FCAL workshop
Tel Aviv
September 18, 2004

A few points about Luminosity

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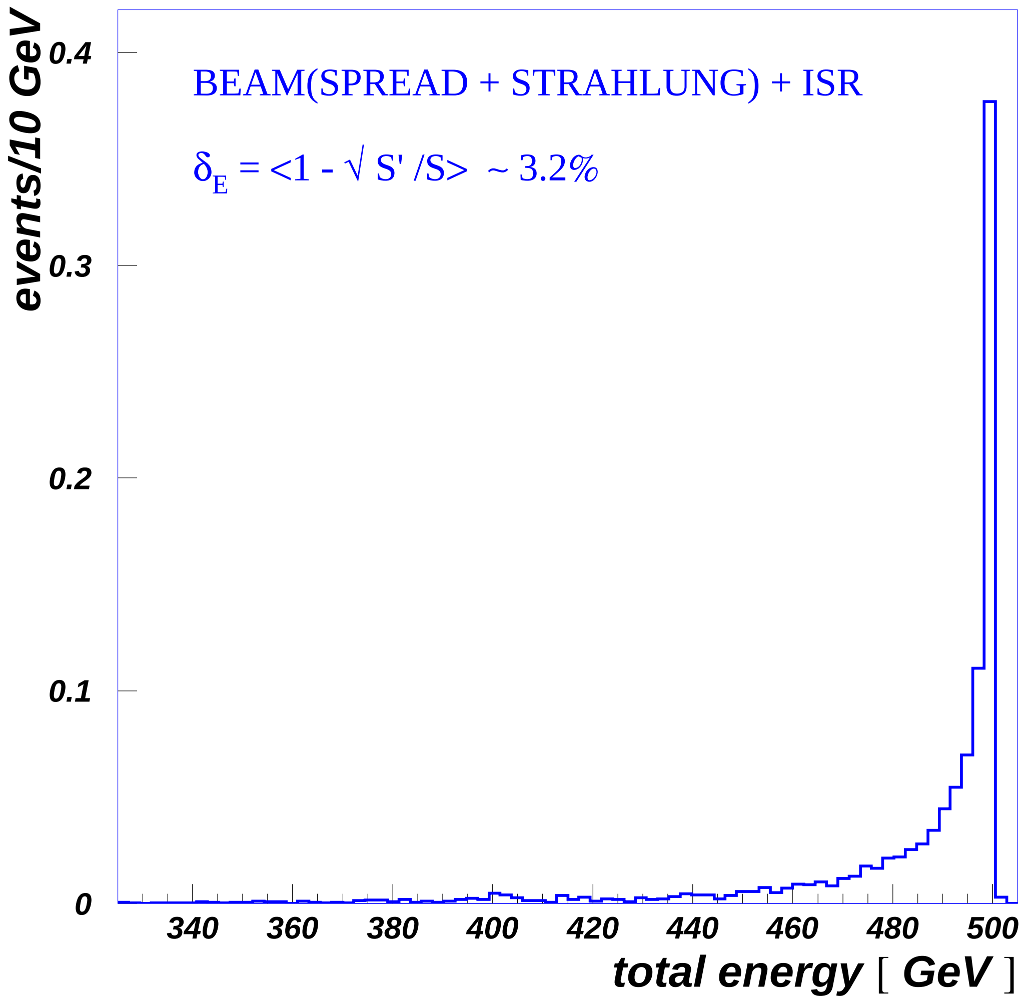
Outline:

- **Introduction**
- **Luminosity**
- **Disruption**
- **Beamstrahlung**

INTRODUCTION

$\sqrt{S}$ = 500 GeV



There are 3 effects causing the collision energy to be shifted from its nominal value.

- Initial State Radiation
- Beamspread
- Beamstrahlung

- **beam-beam interaction ⟶ two main effects**

- **disruption ⟶ particles trajectories are bent by field provided by the oncoming beam**

- **beamstrahlung ⟶ particles radiate due to disruption**

- **impact of disruption → deformation of the beam sizes during collision → luminosity enhacement**

- **impact of beamstrahlung → loss of the energy → degradation of beam energy**

Center of mass energy	$E_{cm} = \sqrt{s}$	GeV	500
Repetion rate	f_{rep}	Hz	5
Bunch charge $[10^{10}]$	N		2
Number of bunches	n_b		2820
Transverse bunch sizes	σ_x/σ_y	nm	553/5
Bunch length	σ_z	μm	300
Geometric emittances	ϵ_x/ϵ_y	μm.μrad	10/0.030
Beta functions	β_x/β_y	mm	15/0.4
Geometric luminosity $[10^{34}]$	$\mathcal{L}_0$	$cm^{-2}s^{-1}$	1.6
Disruption parameters	D_x/D_y		0.22/25

Table 1.1: Beam parameters for the ILC

Luminosity

$R_{ev} = \mathcal{L}_0 \sigma_{int}$

The event rate, R_{ev}, in a collider is proportional to the interaction cross section σ_{int}, and the factor of proportionality is called the *LUMINOSITY*.

Optimal *luminosity* $\rightarrow$ perfect transverse overlap of two equal Gaussian beams.

- $\sigma_x^+ = \sigma_x^- = \sigma_x$ **and** $\sigma_y^+ = \sigma_y^- = \sigma_y$
- $N_+ = N_- = N$ **where** N **is the number of particles per bunch (bunch charge)**

- $\sigma_{x(y)}$ **are the transverse dimensions of the beams,**

The geometric luminosity, $\mathcal{L}_0$,

$$\mathcal{L}_0 = \frac{f_{rep} n_b N^2}{4\pi \sigma_x \sigma_y}$$

- f_{rep} **is the pulse (train) repetition frequency**
- n_b **is the number of bunches per pulse (train).**

Luminosity

- $R = \sigma_x/\sigma_y \rightarrow$ **aspect ratio**

- $R = 1 \rightarrow$ **round beam**

- $R \gg 1 \rightarrow$ **flat beam**

- **The vertical and horizontal beam dimensions can not be chosen arbitrary**

$$\pi\sigma^2_{x,y} = \epsilon_{x,y}\beta^*_{x,y}$$

- $\epsilon \rightarrow$ **transverse emittance, represents the phase-space volume occupied by beam particles.**

- $\beta \rightarrow$ **Currant-Snyder** β^* **function, reflects the beam optics properties.**

Disruption

- **beam transverse sizes, σ_x, σ_y, are in the nanometer (10^{-9}m) range**

- **beam-beam forces are very strong**
$|E| \sim |B| \sim Ne/\sigma$

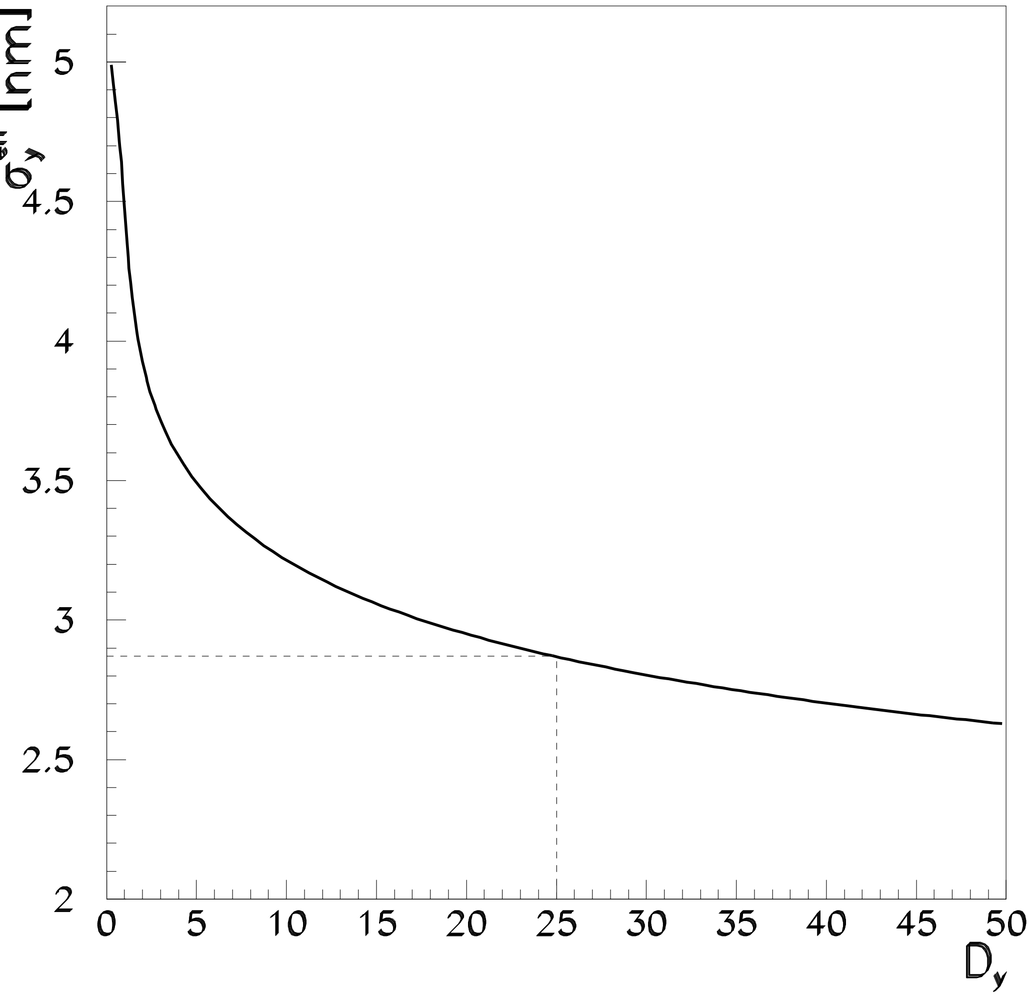
- **as bunches pass through each other, the particles bend inward due to the attraction of opposite charges $\longrightarrow$ DISRUPTION or PINCH effect**

- **The global disruption parameter D characterized the strength of the effect, (dimensionless measure of the amount of pinching)**

$$D_{x(y)} = \frac{2Nr_e}{\gamma} \frac{\sigma_z}{\sigma_{x(y)}(\sigma_x + \sigma_y)}$$

- γ **denote the Lorentz factor of the beam**

- $r_e = 2.82 \times 10^{-13} cm$ **classical electron radius.**



- **beam vertical size, σ_y, decreases with D_y**

$$H_D = \sigma_x \sigma_y / \sigma_x^{eff} \sigma_y^{eff}$$

- **H_D is the luminosity enhancement factor**

- **The effective luminosity of the collider (under disruption)**

$$\mathcal{L} = H_D \mathcal{L}_0$$

- H_D **as a function of a disruption parameter** D

Analytic derivation of H_D **is very difficult**

- **Round beam, small** D **gives**

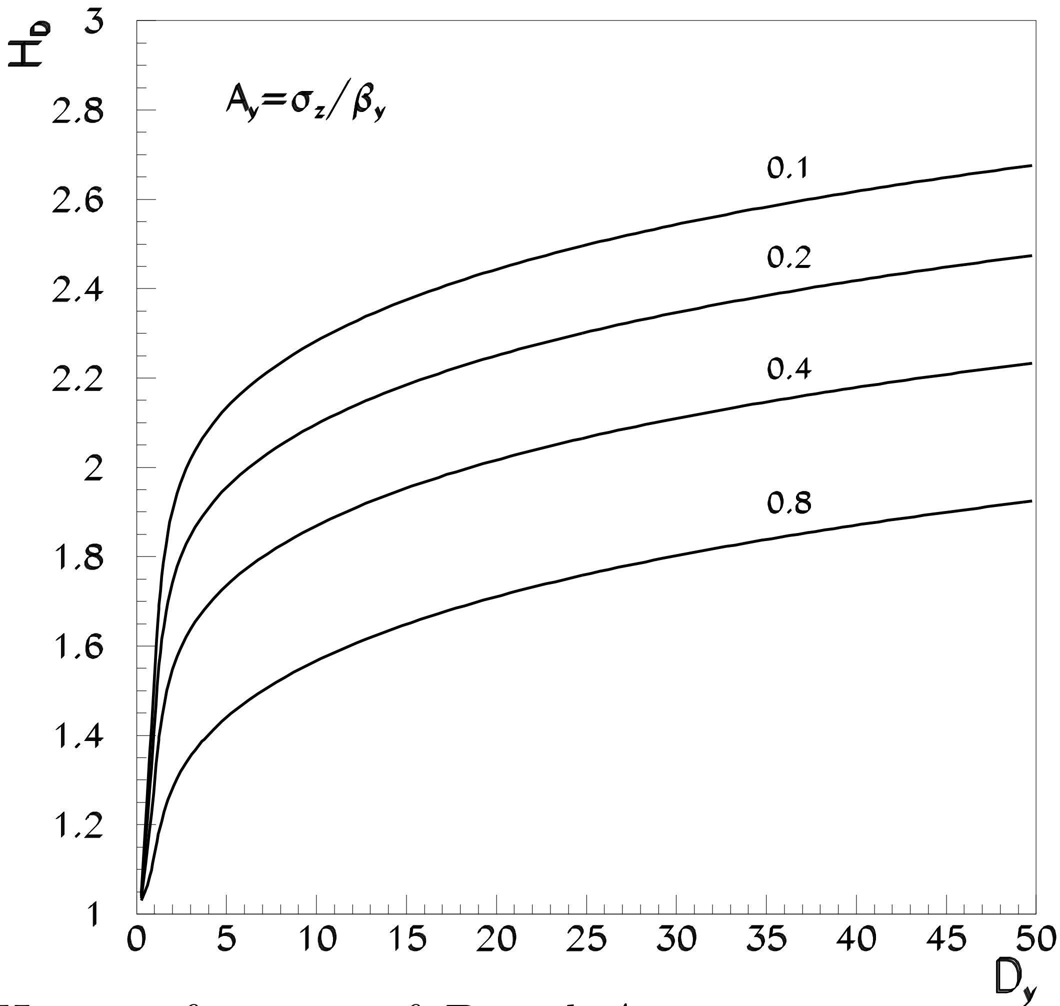
$$H_D = 1 + 2D/3\sqrt{\pi}$$

- $D > 1 \longrightarrow$ **computer simulaitons**

- **Paraxial beam** $\longrightarrow \mathcal{L} \to \infty$

Beam can be focused to a SINGULAR POINT, *BUT* **beam has always an inherent divergence.**

- **Parameter** $A_{x,y} = \sigma_z/\beta^* \sim \epsilon$**, where** ϵ **is emittance for a given beam size** $\sigma_{x,y}$



- $H_D \longrightarrow$ function of D and A

$$H_D \simeq 1 + D^{1/4}(\frac{D^3}{1+D^3})[\ln(\sqrt{D}+1) + 2\ln(0.8/A)]$$

- Flat beam $\longrightarrow$ 2 parameters H_{D_x} and H_{D_y}

$$H_D = (H_{D_x})^1/2 \cdot (H_{D_y})^1/3$$

- Additional parameter $\longrightarrow$ initial beam offset (displacement of the whole bunch), $\Delta_{x,y}$

- $H_D \longrightarrow$ function of D, A and Δ

Gaussian beams for various values of the offset. A=0.4.

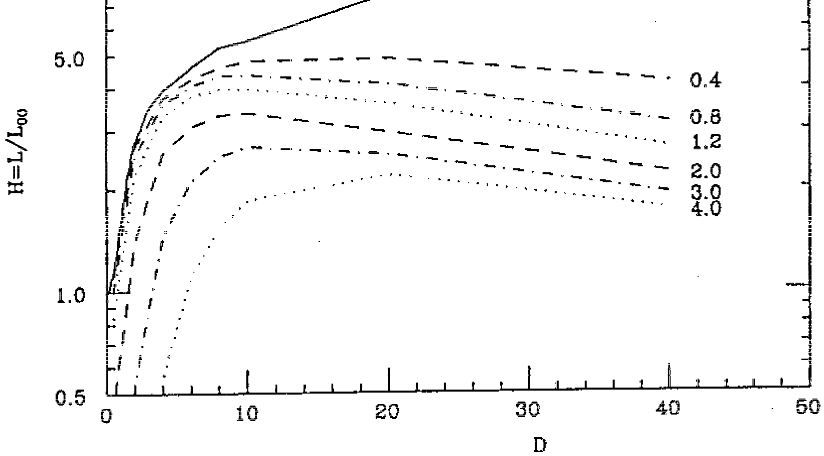
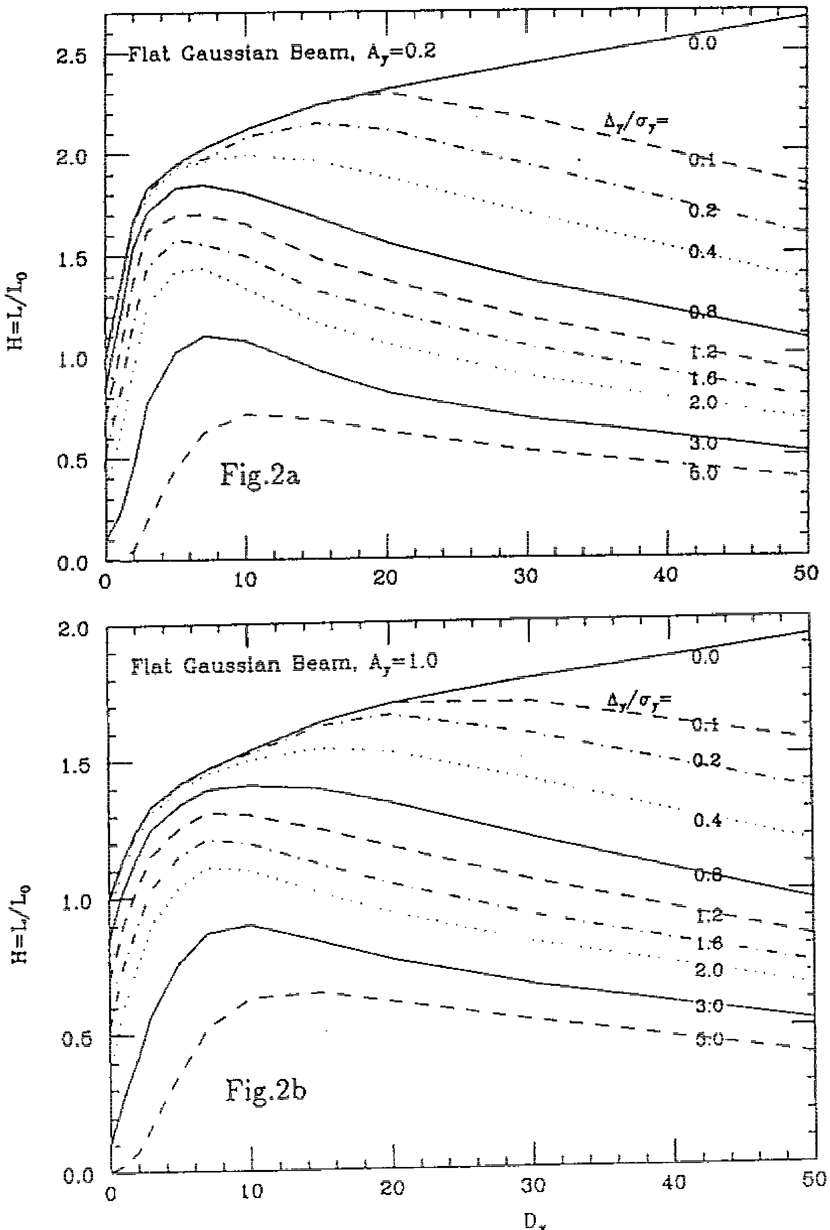
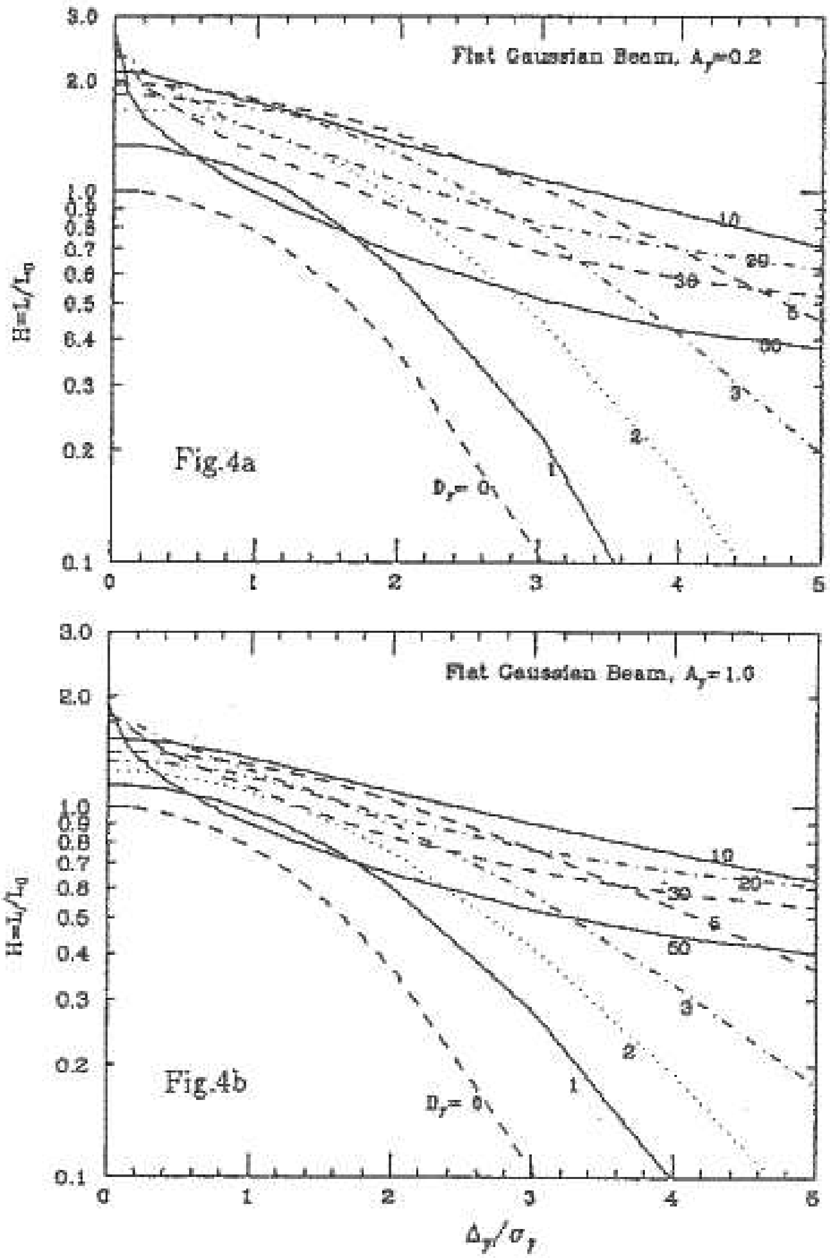


Fig.2. Enhancement factor for flat Gaussian beams for various values of the offset Δ_y. A_y=0.2 in Fig.2a and 1.0 in Fig.2b.



ig.4. Enhancement fac-
or H_D vs. initial off-
et Δ_y for various values
f the disruption param-
ter D_y. Flat Gaussian
eam. A_y=0.2 in Fig.4a
nd 1.0 in Fig.4b. An ana-
ytic expression is used for
he curve D_y=0.

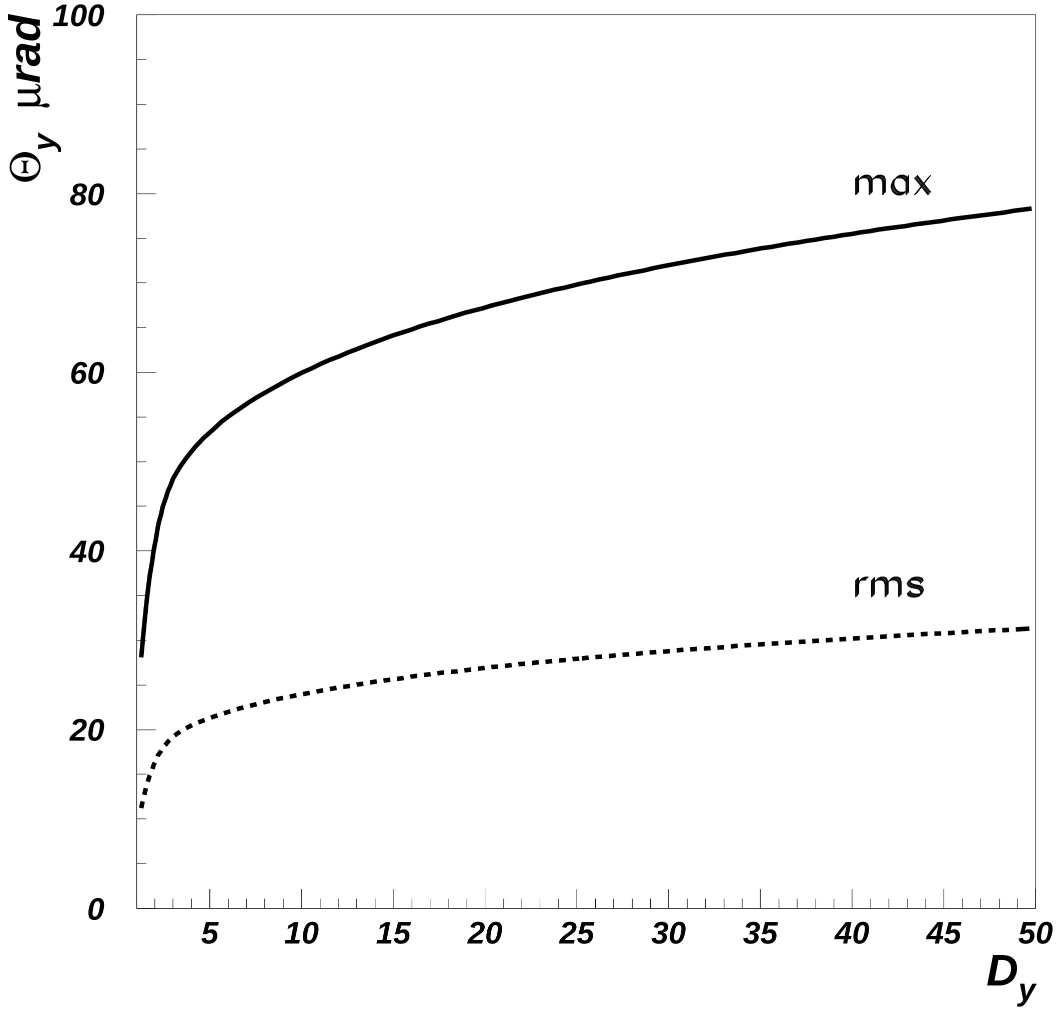


- D_y**=0** $\longrightarrow H_D \propto exp(-\Delta_y^2/4\sigma_y^2)$

- **small** $D_y \longrightarrow H_D$ **rapidly falls off**
- **large** $D_y \longrightarrow H_D$ **goes down slowly**

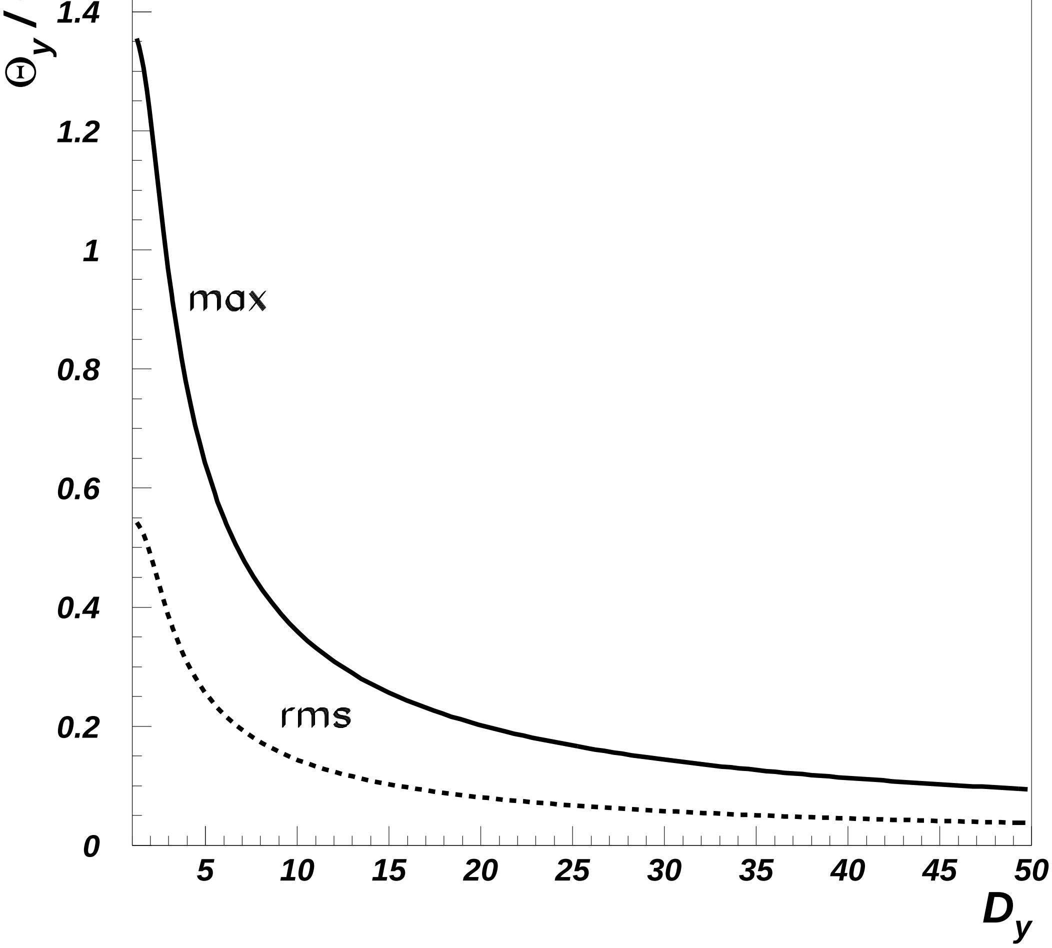
The last two figures are from Kaori Yokoya and Pisin Chen, KEK preprint 91-2

Disruption angle



- **Outgoing angles depend of initial particle position, (x_0, y_0)**
- **small $D_y \longrightarrow$ the rms angles in both directions are the same,**

$$\theta_{x,rms} = \theta_{y,rms} = 0.55\theta_0$$



where $\theta_0 = D_X \sigma_x / \sigma_z = D_y \sigma_y / \sigma_z$

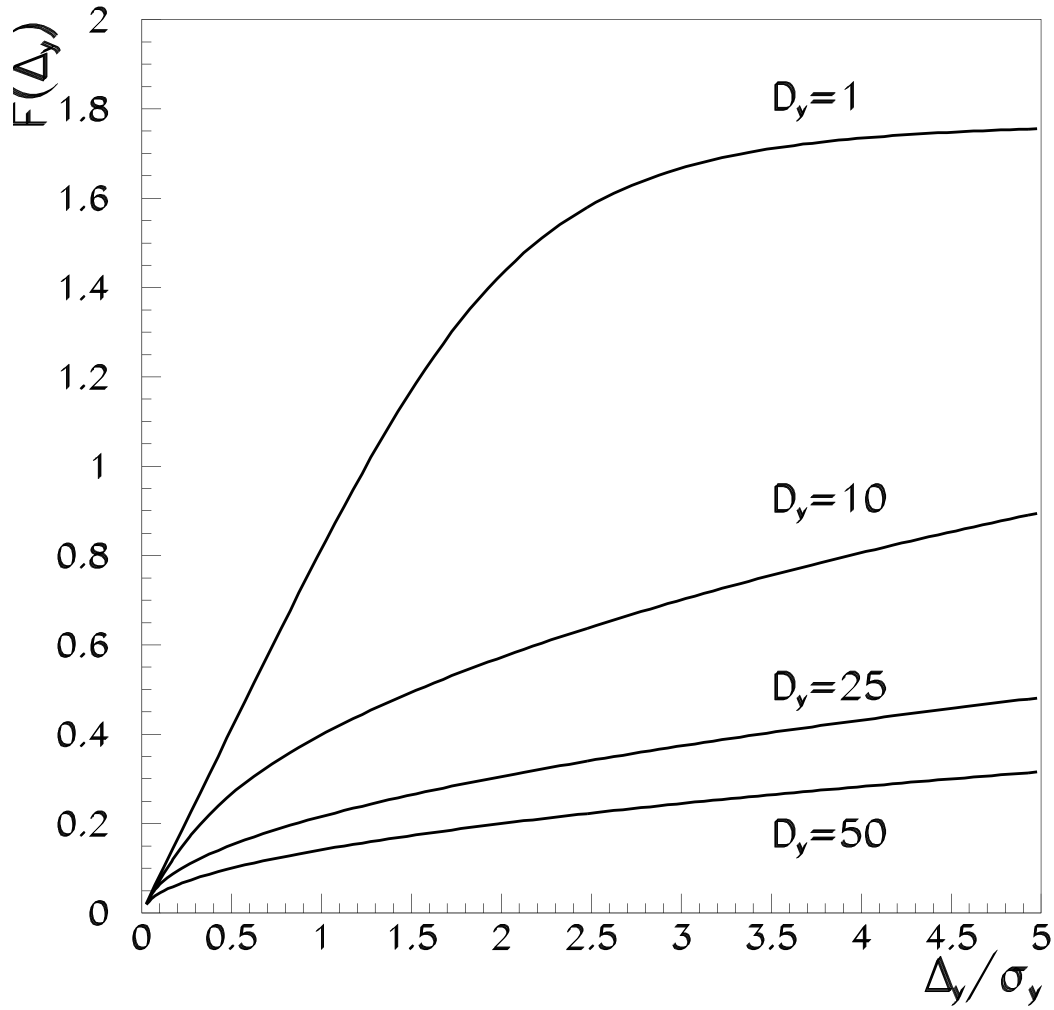
- **large** $D_y \longrightarrow \theta_{y,rms} = 0.55\theta_0 / f(D_y)$

where $f(D_y) = (1 + (0.5D_y)^5)^{1/6}$ **goes up with** D_Y

- **The reason** $\longrightarrow$ **particle trajectories are bent**

- **The max angles are simply** $\theta_{max} = 2.5\theta_{rms}$

Center-of-Mass Deflection



- **The main deflection angle of the entire bunch (in presence of vertical offset Δ_y)**

$$\Theta_y = 0.5\sigma_y/\sigma_z F(D_y, \Delta_y)$$

- **small** Δ **and** $D_y \longrightarrow \Theta_y \propto \theta_0 \Delta_y / \sigma_y$

empirical formula for $F(D_y, \Delta_y)$

$$F = \delta[C_1 + C_2\delta^2 + C_3\delta^4]^{-1/4}$$

where C_1, C_2 **and** C_3 **are the functions of** D_y, A_y **and** $\delta = \Delta_y/\sigma_y$

- **The maximum disruption angle** $\longrightarrow$ **is the sum of center-of-mass deflection angle** Θ_y **(in presence of offsets) and maximum angle in the absence of offsets,** θ_y

Beamstrahlung

- **since the trajectories of the moving particles are bent they emit radiation called beamstrahlung.**
- Υ **parameter** $\longrightarrow$ **dimensionless parameter controls overall radiation intensity**

$$\Upsilon \equiv \frac{2}{3}\frac{< E_c >}{E_{beam}} \simeq \frac{5}{6}\frac{N r_e^2 \gamma}{\alpha \sigma_z (\sigma_x + \sigma_y)}$$

$< E_c >= 3/2 \cdot \gamma^3/\rho$, **average photon critical energy.**
ρ **is radius of curvature**

$$\Upsilon = \gamma B / B_c$$

B **the magnetic field and** B_c **the critical magnetic field,**
(Schwinger field)$B_c \approx$ **4.4GTeslas.**

- Υ **is not a constant during collision**
- **The average value of** Υ **for proposed collider is** $\sim$**0.05.**
- **number of photons emitted per electron,** n_γ**,**

$$n_\gamma \approx 2.59[\frac{\alpha^2 \sigma_z \Upsilon}{r_e \gamma}] U_0(\Upsilon)$$

• **average energy loss,** δ_E

$$\delta_E \approx 1.20[\frac{\alpha^2\sigma_z\Upsilon}{r_e\gamma}]\Upsilon$$

where functions $U_0(\Upsilon) \simeq 1/(1+\Upsilon^{2/3})^{1/2}$ **and** $U_1(\Upsilon) \simeq 1/(1+1.5\Upsilon^{2/3})^2$

• $\Upsilon \ll$ **1** $\longrightarrow U_0$ **and** $U_1 \sim 1$

• **The average photon energy,** $<\omega>$

$$\frac{<\omega>}{E_{beam}} \simeq 0.46\Upsilon$$

• **to keep** $n_\gamma \sim$ **1 and** δ_E **on the level of few percent** $[\alpha^2\sigma_z\Upsilon/r_e\gamma]\sim$ **1**

for proposed ILC is equal to $\simeq$ **0.6.**

• $n_\gamma \simeq$ **1.6 per electron.**

• $\delta_E \sim 4\%$**.**

• $<\omega>\approx$ **2.3%** E_{beam}

• R ~**100** $\longrightarrow \delta_E$ **independent of** σ_y

• $\Upsilon \sim 1/(\sigma_x+\sigma_y) \longrightarrow$ **by decreasing** σ_y **one can increase the luminosity and in the same time to leave the energy loss without change.**