# Searching Inert Scalars at Future e<sup>+</sup>e<sup>-</sup> Colliders

Jan Kalinowski<sup>a</sup>, Jan Klamka<sup>a</sup>, Wojciech Kotlarski<sup>b</sup>, Tania Robens<sup>c</sup>, Dorota Sokolowska<sup>a,d</sup>, Pawel Sopicki<sup>a</sup>, **Aleksander Filip Żarnecki**<sup>a</sup>



<sup>a</sup> Faculty of Physics, University of Warsaw
 <sup>b</sup> Institut für Kern- und Teilchenphysik, TU Dresden
 <sup>c</sup> Theoretical Physics Division, Rudjer Boskovic Institute, Zagreb
 <sup>d</sup> International Institute of Physics, Universidade Federal do Rio Grande do Norte, Brasil

Research supported by



International Workshop on Future Linear Colliders LCWS 2019

### Outline



- Inert Doublet Model
- 2 Benchmark points
- 3 Leptonic analysis (generator level, general)
- 4 Semi-leptonic analysis (fast simulation for CLIC)
- Conclusions

#### For more details:

- on benchmark points:
- results for CLIC report: (leptonic signature)

JHEP 1812 (2018) 081, arXiv:1809.07712

JHEP 1907 (2019) 053, arXiv:1811.06952



One of the simplest extensions of the Standard Model (SM). The scalar sector consists of two doublets:

- $\Phi_S$  is the SM-like Higgs doublet,
- $\Phi_D$  (inert doublet) has four additional scalars H, A,  $H^{\pm}$ .

$$\Phi_{S} = \begin{pmatrix} G^{\pm} \\ \frac{\nu + h + iG^{0}}{\sqrt{2}} \end{pmatrix} \qquad \Phi_{D} = \begin{pmatrix} H^{\pm} \\ \frac{H + iA}{\sqrt{2}} \end{pmatrix}$$

IDM @ e<sup>+</sup>e<sup>-</sup> colliders



One of the simplest extensions of the Standard Model (SM).

The scalar sector consists of two doublets:

- $\Phi_S$  is the SM-like Higgs doublet,
- $\Phi_D$  (inert doublet) has four additional scalars H, A,  $H^{\pm}$ .

$$\Phi_{S} = \begin{pmatrix} G^{\pm} \\ \frac{\nu + h + iG^{0}}{\sqrt{2}} \end{pmatrix} \qquad \Phi_{D} = \begin{pmatrix} H^{\pm} \\ \frac{H + iA}{\sqrt{2}} \end{pmatrix}$$

We assume a discrete  $Z_2$  symmetry under which

- SM Higgs doublet  $\Phi_S$  is *even*:  $\Phi_S \to \Phi_S$  (also other SM $\to$ SM)
- inert doublet  $\Phi_D$  is *odd*:  $\Phi_D \to -\Phi_D$ .
- $\Rightarrow$  Yukawa-type interactions only for Higgs doublet  $(\Phi_S)$ . The inert doublet  $(\Phi_D)$  does not interact with the SM fermions!
- $\Rightarrow$  The lightest inert particle is stable: a natural candidate for dark matter! We assume the neutral scalar H is the dark matter particle.

$$m_H < m_A, m_{H^{\pm}}$$



After EWSB, the model contains a priori seven free parameters.

Two parameters can be fixed from the Standard Model  $(v, m_h)$ .

We are left with five free parameters, which we take as:

- $\Rightarrow$  three inert scalar masses:  $m_H$ ,  $m_A$ ,  $m_{H^{\pm}}$
- $\Rightarrow$  two couplings, eg.  $\lambda_2$  and  $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$



After EWSB, the model contains a priori seven free parameters.

Two parameters can be fixed from the Standard Model  $(v, m_h)$ .

We are left with five free parameters, which we take as:

- $\Rightarrow$  three inert scalar masses:  $m_H$ ,  $m_A$ ,  $m_{H^{\pm}}$
- $\Rightarrow$  two couplings, eg.  $\lambda_2$  and  $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$

Inert scalars couplings to  $\gamma$ ,  $W^{\pm}$  and Z determined by SM parameters

⇒ well established predictions for production and decay rates!

We scanned the IDM parameter space looking for scenarios consistent with current theoretical and experimental constraints, for masses up to 1 TeV.

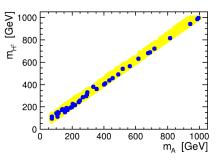
For details and previous IDM parameter scan results see:

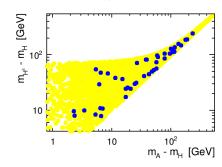
- Agnieszka Ilnicka, Maria Krawczyk, and Tania Robens, Inert Doublet Model in light of LHC Run I and astrophysical data, Phys. Rev. D93(5):055026, 2016, arXiv:1508.01671.
- Agnieszka Ilnicka, Tania Robens, and Tim Stefaniak, Constraining Extended Scalar Sectors at the LHC and beyond, Mod. Phys. Lett. A33(10n11):1830007, 2018, arXiv:1803.03594.

# IDM benchmark points



Out of about 15'000 points consistent with all considered constraints, we chose 41 benchmark points (including 20 "high mass") for detailed studies:





The selection was arbitrary, but we tried to

- cover wide range of scalar masses and the mass splittings
  - get significant contribution to the relic density

For details see: JHEP 1812 (2018) 081, arXiv:1809.07712 For list of benchmark point parameters, see backup slides

# Analysis strategy

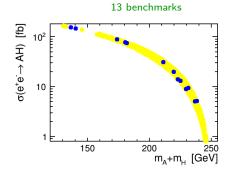


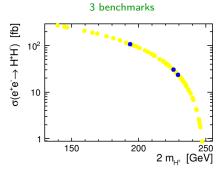
Production of IDM scalars at e<sup>+</sup>e<sup>-</sup> colliders dominated by two processes:

$$e^+e^- \rightarrow A H$$

$$e^+e^- \rightarrow H^+H^-$$

Leading-order cross sections for inert scalar production processes at 250 GeV:





# Analysis strategy

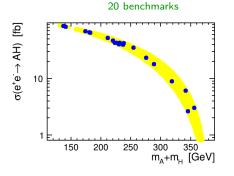


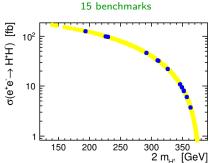
Production of IDM scalars at e<sup>+</sup>e<sup>-</sup> colliders dominated by two processes:

$$e^+e^- \rightarrow A H$$

$$e^+e^- \rightarrow H^+H^-$$

Leading-order cross sections for inert scalar production processes at 380 GeV:





# Analysis strategy

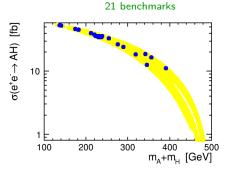


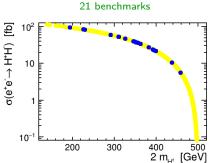
Production of IDM scalars at e<sup>+</sup>e<sup>-</sup> colliders dominated by two processes:

$$e^+e^- \rightarrow A H$$

$$e^+e^- \rightarrow H^+H^-$$

Leading-order cross sections for inert scalar production processes at 500 GeV:





# Leptonic signatures



Same flavour lepton pair production can be considered a signature of the *AH* production process followed by the *A* decay:

$$e^+e^- \rightarrow HA \rightarrow HHZ^{(\star)} \rightarrow HH\mu^+\mu^-$$

while the production of the different flavour lepton pair is the expected signature for  $H^+H^-$  production:

### Leptonic signatures



We consider two possible final state signatures:

- muon pair production,  $\mu^+\mu^-$ , for AH production
- electron-muon pair production,  $\mu^+e^-$  or  $e^+\mu^-$ , for  $H^+H^-$  production

Both channels include contributions from AH and  $H^+H^-$  production! In particular due to leptonic tau decays.

Signal and background samples were generator with WHizard 2.2.8 based on the dedicated IDM model implementation in SARAH, parameter files for benchmark scenarios were prepared using SPheno 4.0.3

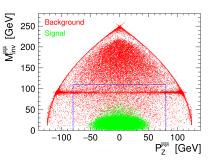
Generator level cuts reflecting detector acceptance:

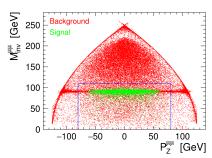
- ullet require lepton energy  $E_{I}>5\,\mathrm{GeV}$  and lepton angle  $\Theta_{I}>100\,\mathrm{mrad}$
- ullet no ISR photon with  $E_{\gamma} > 10\,\mathrm{GeV}$  and  $\Theta_{\gamma} > 100\,\mathrm{mrad}$

No detector resolution/efficiency taken into account (but only electrons and muons in the final state)



Muon pair invariant mass,  $M_{inv}^{\mu\mu}$ , as a function of the lepton pair long. momentum,  $P_Z^{\mu\mu}$ , for IDM signal and SM background, at 250 GeV BP1



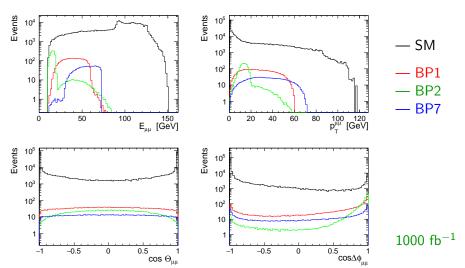


Background dominated by muon pair production ( $e^+e^- \to \mu^+\mu^-$ ) at nominal energy and radiative events ( $e^+e^- \to \mu^+\mu^-\gamma$ )

 $\Rightarrow$  apply pre-selection cuts:  ${\sf M}_{\mu\mu} < 0.33\sqrt{s}$  and  $|{\sf P}_{\sf Z}^{\mu\mu}| < 0.44\sqrt{s}$ 



Distributions of the kinematic variables describing the leptonic final state



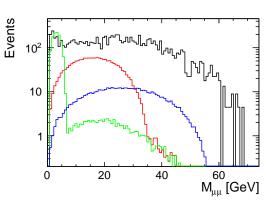


### **Cut based approach**

Lepton pair invariant mass distribution after selection cuts

 $1000 \text{ fb}^{-1}$ 

- pair energy  $E_{\mu\mu} < 75\,\mathrm{GeV}$
- $oldsymbol{ iny p}_{ t T}^{\mu\mu} > 10\, ext{GeV}$
- production angle  $45^{\circ} < \Theta_{\mu\mu} < 135^{\circ}$
- azimuthal distance  $|\Delta \varphi_{uu}| < \frac{\pi}{2}$



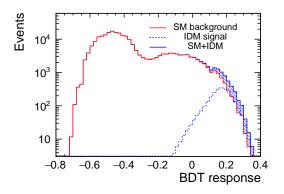
IDM signal would result in the visible excess in  ${\rm M}_{\mu\mu}$  distribution 15.9 $\sigma$ , 11.6 $\sigma$  and 5.4 $\sigma$ , for BP1, BP2 and BP7 (without any cut on  ${\rm M}_{\mu\mu}$ )



#### Multivariate analysis

BDT classifier with 8 input variables used for selection of signal events

Response distribution for  $\mu\mu$  channel: BP1 scenario and SM background unpolarised 1000 fb<sup>-1</sup> at  $\sqrt{s}=250$  GeV



 $\Rightarrow$  signal significance of about 24  $\sigma$  for BDT> 0.11

# Neutral scalar production



### Multivariate analysis commmon approach

We train BDTs to separate the considered signal from the background But we will not know in advance what to look for!
We will not know details of the model (scalar masses)

### Neutral scalar production



### Multivariate analysis commmon approach

We train BDTs to separate the considered signal from the background But we will not know in advance what to look for!
We will not know details of the model (scalar masses)

### Scenario-independent approach

Divide the considered BP scenarios in two groups:

- scenarios with real Z (or real W) production
- ullet scenarios with virtual Z (or virtual W) in intermediate state

For each group: search for given BP (test sample) while using all other scenarios to train BDT (training samples)

Corresponds to the assumption that two independent BDTs will be used in the analysis for the two cases...

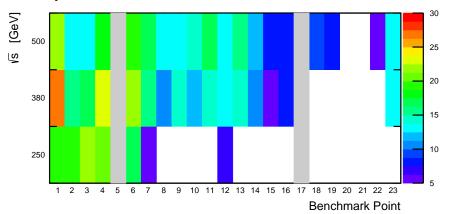
IDM @ e<sup>+</sup>e<sup>-</sup> colliders

# Neutral scalar production



### **Significance of observation**

Summary of results for the considered benchmark scenarios

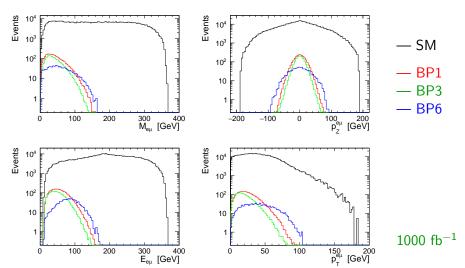


High significance of observation for scenarios accessible at given energy Expected significance mainly related to the AH production cross section

# Charged scalar production @ 380 GeV



Distributions of the kinematic variables describing the leptonic final state

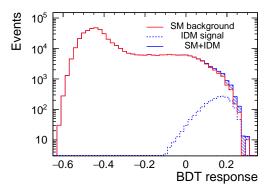


# Charged scalar production @ 380 GeV



#### Multivariate analysis

BDT classifier with 8 input variables used for selection of signal events Response distribution for  $e\mu$  channel: BP1 scenario and SM background unpolarised 1000 fb<sup>-1</sup> at  $\sqrt{s}=380$  GeV



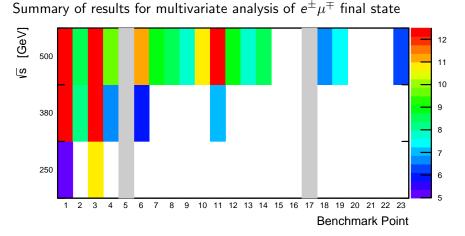
 $\Rightarrow$  signal significance of about 17  $\sigma$  for BDT> 0.12

# Charged scalar production



### Significance of observation

scenario-independent approach



Fewer scenarios can be observed, clear need for 500 GeV Significance reduced by about 10% by scenario-independent BDT training

# Low energy summary



### **Expected significance**

Search for pair-production of IDM scalars, for different  $\sqrt{s}$ 

AH signature 
$$(\mu^+\mu^-)$$

250 GeV

380 GeV

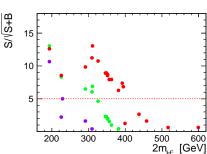
500 GeV

10

150 200 250 300 350 400

 $m_A + m_H$  [GeV]

$$H^+H^-$$
 signature  $(\mu^\pm e^\mp)$ 



Discovery reach mainly depends on the scalar masses!

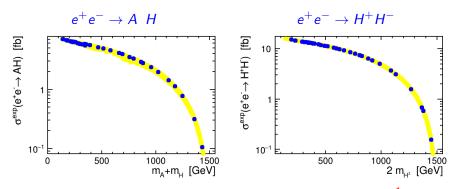
- $m_A + m_H < 220$ , 300, 330 GeV
- $m_{H^{\pm}} < 110$ , 160, 200 GeV

for 1000 fb $^{-1}$  at  $\sqrt{s}=$  250, 380, 500 GeV



Production of IDM scalars considered also for high energy stages of CLIC JHEP 1907 (2019) 053, arXiv:1811.06952: results submitted to CLIC Physics Potential report

Leading-order cross sections for inert scalar production at 1.5 TeV:



Much smaller cross sections for light IDM scalar production  $(\sim \frac{1}{s})!$ 

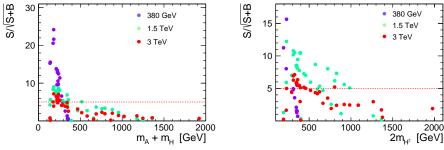


### **IDM study for CLIC**

including luminosity spectra

Comparing CLIC running scenarios:

$$1000 \, {\rm fb^{-1}} \,$$
 at  $380 \, {\rm GeV} \,$   $2500 \, {\rm fb^{-1}} \,$  at  $1.5 \, {\rm TeV} \,$   $5000 \, {\rm fb^{-1}} \,$  at  $3 \, {\rm TeV} \,$   $AH \,$  signature  $(\mu^+ \mu^-) \,$   $H^+ H^- \,$  signature  $(\mu^\pm e^\mp) \,$ 



Only moderate increase in discovery reach for 1.5 TeV:

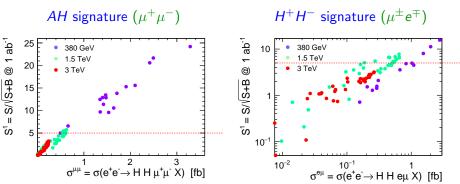
- neutral scalar production:  $m_A + m_H < 450 \,\text{GeV}$  (290 GeV @ 380 GeV)
- charged scalar production:  $m_{H^{\pm}} < 500 \,\mathrm{GeV}$  (150 GeV @ 380 GeV)



### **IDM study for CLIC**

including luminosity spectra

Significance scaled to the same integrated luminosity of 1000 fb<sup>-1</sup> as a function of the signal channel cross section



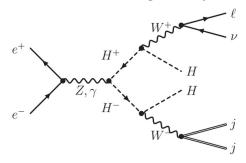
Expected significance mainly related to the signal channel cross section!  $\sim 0.5\ \text{fb}$  required for the discovery...



### Semi-leptonic signature

Much higher significance can be expected for  $H^+H^-$  production in the semi-leptonic final state (isolated lepton and two jets)

- energy and invariant mass recontruction for one of W bosons
   ⇒ better signal-background separation
- much larger branching fraction compared to  $e\mu$ : 2.25%  $\Rightarrow$  28.6%  $\Rightarrow$  discovery reach should increase significantly





### **Analysis framework**

Event samples generated with Whizard 2.7.0

based on the dedicated IDM model implementation in SARAH, parameter files prepared using SPheno 4.0.3 (as before) fragmentation and hadronisation is simulated using PYTHIA 6.4

CLIC beam energy spectra taken into account Consider running with -80% electron beam polarisation, with  $2\,\mathrm{ab^{-1}}$  collected at  $1.5\,\mathrm{TeV}$  and  $4\,\mathrm{ab^{-1}}$  collected at  $3\,\mathrm{TeV}$ 

Fast simulation of CLIC detector response with DELPHES dedicated CLICdet model cards

beam related backgrounds taken into account by additional jet energy-momentum smearing





### **Signal signature** $e^+e^- \rightarrow H^+H^- \rightarrow HHW^+W^- \rightarrow HHqq' l\nu$

- two hadronic jets consistent with (real or virtual) W decay
- single lepton from leptonic W decay
- large missing (transverse) energy/momentum/mass (two invisible scalars H produced)

### Analysis flow

Event reconstruction in DELPHES

- jets reconstructed with VLC algorithm (two exclusive jets)
- isolated leptons ( $e^{\pm}$  and  $\mu^{\pm}$ ) and photons identified require single leptons and no hard isolated photons (above 10 GeV)

IDM @ e<sup>+</sup>e<sup>-</sup> colliders

• require no additional energy-flow in the detector (20 GeV cut)



### **Backgrounds**

Backgrounds which could result in the same final state simulated Main contributions coming from  $qql\nu$ , qqll,  $qql\nu l\nu$ ,  $qql\nu \nu \nu$ 

Event pre-selection cuts applied on  $M_{qq}$ ,  $\Theta_{qq}$ ,  $E_l$ ,  $p_T^l$ ,  $\Theta_l$ Preselection results for 3 TeV: two example BPs included

channel	all exp. ev.	exp. ev. after preselec.	eff.
$H^{+}H^{-}$ (BP23)	22716	8872	39.1%
$H^{+}H^{-} \text{ (HP15)}$	11963	6063	50.7%
tot. backg.	74877722	625494	0.84%
$qq\ell\ell$	12877040	78382	0.61%
$qq\ell  u$	35326320	399470	1.13%
$qq\ell  u\ell  u$	317914	30742	9.67%
$qq\ell u u u$	360848	63581	17.62%
signal/backg. (BP23)	0.0003	0.014	
signal/backg. (HP15)	0.00016	0.0097	

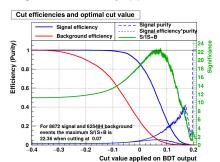


### Multivariate analysis

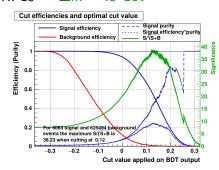
Final selection/significance estimate based on BDT with 11 input variables Two BDTs trained: for scenarios with virtual and real W production.

We do not optimise the selection for each particular scenario!

BP23  $\Delta m = 128 \text{ GeV}$ 



HP15  $\Delta m = 45 \text{ GeV}$ 

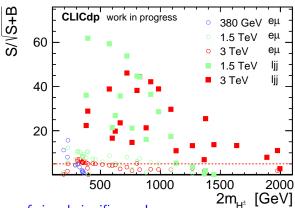


$$\Delta m = m_{H^{\pm}} - m_H$$



#### Results

Summary of results obtained for the semi-leptonic channel compared with leptonic channel results presented earlier



Huge increase of signal significance!

Discovery reach extended up to  $m_{H^\pm} \sim 1 \; {
m TeV}$ 

#### Conclusions



Inert Doublet Model is one of the simplest SM extensions providing natural candidate for dark matter

Light IDM scenarios (masses in 0(100 GeV) range) are still not excluded

Low mass IDM scenarios can be observed with high significance in the di-lepton channels already with  $250\,\text{GeV}$  e<sup>+</sup>e<sup>-</sup> collider

Discovery reach increases for higher  $\sqrt{s}$ . Significant improvement when looking at semi-leptonic final state!

Full simulation study starting, to confirm fast simulation results for selected BPs

# Thank you!

### Additional references



#### Inert Doublet Model

- A. Ilnicka, M. Krawczyk, T. Robens, Inert Doublet Model in light of LHC Run I and astrophysical data, Phys. Rev. D93:055026, 2016, 1508.01671.
- Nilendra G. Deshpande and Ernest Ma, Pattern of Symmetry Breaking with Two Higgs Doublets, Phys. Rev. D18:2574, 1978.
- Laura Lopez Honorez and Carlos E. Yaguna, The inert doublet model of dark matter revisited, JHEP 09:046, 2010, 1003.3125.
- Ethan Dolle, Xinyu Miao, Shufang Su, and Brooks Thomas, Dilepton Signals in the Inert Doublet Model, Phys. Rev. D81:035003, 2010, 0909.3094.
- A. Goudelis, B. Herrmann, and O. Stål, Dark matter in the Inert Doublet Model after the discovery of a Higgs-like boson at the LHC, JHEP 09:106, 2013, 1303.3010.

#### **Software**

- Wolfgang Kilian, Thorsten Ohl, and Jurgen Reuter, WHIZARD: Simulating Multi-Particle Processes at LHC and ILC, Eur. Phys. J. C71:1742, 2011, arXiv:0708.4233.
- Florian Staub, Exploring new models in all detail with SARAH, Adv. High Energy Phys. 2015:840780, 2015, arXiv:1503.04200.
- Werner Porod, SPheno, a program for calculating supersymmetric spectra, SUSY particle decays and SUSY particle production at e+ e- colliders, Comput. Phys. Commun. 153:275-315, 2003, hep-ph/0301101.
- Andreas Hoecker, Peter Speckmayer, Joerg Stelzer, Jan Therhaag, Eckhard von Toerne, and Helge Voss, TMVA: Toolkit for Multivariate Data Analysis, PoS ACAT:040, 2007, physics/0703039.



### **IDM** benchmark points

Constraints on inert scalar masses and couplings

- Theoretical
  - vacuum stability at tree level
  - perturbative unitarity
  - global minimum of the potential

#### Experimental

- (SM-like) Higgs boson mass and signal strenghts from LHC
- Total widths of W and Z boson
- Agreement with electroweak precision observables
- Exclusion from SUSY searches at LEP and LHC experiments we use whatever is available, but not all recasts are done yet
- ullet Lower limit on  $H^\pm$  width from long-lived charged particle searches
- Direct bound by the dark matter nucleon scattering (LUX, XENON1T)
- Planck upper limit on relic density



### Low mass IDM benchmark points

No.	Мн	$M_A$	$M_{H^{\pm}}$	$\lambda_2$	$\lambda_{345}$	$\Omega_c h^2$
BP1	72.77	107.8	114.6	1.445	-0.004407	0.1201
BP2	65	71.53	112.8	0.7791	0.0004	0.07081
BP3	67.07	73.22	96.73	0	0.00738	0.06162
BP4	73.68	100.1	145.7	2.086	-0.004407	0.08925
BP6	72.14	109.5	154.8	0.01257	-0.00234	0.1171
BP7	76.55	134.6	174.4	1.948	0.0044	0.0314
BP8	70.91	148.7	175.9	0.4398	0.0051	0.124
BP9	56.78	166.2	178.2	0.5027	0.00338	0.08127
BP10	76.69	154.6	163	3.921	0.0096	0.02814
BP11	98.88	155	155.4	1.181	-0.0628	0.002737
BP12	58.31	171.1	173	0.5404	0.00762	0.00641
BP13	99.65	138.5	181.3	2.463	0.0532	0.001255
BP14	71.03	165.6	176	0.3393	0.00596	0.1184
BP15	71.03	217.7	218.7	0.7665	0.00214	0.1222
BP16	71.33	203.8	229.1	1.03	-0.00122	0.1221
BP18	147	194.6	197.4	0.387	-0.018	0.001772
BP19	165.8	190.1	196	2.768	-0.004	0.002841
BP20	191.8	198.4	199.7	1.508	0.008	0.008494
BP21	57.48	288	299.5	0.9299	0.00192	0.1195
BP22	71.42	247.2	258.4	1.043	-0.00406	0.1243
BP23	62.69	162.4	190.8	2.639	0.0056	0.06404

Note that BP5 and BP17 were excluded by the updated XENON1T limits, arXiv:1805.12562



### **High mass IDM benchmark points**

No.	$M_H$	$M_A$	$M_{H^{\pm}}$	$\lambda_2$	$\lambda_{345}$	$\Omega_c h^2$
HP1	176	291.4	312	1.49	-0.1035	0.0007216
HP2	557	562.3	565.4	4.045	-0.1385	0.07209
HP3	560	616.3	633.5	3.38	-0.0895	0.001129
HP4	571	676.5	682.5	1.98	-0.471	0.0005635
HP5	671	688.1	688.4	1.377	-0.1455	0.02447
HP6	713	716.4	723	2.88	0.2885	0.03515
HP7	807	813.4	818	3.667	0.299	0.03239
HP8	933	940	943.8	2.974	-0.2435	0.09639
HP9	935	986.2	988	2.484	-0.5795	0.002796
HP10	990	992.4	998.1	3.334	-0.051	0.1248
HP11	250.5	265.5	287.2	3.908	-0.1501	0.00535
HP12	286.1	294.6	332.5	3.292	0.1121	0.00277
HP13	336	353.3	360.6	2.488	-0.1064	0.00937
HP14	326.6	331.9	381.8	0.02513	-0.06267	0.00356
HP15	357.6	400	402.6	2.061	-0.2375	0.00346
HP16	387.8	406.1	413.5	0.8168	-0.2083	0.0116
HP17	430.9	433.2	440.6	3.003	0.08299	0.0327
HP18	428.2	454	459.7	3.87	-0.2812	0.00858
HP19	467.9	488.6	492.3	4.122	-0.252	0.0139
HP20	505.2	516.6	543.8	2.538	-0.354	0.00887



# Signal processes for $\mu^+\mu^-$ final state

$$\begin{array}{lll} \mathrm{e^{+}e^{-}} & \rightarrow & \mu^{+}\mu^{-} \; HH, \\ & \rightarrow & \mu^{+}\mu^{-}\nu_{\mu}\bar{\nu}_{\mu} \; HH, \\ & \rightarrow & \tau^{+}\mu^{-}\nu_{\tau}\bar{\nu}_{\mu} \; HH, \; \; \mu^{+}\tau^{-}\nu_{\mu}\bar{\nu}_{\tau} \; HH, \\ & \rightarrow & \tau^{+}\tau^{-} \; HH, \; \; \tau^{+}\tau^{-}\nu_{\tau}\bar{\nu}_{\tau} \; HH. \\ & & \mathrm{with}\tau^{\pm} \rightarrow \mu^{\pm}\nu\nu \end{array}$$

# Signal processes for $e^{\pm}\mu^{\mp}$ final state

$$\begin{array}{lll} e^{+}e^{-} & \to & \mu^{+}\nu_{\mu}\;e^{-}\bar{\nu}_{e}\;HH,\;\; e^{+}\nu_{e}\;\mu^{-}\bar{\nu}_{\mu}\;HH,\\ & \to & \mu^{+}\nu_{\mu}\;\tau^{-}\bar{\nu}_{\tau}\;HH,\;\; \tau^{+}\nu_{\tau}\;\mu^{-}\bar{\nu}_{\mu}\;HH,\\ & \to & e^{+}\nu_{e}\;\tau^{-}\bar{\nu}_{\tau}\;HH,\;\; \tau^{+}\nu_{\tau}\;e^{-}\bar{\nu}_{e}\;HH,\\ & \to & \tau^{+}\;\tau^{-}\;HH,\;\; \tau^{+}\nu_{\tau}\;\tau^{-}\bar{\nu}_{\tau}\;HH, \end{array}$$