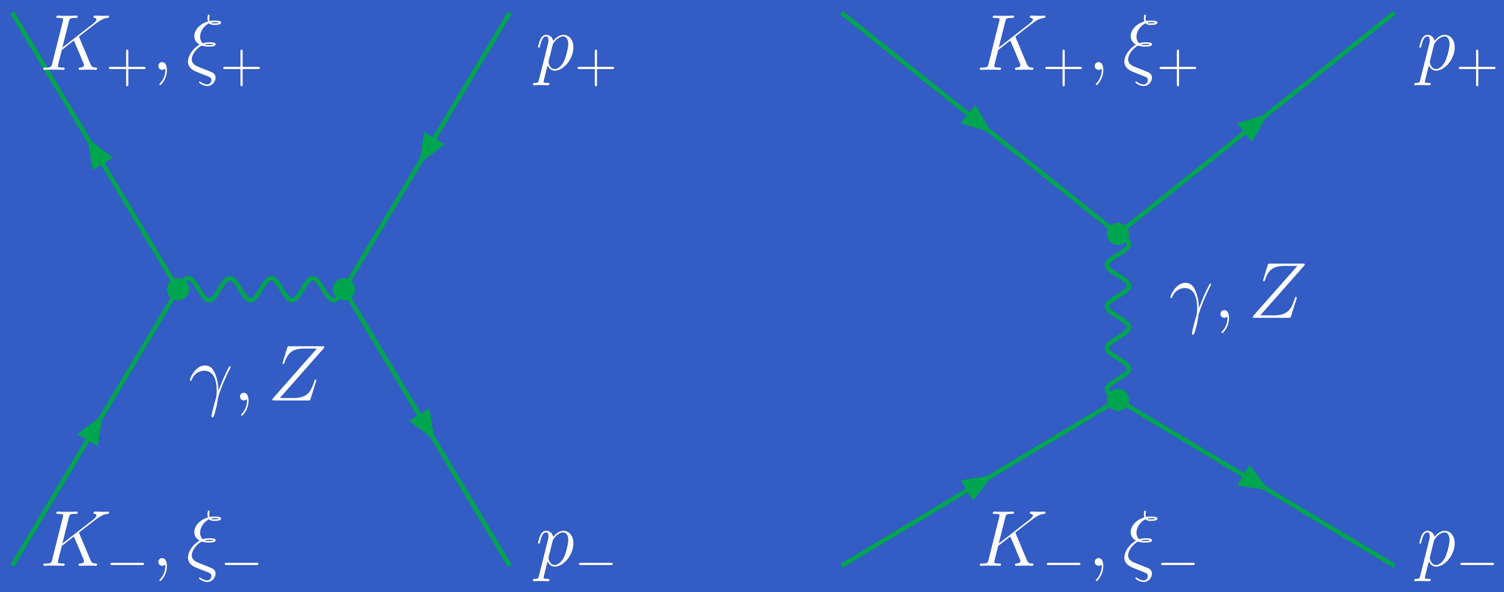
The impact of beam polarisations on the Bhabha cross section

N M Shumeiko, P M Starovoitov, J G Suarez

National Centre for Particle and High Energy Physics BSU

Definitions

$$e^-(K_-,\xi_-) + e^+(K_+,\xi_+) \to e^-(p_-) + e^+(p_+)$$



Definitions

$$e^-(K_-,\xi_-) + e^+(K_+,\xi_+) \to e^-(p_-) + e^+(p_+)$$

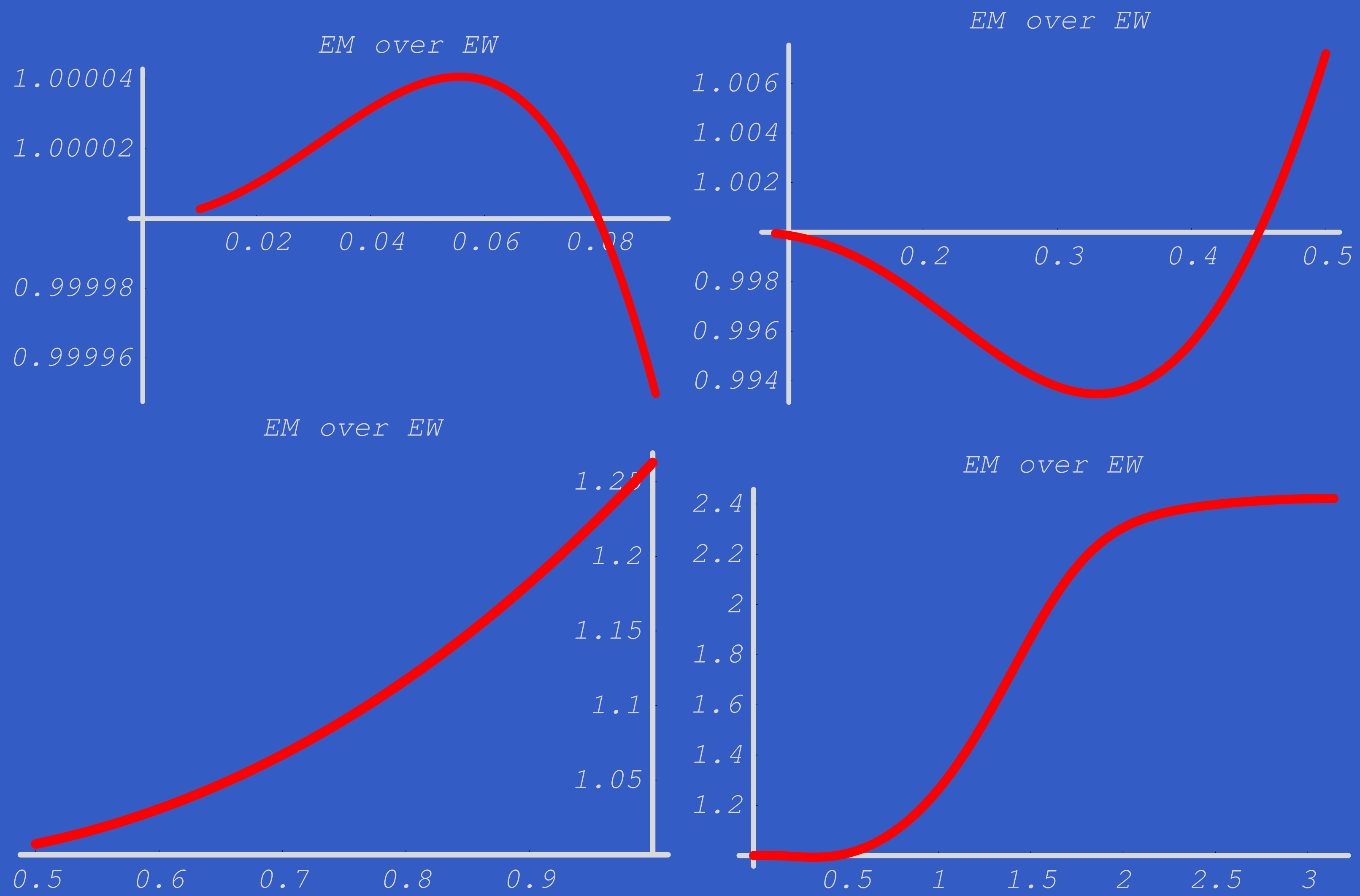
Mandelstam variables

$$S = (K_- + K_+)^2 = (p_- + p_+)^2,$$
$$T = (K_+ - p_+)^2 = (K_- - p_-)^2,$$
$$U = (K_+ - p_-)^2 = (K_- - p_+)^2$$

Polarisation vectors

$$\xi_-^2 = \xi_+^2 = -1, \quad (\xi_- \cdot K_-) = 0, (\xi_+ \cdot K_+) = 0$$

Ratio $\sigma^{O}_{EM}/\sigma^{O}_{EW}$



Polarisation vector parametrisation

$$\xi_-^{L\mu} = \frac{P_-}{\sqrt{S}\sqrt{S-4m^2}}\left(\frac{S-2m^2}{m}K_-^\mu - 2m{K_+}^\mu\right)$$

$$\xi_+^{L\mu} = \frac{P_+}{\sqrt{S}\sqrt{S-4m^2}}\left(\frac{S-2m^2}{m}K_+^\mu - 2m{K_-}^\mu\right)$$

$${\xi_{-,+}^T}^\mu = \frac{P_{-,+}}{\sqrt{S-4m^2}}\left(\frac{\sqrt{-U}}{\sqrt{-T}}K_-^\mu + \frac{\sqrt{-T}}{\sqrt{-U}}K_+^\mu - \frac{S-4m^2}{\sqrt{-T}\sqrt{-U}}p_-^\mu\right)$$

$${\xi_{-,+}^N}^\mu = \frac{2P_{-,+}}{\sqrt{-U}\sqrt{-T}\sqrt{S}}\epsilon^{\mu K_- p_- K_+}$$

$${\xi_{-,+}^L}^2 = {\xi_{-,+}^T}^2 = {\xi_{-,+}^N}^2 = -1 \quad \xi_{-,+}^L \cdot K_{-,+} = \xi_{-,+}^T \cdot K_{-,+} = \xi_{-,+}^N \cdot K_{-,+} = 0$$

Electromagnetic Asymmetry

$$\sigma_{EM}(P_-, P_+) = \frac{d\sigma_{EM}}{d\Omega} = \frac{d\sigma^O_{EM}}{d\Omega}\left(1 + \sum_{i,j=L,T,N} P_-{}^i P_+{}^j A_{ij}\right)$$

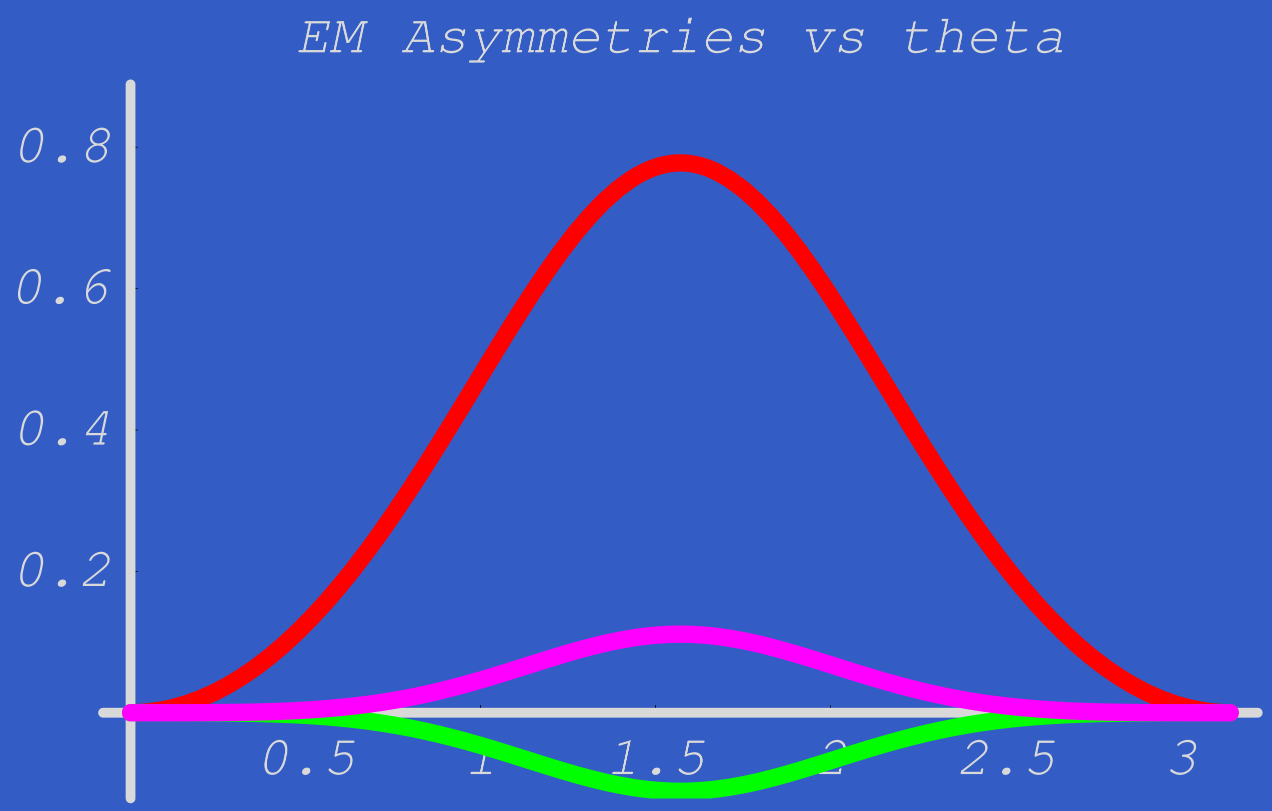
$$A_{ij} = \frac{\sigma^{ij}(-,+) - \sigma^{ij}(-,+)}{\sigma^{ij}(-,+) + \sigma^{ij}(-,+)}$$

in electromagnetic Bhabha scattering

$$A_{TT} = -A_{NN} \quad A_{LT} = A_{LN} = A_{TN} = A_{TL} = A_{NL} = A_{NT} = 0$$

Electromagnetic Asymmetry

$\mathbf{A_{LL}}$ A_{TT} A_{NN}



Electroweak Asymmetry

$$\sigma_{EW}(P_-, P_+) = \frac{d\sigma_{EW}}{d\Omega} = \frac{d\sigma^O_{EW}}{d\Omega}\left(1 + \sum_{i,j=L,T,N}\left(P_-{}^i A^-_{ij} + P_+{}^j A^+_{ij} + P_-{}^i P_+{}^j A^{-+}_{ij}\right)\right)$$

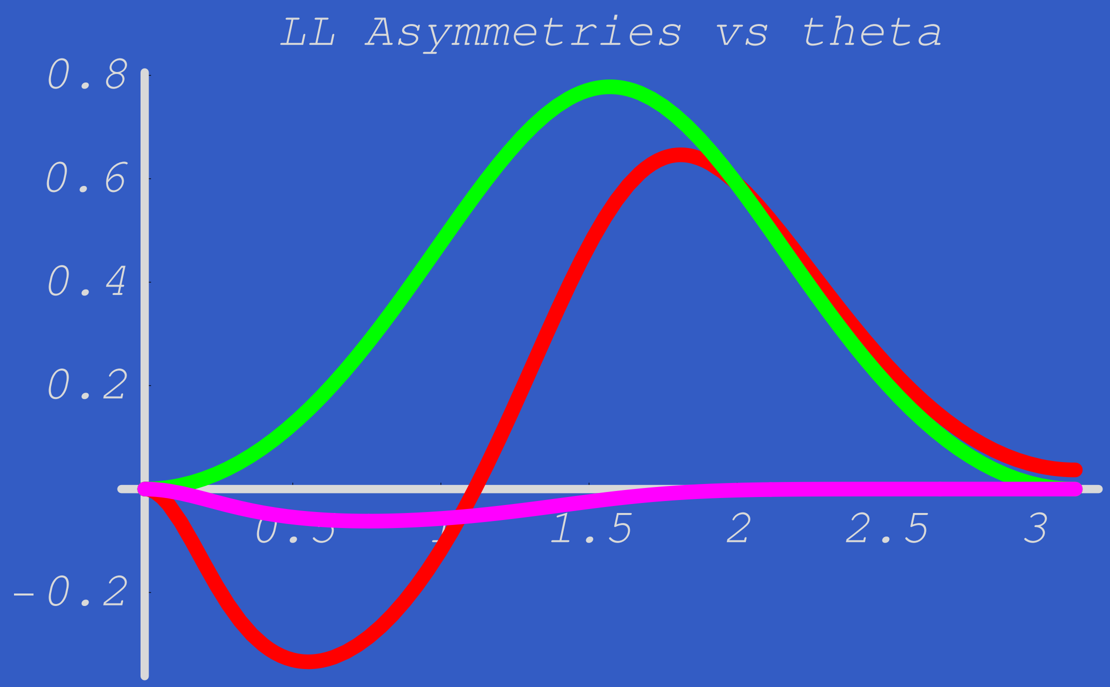
$$A^+_{ij} = \frac{\sigma^{ij}(+,+) - \sigma^{ij}(-,+) + \sigma^{ij}(+,-) - \sigma^{ij}(-,-)}{\sigma^{ij}(+,+) + \sigma^{ij}(-,+) + \sigma^{ij}(+,-) + \sigma^{ij}(-,-)}$$

$$A^-_{ij} = \frac{\sigma^{ij}(+,+) + \sigma^{ij}(-,+) - \sigma^{ij}(+,-) - \sigma^{ij}(-,-)}{\sigma^{ij}(+,+) + \sigma^{ij}(-,+) + \sigma^{ij}(+,-) + \sigma^{ij}(-,-)}$$

$$A^{-+}_{ij} = \frac{\sigma^{ij}(+,+) - \sigma^{ij}(-,+) - \sigma^{ij}(+,-) + \sigma^{ij}(-,-)}{\sigma^{ij}(+,+) + \sigma^{ij}(-,+) + \sigma^{ij}(+,-) + \sigma^{ij}(-,-)}$$

LongLong Asymmetry

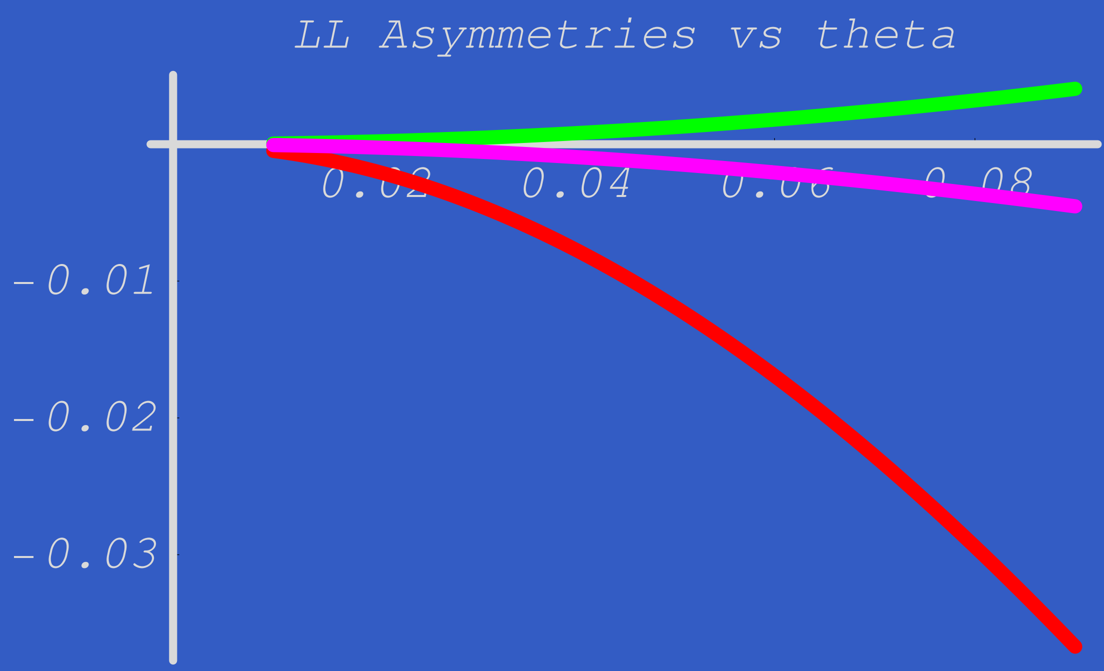
$$\sigma_{EW}^{LL}(P_-,P_+) = \frac{d\sigma_{EW}^{O}}{d\Omega}\left(1+\left(P_-{}^L - P_+{}^L\right)A_{LL}^{-} + P_-{}^L P_+{}^L A_{LL}^{-+}\right)$$



$\mathbf{A_{LL}^{-+}}$ A_{LL}^{EM} A_{LL}^{-}

LongLong Asymmetry

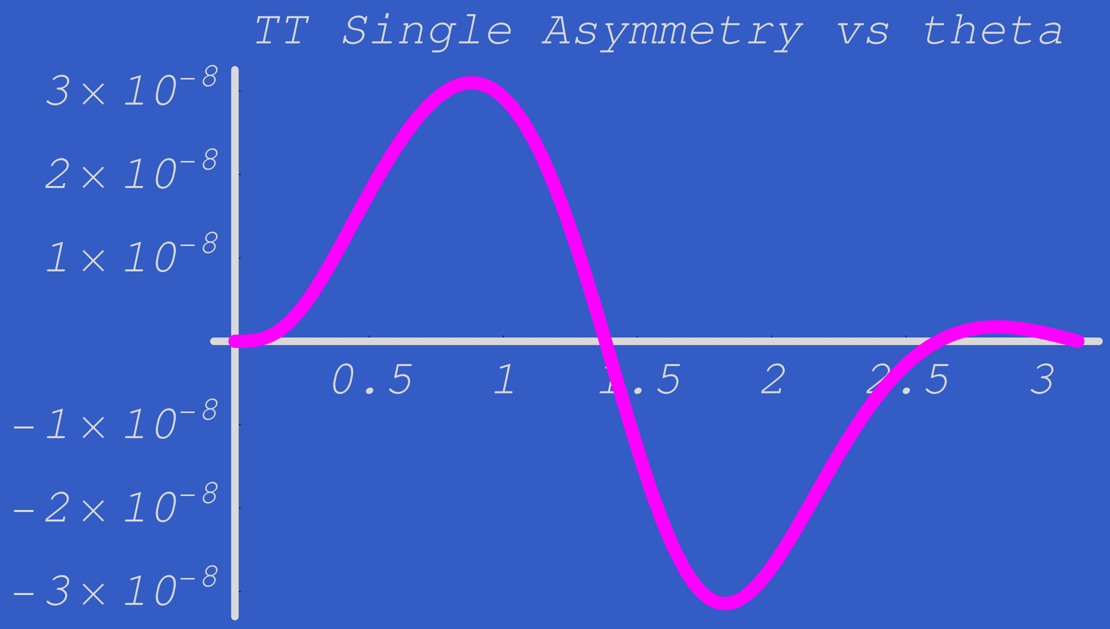
$$\sigma^{LL}_{EW}(P_-, P_+) = \frac{d\sigma^{O}_{EW}}{d\Omega}\left(1 + \left(P_-{}^{L} - P_+{}^{L}\right) A^{-}_{LL} + P_-{}^{L} P_+{}^{L} A^{-+}_{LL}\right)$$



$\mathbf{A^{-+}_{LL}}$ $\quad A^{EM}_{LL}$ $\quad A^{-}_{LL}$

TranTran Asymmetry

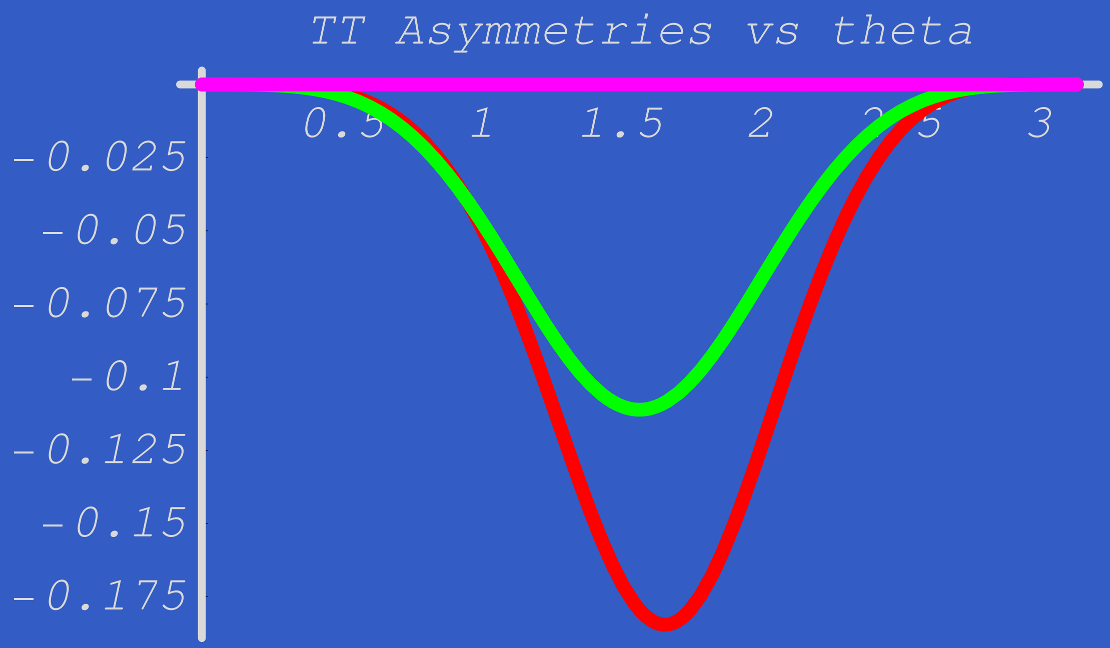
$$\sigma^{TT}_{EW}(P_-,P_+) = \frac{d\sigma^O_{EW}}{d\Omega}\left(1 + \left(P_-{}^T + P_+{}^T\right) A^-_{TT} + P_-{}^T P_+{}^T A^{-+}_{TT}\right)$$



$\mathbf{A^{-+}_{TT}}$ A^{EM}_{TT} A^-_{TT}

TranTran Asymmetry

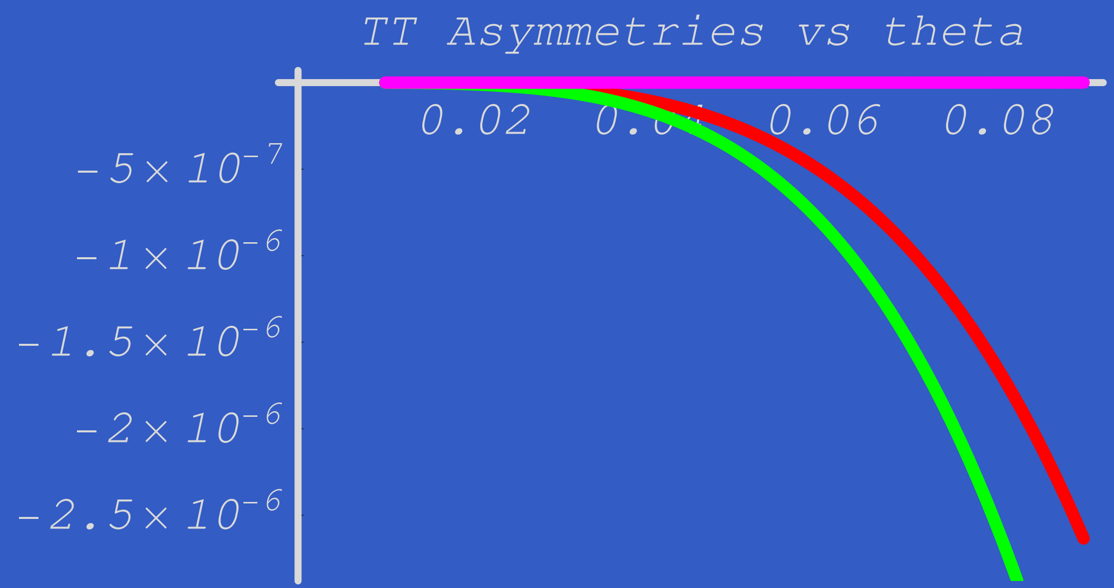
$$\sigma^{TT}_{EW}(P_-, P_+) = \frac{d\sigma^O_{EW}}{d\Omega}\left(1 + \left({P_-}^T + {P_+}^T\right) A^{-}_{TT} + {P_-}^T {P_+}^T A^{-+}_{TT}\right)$$



$\mathbf{A^{-+}_{TT}}$ A^{EM}_{TT} A^{-}_{TT}

TranTran Asymmetry

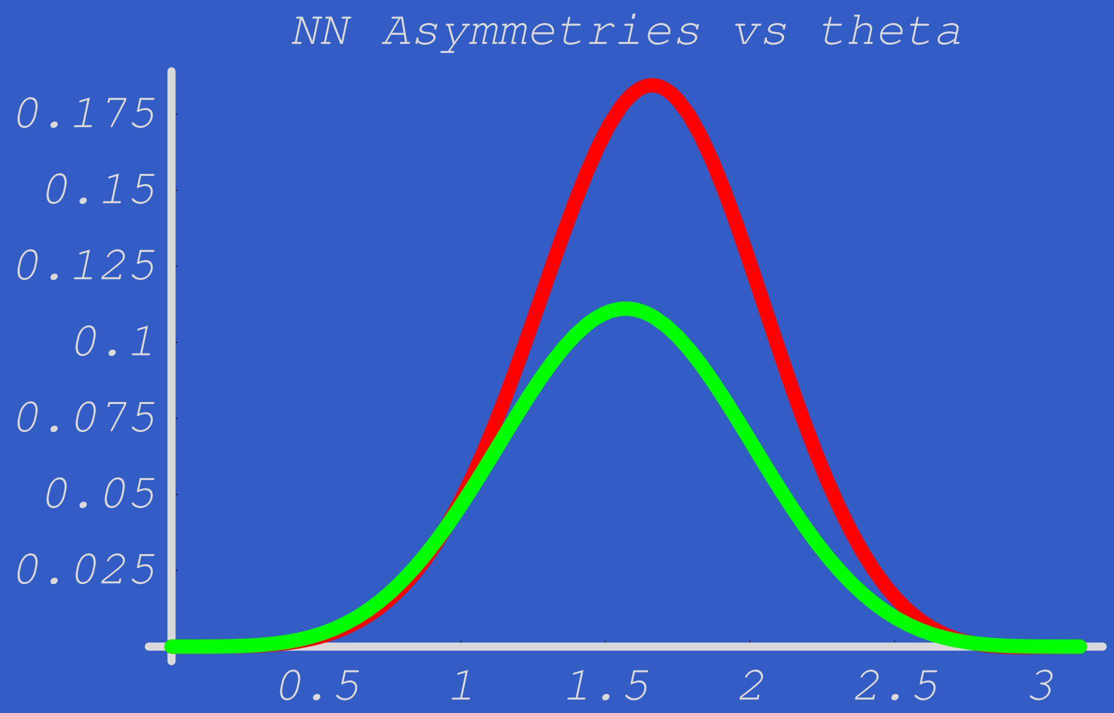
$$\sigma_{EW}^{TT}(P_-, P_+) = \frac{d\sigma_{EW}^{O}}{d\Omega}\left(1 + \left(P_-{}^T + P_+{}^T\right) A_{TT}^{-} + P_-{}^T P_+{}^T A_{TT}^{-+}\right)$$



$\mathbf{A_{TT}^{-+}}$ A_{TT}^{EM} A_{TT}^{-}

NormNorm Asymmetry

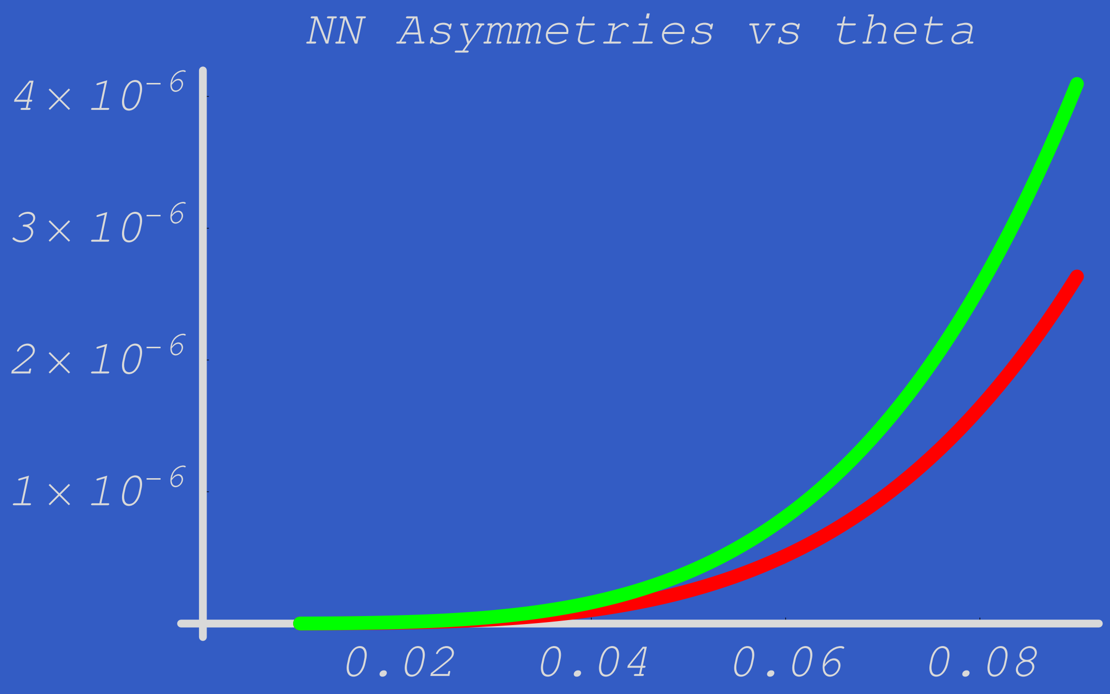
$$\sigma^{LL}_{EW}(P_-, P_+) = \frac{d\sigma^O_{EW}}{d\Omega}\left(1 + P_-{}^N P_+{}^N A^{-+}_{NN}\right)$$



$\mathbf{A^{-+}_{NN}}$ A^{EM}_{NN}

NormNorm Asymmetry

$$\sigma^{LL}_{EW}(P_-, P_+) = \frac{d\sigma^{O}_{EW}}{d\Omega}\left(1 + P_-{}^N P_+{}^N A^{-+}_{NN}\right)$$

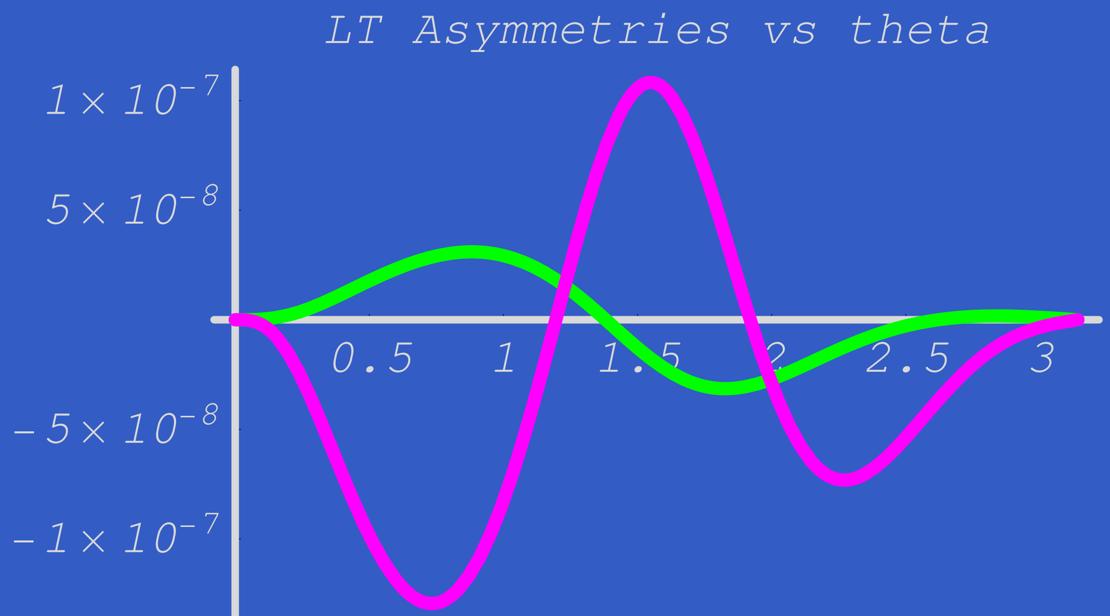


$\mathbf{A^{-+}_{NN}}$ A^{EM}_{NN}

LongTran & TranLong Asymmetry

$$\sigma_{EW}^{LT}(P_-, P_+) = \frac{d\sigma_{EW}^{O}}{d\Omega}\left(1 + P_-{}^{L} A_{LT}^{-} + P_+{}^{T} A_{LT}^{+} + P_-{}^{L} P_+{}^{T} A_{LT}^{-+}\right)$$

$$\sigma_{EW}^{TL}(P_-, P_+) = \frac{d\sigma_{EW}^{O}}{d\Omega}\left(1 + P_-{}^{T} A_{LT}^{+} - P_+{}^{L} A_{LT}^{-} - P_-{}^{T} P_+{}^{L} A_{LT}^{-+}\right)$$

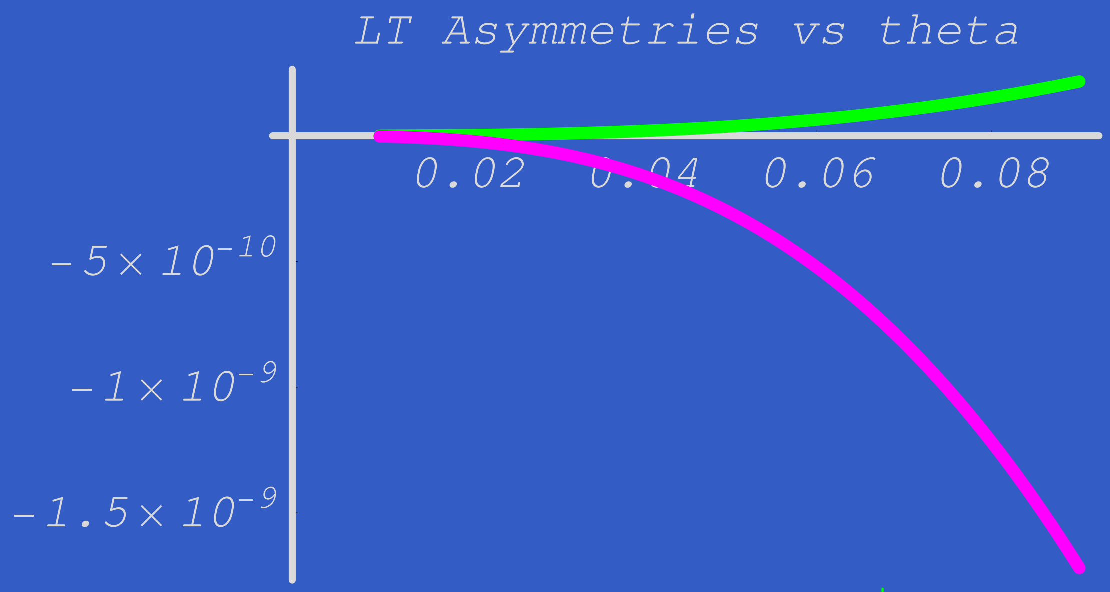


$\mathbf{A_{LT}^{-}}$ A_{LT}^{+} A_{LT}^{-+}

LongTran & TranLong Asymmetry

$$\sigma^{LT}_{EW}(P_-,P_+) = \frac{d\sigma^O_{EW}}{d\Omega}\left(1 + P_-{}^L A^-_{LT} + P_+{}^T A^+_{LT} + P_-{}^L P_+{}^T A^{-+}_{LT}\right)$$

$$\sigma^{TL}_{EW}(P_-,P_+) = \frac{d\sigma^O_{EW}}{d\Omega}\left(1 + P_-{}^T A^+_{LT} - P_+{}^L A^-_{LT} - P_-{}^T P_+{}^L A^{-+}_{LT}\right)$$

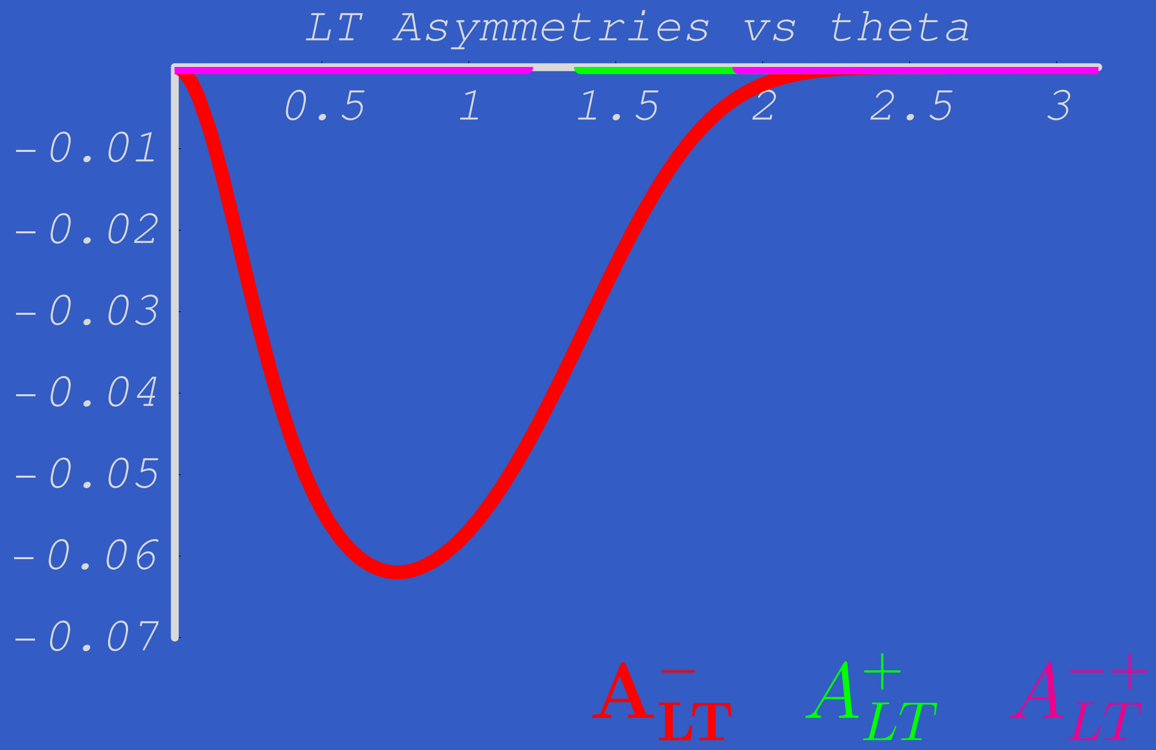


$\mathbf{A^-_{LT}}$ A^+_{LT} A^{-+}_{LT}

LongTran & TranLong Asymmetry

$$\sigma^{LT}_{EW}(P_-, P_+) = \frac{d\sigma^{O}_{EW}}{d\Omega}\left(1 + P_-{}^{L} A^{-}_{LT} + P_+{}^{T} A^{+}_{LT} + P_-{}^{L} P_+{}^{T} A^{-+}_{LT}\right)$$

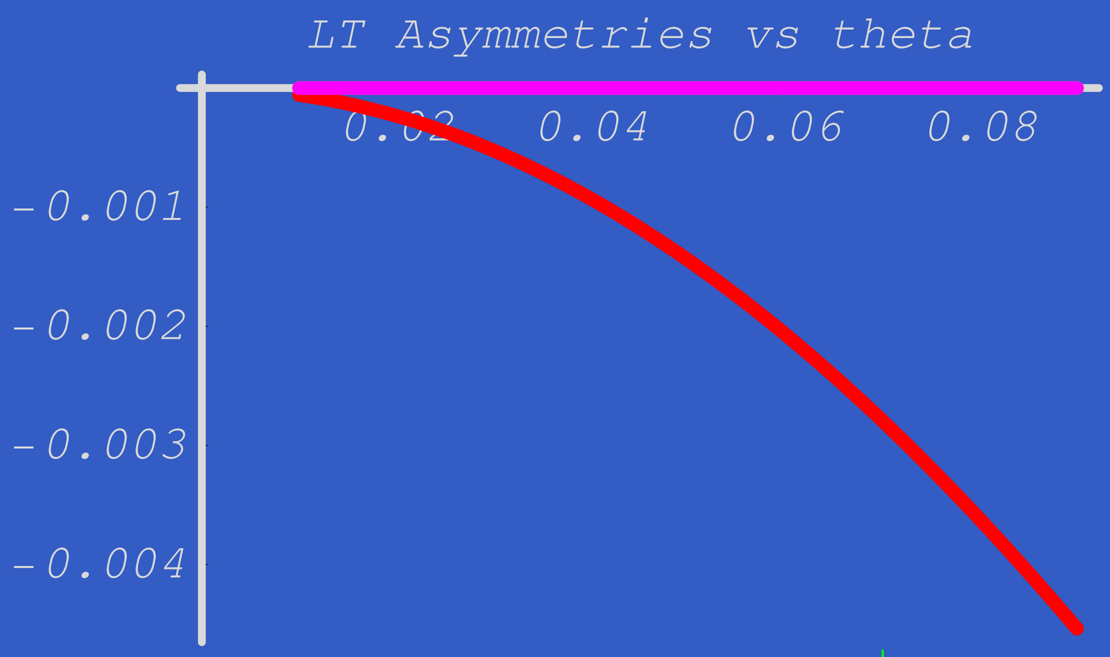
$$\sigma^{TL}_{EW}(P_-, P_+) = \frac{d\sigma^{O}_{EW}}{d\Omega}\left(1 + P_-{}^{T} A^{+}_{LT} - P_+{}^{L} A^{-}_{LT} - P_-{}^{T} P_+{}^{L} A^{-+}_{LT}\right)$$



LongTran & TranLong Asymmetry

$$\sigma_{EW}^{LT}(P_-, P_+) = \frac{d\sigma_{EW}^{O}}{d\Omega}\left(1 + P_-{}^{L} A_{LT}^{-} + P_+{}^{T} A_{LT}^{+} + P_-{}^{L} P_+{}^{T} A_{LT}^{-+}\right)$$

$$\sigma_{EW}^{TL}(P_-, P_+) = \frac{d\sigma_{EW}^{O}}{d\Omega}\left(1 + P_-{}^{T} A_{LT}^{+} - P_+{}^{L} A_{LT}^{-} - P_-{}^{T} P_+{}^{L} A_{LT}^{-+}\right)$$

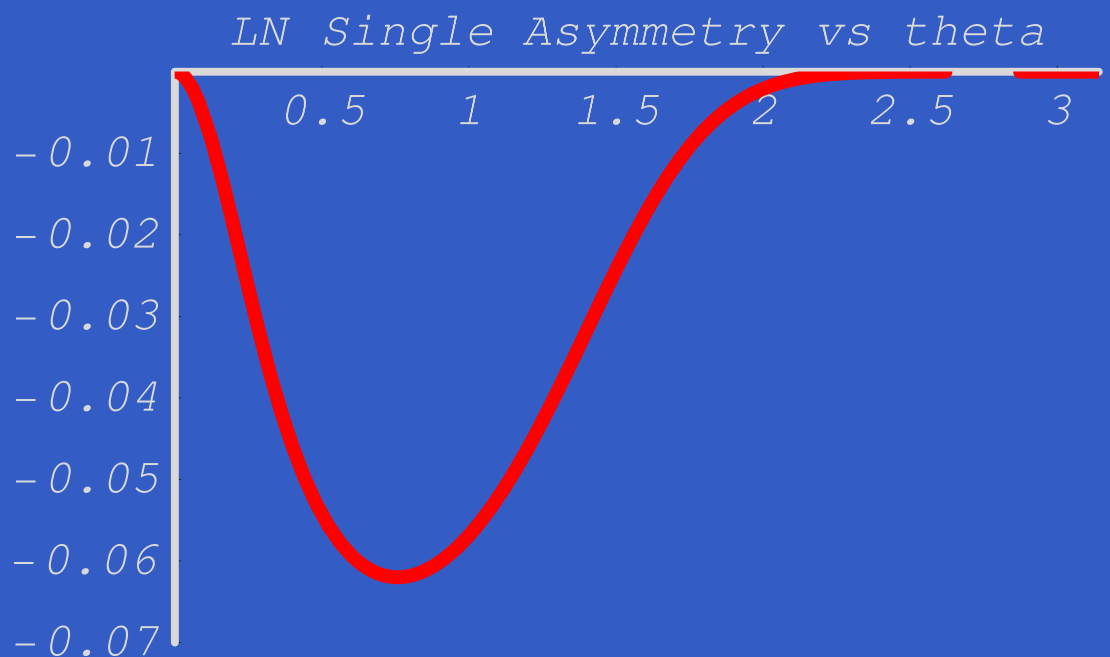


$\mathbf{A_{LT}^{-}}$ A_{LT}^{+} A_{LT}^{-+}

LongNorm & NormLong Asymmetry

$$\sigma^{LN}_{EW}(P_-, P_+) = \frac{d\sigma^O_{EW}}{d\Omega}\left(1 + P_-{}^L A^-_{LN}\right)$$

$$\sigma^{NL}_{EW}(P_-, P_+) = \frac{d\sigma^O_{EW}}{d\Omega}\left(1 - P_+{}^L A^-_{LN}\right)$$

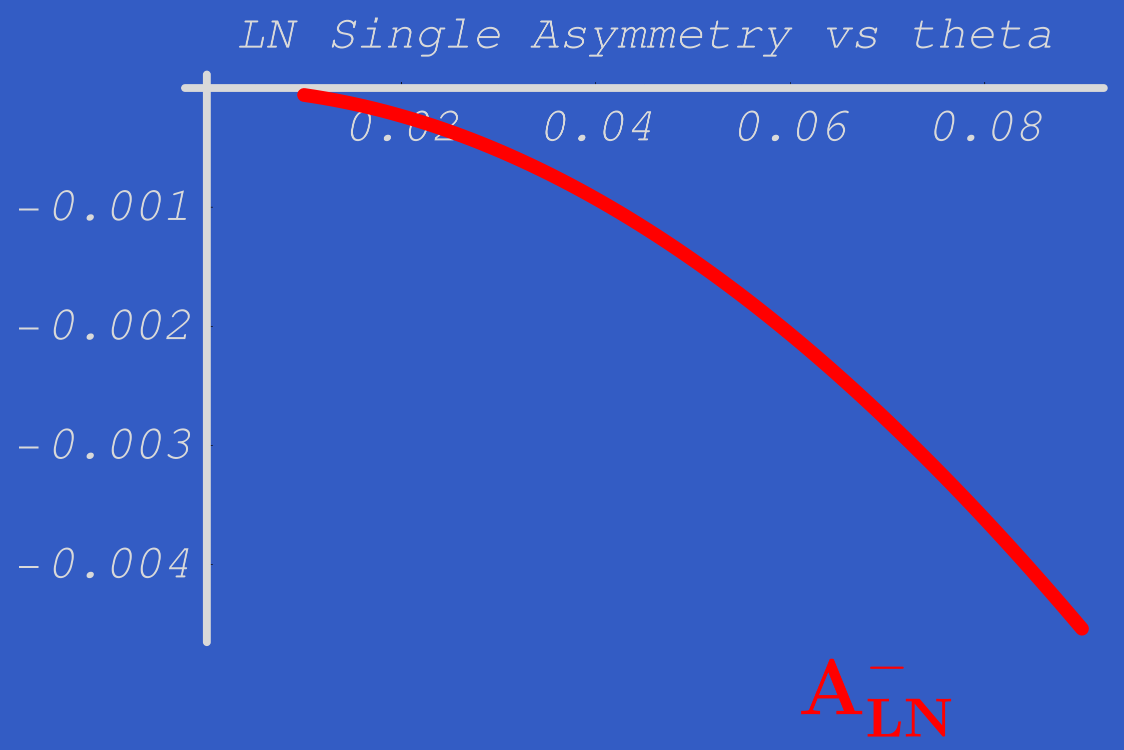


$\mathbf{A^-_{LN}}$

LongNorm & NormLong Asymmetry

$$\sigma_{EW}^{LN}(P_-, P_+) = \frac{d\sigma_{EW}^{O}}{d\Omega}\left(1 + P_-{}^{L}A_{LN}^{-}\right)$$

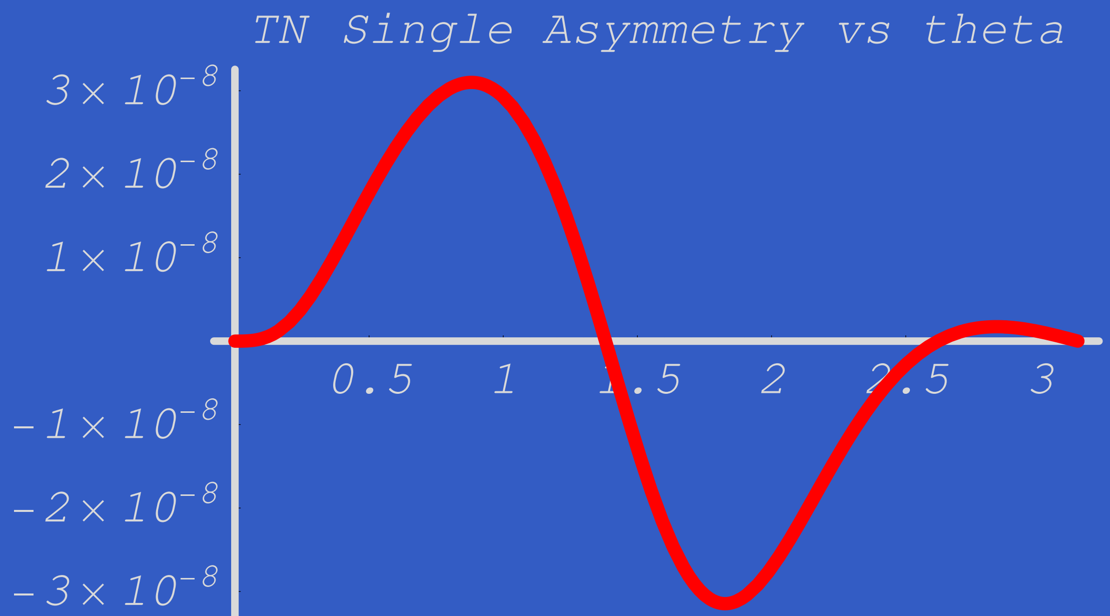
$$\sigma_{EW}^{NL}(P_-, P_+) = \frac{d\sigma_{EW}^{O}}{d\Omega}\left(1 - P_+{}^{L}A_{LN}^{-}\right)$$



NormTran & TranNorm Asymmetry

$$\sigma_{EW}^{NT}(P_-, P_+) = \frac{d\sigma_{EW}^{O}}{d\Omega}\left(1 + P_+{}^T A_{NT}^+\right)$$

$$\sigma_{EW}^{TN}(P_-, P_+) = \frac{d\sigma_{EW}^{O}}{d\Omega}\left(1 + P_-{}^T A_{NT}^+\right)$$

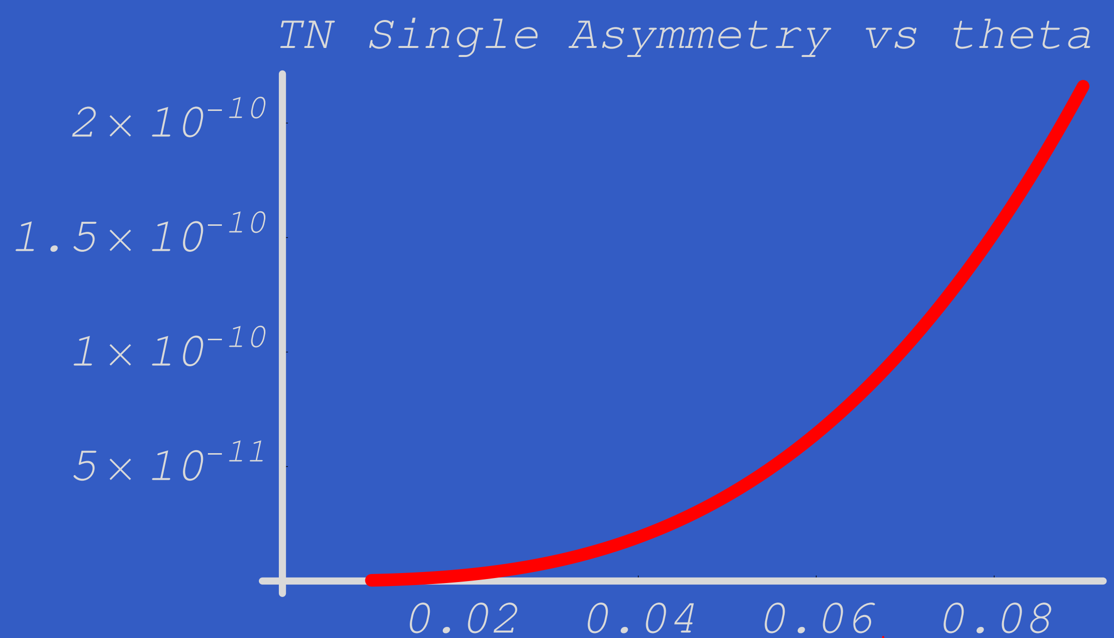


$\mathbf{A_{NT}^{+}}$

NormTran & TranNorm Asymmetry

$$\sigma^{NT}_{EW}(P_-, P_+) = \frac{d\sigma^{O}_{EW}}{d\Omega}\left(1 + P_+{}^{T} A^{+}_{NT}\right)$$

$$\sigma^{TN}_{EW}(P_-, P_+) = \frac{d\sigma^{O}_{EW}}{d\Omega}\left(1 + P_-{}^{T} A^{+}_{NT}\right)$$



$\mathbf{A^{+}_{NT}}$

Conclusions

- The full set of EW asymmetries for polarised Bhabha scattering has been calculated at the Born level. Results obtained give a clear picture of the potential of polarised asymmetry measurements at ILC.
- EW LL asymmetry exceeds EM one in forward region, providing an interesting possibility for dedicated measurements at small angles.
- EW asymmetries for combined beam polarisations (LT, LN, etc.) in contrast to EM are non zero. This opens possibility for precise measurements of pure EW effects (parity violation, $\sin(\theta_W)$).
- Importance of the RC has been stressed on the example of the EM Möller scattering.