Kinematic Edge Detection Using Finite Impulse Response Filters

Publication Draft

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Overview

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Kinematic Edge Detection Using Finite Impulse Response Filters

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* *DESY*

- Publication based on two PhD theses (Univ. Hamburg & DESY):
  - S. Caiazza – “The *GridGEM module*: a new GEM based readout module for a large TPC & A new algorithm for the determination of the position of kinematic edges in SUSY decays”, in course of publication
  - M. Chera – “Particle Flow: From First Principles to Gaugino Property Determination at the ILC”, DOI: 10.3204/PUBDB-2018-01897
- To be submitted to “***Nuclear Instruments and Methods in Physics - A***”
- ILD internal reviewers: Daniel Jeans and Remi Ete – many thanks!

Outline

1. Introduction
2. FIR Filters in Kinematic Edge Detection
3. Study Cases
4. Application in the “STC4” Scenario
5. Application in the “Point 5” Scenario
6. Conclusions

1. Introduction

Motivation: why measure kinematic edges

- Considering decays characterised by large amount of missing energy:
  - e.g., direct searches for SUSY particles in R-parity conserving scenarios
  - **two body decay**: $\tilde{X} \rightarrow \tilde{Y}U$, where $\tilde{Y}$ is the stable LSP and U is a SM particle
  - momentum and energy conservation in rest frame of U:

$$p_U^2 = \frac{1}{4M_{\tilde{X}}^2}\left(\left(M_{\tilde{X}}^2 - M_{\tilde{Y}}^2 + M_U^2\right)^2 - 4M_{\tilde{X}}^2 M_U^2\right)$$

$$E_U = \frac{M_{\tilde{X}}^2 - M_{\tilde{Y}}^2 + M_U^2}{2M_{\tilde{X}}}$$

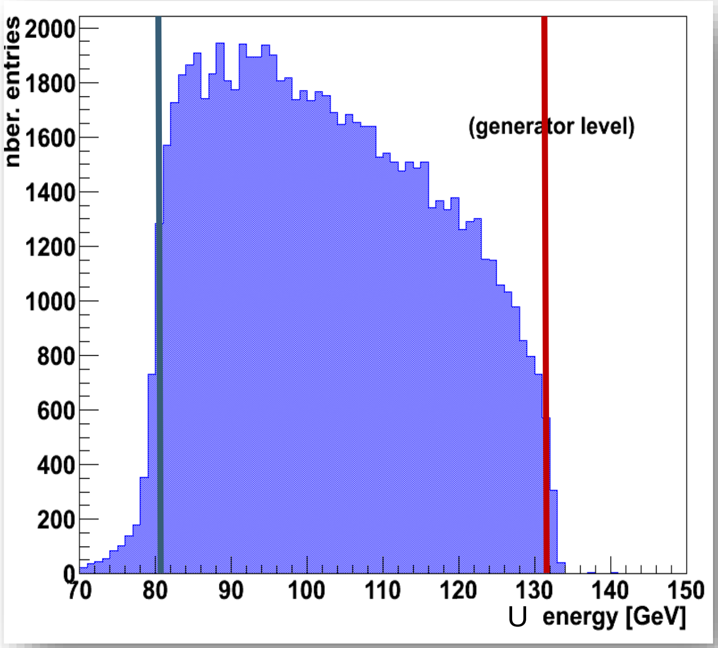
  - boost to lab frame:

$$E_U^{lab} = \gamma E_U + \beta\gamma \vec{p}_{U,\parallel}$$

$$= \gamma E_U + \beta\gamma |\vec{p}_U| \cos\theta'$$

$$\theta' = 0 \rightarrow E_U^{lab} = \gamma E_V + \beta\gamma\sqrt{E_V^2 - M_V^2}$$

$$\theta' = \pi \rightarrow E_U^{lab} = \gamma E_V - \beta\gamma\sqrt{E_V^2 - M_V^2}$$



  - spin of involved particles determines shape of box top

Kinematic edges: key ingredient in particle property determination

- Relevant observable: energy (momentum) spectrum of visible decay product
- Goal: measure positions of kinematic edges as precisely as possible
- "Traditional" approaches:
  - describe measured spectrum with well motivated function ↔ requires a priori assumptions and approximations
  - compute (numerically) first derivative of distribution ↔ highly sensitive to noise (e.g., detector resolution, beam energy spectrum, intrinsic particle width)
- We propose a new method that circumvents these issues:
  apply a Finite Impulse Response (FIR) filter on the measured spectra

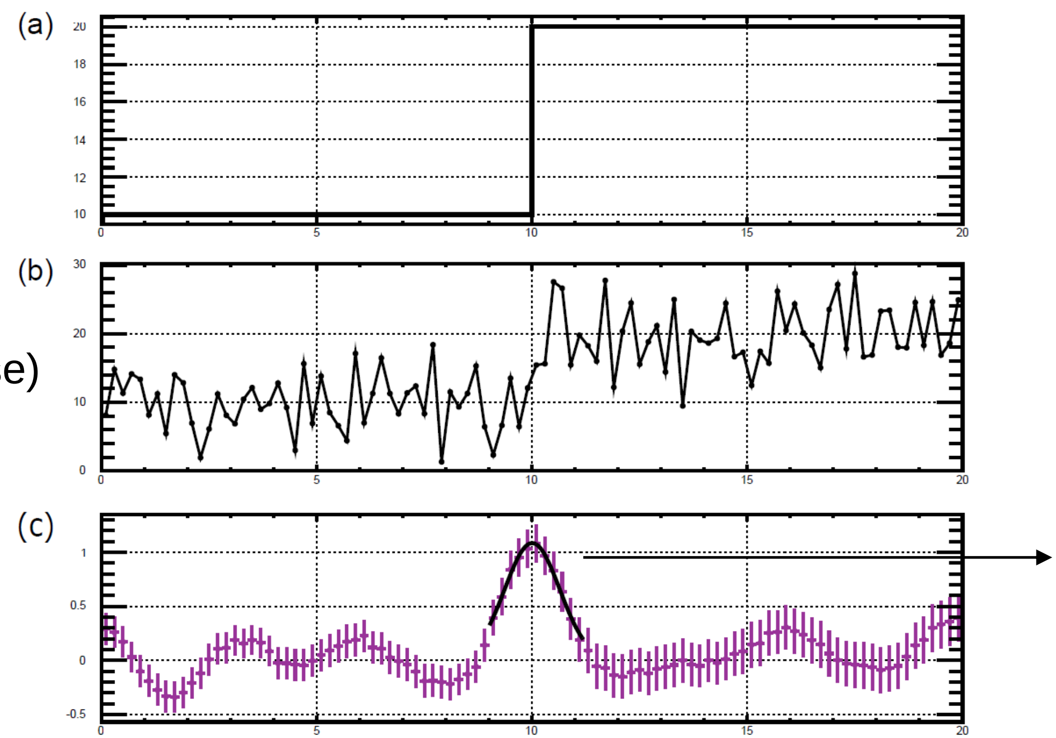
Proposed new approach

- FIR filters typically used in signal processing for noise reduction and enhancing relevant features: e.g. in gravitational waves detection, image processing, etc.
- Illustration of using an FIR filter for edge detection:

Ideal edge

Realistic edge
(added Gaussian noise)

Filter response
(output of FIR filter)



Peak indicates
edge position

2. FIR Filters For Kinematic Edge Detection

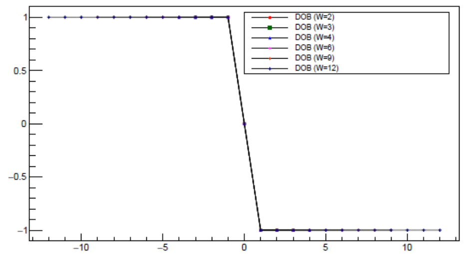
Key elements of an FIR filter for binned input

- FIR filter of order N_f = discrete and finite set of N_f numbers (coefficients)
- Coefficients usually determined from sampling a chosen function (filter kernel)
- Applying FIR filter on measured data = convolution $\Longrightarrow$ filter response
- Ingredients of an FIR filter
  - The function defining the filter kernel
  - The values of the function parameters
  - The size/length of the filter, e.g.:
    - value interval (val_{min}, val_{max}) over which function is computed (continuous input)
    - number of filter coefficients $c_{1...n}$ = number of times the kernel is sampled (discrete input)
  - For discrete data: the binning is also important

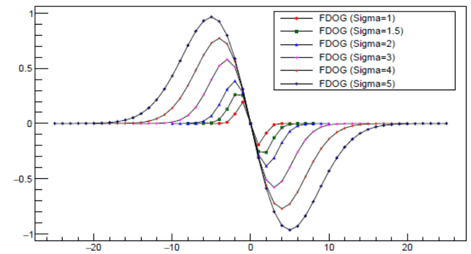
$\Longrightarrow$ Optimisation needed

FIR filter kernel for edge detection on binned distributions

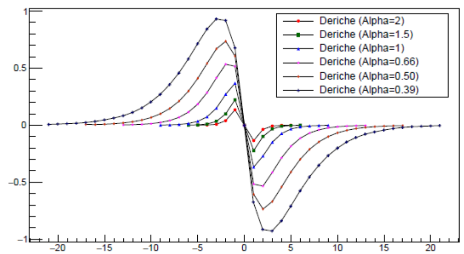
- For continuous distributions: optimal filter ≈ **first derivative of Gaussian** (J.F. Canny, doi:10.1109/TPAMI.1986.4767851)
- Is this true for discrete distributions (i.e., histograms)? → S. Caiazza study



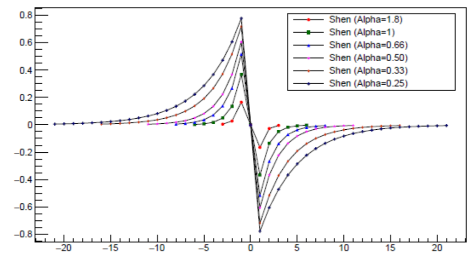
(a) DOB filter DIR



(b) FDOG Filter DIR



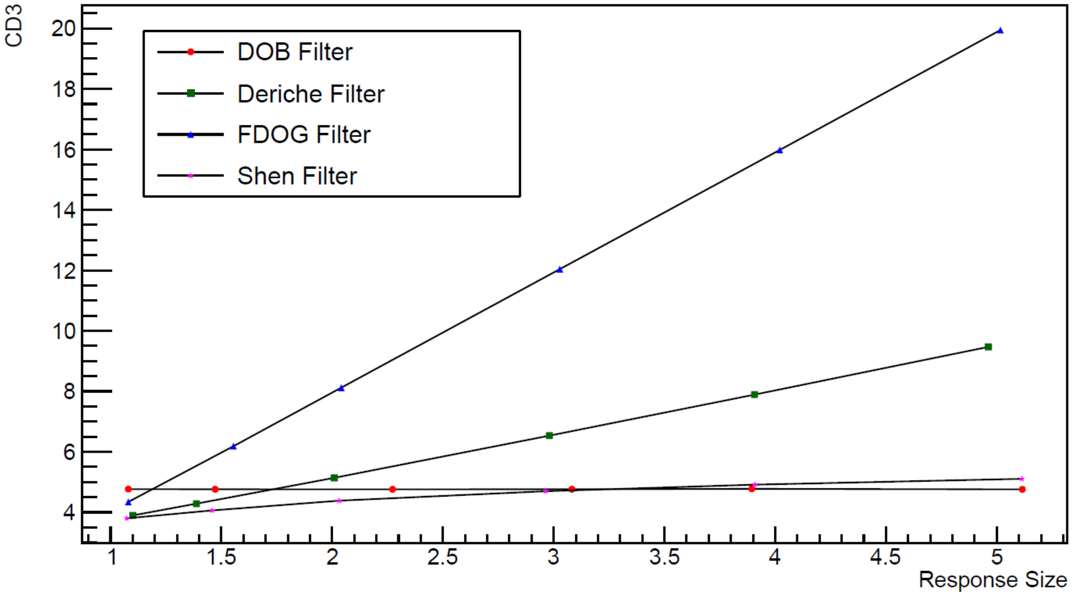
(c) Deriche Filter DIR



(d) Shen Filter DIR

Evaluation of the considered kernels: CD3

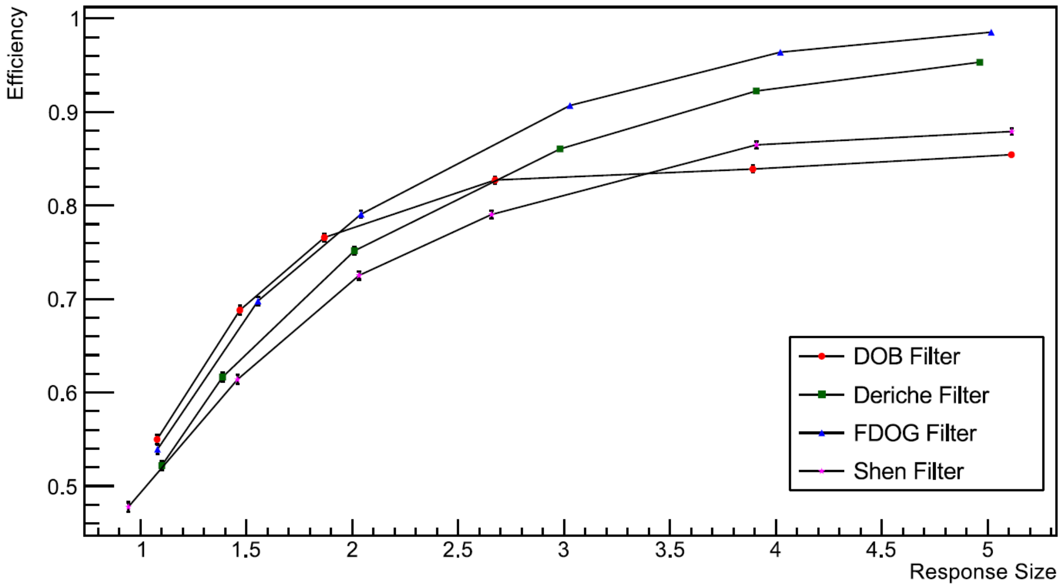
- Evaluate performance of the 4 considered kernels in terms of:
  - Multiple response criterion (CD3) = mean "distance" between detected edges (maxima in filter response) on a constant noisy function



Trade-off: if CD3 too small, filter can be highly sensitive to noise

Kernel evaluation in terms of detection efficiency

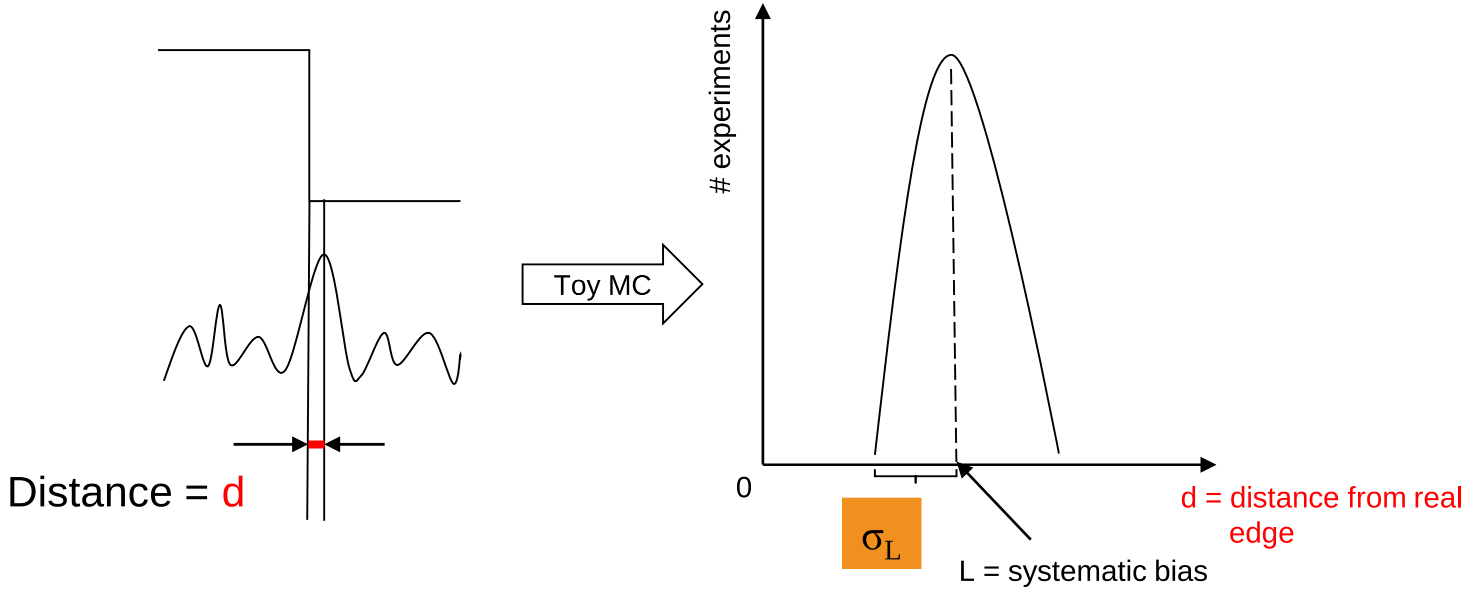
- Evaluate performance of the 4 considered kernels in terms of:
  - Detection efficiency = probability that the largest peak in the filter response is the one closest to the true edge position (toy Monte Carlo experiment)



- FDOG kernel provides best efficiency
- For all kernels, the efficiency increases with the filter size

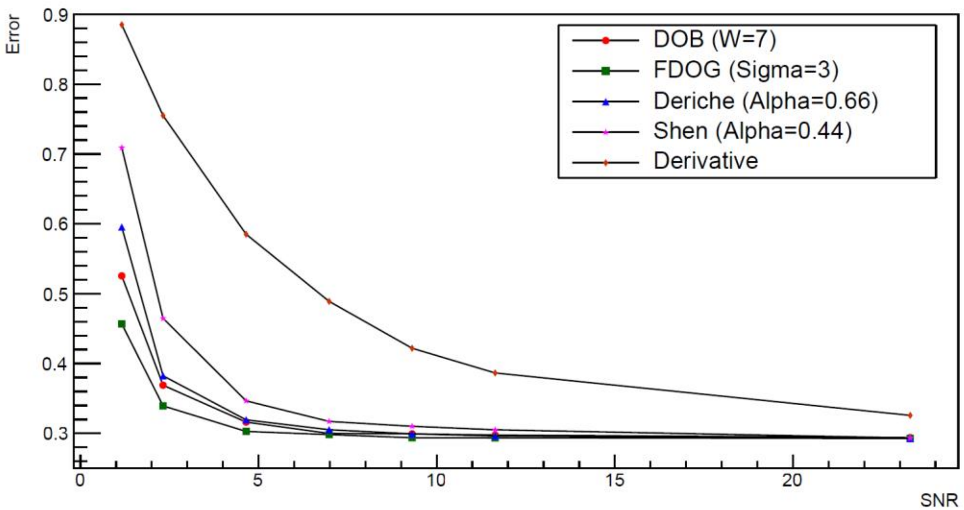
Localisaion error and bias

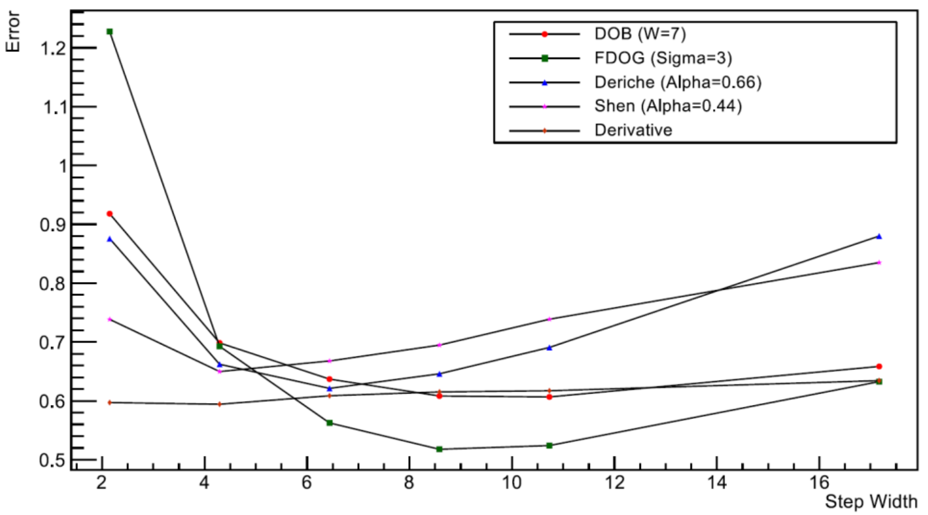
- Evaluate performance of the 4 considered kernels in terms of:
  - Localisation error: σ_L



Kernel evaluation in terms of σ_L

- Evaluate performance of the 4 considered kernels in terms of:
  - Localisation error: σ_L





- FDOG kernel has smallest error even for small S/N ratio
- For narrow edges Shen kernel performs better
- For wider edges FDOG kernel has smallest error

⇒ Use FDOG kernel

3. Study Cases

Two SUSY scenarios accessible at the ILC

- Illustrate the FIR filter method in the context of the ILC, using ILD simulated data:
  - Well known initial state → well defined kinematics
  - Handling of missing energy
  - High precision ↔ Particle Flow reconstruction

Investigating the FIR filter performance in two SUSY scenarios:

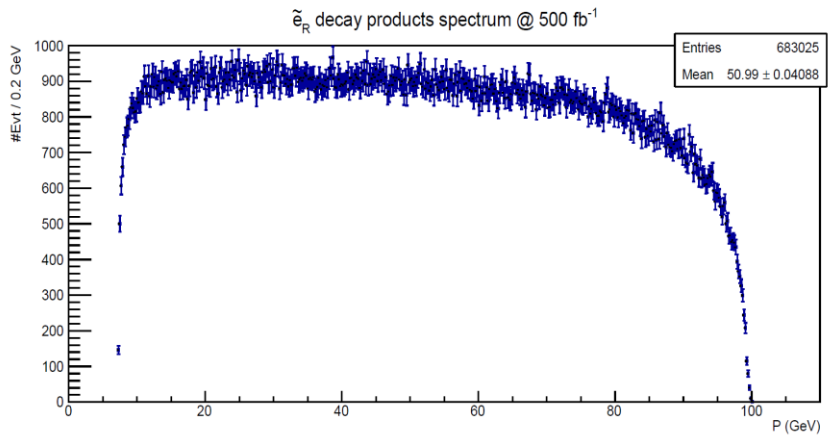
- selectron & gaugino pair production @ ILC: $\sqrt{s}$ = 500 GeV, $P(e^-)$ = -80%, $P(e^+)$ = +30%, 500 fb^{-1}
- Data sets
  - → **selectron analysis**: signal + SM & SUSY background simulated with SGV
  - → **gaugino analysis**: signal + SM & SUSY background simulated and reconstructed with DBD version of full simulation (ILD_o1_v5)

The “STC4” and “Point 5” scenarios

- **STC4:** Phys. Rev. D. 88, 055004
- Relevant processes:

$$e^+e^- \longrightarrow \tilde{e}\bar{\tilde{e}} \longrightarrow e^{\pm}\tilde{\chi}_1^0 e^{\mp}\tilde{\chi}_1^0$$

- Signal topology: isolated and uncorrelated electron and positron pair and large amount of missing energy
- Relevant observables

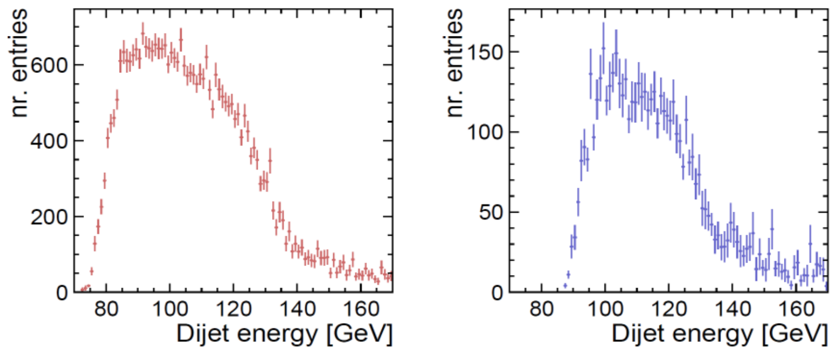


- selectrons (sfermions) decay isotropically ⇒ spectrum with flat top

- **“Point 5”:** arXiv:hep-ex/0603010v1
- Relevant processes:

$$e^+e^- \longrightarrow \tilde{\chi}_1^+\tilde{\chi}_1^- \longrightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0 W^+W^- \longrightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0 qqqq$$
$$e^+e^- \longrightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0 \longrightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0 Z^0Z^0 \longrightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0 qqqq$$

- Signal topology: 4 hadronic jets and large amounts of missing energy
- Relevant observables (after sample separation)



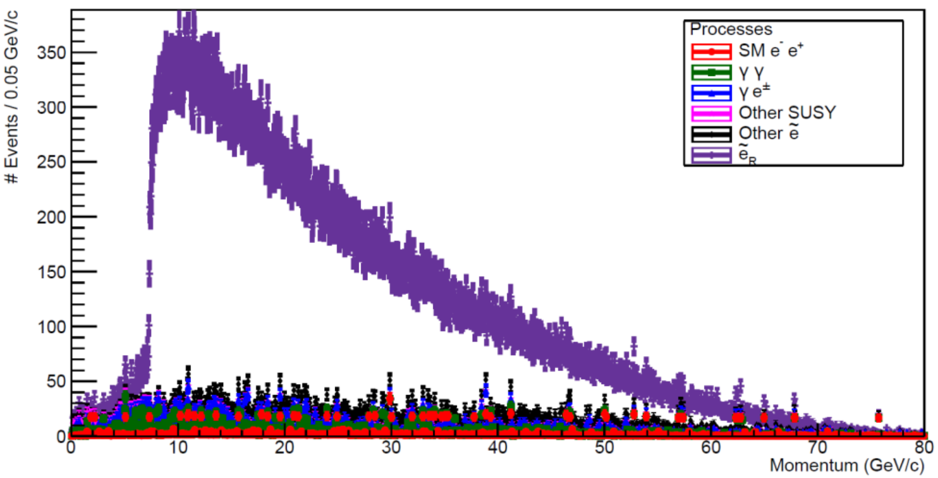
(a) $\tilde{\chi}_1^{\pm}$ candidates sample **(b)** $\tilde{\chi}_2^0$ candidates sample

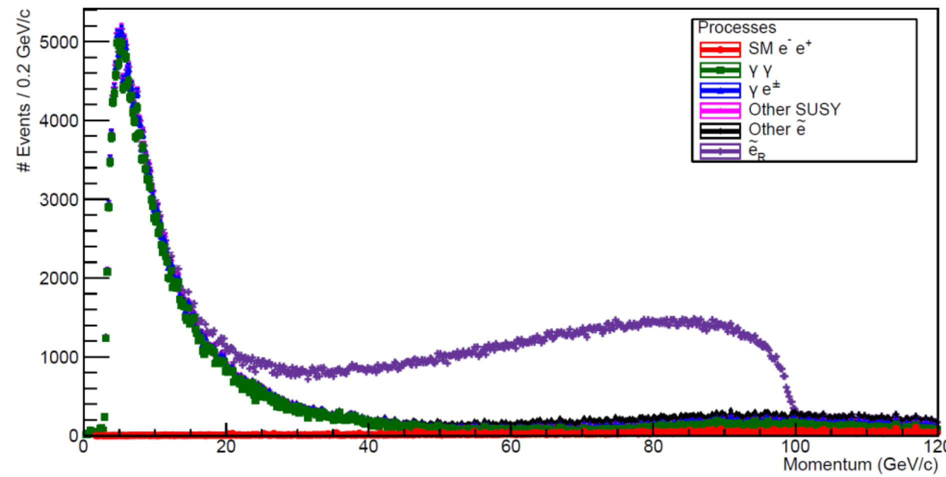
- gauginos ⇒ complex spectrum top shape
- intrinsic width of W/Z affects edge position

FIR Filters Applied in the STC4 Scenario

Input data for FIR filter in STC4 scenario

- Background and signal data simulated with SGV (ILD_o1_v5)
- Measure lower and upper edge independently:





- Details concerning event selection & efficiency → S. Caiazza, PhD Thesis

Kernel parameter optimisation & Measured edges

- Optimise FIR filter parameters: kernel size and input data binning

- Perform toy Monte Carlo study:
  1. Use edge distributions (pg. 15) as input and randomly generate a large number of new histograms with different bin sizes
  2. Apply FDOG with different filter sizes on each new batch of histograms
  3. Determine optimal parameter values = they minimise σ_L (localisation error)

Concluded:
- optimal bin size: 50 MeV/bin (lower edge) & 200 MeV/bin (upper edge)
- optimal FDOG filter size: 5 bins (lower edge) & 6 bins (upper edge)

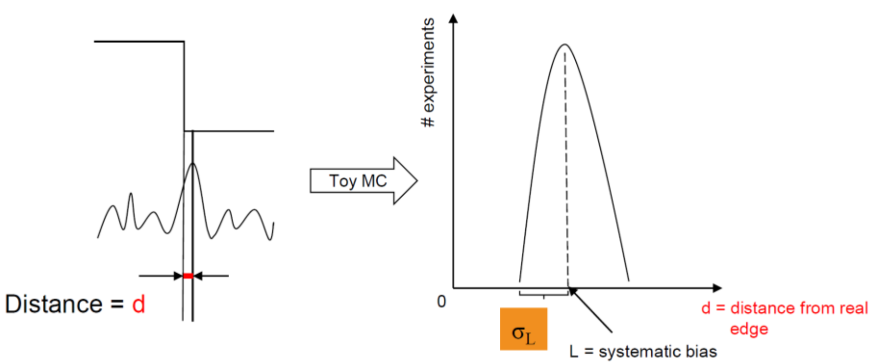
- Measured edges:

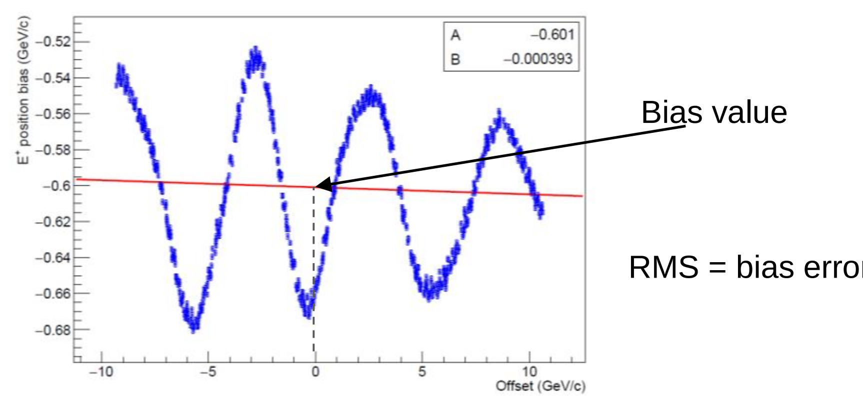
	Lower edge [GeV]	Upper Edge [GeV]
Calculated	7.298	99.362
Generator level	7.417 ± 0.008	98.974 ± 0.024
Measured	7.409 ± 0.012	98.748 ± 0.043

- Statistical uncertainty evaluated in separate toy Monte Carlo study

Edge Calibration

- Evaluate beam-spectrum effects (bias on edge position)
- Perform toy Monte Carlo study:
  1. Introduce a random offset to the lower and upper edges
     - carried out ONLY for signal (modify the momentum of particles in event)
     - leave SM and SUSY background unchanged
  2. Randomly generate large number of histograms based on previously obtained one
  3. Apply FDOG with optimal parameter values ⇒ extract localisation bias and error
  4. Repeat steps 1-3 for a different offset value
- Obtained calibration line:





(b) High-momentum edge calibration

Results

- Calibrated edge values:

	Lower edge [GeV]	Upper Edge [GeV]
Calculated	7.298	99.362
Generator level	7.417 ± 0.008	98.974 ± 0.024
Measured	7.409 ± 0.012	98.748 ± 0.043
Calibrated	7.300 ± 0.012 $\oplus$ 0.042	99.349 ± 0.043 $\oplus$0.008

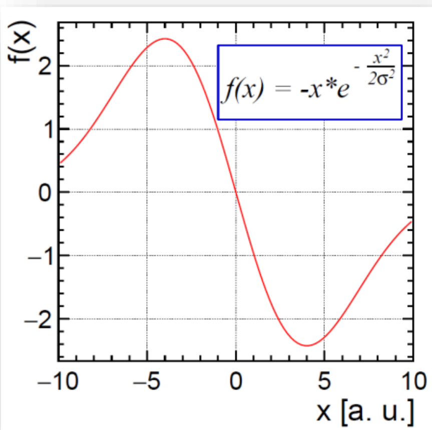
- Determined masses:

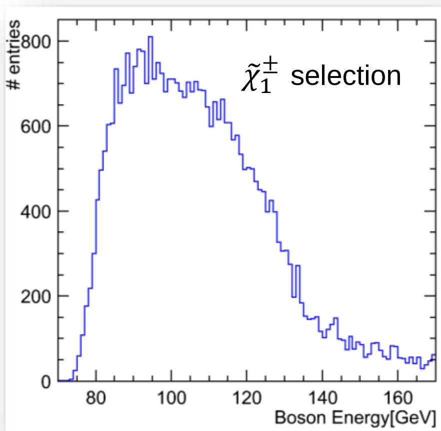
	Mass $\tilde{\chi}_1^0$ [GeV]	Mass $\tilde{e}$ [GeV]
FIR filter edges	95.56 ± 0.09	126.20 ± 0.11
arxiv: 1508.04383	95.47 ± 0.16	126.20 ± 0.21
STC4 model	95.58	126.24

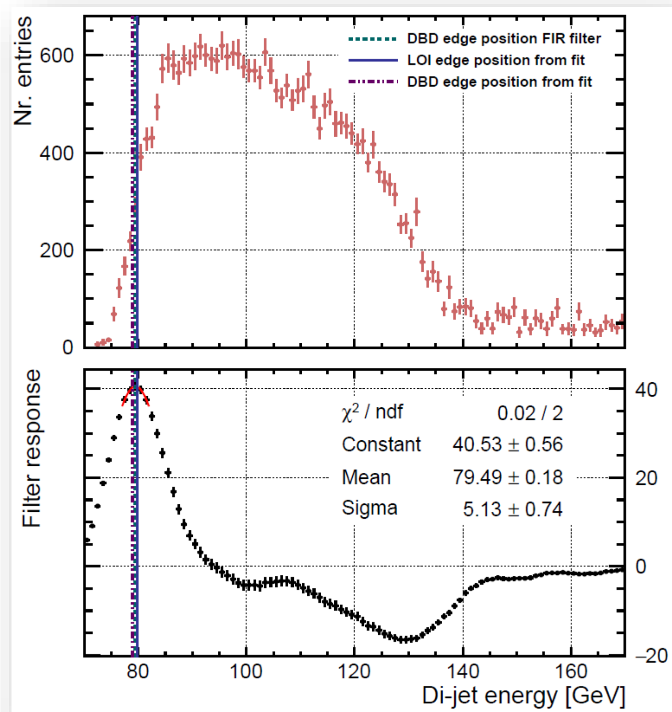
FIR Filters Applied in the Point 5 Scenario

Overview

- **Two signal processes**: $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^0$ pair production → selection and sample separation (M.C. PhD thesis)
- Data simulated and reconstructed with full simulation of ILD_o1_v5
- FIR kernel choice: optimal filter → ≈ **first derivative of a Gaussian**
- Initial implementation: S. Caiazza → further developed and optimised
- Convolute FIR filter with input histo.: both lower and upper edge considered simultaneously
  → Increasing edge = positive extremum (peak)
  → Decreasing edge = negative extremum
  → Edge position extracted from fitting extrema



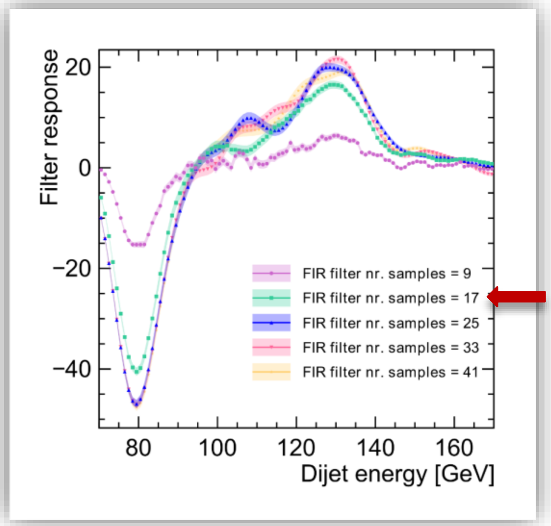




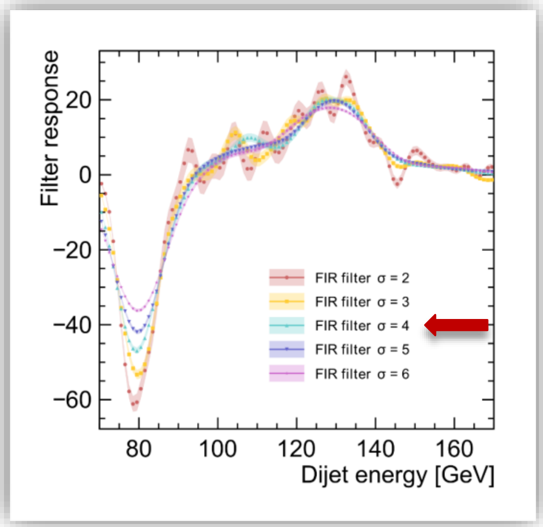
Tuning of Filter Parameters

- There are 3 filter parameters that can be optimised:
  kernel size, Gaussian σ, input histogram binning
- **Criteria for optimum**: clear and narrow peaks in filter response
- **Data samples:** new randomised $\tilde{\chi}_2^0$ Monte Carlo signal

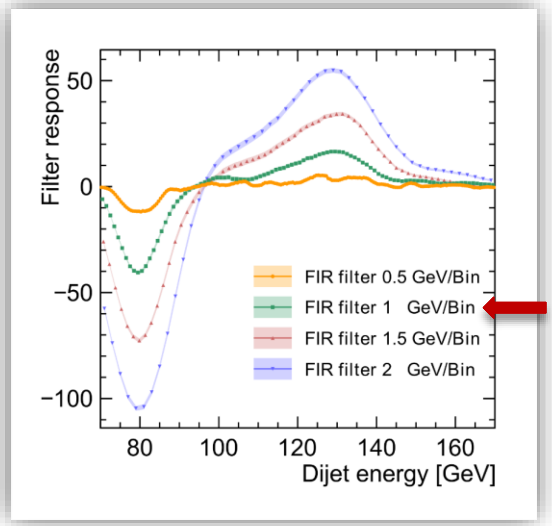
Vary kernel size
(# filter coefficients)



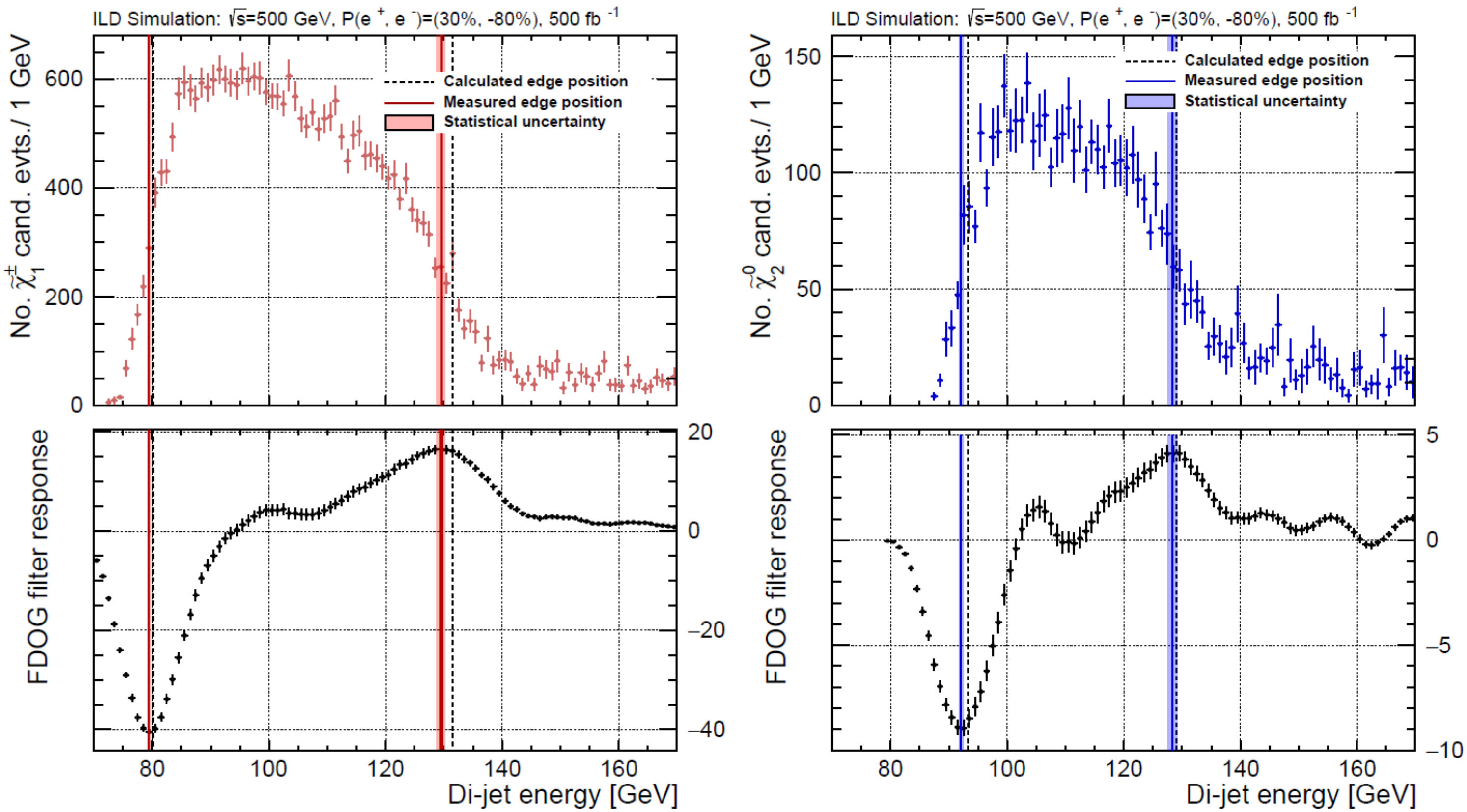
Vary Gaussian width(σ)



Vary binning of input histogram



Measured Edge Positions



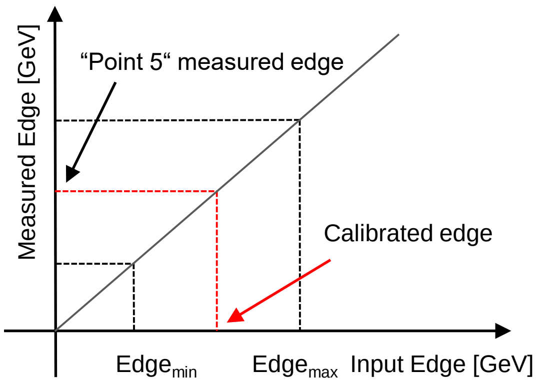
Statistical uncertainties from Toy Monte Carlo study.

	$\tilde{\chi}_1^\pm$ low [GeV]	$\tilde{\chi}_1^\pm$ high [GeV]	$\tilde{\chi}_2^0$ low [GeV]	$\tilde{\chi}_2^0$ high [GeV]
Calculated	80.17	132.76	93.09	129.92
FIR filter	79.5±0.2	129.5±0.7	92.1±0.3	128.4±0.8
Fit	78.9±0.3	130.2±0.7	91.7±0.4	137.2±5.4

Fit highly sensitive to fluctuations!

Edge Calibration

- Mass calculation formulae do not account for beam energy spectrum, gauge boson width, etc. effects on edge positions → **perform edge calibration**:
  - Vary input masses ↔ different edge positions
    → $\tilde{\chi}_1^{\pm}$ & $\tilde{\chi}_2^0$ varied simultaneously (210 GeV ↔ 225 GeV, 3 GeV step)
    → LSP mass fixed!
  - Measure edges for each new Monte Carlo sample
    → obtain **calibration curve**



- Investigated 3 different aspects: calibrate edges measured on
  1. **generator level ↔ calculated edges:**
     effects of ISR emission, gauge boson width = 0.8% → 1.8%
  2. **reconstruction level** ↔ **generator level**
     simulation and reconstruction effects = **0.2% → 0.9%**
  3. **reconstruction level** ↔ **calculated edges**
     take all effects into account = 1.1% → 2%

	Mass $\tilde{\chi}_1^{\pm}$ [GeV]	Mass $\tilde{\chi}_2^0$ [GeV]	Mass $\tilde{\chi}_1^0$ [GeV]
Model	**216.5**	**216.7**	**115.7**
Mass w. calibration	214.1±4.8	216.9±3.4	115.5±1.8
Mass <u>no</u> calibration	216.7±3.1	220.4±1.3	118.1±0.9

6. Conclusions

- Kinematic edge detection is crucial for determining particle masses in cases with large amounts of missing energy
- We propose a new method for kinematic edge detection: FIR filters
- The FDOG filter kernel was found to be optimal in the discrete case as well
- The FIR filter method was applied in the context of two different SUSY scenarios accessible at the ILC: STC4 and "Point 5"
- For each study case the kernel parameters were optimised
- Sparticles masses obtained from the FIR filter detected edges highly compatible with model masses & other methods
- FIR filter significantly more robust and stable

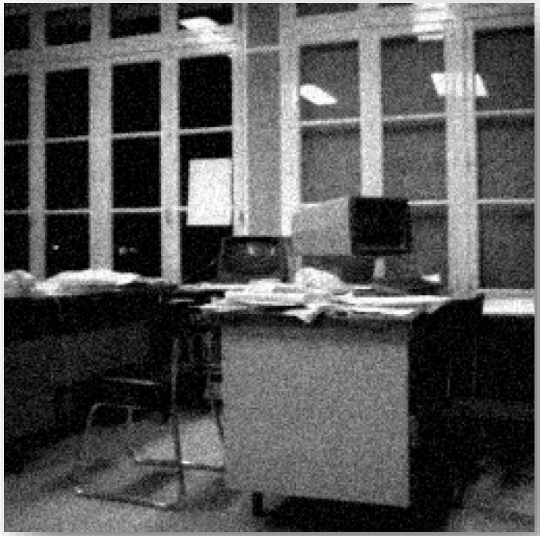
Thank you

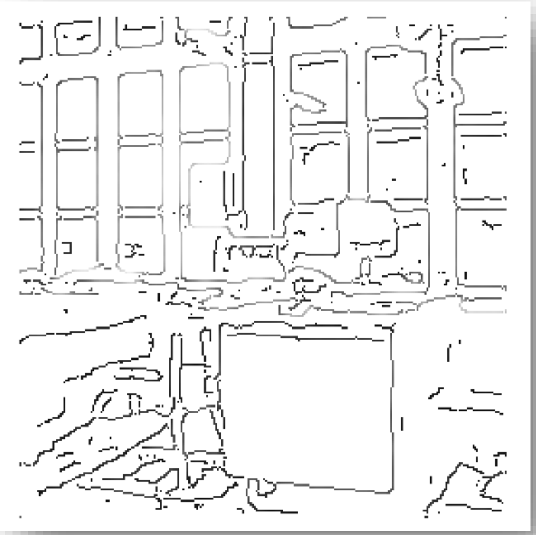
2. FIR Filters for Kinematic Edge Detection

Proposed new approach

- FIR filters typically used in signal processing for noise reduction and enhancing relevant features: e.g. in gravitational waves detection, image processing, etc.

D. Demigny, T. Kamlé





Toy Monte Carlo Study – Fit Stability

Investigating the Stability of LOI Fit to Gauge Boson Energy Spectrum

- Randomly generated 10^4 $\tilde{\chi}_1^+$ and $\tilde{\chi}_2^0$ distributions + bgrd.
- Applied fit 10^4 times