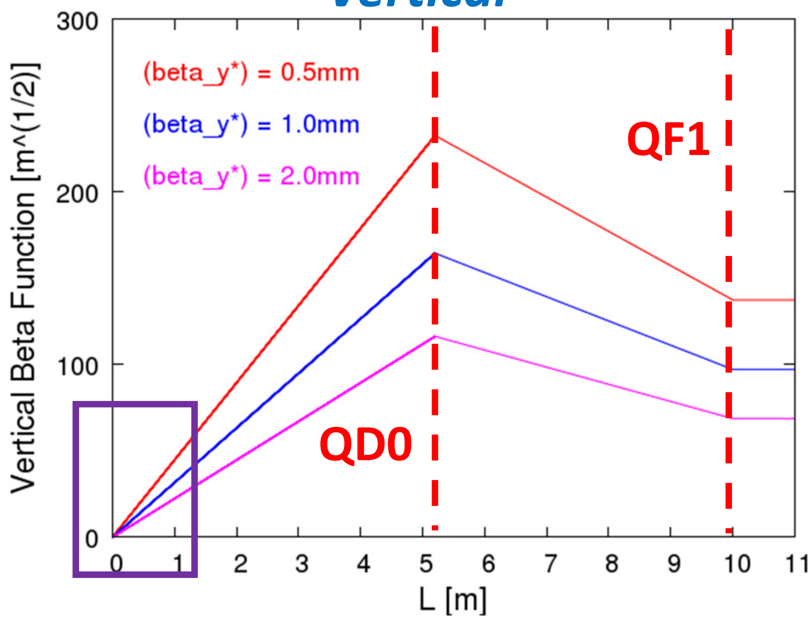




Horizontal



Vertical



Chromaticity of Final Doublet

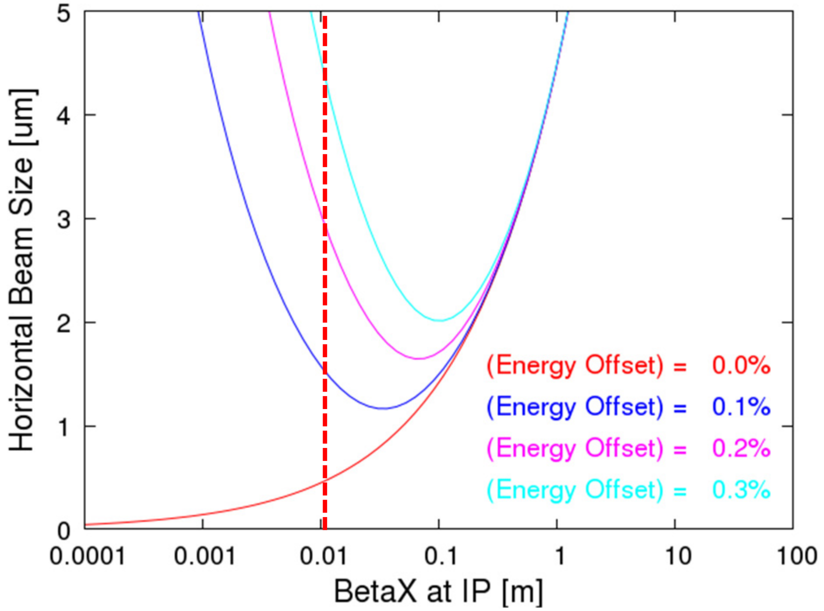
The aberration of off-momentum particle is called to "chromaticity".

$$K_F = \frac{1}{\sqrt{L(L+L^*)}} \Rightarrow K_F = \frac{1}{\boxed{(1+\delta)}\sqrt{L(L+L^*)}}$$

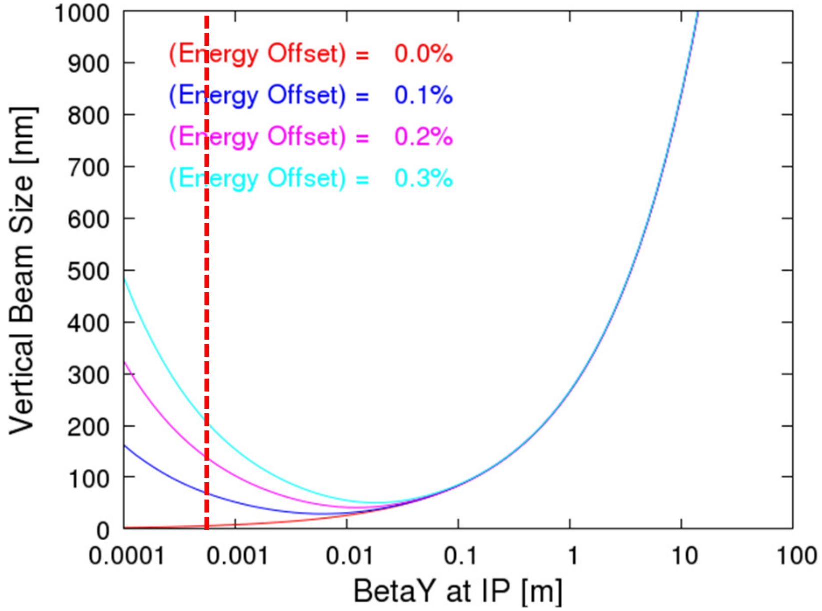
$$K_D = \frac{1}{L^*}\sqrt{\frac{L+L^*}{L}} \Rightarrow K_D = \frac{1}{\boxed{(1+\delta)}L^*}\sqrt{\frac{L+L^*}{L}}$$

Focusing strength is different for off-momentum particle

Horizontal



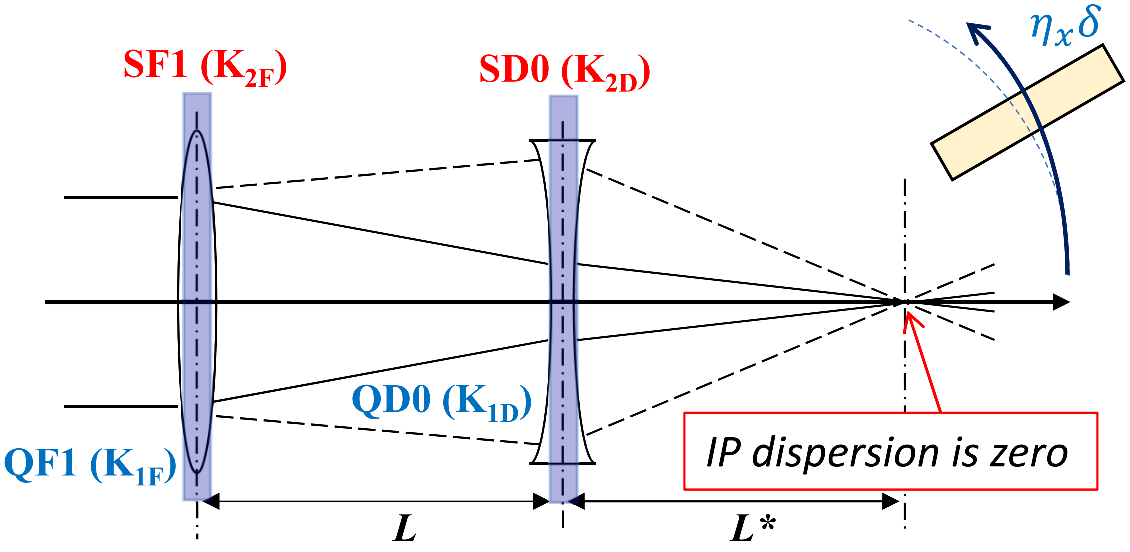
Vertical



Limit of IP beta function w/o chromaticity correction.
(ILC 500 GeV design BetaX=0.011m, BetaY*=0.00048m)*

⇨ *We need correction !*

Geometrical Aberration and 2nd order Dispersion



Sextupole Magnet

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ -2\eta_x K_2 \sqrt{\beta_x}\, u_x \delta \\ 0 \\ +2\eta_x K_2 \sqrt{\beta_y}\, u_y \delta \end{pmatrix}}_{\text{Chromaticity}} + K_2 \underbrace{\begin{pmatrix} 0 \\ \beta_y u_y^2 - \beta_x u_x^2 - \eta_x^2 \delta^2 \\ 0 \\ 2\sqrt{\beta_x \beta_y}\, u_x u_y \end{pmatrix}}_{\text{2nd order aberrations}}$$

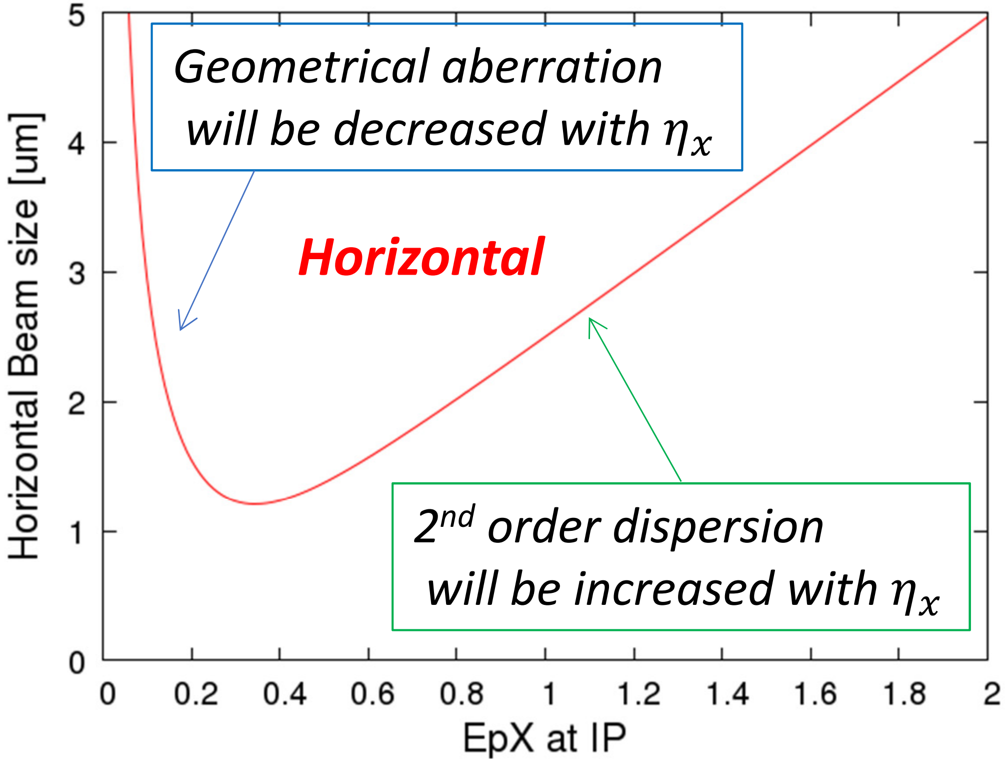
Chromaticity

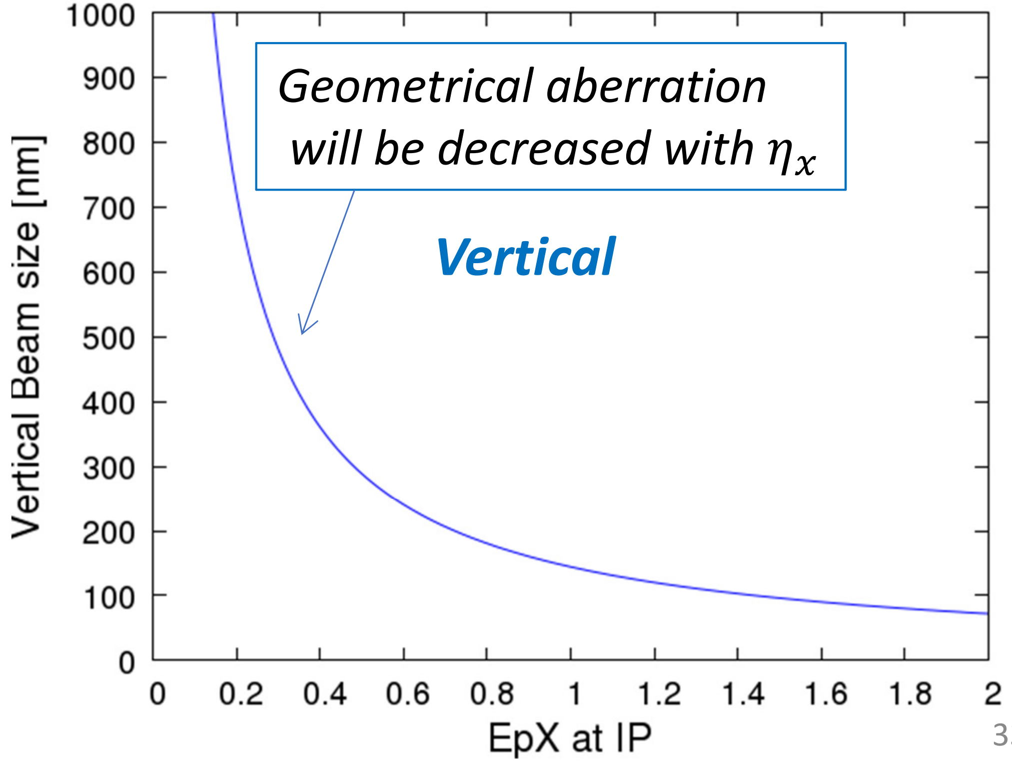
$$\xi_x = -2\eta_x K_2 \beta_x$$

$$\xi_y = +2\eta_x K_2 \beta_y$$

2nd order aberrations

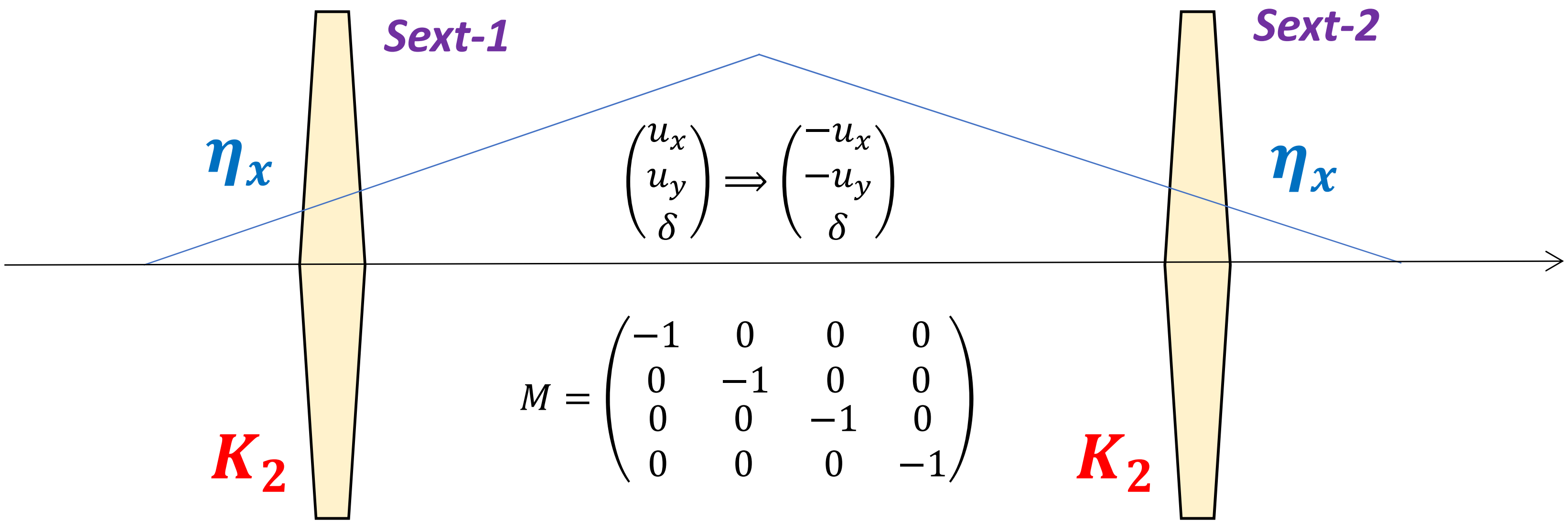
Calculation of chromaticity correction with sextupoles (SF1, SD0) for with ILC $E_{CM} = 500\ GeV$





Idea to avoid 2nd order aberration 1

2 sextupoles are put to the following condition.



$$\begin{pmatrix} u_x \\ u_y \\ \delta \end{pmatrix} \Rightarrow \begin{pmatrix} -u_x \\ -u_y \\ \delta \end{pmatrix}$$

$$M = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Sext-1

$$\begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix} = \begin{pmatrix} -2\eta_x K_2 \sqrt{\beta_x}\, u_x \delta \\ +2\eta_x K_2 \sqrt{\beta_y}\, u_y \delta \end{pmatrix} + K_2 \begin{pmatrix} \beta_y u_y^2 - \beta_x u_x^2 - \eta_x^2 \delta^2 \\ 2\sqrt{\beta_x \beta_y}\, u_x u_y \end{pmatrix} \Rightarrow \begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix} = -\begin{pmatrix} -2\eta_x K_2 \sqrt{\beta_x}\, u_x \delta \\ +2\eta_x K_2 \sqrt{\beta_y}\, u_y \delta \end{pmatrix} - K_2 \begin{pmatrix} \beta_y u_y^2 - \beta_x u_x^2 - \eta_x^2 \delta^2 \\ 2\sqrt{\beta_x \beta_y}\, u_x u_y \end{pmatrix}$$

$$+$$

Sext-2 $$\begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix} = -\begin{pmatrix} -2\eta_x K_2 \sqrt{\beta_x}\, u_x \delta \\ +2\eta_x K_2 \sqrt{\beta_y}\, u_y \delta \end{pmatrix} + K_2 \begin{pmatrix} \beta_y u_y^2 - \beta_x u_x^2 - \eta_x^2 \delta^2 \\ 2\sqrt{\beta_x \beta_y}\, u_x u_y \end{pmatrix}$$

$$\Downarrow$$

Total $$\begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix} = \underline{\underline{\begin{pmatrix} +4\eta_x K_2 \sqrt{\beta_x}\, u_x \delta \\ -4\eta_x K_2 \sqrt{\beta}\, u_y \delta \end{pmatrix}}}$$

Chromaticity

Only chromaticities are generated as total system.

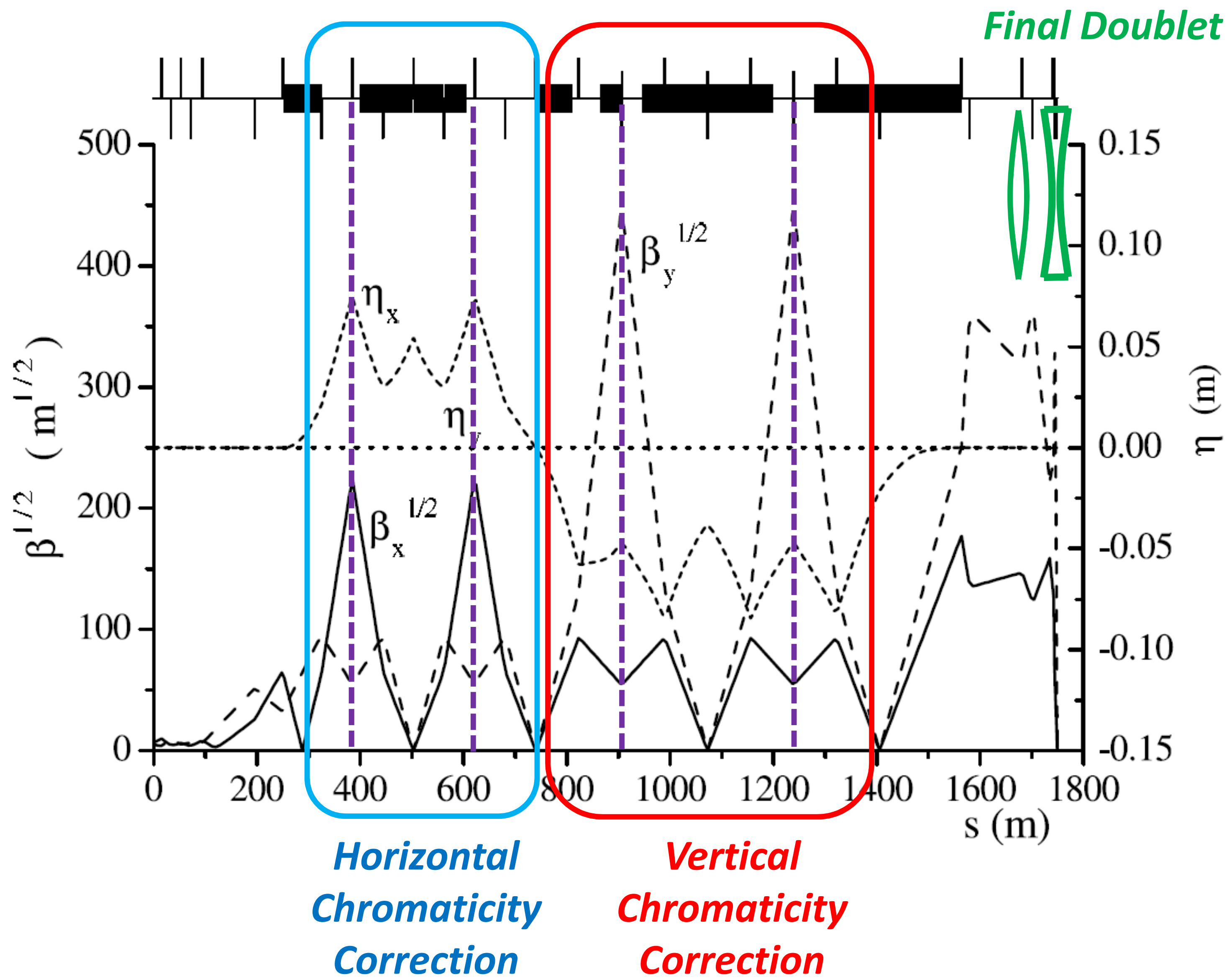
Global Chromaticity Correction System

3 independent beamline are connected to be in series.

Horizontal chrotaticity correction section

Vertical chromaticity correction section

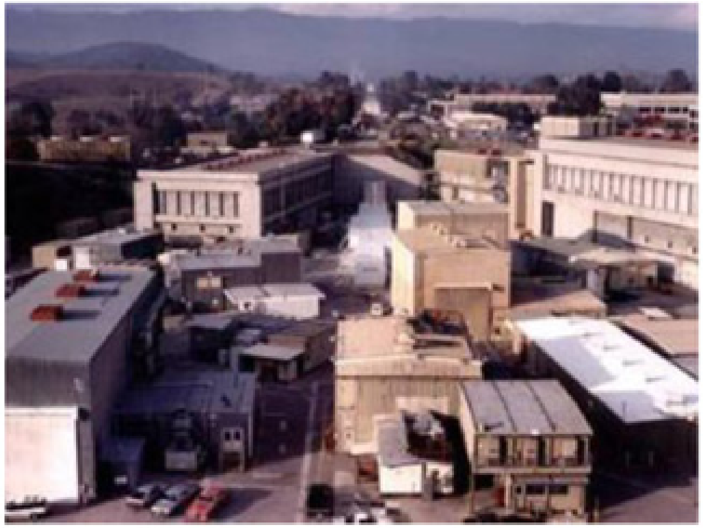
Final focus beamline

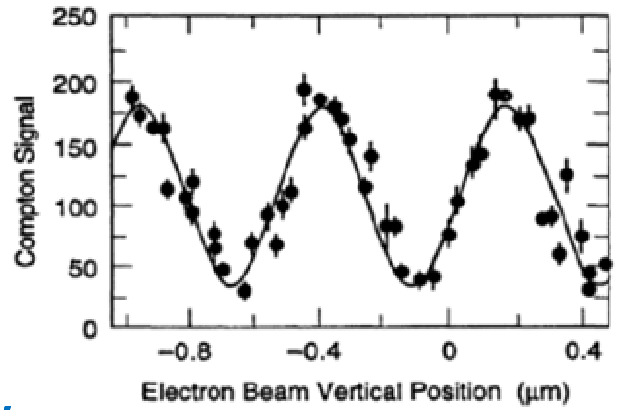


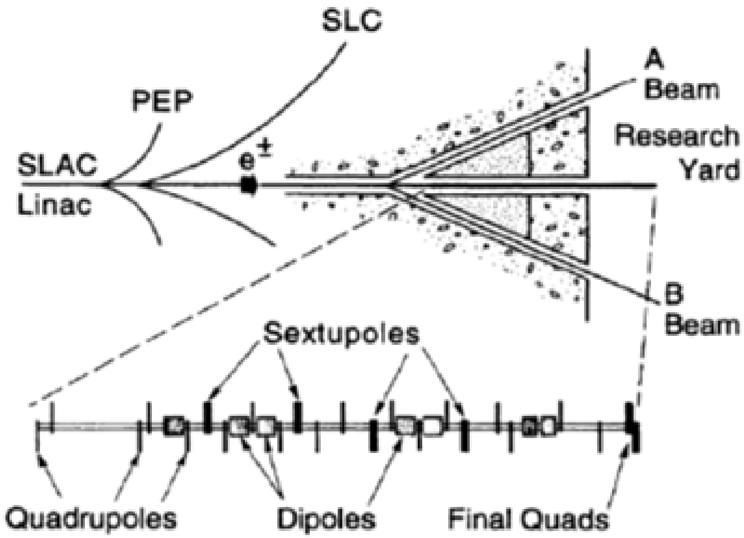
Final Focus Test Beam (FFTB)

*FFTB is built in SLC research yard (SLAC) for the LC final focus test with **global chromaticity correction system**.*

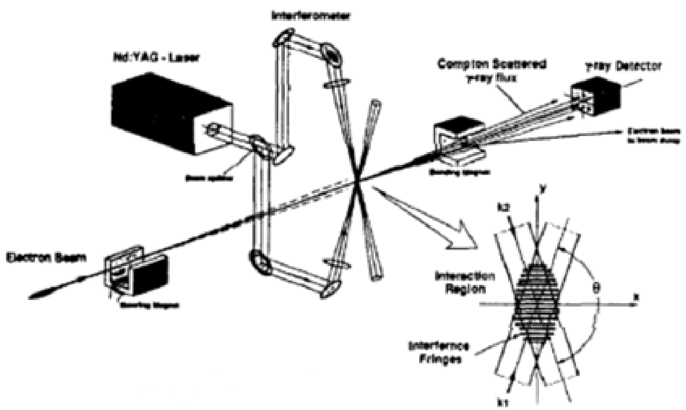
	IP beam size
Design	45 nm
Achieved	70 nm

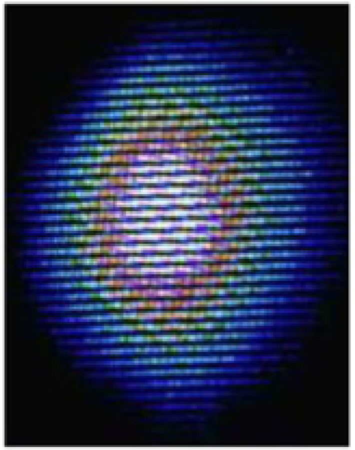






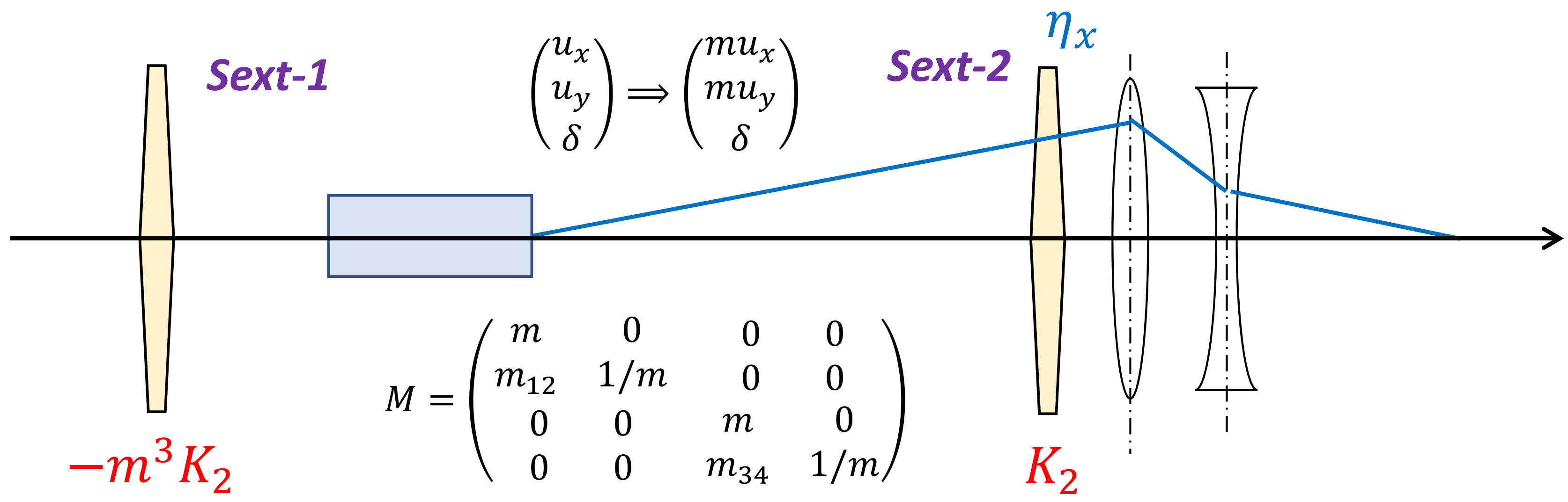
Shintake Monitor





detail will be shown in later.

Idea to avoid 2nd order aberration 2



Sext-1 $\begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix} = -m^3 K_2 \begin{pmatrix} \beta_y u_y^2 - \beta_x u_x^2 \\ 2\sqrt{\beta_x \beta_y}\, u_x u_y \end{pmatrix} \Rightarrow \begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix} = -m^2 K_2 \begin{pmatrix} \beta_y u_y^2 - \beta_x u_x^2 \\ 2\sqrt{\beta_x \beta_y}\, u_x u_y \end{pmatrix}$

$+$

Sext-2 $\begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix} = \begin{pmatrix} -2m\, \eta_x K_2 \sqrt{\beta_x}\, u_x \delta \\ +2m\, \eta_x K_2 \sqrt{\beta}\ \ u_y \delta \end{pmatrix} + m^2 K_2 \begin{pmatrix} \beta_y u_y^2 - \beta_x u_x^2 - \eta_x^2 \delta^2 / m^2 \\ 2\sqrt{\beta_x \beta_y}\, u_x u_y \end{pmatrix}$

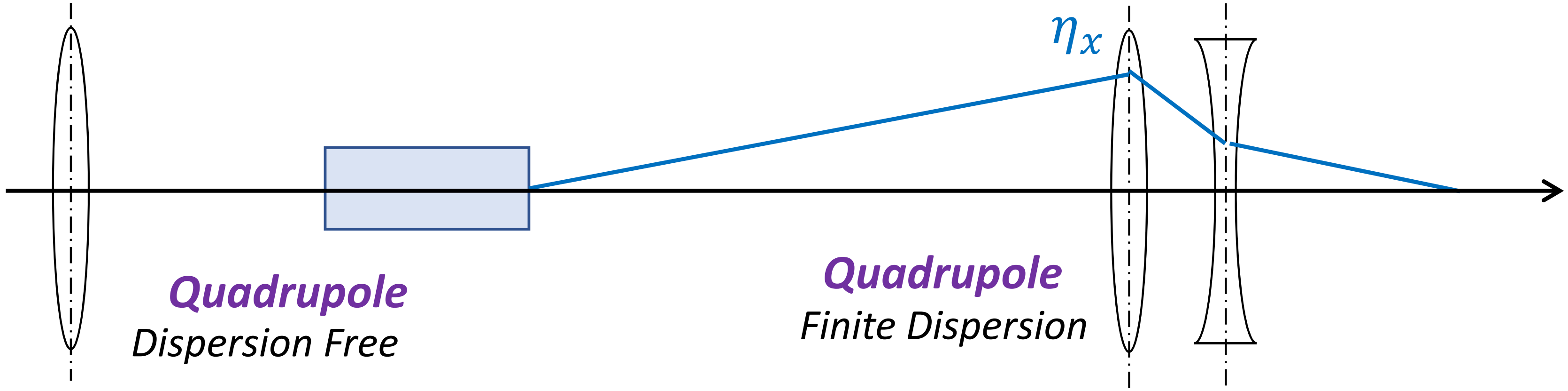
$\Downarrow$

Geonetrical aberration was cancelled.

Total $\begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix} = \underbrace{\begin{pmatrix} -2m\, \eta_x K_2 \sqrt{\beta_x}\, u_x \delta \\ +2m\, \eta_x K_2 \sqrt{\beta_y}\, u_y \delta \end{pmatrix}}_{\text{Chromaticity}} + \underbrace{\begin{pmatrix} -K_2 \eta_x^2 \delta^2 \\ 0 \end{pmatrix}}_{\text{2nd order dispersion in horizontal}}$

Local Dispersion Correction System

Quadrupole at dispersive area generate not only chromaticity but also 2nd order dispersion.



Quadrupole
Dispersion Free

$$\begin{pmatrix}\Delta x_{IP}\\ \Delta y_{IP}\end{pmatrix} = \underbrace{\begin{pmatrix}+K\beta_x\sqrt{\beta_x^*}\,u_x\delta\\ -K\beta_y\sqrt{\beta_y^*}\,u_y\delta\end{pmatrix}}_{\text{Chromaticity}}$$

Quadrupole
Finite Dispersion

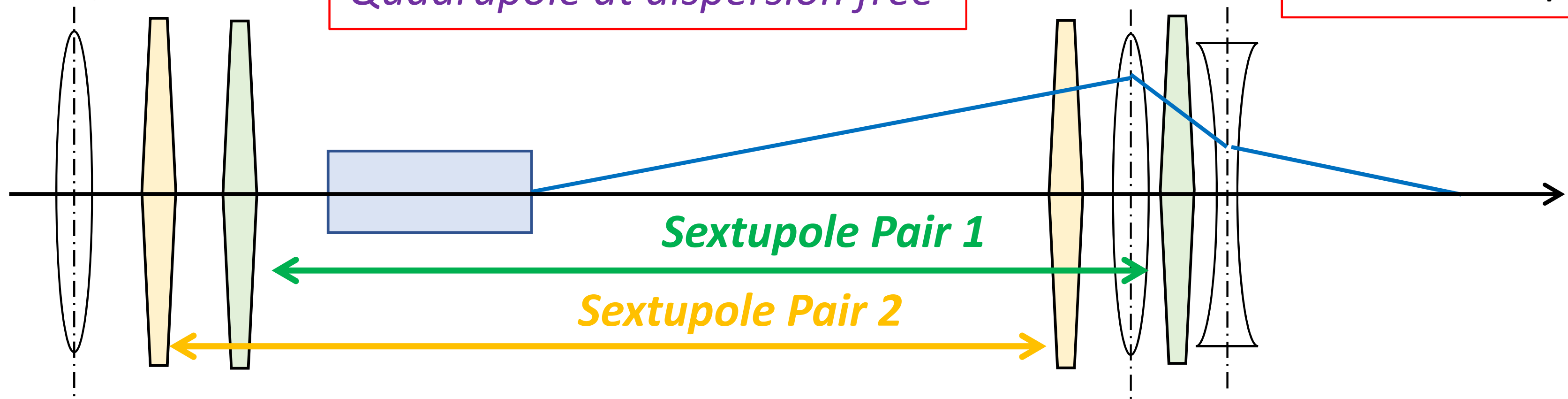
$$\begin{pmatrix}\Delta x_{IP}\\ \Delta y_{IP}\end{pmatrix} = \underbrace{\begin{pmatrix}+K\beta_x\sqrt{\beta_x^*}\,u_x\delta\\ -K\beta_y\sqrt{\beta_y^*}\,u_y\delta\end{pmatrix}}_{\text{Chromaticity}} + \underbrace{\begin{pmatrix}+K\eta_x\sqrt{\beta_x\beta_x^*}\,\delta^2\\ 0\end{pmatrix}}_{\text{2nd order dispersion}}$$

Sextupole Pair 1
Sextupole Pair 2
Quadrupole at dispersion free

Correction

X Chromaticity
Y Chromaticity
2nd order Dispersion

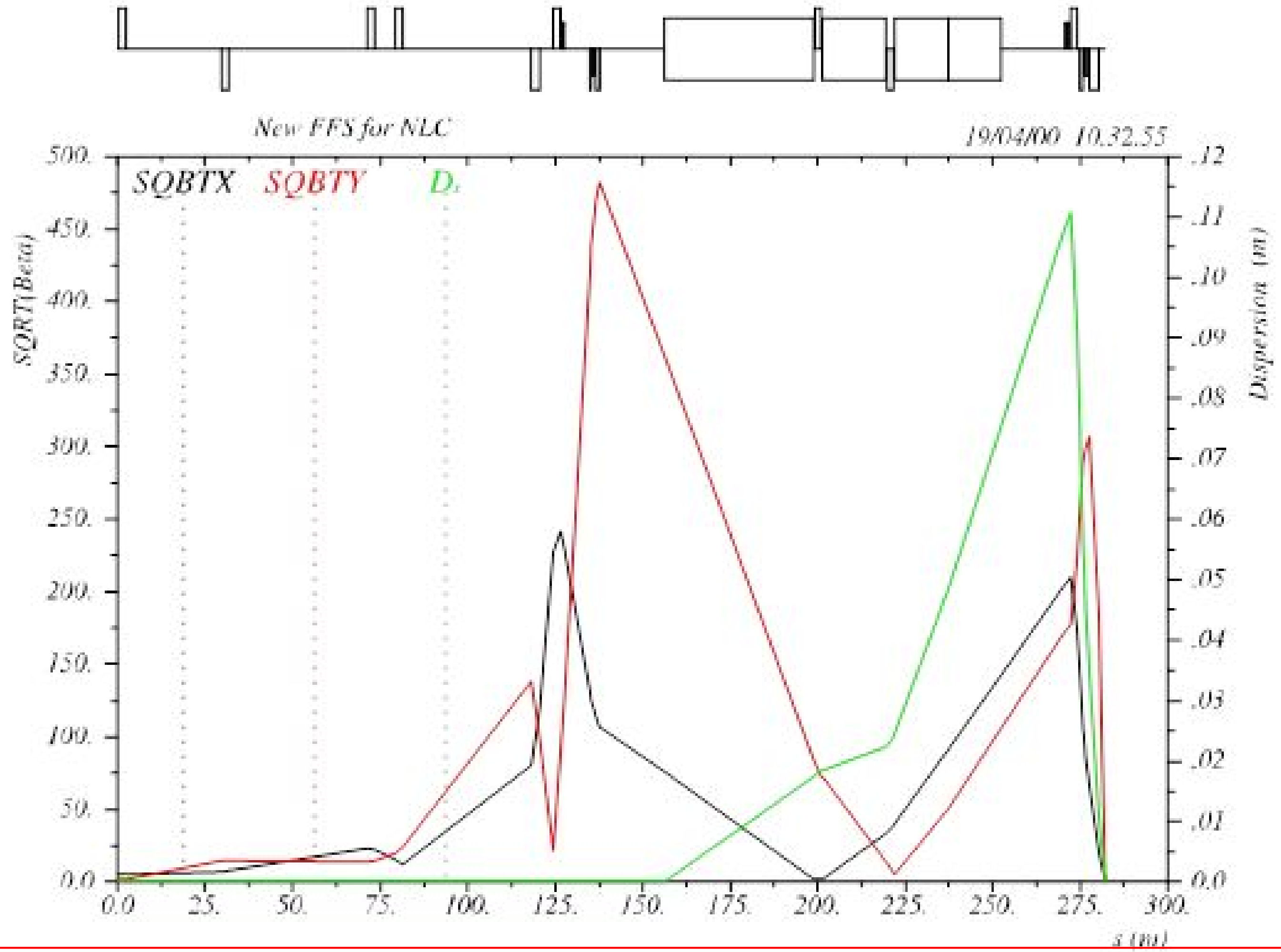
Quadrupole
Dispersion Free



Beam Optics of LC Final Focus System with Local Chromaticity Correction Method

The sextupoles to correct the chromaticities are interleaved in final focus beamline.

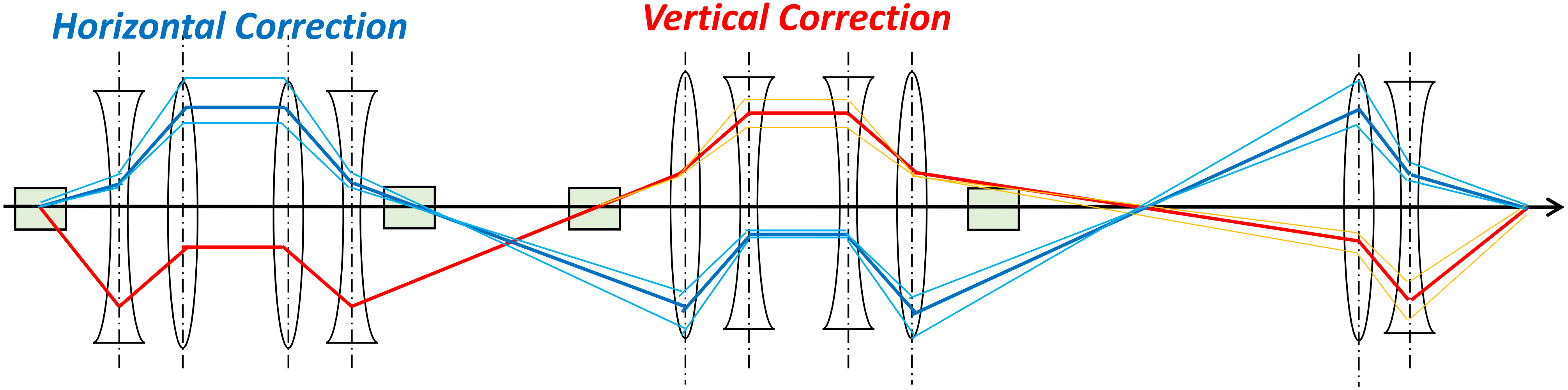
proposed by P. Raimondi and A. Seryi, PRL Vol. 86 3779 (2001)



6times shorter than LC FF beamline with Global Chromaticity Correction.

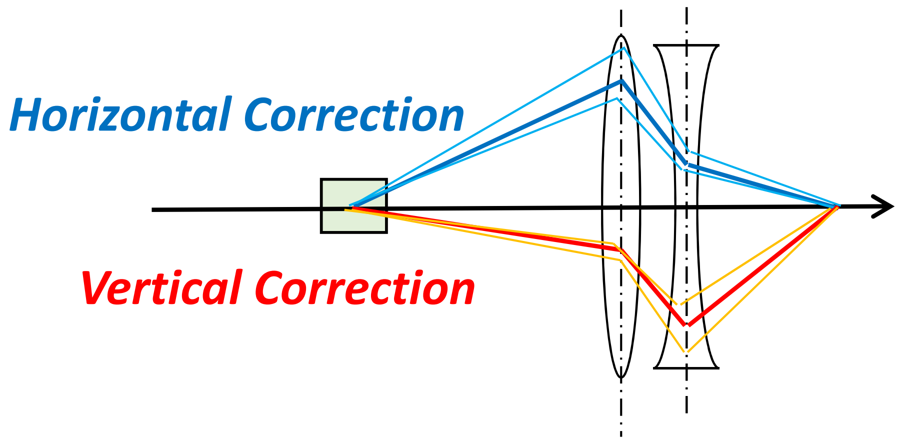
Orbit Distortion of off-momentum particle

Global Chromaticity Correction



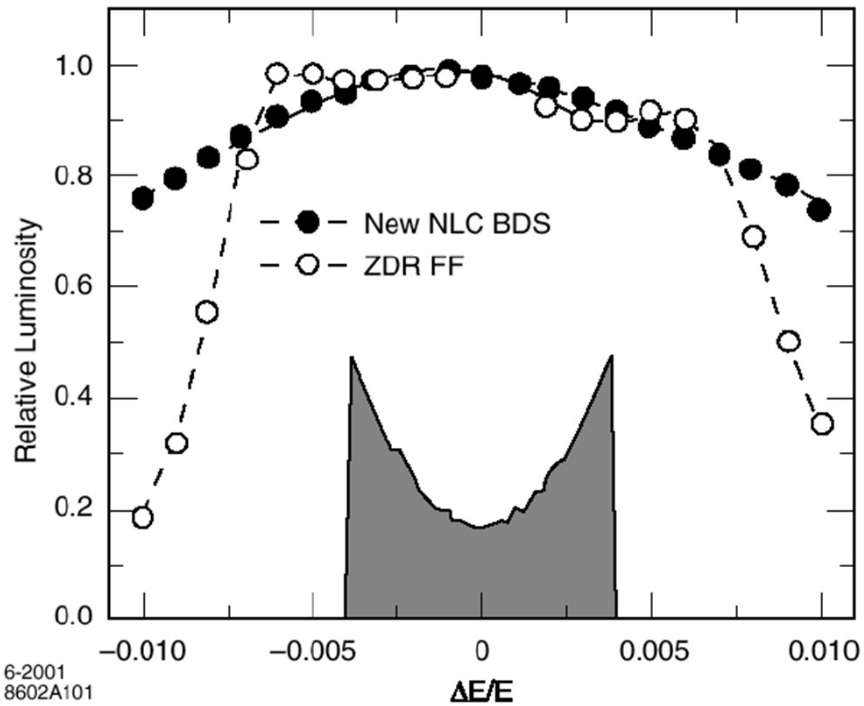
Orbit distortion through long beamline
of off-momentum particle exists
for global chromaticity correction beamline.

Local Chromaticity Correction

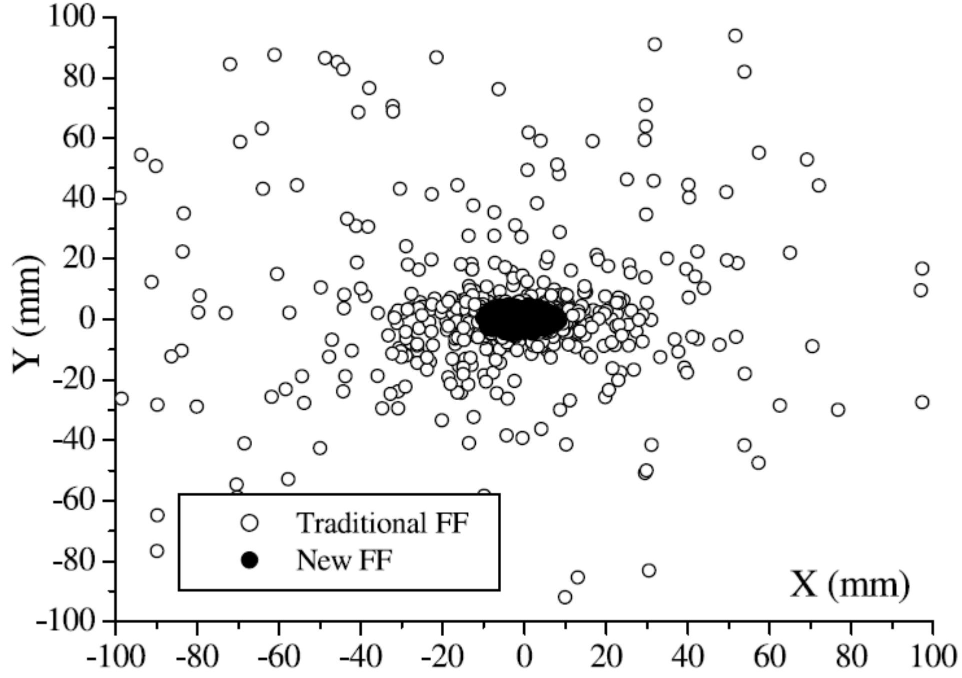


IP Energy Bandwidth

Luminosity bandwidth



Beam Tail at IP



Benefits of local chromaticity correction scheme

Shorter beamline

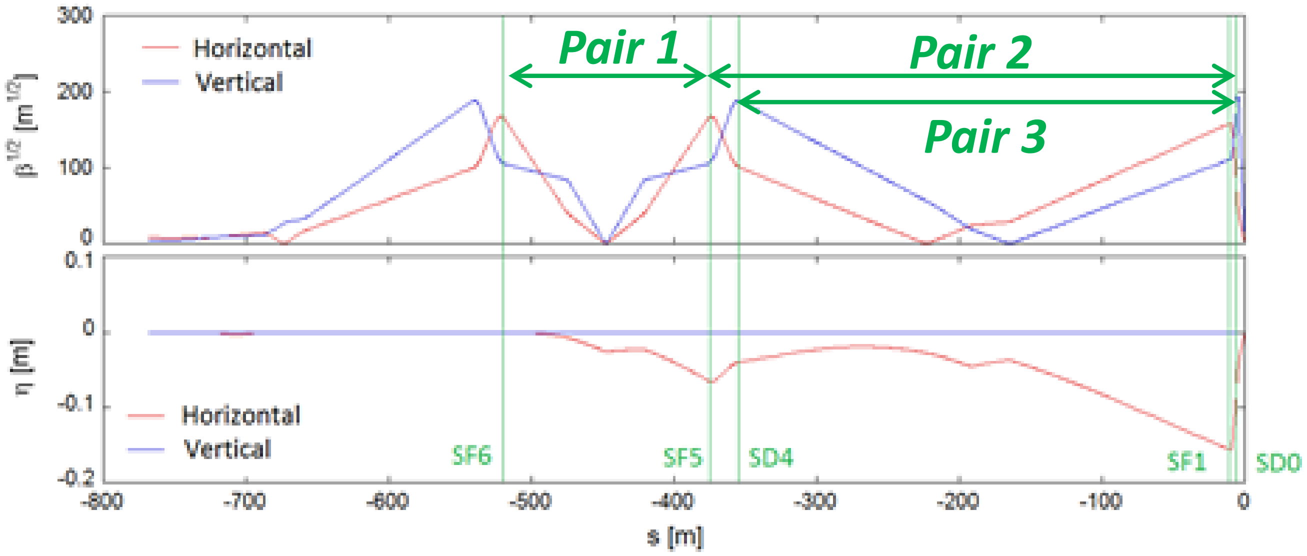
Wider energy bandwidth

Smaller beam halo

But, tuning is complex.

Present ILC Final Focus Optics

We adopted the modified Local Chromaticity Correction Optics for ILC.



Name	X	Y
QD10B	-131.9	757.6
QD10A	-168.7	673.4
QF9B	437.4	-377.5
SF6	0.0	0.0
QF9A	460.6	-295.4
QD8	-45.0	379.0
QF7B	0.2	-1.2
QF7A	0.2	-1.2
QD6	-45.0	379.0
QF5B	460.9	-295.6
SF5	155.6	-112.9
QF5A	437.6	-377.8
QD4B	-162.6	650.6
SD4	1238.1	-6089.7
QD4A	-126.0	736.6
QD2B	0.0	-3.9
QF3	5.8	-7.5
QD2A	-13.7	0.1
SF1	-9095.3	4954.9
QF1	4830.8	-2934.4
SD0	2497.5	-12835.6
QD0	-1002.9	14564.7
Total	-266.5	-236.9

3 parameters to be corrected

- *X Chromaticity*
- *Y Chromaticity*
- *2^{nd} order Dispersion*

3 sext. pair for correction

- *SF6 and SF5*
- *SF5 and SF1*
- *SD4 and SD0*

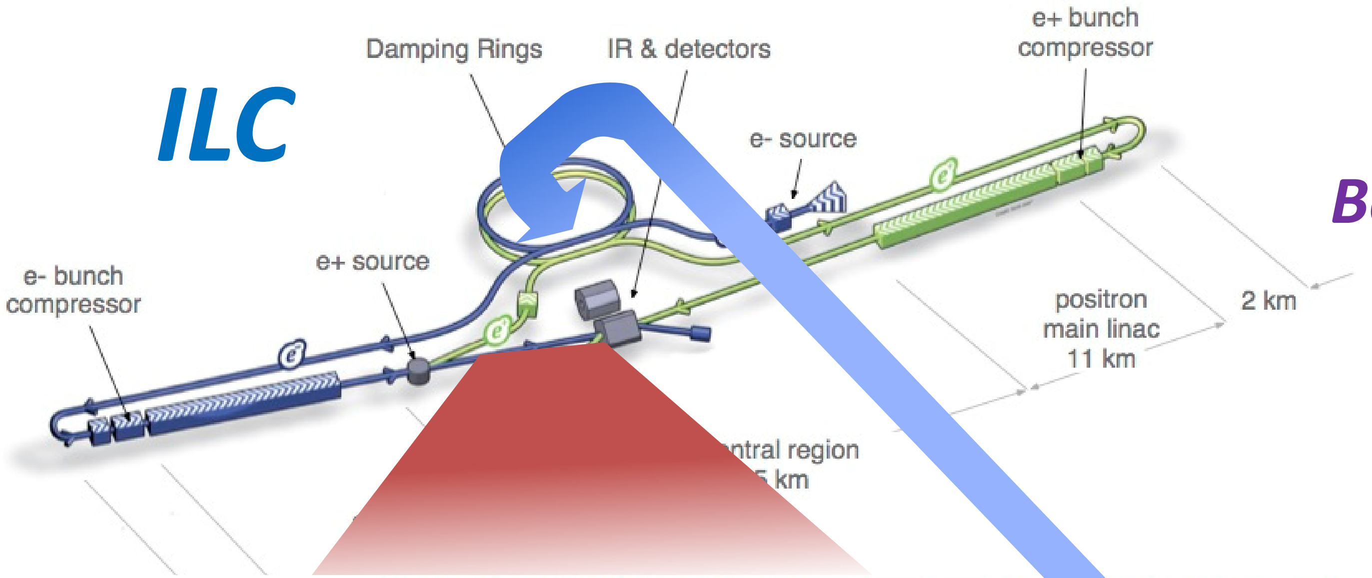
We can correct 2^{nd} order aberration
- *only with sextupole magnet.*
- *without linear optics change.*

Since SF5 is common for 2 pairs, total 5 sextupole is used for correction.

ATF2 Project

Final focus test with ATF low emittance beam.
ATF2 project was proposed at 1st LCWS (2004 November).

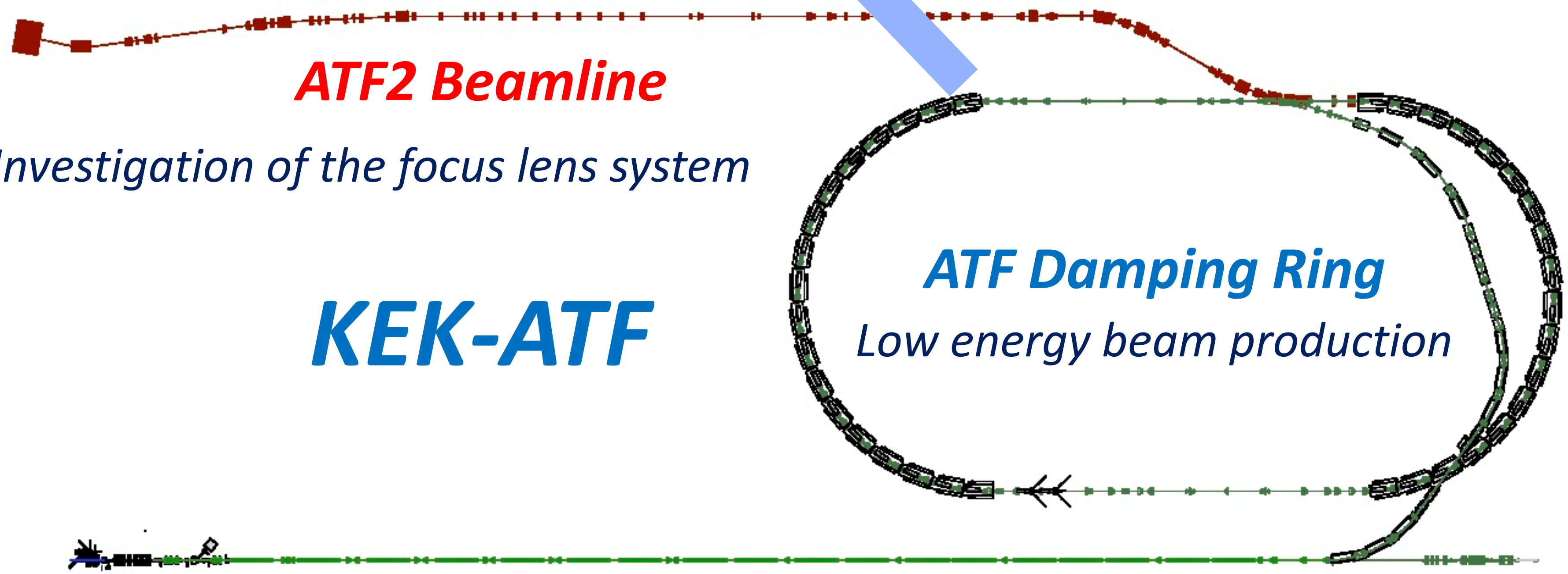
ILC



Beam focusing of ILC

Low emittance beam
⇒ Damping Ring

Beam Focusing
⇒ Final Focus System



ATF2 Beamline

Investigation of the focus lens system

ATF Damping Ring

Low energy beam production

KEK-ATF

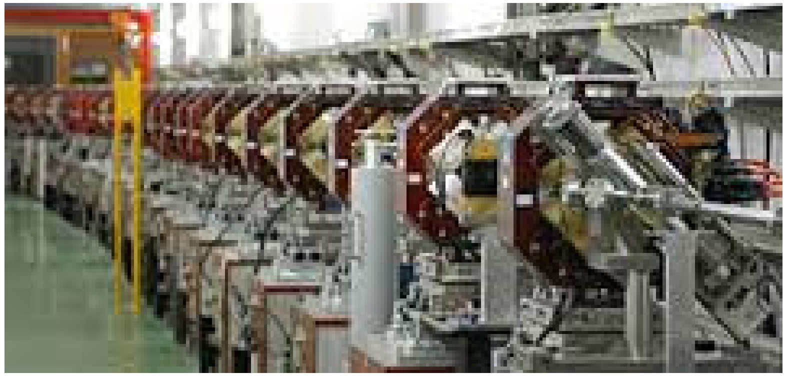
ATF2 Beamline

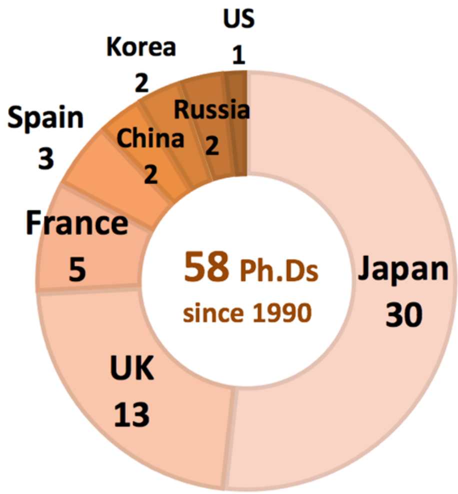
Test beamline for ILC final focus test

Start construction at 2007

Design and construction were done by international collaboration.

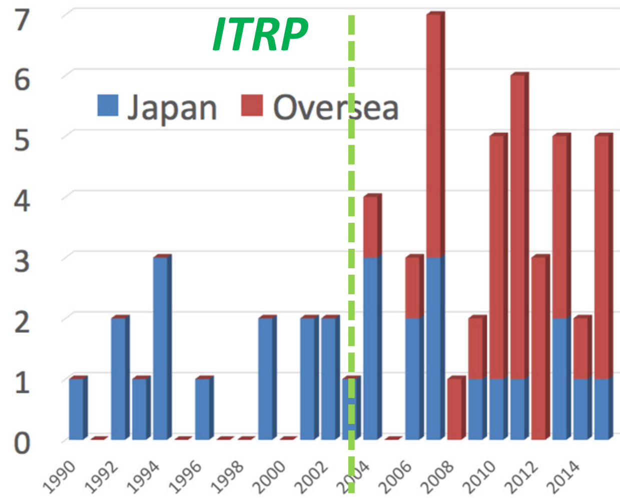
ATF has been operating by international collaboration.



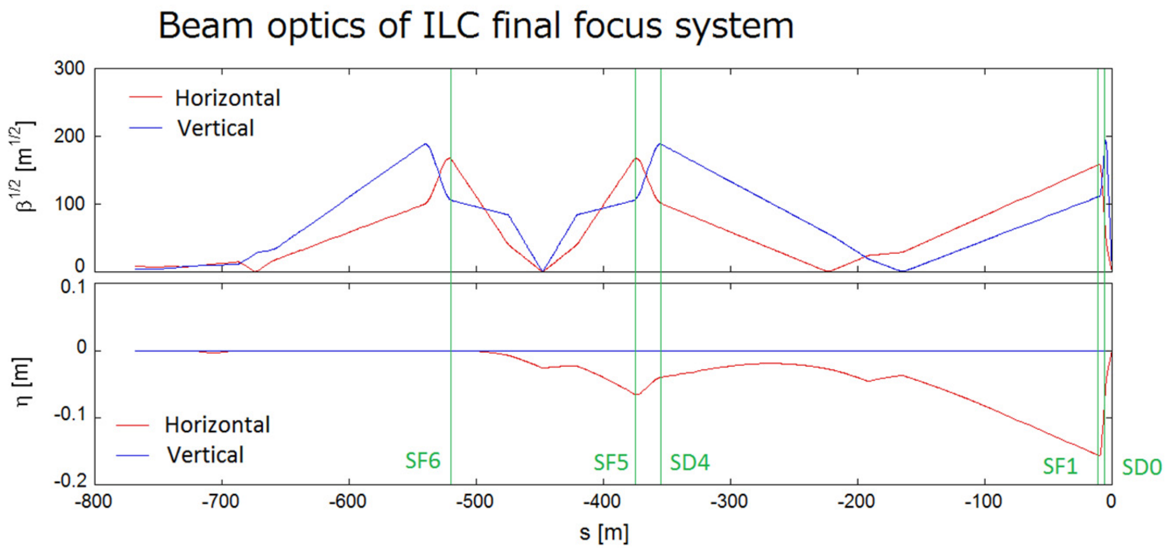


Distribution of Ph.D

Number of Ph.D at ATF



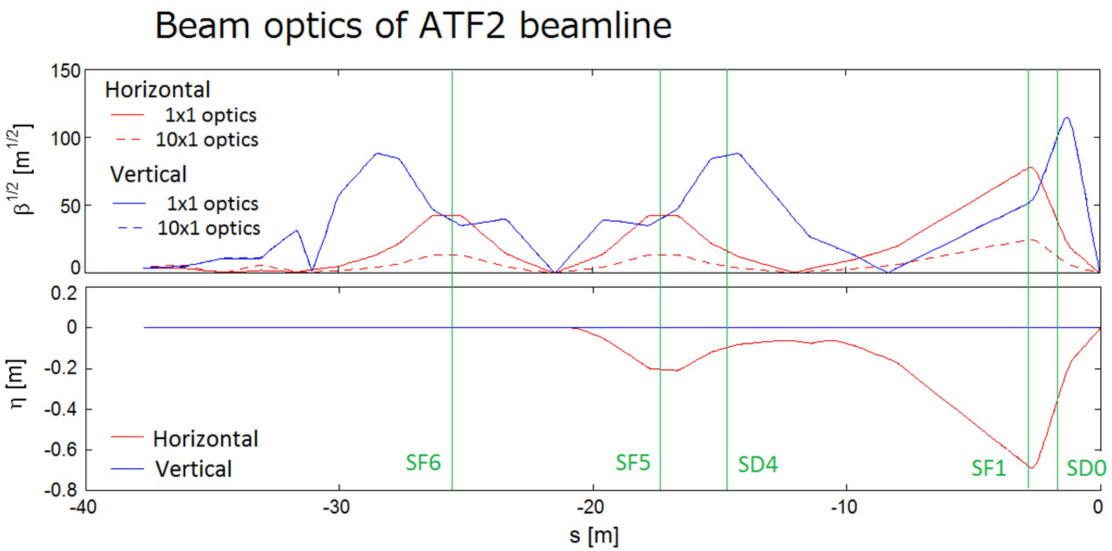
Beam Optics of ILC & ATF2



ILC final Focus System

- *ILC final focus systemand ATF2 beamline are both based on* *the Local Chromaticity Correction.*

- *Same magnet arrangement*



ATF2 Beam Optics

1x1 optics

X&Y chromaticities are comparable to ILC final focus system.

10x1 optics

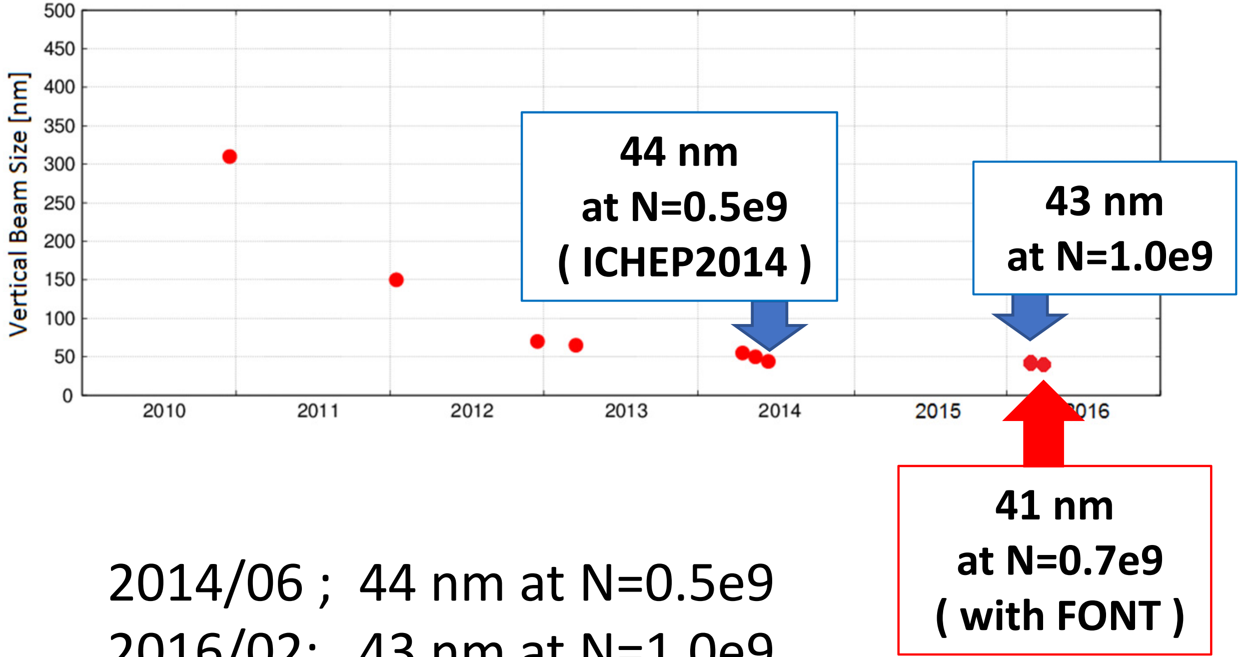
Since betax is 10 times larger than 1x1 optics, X chromaticity is one order smaller than ILC .*

Same concept of beamline design to ILC !

IP beam size trend of ATF2 beam size

Minimum beam size of 41 nm was measured on 2016/03/10 by using FONT orbit jitter correction at N=0.7e9.

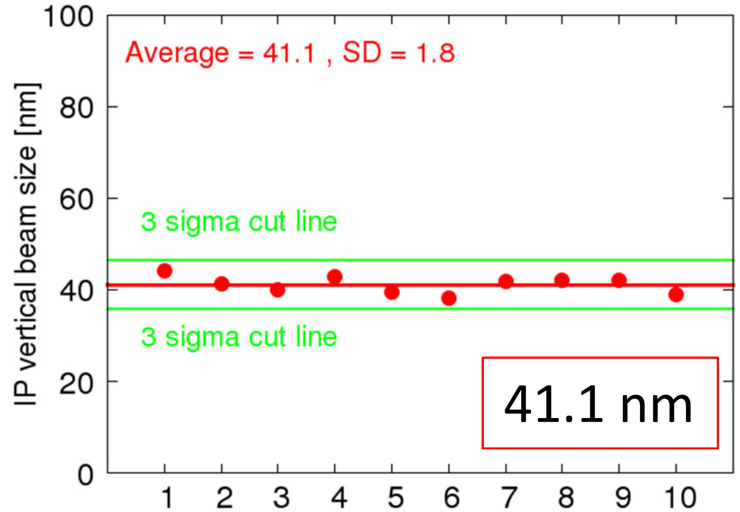
ATF design beam size is 37 nm.



2014/06 ; 44 nm at N=0.5e9
2016/02; 43 nm at N=1.0e9
2016/03; 41 nm at N=0.7e9 with FONT

Minimum beam size

presented by Y. Kano and T. Okugi at ECFA LC workshop 2016.



The beam jitter was subtracted with FONT(*) feedback

	FFTB	ATF2
Design	45 nm	37 nm
Achieved	70 nm	**41 nm**

Thank you for your attention

今回は指定された題目が多かったので、
物理や測定器と関連がある
ILC で絞れるビームサイズの限界や、その時どんなビームになるか
などの話はする時間がありませんでした。
またの機会（があったら）に！