Moliere radius of electron showers in MC simulated structures

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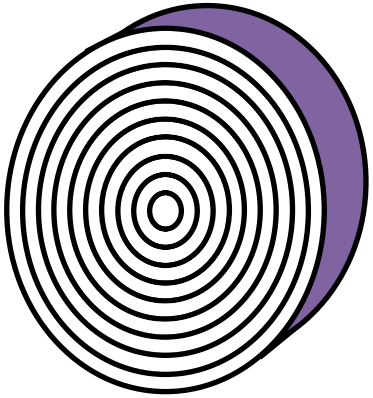
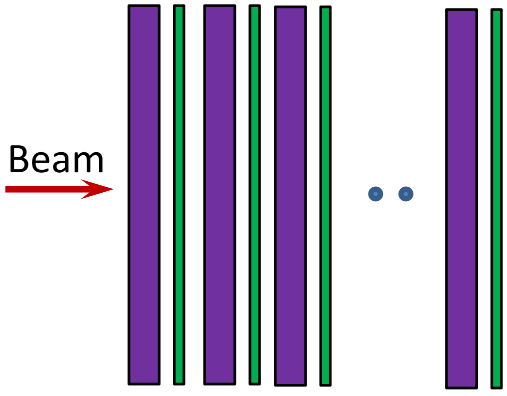




Moliere radius definition

Calorimetry

Energy Measurement in Particle Physics

Second Edition

Richard Wigmans

Paul Friederich Gaspard Gert Molière (1909–64)

2.1.5.2 *The Molière radius.* This quantity does not have a physics meaning equal in precision to that of the radiation length. The Molière radius is frequently used to describe the transverse development of em showers in an *approximately* material independent way. It is defined in terms of the radiation length X_0 and the critical energy ϵ_c (Section 2.1.1), as follows:

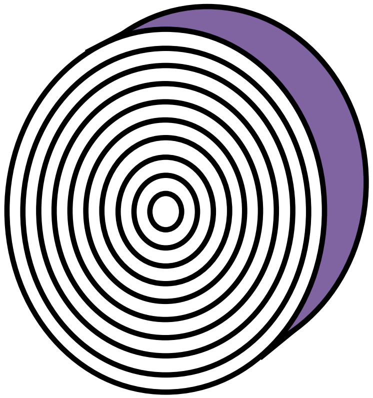
$$\rho_M = E_s \frac{X_0}{\epsilon_c} \tag{2.15}$$

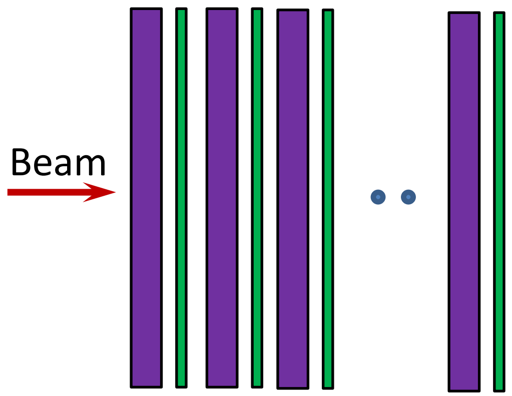
in which the scale energy E_s, defined as $m_e c^2 \sqrt{4\pi/\alpha}$, equals 21.2 MeV. Typically, $\sim$85–90% of the shower energy is deposited in a cylinder with radius ρ_M around the shower axis (see Figure 2.18 for an example).

In PDG – on average 90% of the shower energy

EM showers in MC (GEANT3,4)

Typical regular structure: absorber (passive) disks + radially segmented sensors in air gaps. Electrons (5 – 100 GeV) sent along the structure axis.





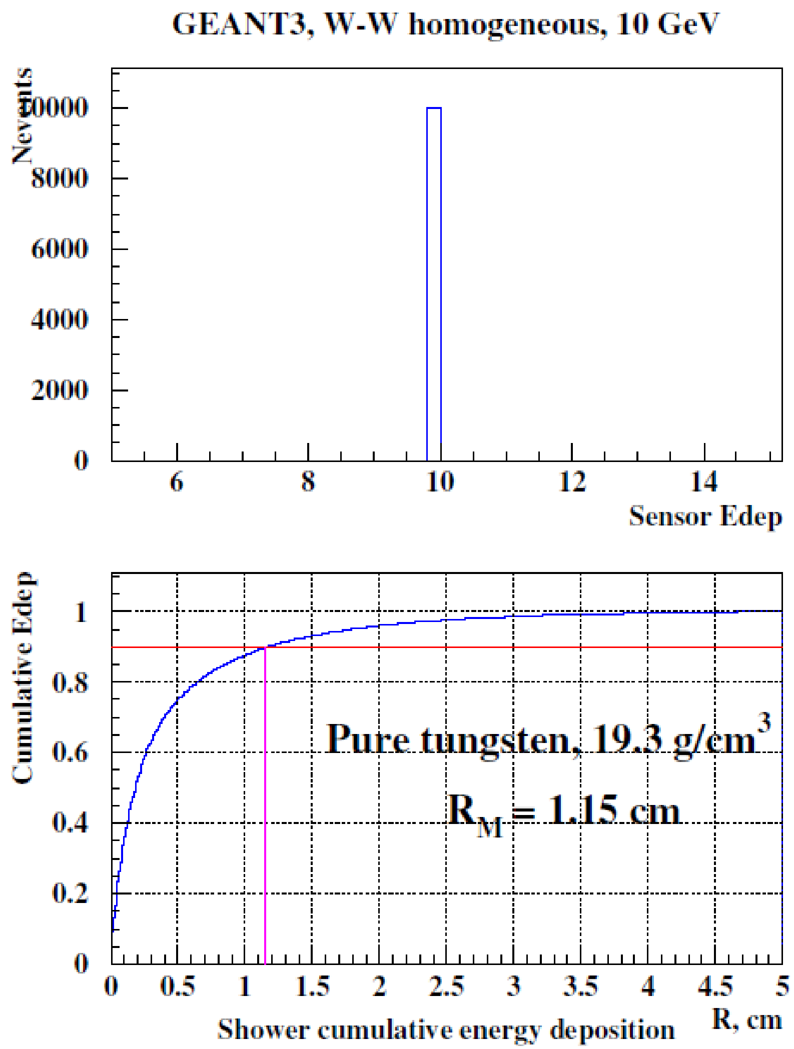
Segmentation: dR = 0.1 mm, dZ ~ 1 Xo

Full shower containment

Tungsten ρ = 19.3 g/cm³ R_M = 0.9327 cm from PDG

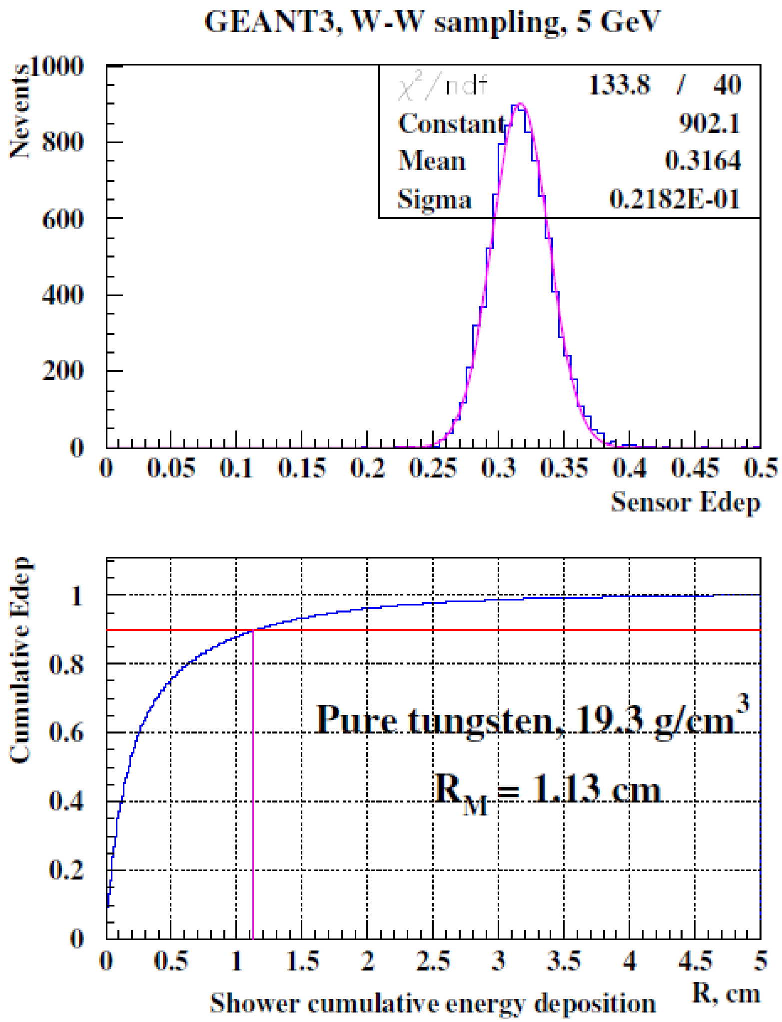
Other materials used: Pb, Si, Air

Simulation results - 1



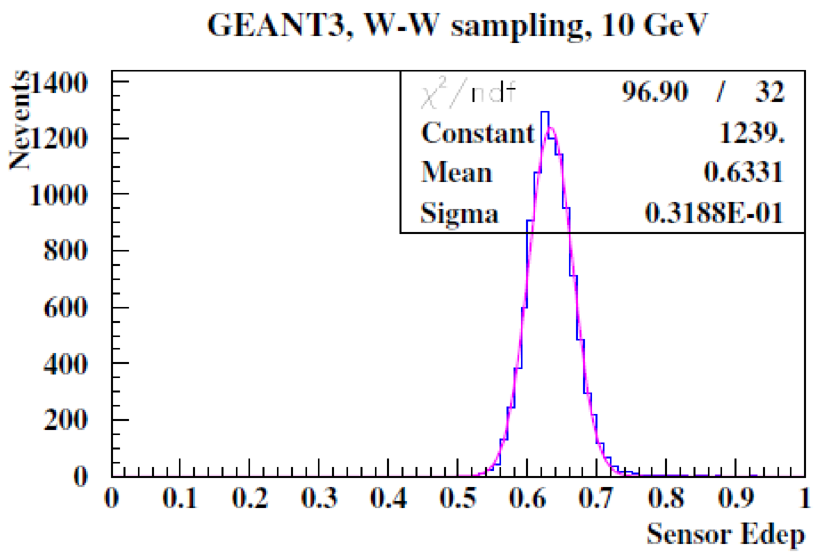
- Cylinder made of pure tungsten
- Whole volume is sensitive

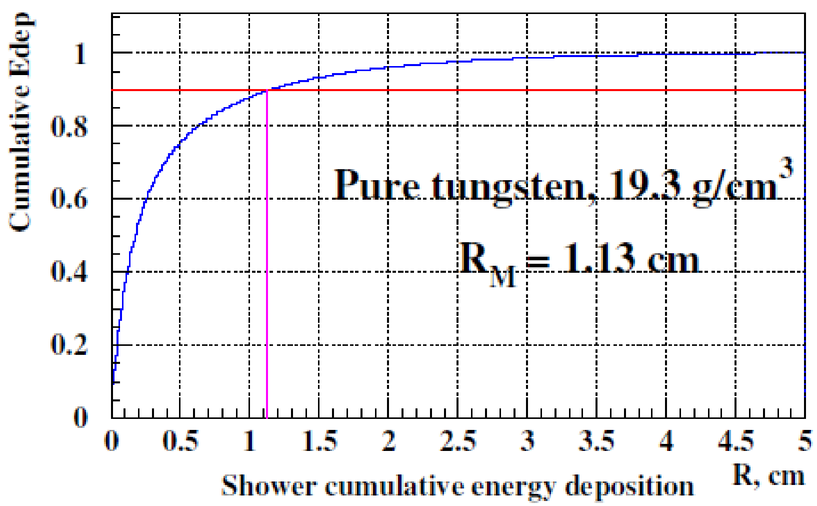
Simulation results - 2



- Cylinder made of pure tungsten
- Sensitive W layers 300 μm thick every 4.5 mm
- No air gaps
- Electron energy = 5 GeV

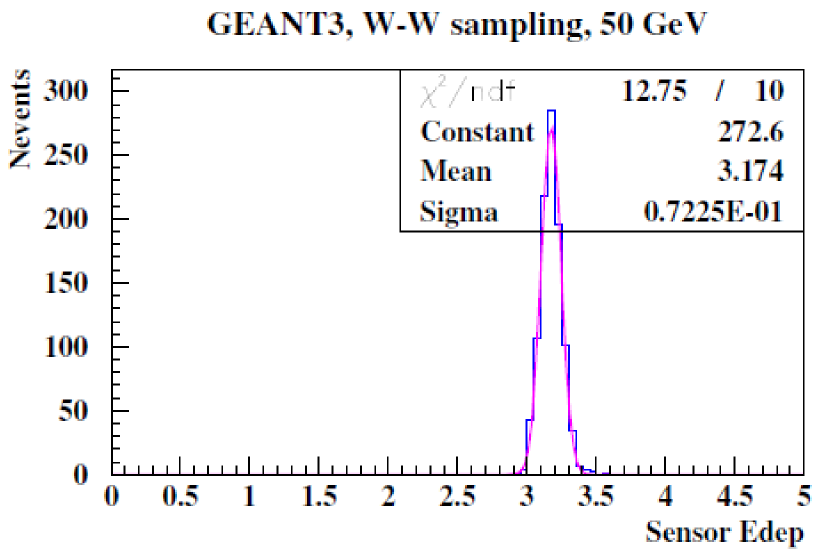
Simulation results - 3

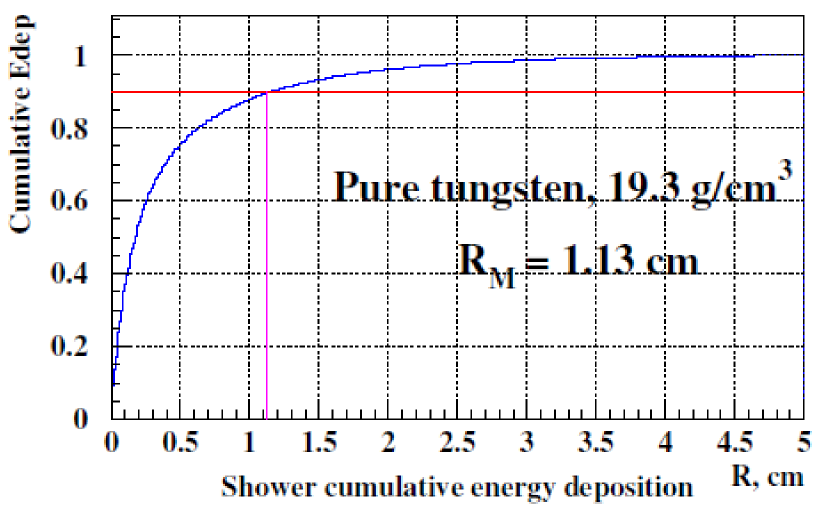




- Cylinder made of pure tungsten
- Sensitive W layers 300 µm thick every 4.5 mm
- No air gaps
- Electron energy = 10 GeV

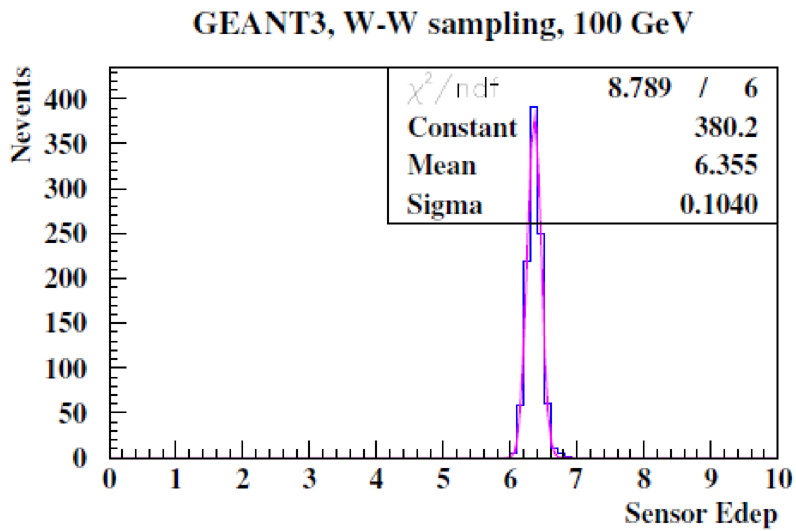
Simulation results - 4

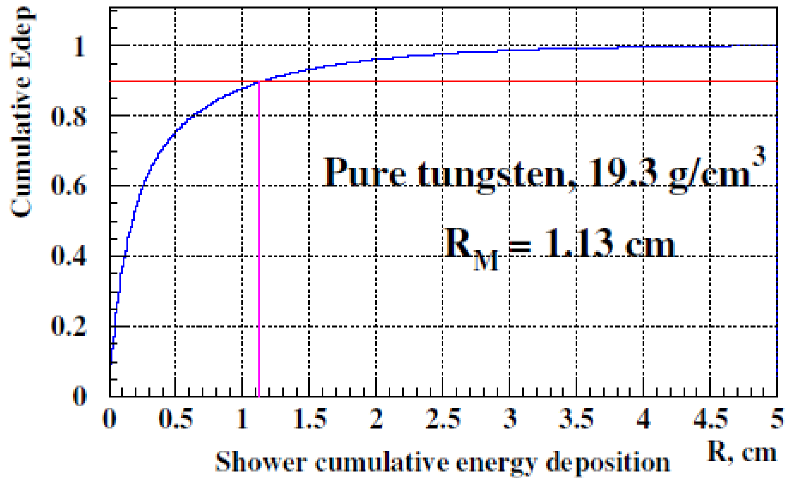




- Cylinder made of pure tungsten
- Sensitive W layers 300 µm thick every 4.5 mm
- No air gaps
- Electron energy = 50 GeV

Simulation results - 5

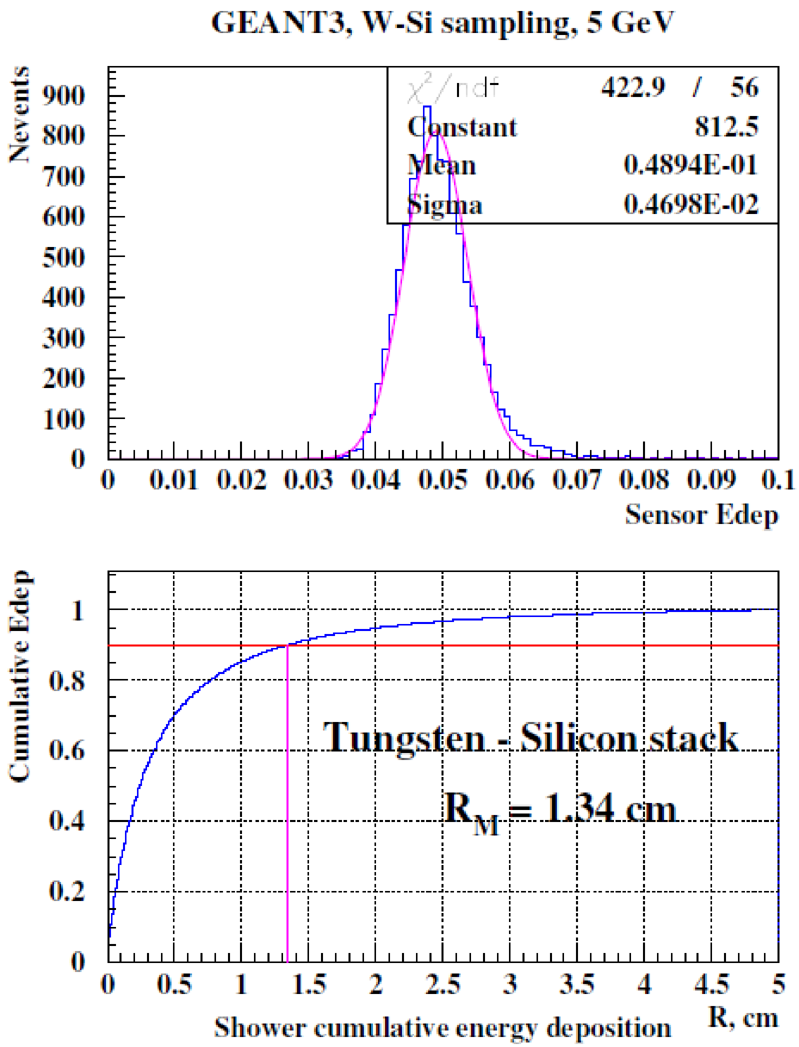




- Cylinder made of pure tungsten
- Sensitive W layers 300 µm thick every 4.5 mm
- No air gaps
- Electron energy = 100 GeV

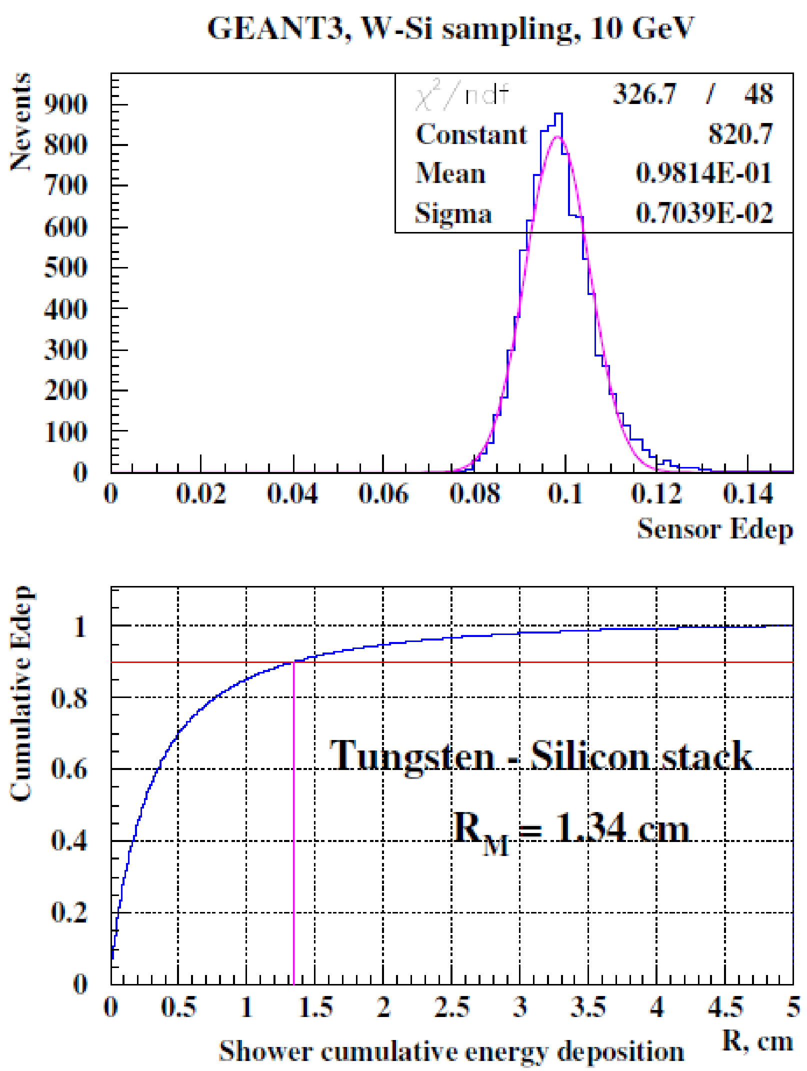
- R_M doesn't depend on shower energy

Simulation results - 6



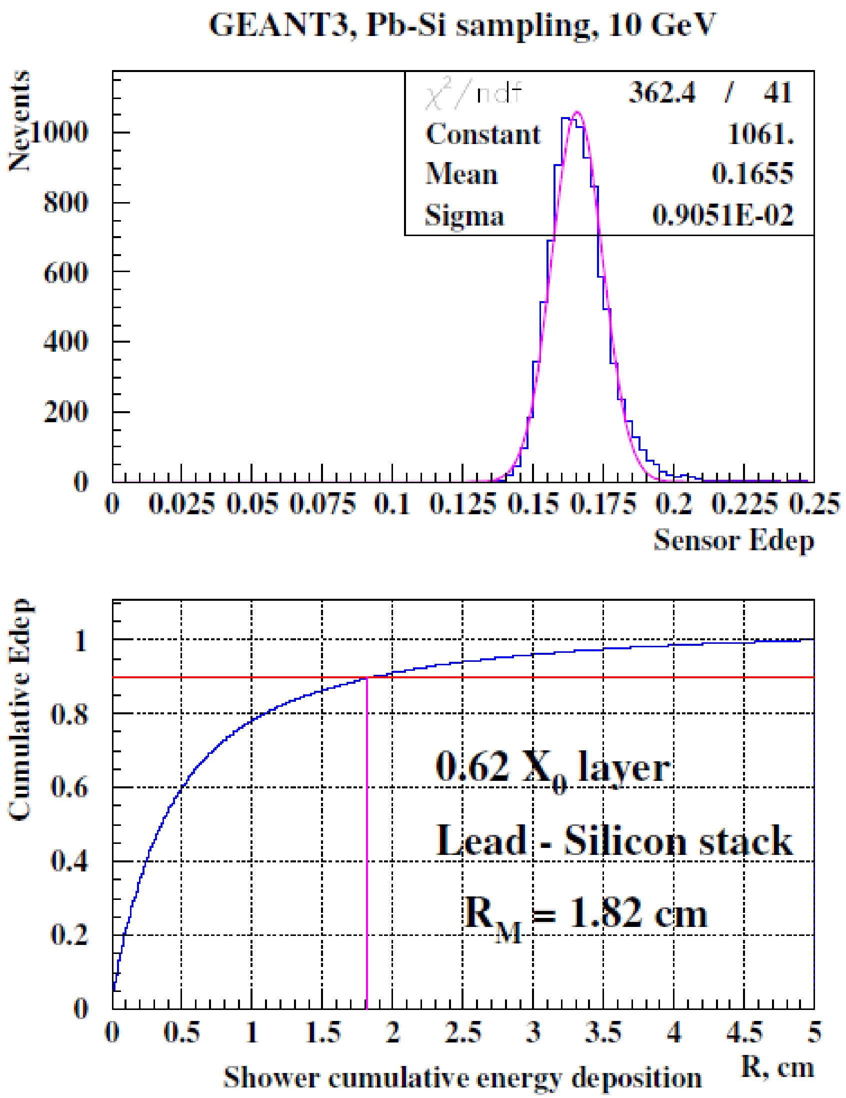
- Sampling structure of pure tungsten plates 3.5 mm thick with 1 mm gaps
- Sensitive Si layers 300 µm thick in the gaps
- 35 layers
- Electron energy = 5 GeV

Simulation results - 7



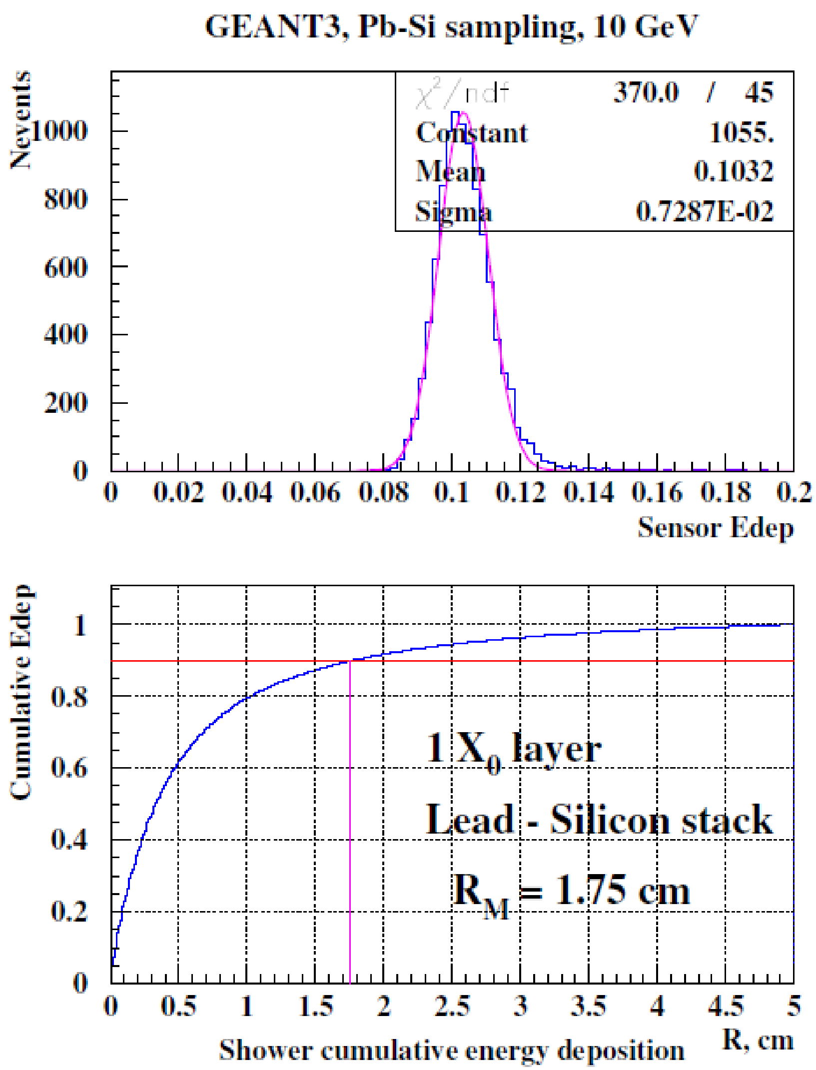
- Sampling structure of pure tungsten plates 3.5 mm thick with 1 mm gaps
- Sensitive Si layers 300 µm thick in the gaps
- 35 layers
- Electron energy = 10 GeV

Simulation results - 8



- Sampling structure of lead plates 3.5 mm thick with 1 mm gaps
- Sensitive Si layers 300 µm thick in the gaps
- 50 layers 0.62 Xo each
- Electron energy = 10 GeV

Simulation results - 9



- Sampling structure of lead plates 5.6 mm thick with 1 mm gaps
- Sensitive Si layers 300 µm thick in the gaps
- 35 layers 1 Xo each
- Electron energy = 10 GeV

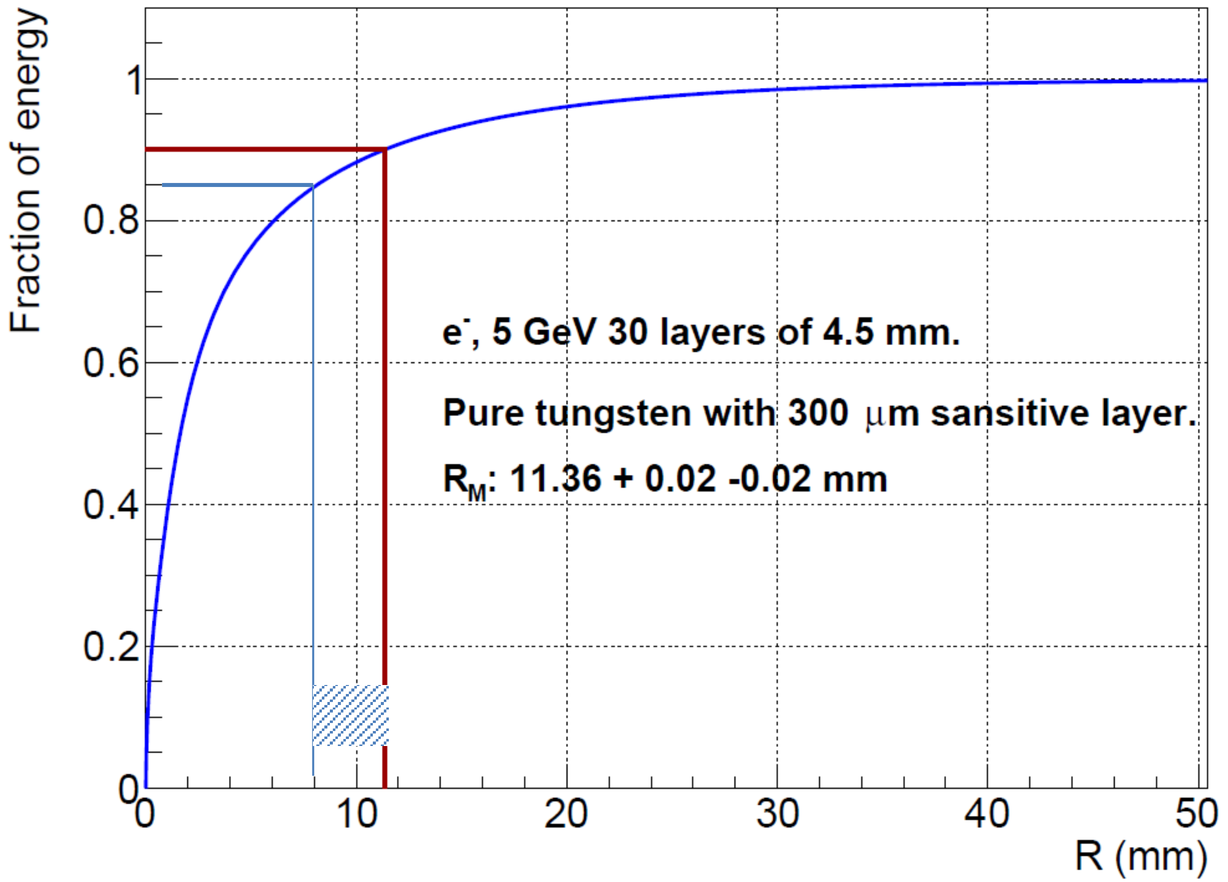
GEANT3 Simulation results

← Electron energy →

		5 GeV	10 GeV	50 GeV	100 GeV	
W-W homogeneous			1.15 cm			
			<2%			
W-W sampling		1.13 cm	1.13 cm	1.13 cm	1.13 cm	
		6.9%	5.04%	2.28%	1.64%	
W-Si sampling		1.34 cm	1.34 cm			
		9.6%	7.2%			
Pb-Si sampling	0.62 Xo		1.82 cm			
			5.4%			
Pb-Si sampling	1 Xo		1.75 cm			- Eff. Moliere radius
			7.1%			- Energy resolution

GEANT4: $R_M(W) = 1.14$ cm

GEANT4 simulations



The same result as for GEANT3

Depending on definition $R_M(W)=8.8 \div 11.4$ mm

Compatible with PDG $R_M=9.33$ mm

Conclusions

Energetic (5-100 GeV) electron showers were simulated in some calorimeter models, including homogeneous media and sampling structures of several types. Special attention was paid on the transverse development of the showers

Results of MC simulations of energetic electron showers are fully compatible between GEANT3 and GEANT4
(trivial statement, but anyway...) → fast GEANT3 may still be used

Transverse width of the showers in sampling structures is compatible with the one in the homogeneous mixture of materials. RM doesn’t depend on shower energy

PDG statement of 90% shower energy containment in 1 R_M cylinder around shower axis is valid with only limited precision (R_M defined this way may differ by ~20% from table value)