Precision b-quark mass determination

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☆Plan of Talk

1. What's quark mass(+Yukawa coupling)? [Introduction]
2. Current status of bottom mass
3. Bottom mass determination from bottomonium 1S energy levels
4. Summary and future prospects

Quark mass definitions

- *What are quark masses?*
- *Why do we measure them?*

SM parameters at weak scale:

$\mathcal{L}_{SM}(e, g_w, g_s, v, \lambda, y_{ij}, \cdots)$ ⟶ *connected to fundamental physics*
GUT models

Yukawa couplings at weak scale: $y_q \bar{Q}_L H q_R + h.c.$

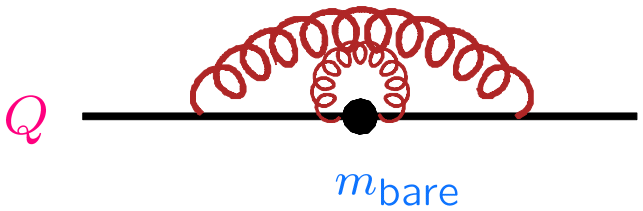
↕ Relation free from IR contaminations

Short-distance quark masses (e.g. $\overline{\mathrm{MS}}$ mass)

Definitions of quark mass in pert. QCD

Pole mass m_{pole}

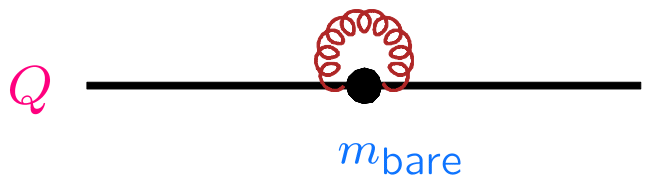
$0 < \lambda_g < \infty$



Not defined beyond pert. theory
Perturbative uncertainty
$O(\Lambda_{\text{QCD}}) \lesssim 1$ GeV

$\overline{\text{MS}}$ mass $\overline{m} \equiv m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$

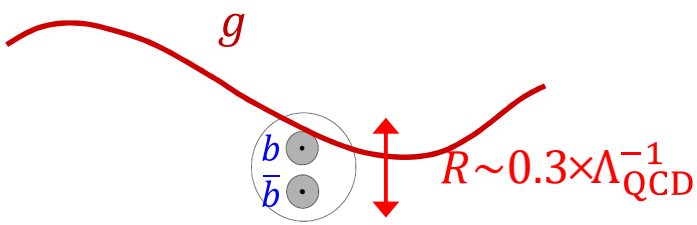
$0 < \lambda_g < 1/\overline{m}$



Conceptually close to
Yukawa coupling at scale $\mu \sim \overline{m}$

b-flavor hadrons

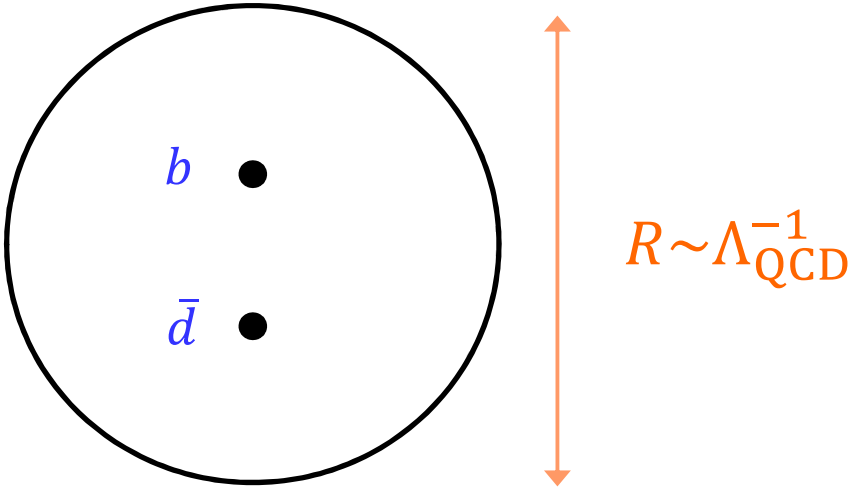
Bottomonium



IR gluons decouple

color-singlet
small size

B meson



color singlet
typical hadron size

Particle Data Group 2019

b

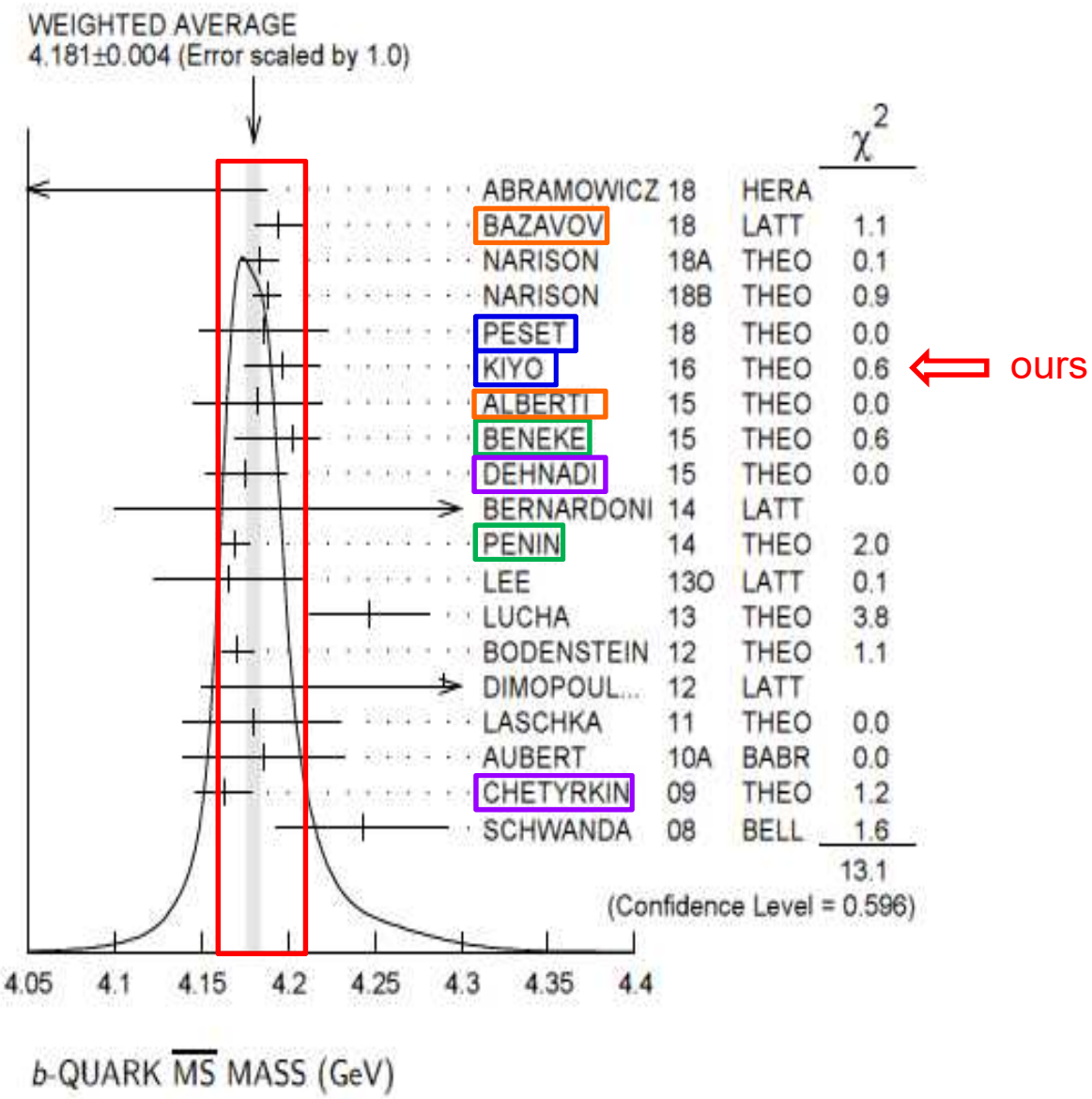
$I(J^P) = 0(\frac{1}{2}^+)$

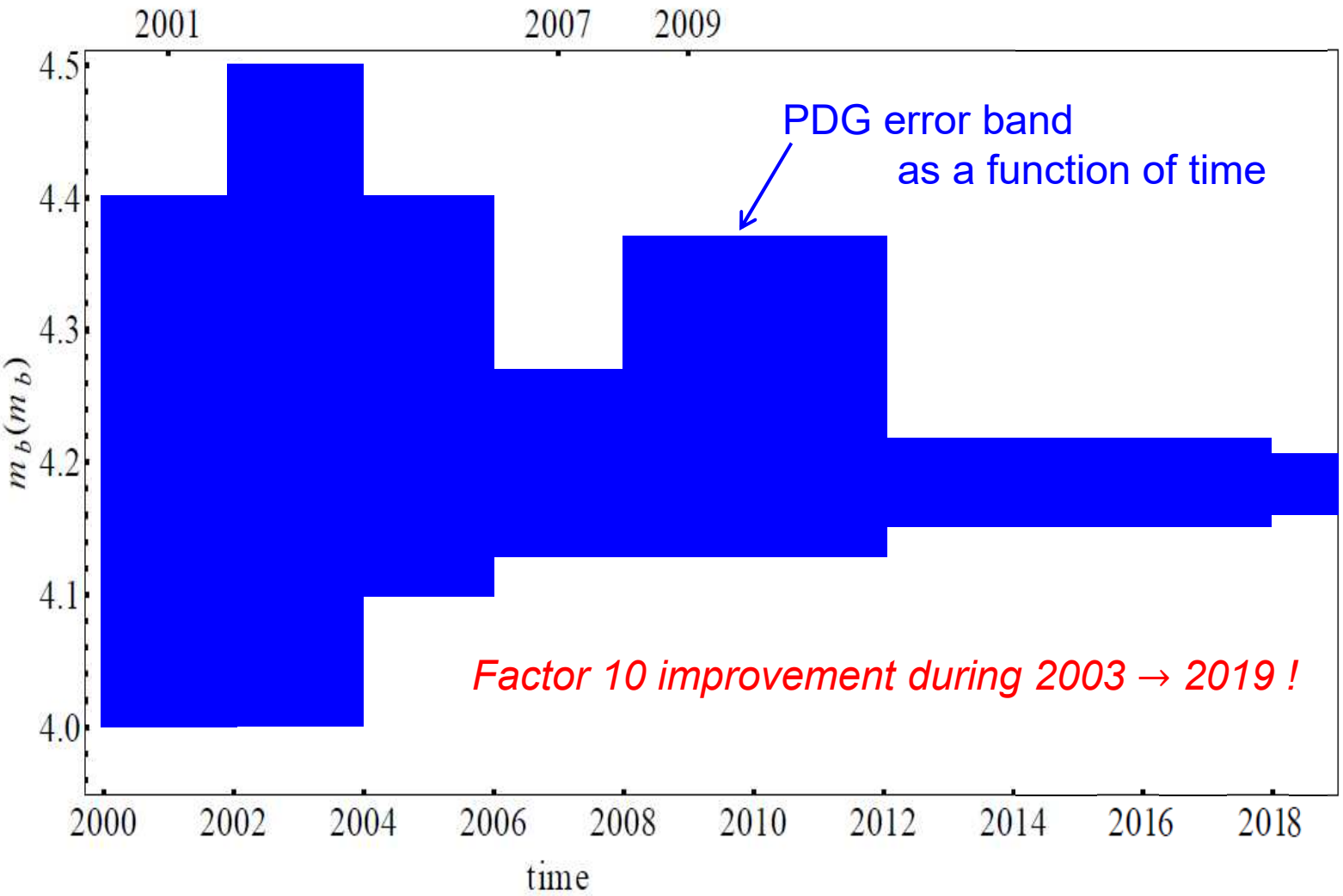
Charge $= -\frac{1}{3}\ e$ Bottom $= -1$

b-QUARK MASS

b-quark mass corresponds to the "running mass" $\overline{m}_b(\mu = \overline{m}_b)$ in the $\overline{\text{MS}}$ scheme. We have converted masses in other schemes to the $\overline{\text{MS}}$ mass using two-loop QCD perturbation theory with $\alpha_s(\mu = \overline{m}_b) = 0.223 \pm 0.008$

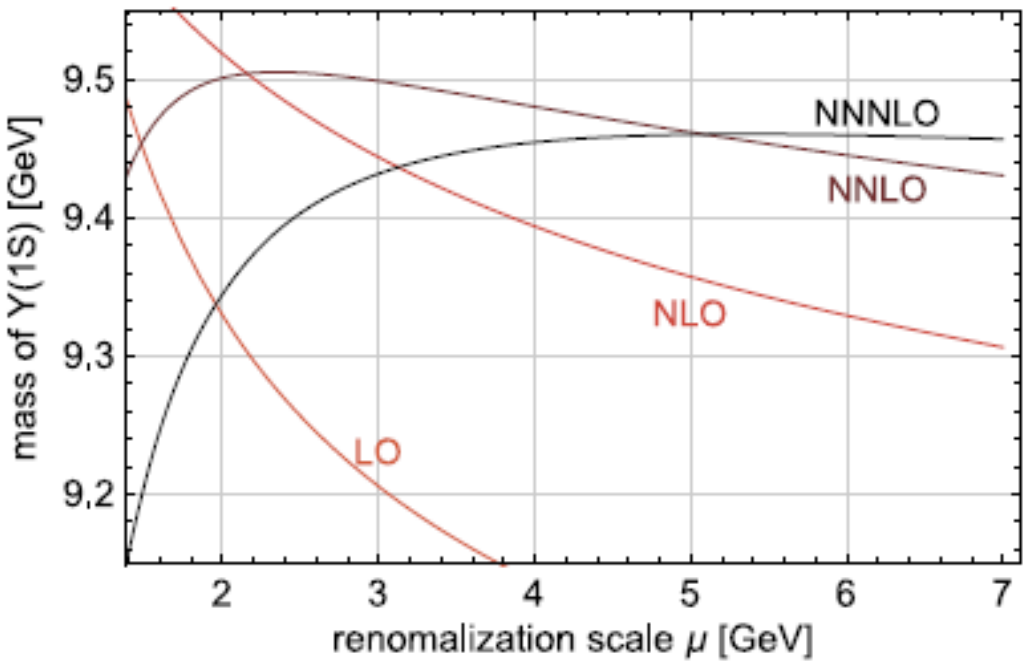
$\overline{\text{MS}}$ MASS (GeV)	DOCUMENT ID		TECN
$\boxed{4.18\ ^{+0.03}_{-0.02}}$ **OUR EVALUATION**	of $\overline{\text{MS}}$ Mass. See the ideogram below.		
$4.049\ ^{+0.138}_{-0.118}$	[1] ABRAMOWICZ	18	HERA
4.195 ± 0.014	[2] BAZAVOV	18	LATT
4.184 ± 0.011	[3] NARISON	18A	THEO
4.188 ± 0.008	[4] NARISON	18B	THEO
4.186 ± 0.037	[5] PESET	18	THEO
4.197 ± 0.022	[6] KIYO	16	THEO
4.183 ± 0.037	[7] ALBERTI	15	THEO
$4.203\ ^{+0.016}_{-0.034}$	[8] BENEKE	15	THEO
4.176 ± 0.023	[9] DEHNADI	15	THEO
4.21 ± 0.11	[10] BERNARDONI	14	LATT
$4.169 \pm 0.002 \pm 0.008$	[11] PENIN	14	THEO
4.166 ± 0.043	[12] LEE	13O	LATT
4.247 ± 0.034	[13] LUCHA	13	THEO
4.171 ± 0.009	[14] BODENSTEIN	12	THEO
4.29 ± 0.14	[15] DIMOPOUL...	12	LATT
$4.18\ ^{+0.05}_{-0.04}$	[16] LASCHKA	11	THEO
$4.186 \pm 0.044 \pm 0.015$	[17] AUBERT	10A	BABR
4.163 ± 0.016	[18] CHETYRKIN	09	THEO
4.243 ± 0.049	[19] SCHWANDA	08	BELL


WEIGHTED AVERAGE
4.181±0.004 (Error scaled by 1.0)
χ2
ABRAMOWICZ 18 HERA
BAZAVOV 18 LATT 1.1
NARISON 18A THEO 0.1
NARISON 18B THEO 0.9
PESET 18 THEO 0.0
KIYO 16 THEO 0.6
ours
ALBERTI 15 THEO 0.0
BENEKE 15 THEO 0.6
DEHNADI 15 THEO 0.0
BERNARDONI 14 LATT
PENIN 14 THEO 2.0
LEE 13O LATT 0.1
LUCHA 13 THEO 3.8
BODENSTEIN 12 THEO 1.1
DIMOPOUL... 12 LATT
LASCHKA 11 THEO 0.0
AUBERT 10A BABR 0.0
CHETYRKIN 09 THEO 1.2
SCHWANDA 08 BELL 1.6
13.1
(Confidence Level = 0.596)
4.05 4.1 4.15 4.2 4.25 4.3 4.35 4.4
b-QUARK MS MASS (GeV)


2001
2007
2009
4.5
4.4
4.3
4.2
4.1
4.0
$m_b(m_b)$
PDG error band
as a function of time
Factor 10 improvement during 2003 → 2019 !
2000
2002
2004
2006
2008
2010
2012
2014
2016
2018
time

Determination of $\overline{m}_b$ from $\Upsilon(1S)$ and $\eta_b(1S)$ energy levels

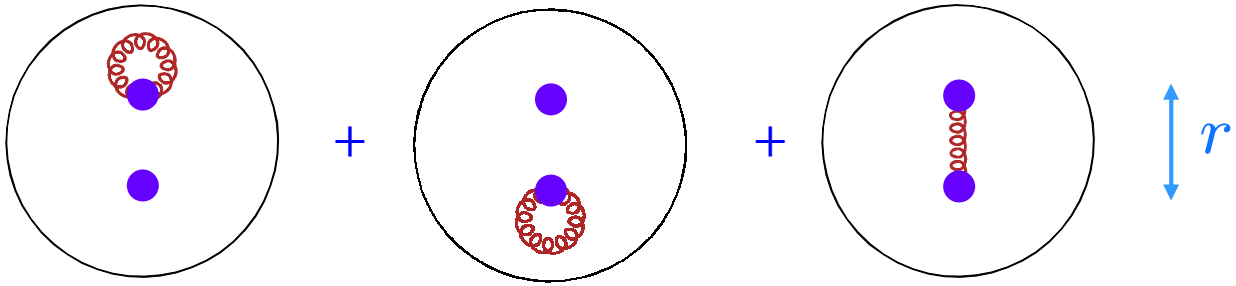
Kiyo, Mishima, YS



Use $\overline{\mathrm{MS}}$ mass $\overline{m}_b \equiv m_b^{\overline{\mathrm{MS}}}\left(m_b^{\overline{\mathrm{MS}}}\right)$

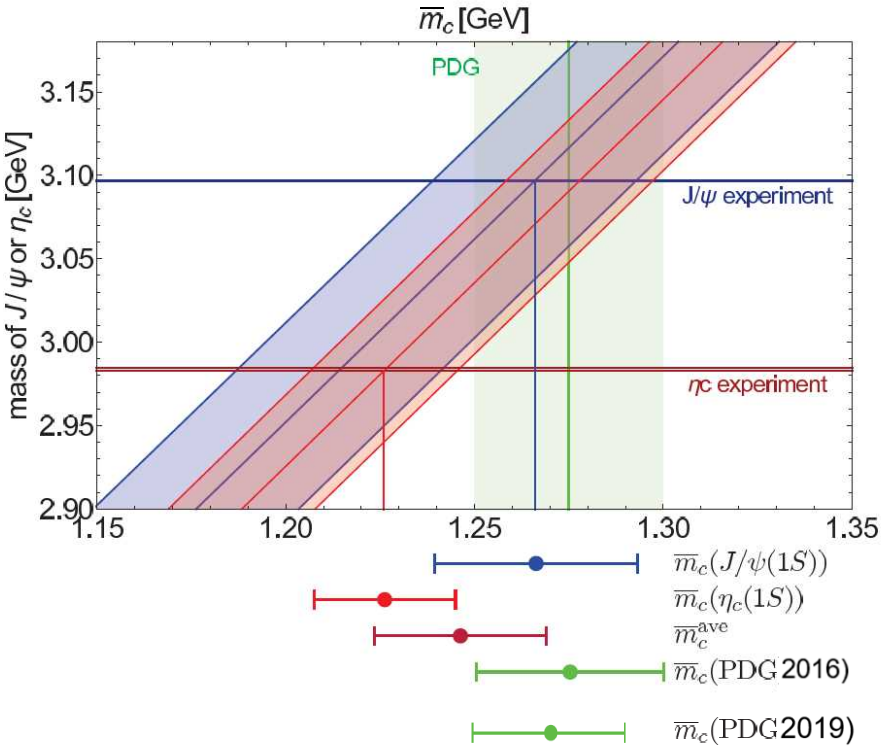
$$M_n = 2\,\overline{m}_b\,(1 + c_{n,1}\alpha_s + c_{n,2}\alpha_s^2 + \cdots)$$

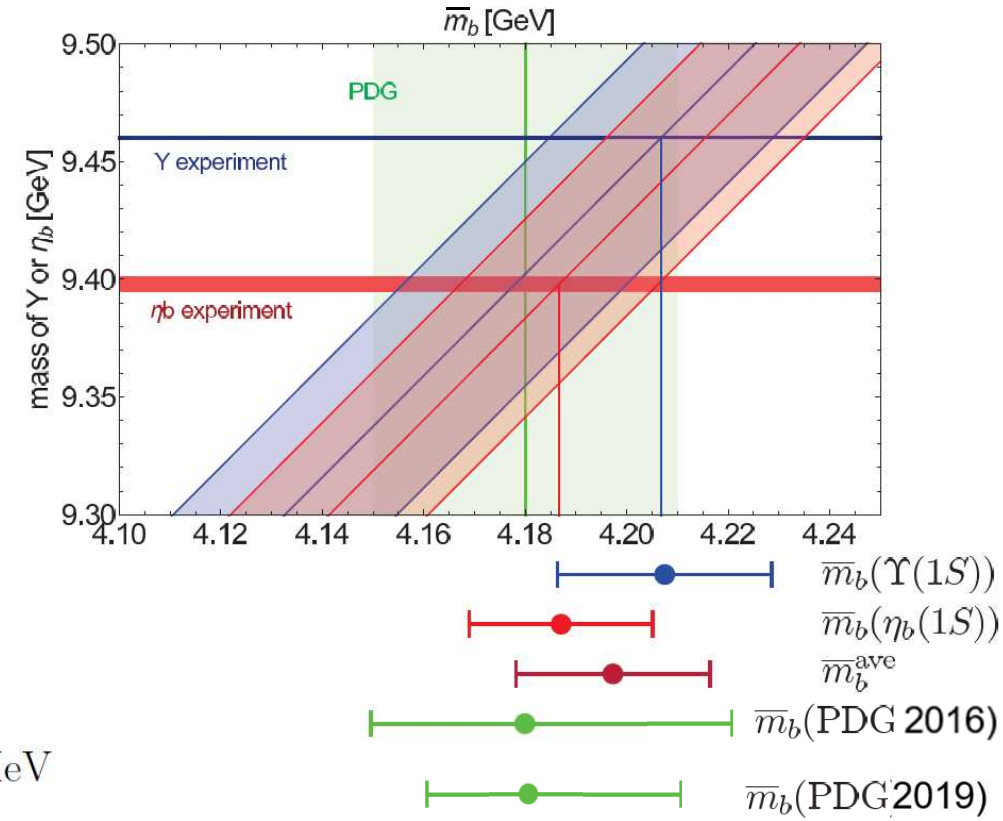
$M_{\Upsilon(1S)} =$



$\overline{m}_b, \overline{m}_c$ determination

Kiyo, Mishima, YS





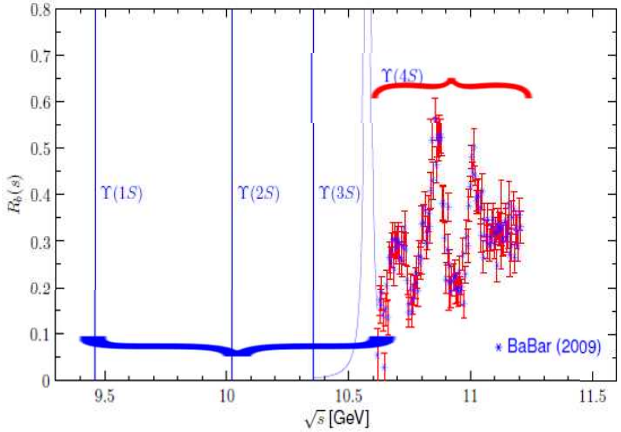
$$\overline{m}_c^{\rm ave} = 1246 \pm 2\ (d_3)\ \pm 4\ (\alpha_s)\ \pm 23\ (\text{h.o.})\quad \text{MeV}$$

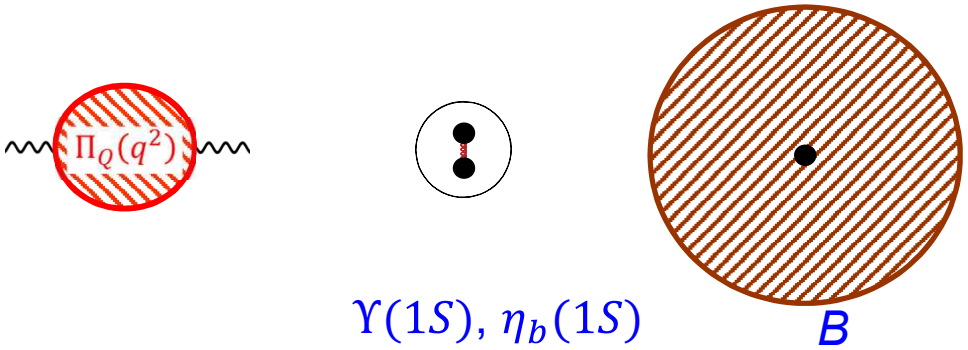
$$\overline{m}_b^{\rm ave} = 4197 \pm 2\ (d_3)\ \pm 6\ (\alpha_s)\ \pm 20\ (\text{h.o.}) \pm 5\ (m_c)\quad \text{MeV}$$

Summary and future prospects

- Current theoretical analyses can extract the $\overline{\mathrm{MS}}$ masses $\overline{m}_b$ (and $\overline{m}_c$) consistently with 20-30 MeV accuracy from different observables.

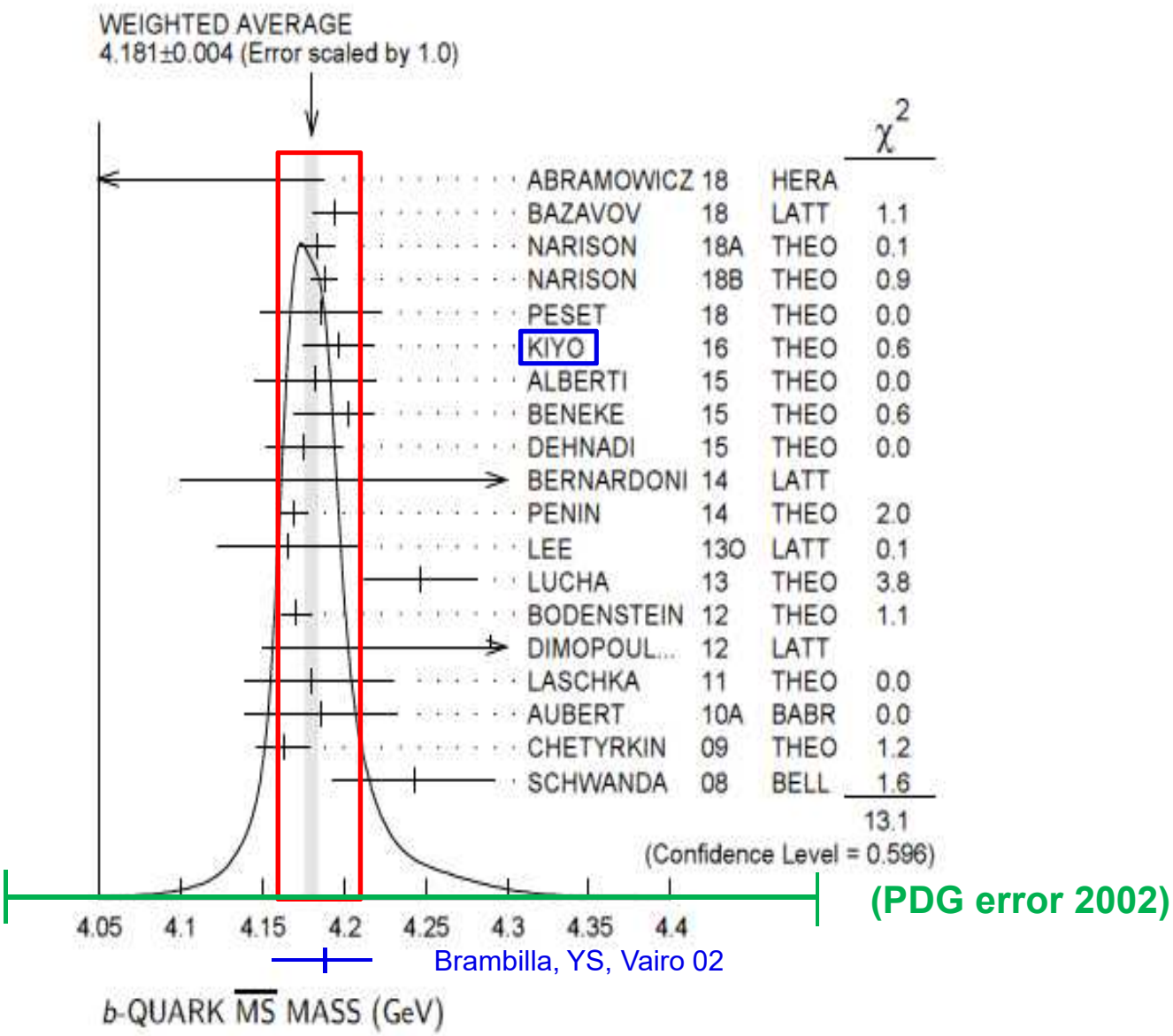
  Relativistic/Non-relativistic sum rules, Quarkonium 1S energy levels, *B,D* masses+HQET OPE, Inclusive observables in semileptonic *B* decays, $\cdots$





- Towards high precision QCD predictions
  Theoretical tools: pert. QCD, EFT, OPE, lattice QCD, $\cdots$

  Higher-order computation, eliminate IR contamination (renormalons)



(PDG error 2002)

Sum rules

Shifman, Vainshtein, Zakharov

Moments of *R*-ratio

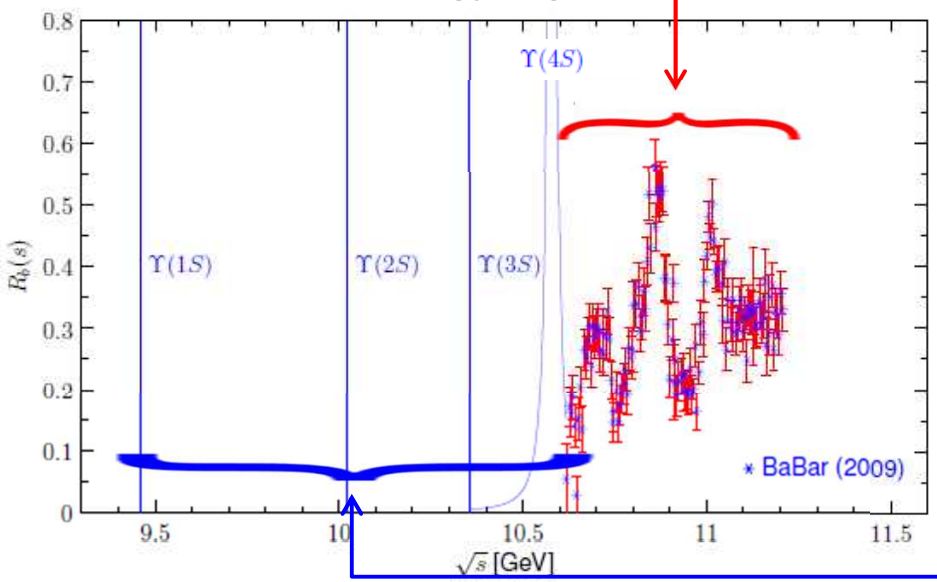
$$\mathcal{M}_n = \int_{s_0}^{\infty} ds \, \frac{R_Q(s)}{s^{n+1}} = \frac{12\pi^2}{n^2}\left(\frac{\partial}{\partial q^2}\right)^n \Pi_Q(q^2)\Big|_{q^2=0}$$

$\boldsymbol{Q = b}$ or $\boldsymbol{c}$

$$\mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}(\text{NNNLO})$$

$$R_Q(s) = R_{tot}^{\text{exp}}(s) - R_{bkg}^{\text{th}}(s)$$

Sensitive energy regions



$1 \le n \le 4$ Relativistic sum rule

relativistic pert. quarks

$n \gg 1$ Non-relativistic sum rule

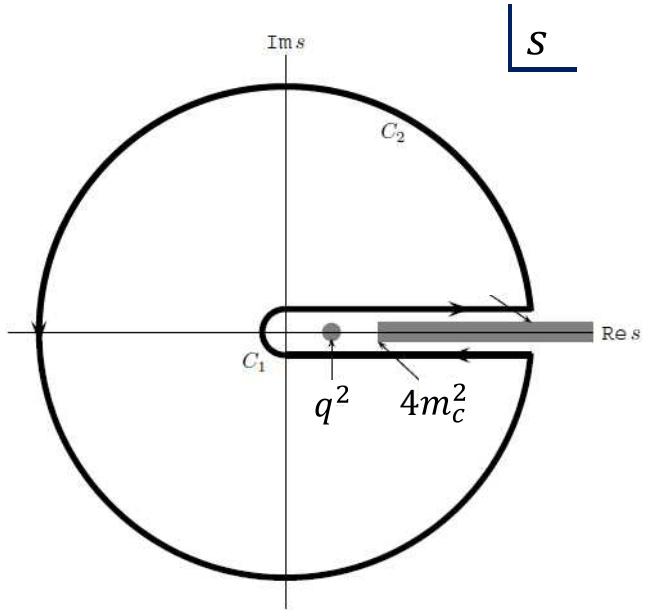
non-rel. bound-state theory

$$R_Q(s) \propto \left|\text{Im}\sum_n \frac{|\psi_n(0)|^2}{\sqrt{s} - M_n + i\Gamma_n/2}\right.$$

Dispersion integral relating Π_c and R_c:

$$\Pi_c(q^2) = \frac{q^2}{12\pi^2}\int \mathrm{d}s \frac{R_c(s)}{s(s-q^2)} + Q_c^2 \frac{3}{16\pi^2}\bar{C}_0,$$

$$R_c(s) \propto \mathrm{Im}\,\Pi_c(s)$$



Integral path of $\int_C ds \frac{\Pi_c(s)}{s(s-q^2)}$

$$\Pi_c(q^2) = Q_c^2 \frac{3}{16\pi^2}\sum_{n\geqslant 0}\bar{C}_n z^n, \tag{17}$$

with $Q_c = 2/3$ and $z = q^2/(4m_c^2)$ where $m_c = m_c(\mu)$ is the $\overline{\mathrm{MS}}$ charm quark mass at the scale μ.

Sum rules

Shifman, Vainshtein, Zakharov

Moments of *R*-ratio

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$\boldsymbol{Q = b}$ or $\boldsymbol{c}$

$$\mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}(\text{NNNLO})$$

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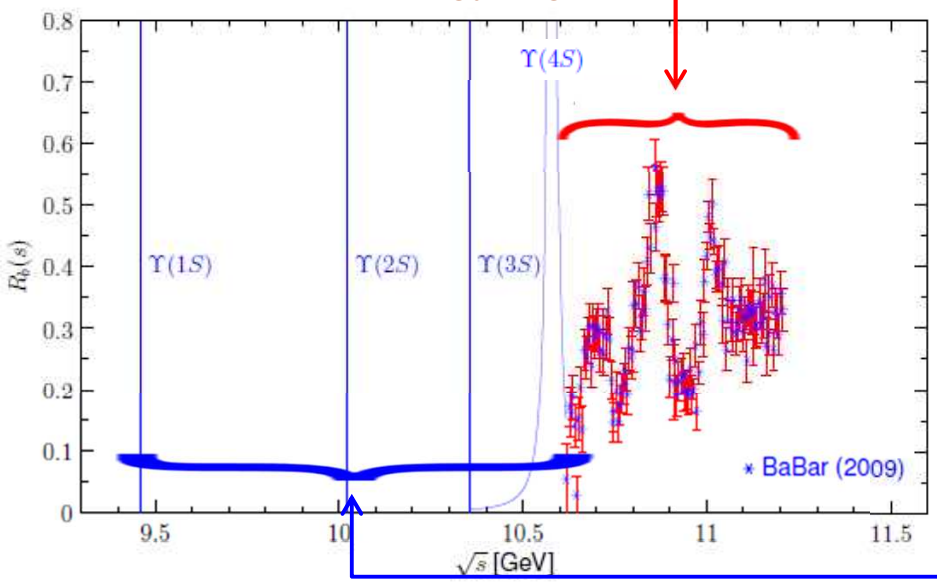
Sensitive energy regions



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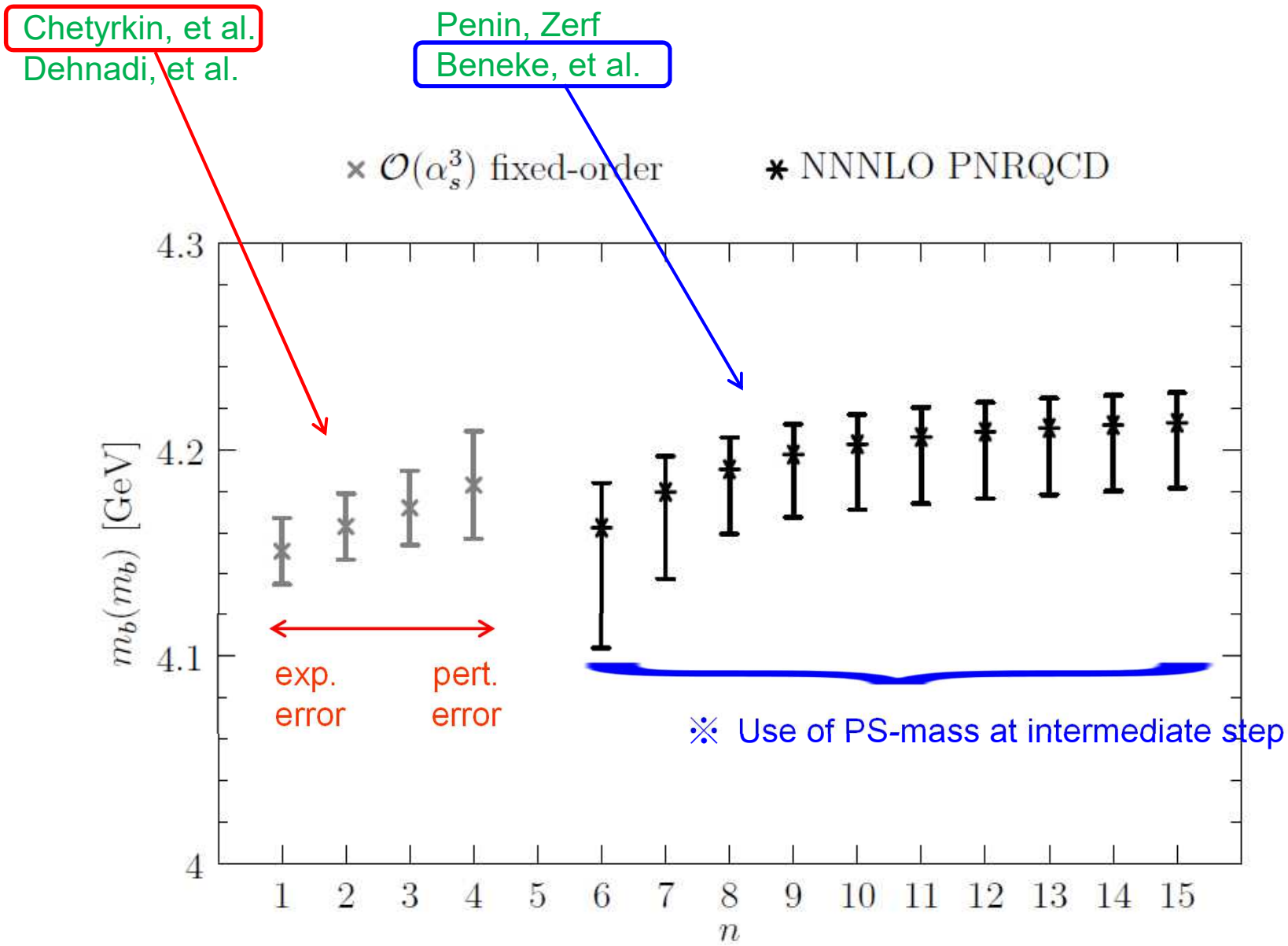
Original slide by A. Maier @Hansfest2016

Relativistic vs. non-relativistic sum rules



Heavy quarkonium states ($t\bar{t}, b\bar{b}, c\bar{c}, b\bar{c}$)

Unique system: Properties of individual hadrons predictable in pert. QCD

Two theoretical foundations for computing higher-order corr. systematically

- EFT (pNRQCD, vNRQCD) — Pineda, Soto, Brambilla, Vairo; Luke, Manohar, Rothstein
- Threshold expansion — Beneke, Smirnov

Computation of full spectrum up to NNNLO

Kiyo, YS: 1408.5590

$$\mathcal{L}_{\mathrm{pNRQCD}} = S^\dagger\left(i\partial_t - \hat{H}_S\right)S + O^{a\dagger}\left(iD_t - \hat{H}_O\right)^{ab}O^b + g\, S^\dagger \vec{r}\cdot\vec{E}^a O^a + \cdots$$

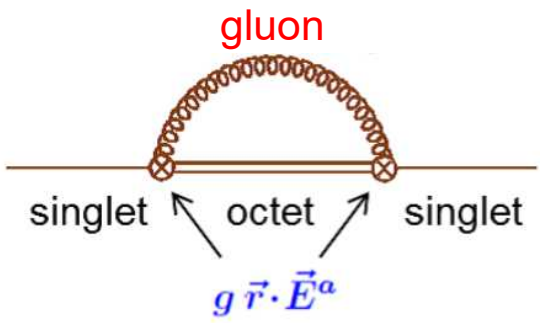
S, O^a: color singlet and octet composite-state fields

Energy levels given by poles of the full propagator of S in pNRQCD.

$$\frac{1}{E - \hat{H}_S + i\varepsilon}$$

$\downarrow$

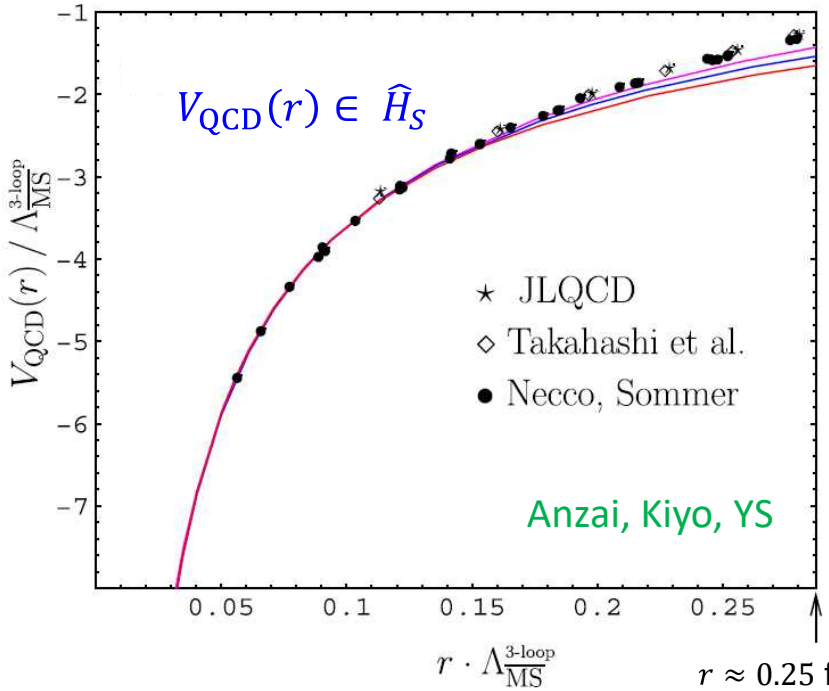
Pert. theory in Quantum Mech.



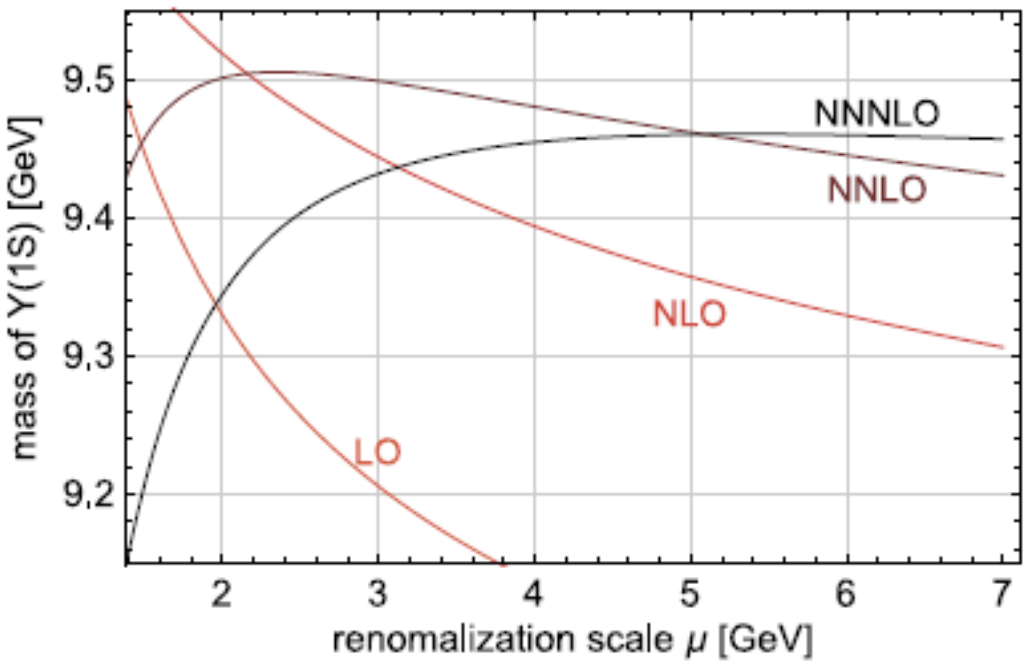
NNNLO

same as Lamb shift

3-loop pert. QCD vs. lattice comp.



Scale dependence

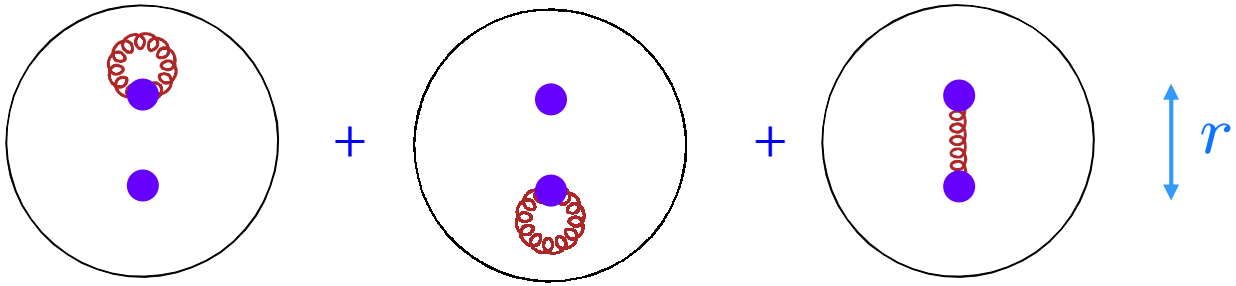


Kiyo, Mishima, YS

Use $\overline{\mathrm{MS}}$ mass $\bar{m}_b \equiv m_b^{\overline{\mathrm{MS}}}\left(m_b^{\overline{\mathrm{MS}}}\right)$

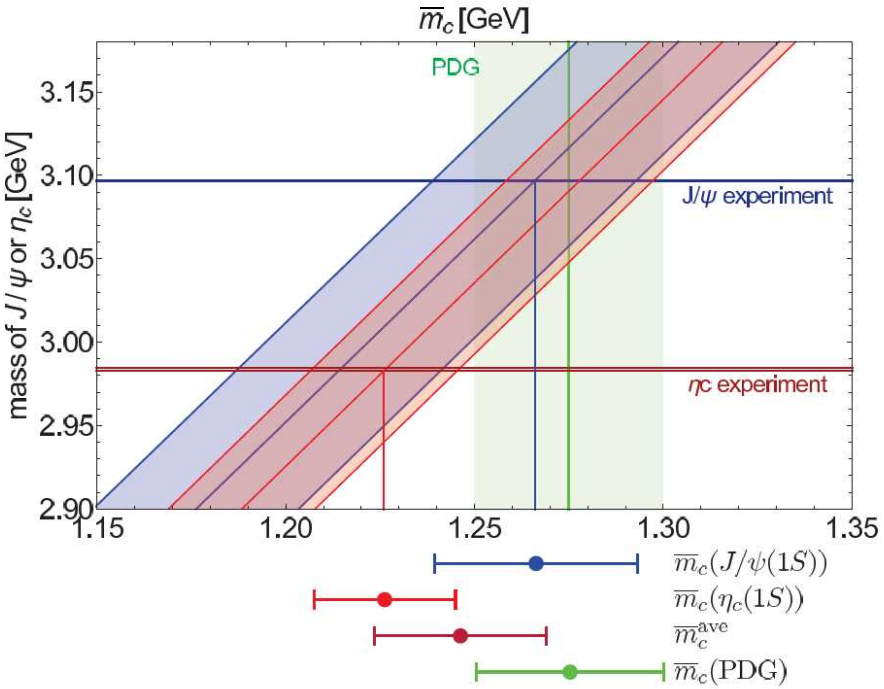
$$M_n = 2\,\bar{m}_b\,(1 + c_{n,1}\alpha_s + c_{n,2}\alpha_s^2 + \cdots)$$

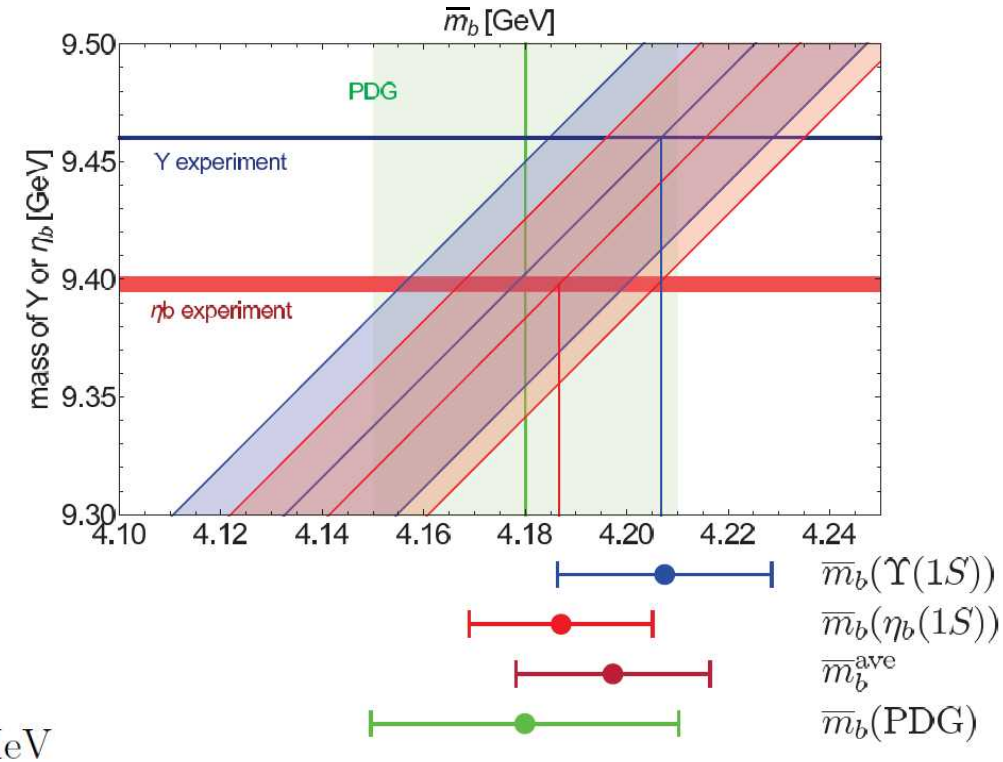
$M_{\Upsilon(1S)} =$



$\overline{m}_b, \overline{m}_c$ determination

Kiyo, Mishima, YS



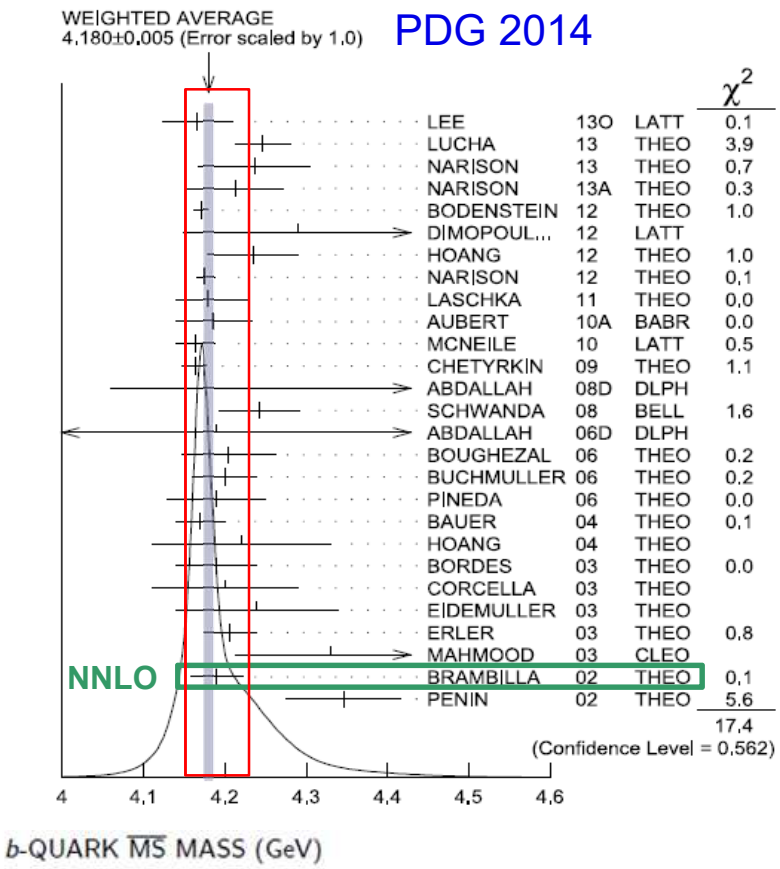


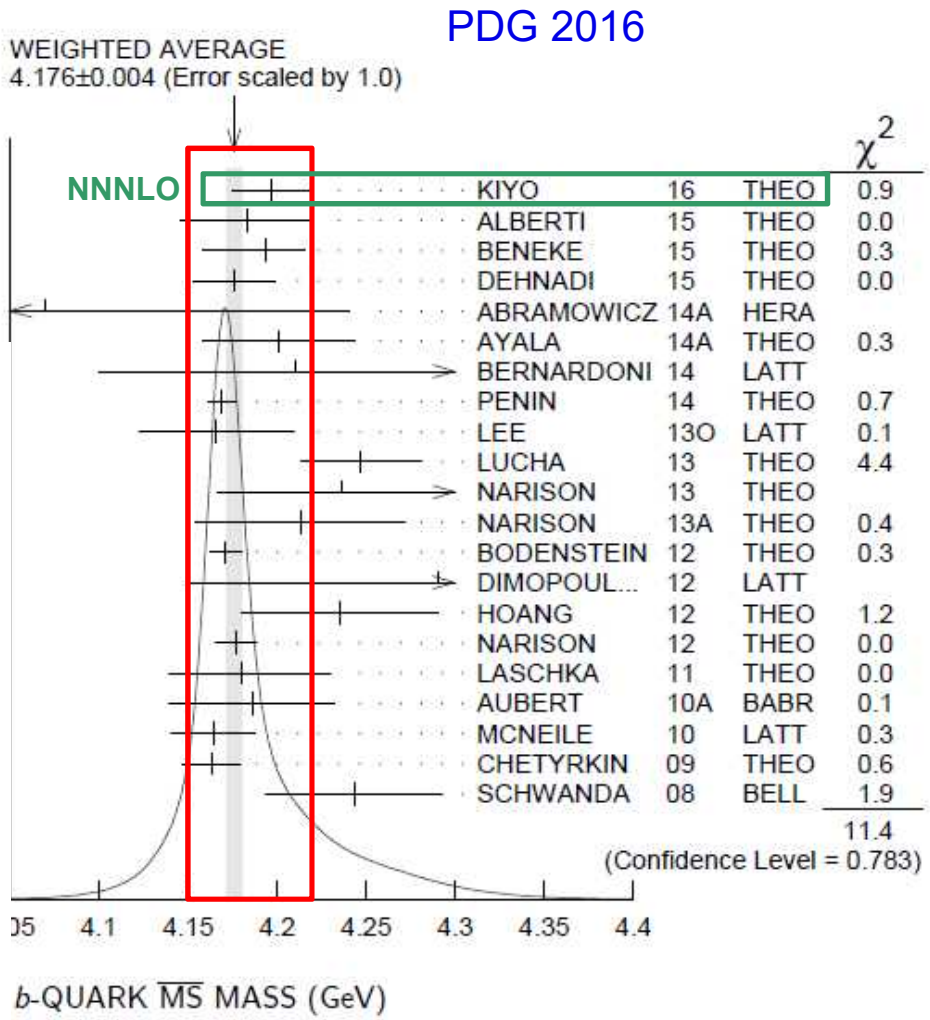
$$\overline{m}_c^{\rm ave} = 1246 \pm 2\ (d_3)\ \pm 4\ (\alpha_s)\ \pm 23\ (\text{h.o.})\quad \text{MeV}$$

PDG value $\overline{m}_c = 1275 \pm 25$ MeV

$$\overline{m}_b^{\rm ave} = 4197 \pm 2\ (d_3)\ \pm 6\ (\alpha_s)\ \pm 20\ (\text{h.o.}) \pm 5\ (m_c)\quad \text{MeV}$$

PDG value $\overline{m}_b = 4.18 \pm 0.03$ GeV





Uncertainty is nearly saturated by renormalon ~ $\Lambda_{\mathrm{QCD}} \cdot \left(\Lambda_{\mathrm{QCD}}\, r_{1S}\right)^2$

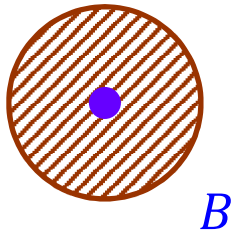
Simultaneous determination of $|V_{cb}|$ and $m_b^{\overline{\mathrm{MS}}}(m_b^{\overline{\mathrm{MS}}})$ from inclusive semileptonic B decays

Alberti, et al

Observables in inclusive $B \to X_c \ell \nu$ decays

$$\langle E_\ell^n \rangle = \frac{1}{\Gamma_{E_\ell > E_{\rm cut}}} \int_{E_\ell > E_{\rm cut}} E_\ell^n \frac{d\Gamma}{dE_\ell} dE_\ell,$$

$$\langle m_X^{2n} \rangle = \frac{1}{\Gamma_{E_\ell > E_{\rm cut}}} \int_{E_\ell > E_{\rm cut}} m_X^{2n} \frac{d\Gamma}{dm_X^2} dm_X^2,$$



m_X: invariant hadronic mass

OPE in $1/m_b$ expansion

$$\Gamma_{\rm sl} = \Gamma_0 \Bigg[1 + a^{(1)} \frac{\alpha_s(m_b)}{\pi} + a^{(2,\beta_0)} \beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 + a^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \left(-\frac{1}{2} + p^{(1)} \frac{\alpha_s}{\pi}\right) \frac{\mu_\pi^2}{m_b^2} + \left(g^{(0)} + g^{(1)} \frac{\alpha_s}{\pi}\right) \frac{\mu_G^2(m_b)}{m_b^2} + d^{(0)} \frac{\rho_D^3}{m_b^3} - g^{(0)} \frac{\rho_{\rm LS}^3}{m_b^3} + \text{higher orders} \Bigg],$$

$$\mu_\pi^2 = \frac{1}{2M_B} \langle \bar{B} | \bar{b}_v (i\vec{D})^2 b_v | \bar{B} \rangle$$

$$\mu_G^2 = -\frac{1}{2M_B} \langle \bar{B} | \bar{b}_v \frac{g_s}{2} G_{\mu\nu} \sigma^{\mu\nu} b_v | \bar{B} \rangle$$

Observables are mostly sensitive to $\approx m_b - 0.8\, m_c$

⇨ Input $\overline{m}_c(3\ \mathrm{GeV}) = 0.986(13)\ \mathrm{GeV}$

Fit experimental data of the observables and determine $\mu_\pi^2, \rho_D^3, \mu_G^2, \rho_{\mathrm{LS}}^3, |V_{cb}|, \overline{m}_b$.

Results:

- $\chi^2/\text{d.o.f.} \approx 0.4$
- $|V_{\mathrm{cb}}| = (42.21 \pm 0.78) \times 10^{-3}$,

  c.f. Determination from exclusive $B \to D^* \ell \nu$ decays

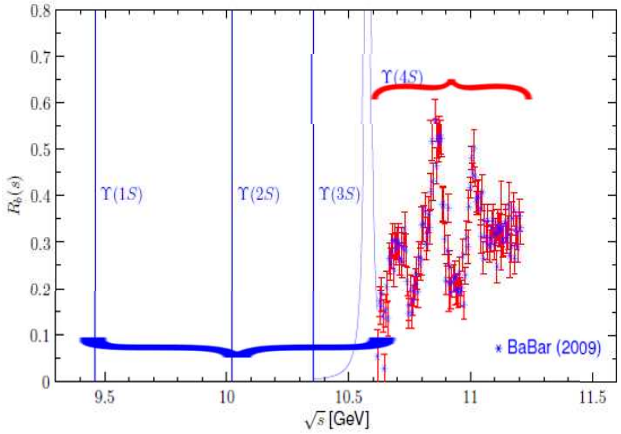
  $$|V_{\mathrm{cb}}| = (39.04 \pm 0.49_{\mathrm{exp}} \pm 0.53_{\mathrm{lat}} \pm 0.19_{\mathrm{QED}}) \times 10^{-3}$$

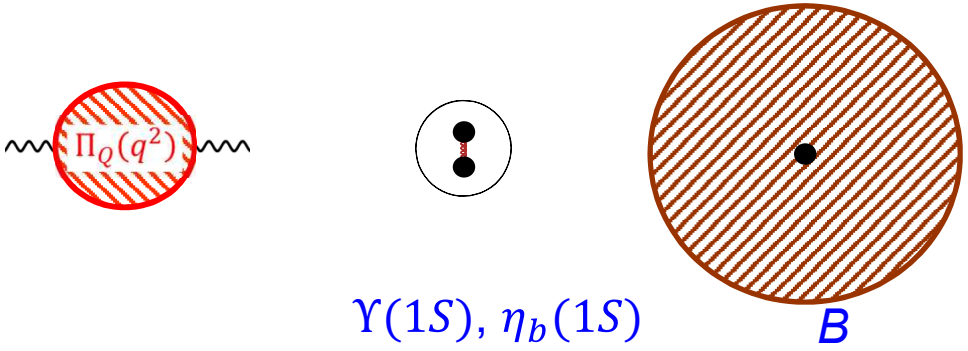
- $\overline{m}_b \equiv m_b^{\overline{\mathrm{MS}}}\left(m_b^{\overline{\mathrm{MS}}}\right) = 4183 \pm 37$ MeV

Summary

- Current theoretical analyses can extract the $\overline{\mathrm{MS}}$ masses $\overline{m}_b$ and $\overline{m}_c$ consistently with ~30 MeV accuracy from different observables.

  Relativistic/Non-relativistic sum rules, Quarkonium 1S energy levels, Inclusive observables in semileptonic *B* decays, ⋯





- Theoretical tools: pert. QCD, EFT, OPE, lattice QCD, ⋯

  Higher-order computations, renormalons vs. non-pert. matrix element