# Two-loop corrections to the Higgs trilinear coupling in models with extended scalar sectors

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*based on*
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and work in preparation
with **Shinya Kanemura**

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# Outline

1. Introduction: why we study $\lambda_{hhh}$
2. Non-decoupling effects at one loop
3. Our two-loop calculation in the 2HDM
4. Some numerical results

# Introduction

# Investigating the Higgs trilinear coupling $\lambda_{hhh}$

## Probing the shape of the Higgs potential

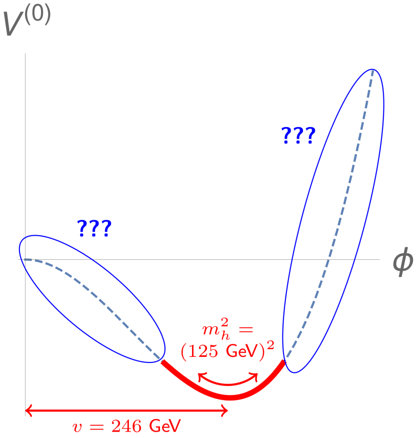
- Since the Higgs discovery, the existence of the Higgs potential is confirmed, but at the moment we only know:

  $\rightarrow$ the location of the EW minimum: $v \simeq 246$ GeV
  $\rightarrow$ the curvature of the potential around the EW minimum: $m_h \simeq 125$ GeV

  However what we still don't know is the **shape** of the Higgs potential, which **depends on $\lambda_{hhh}$**

- $\lambda_{hhh}$ determines the nature of the EWPT!

  $\Rightarrow \mathcal{O}(20\%)$ deviation of $\lambda_{hhh}$ from its SM prediction needed to have a strong first-order EWPT
  $\rightarrow$ necessary for EWBG
  [Grojean, Servant, Wells '04], [Kanemura, Okada, Senaha '04]



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## Alignment with or without decoupling

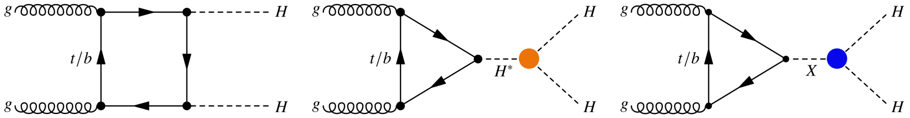
- Aligned scenarios already seem to be favoured $\rightarrow$ Higgs couplings are SM-like at **tree-level**
- Non-aligned scenarios ($e.g.$ in 2HDMs) could be almost entirely excluded in the close future using synergy of HL-LHC and ILC!

  $\rightarrow$ Alignment **through decoupling**? or alignment **without** decoupling?
- If alignment without decoupling, Higgs couplings like $\lambda_{hhh}$ can still exhibit **large deviations** from SM predictions because of **BSM loop effects** $\rightarrow$ still allowed by experimental results

Current limits (LHC) on $\kappa_\lambda \equiv \lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$ are (at 95% CL)

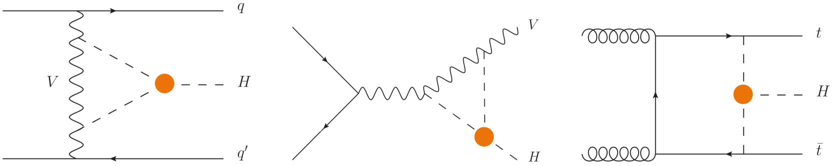
- Double $h$ production: $-5.0 < \kappa_\lambda < 12.1$ (ATLAS) and $-11 < \kappa_\lambda < 17$ (CMS)

  see [ATL-PHYS-PROC-2018-117] (ATLAS), [CMS-HIG-17-008] (CMS)



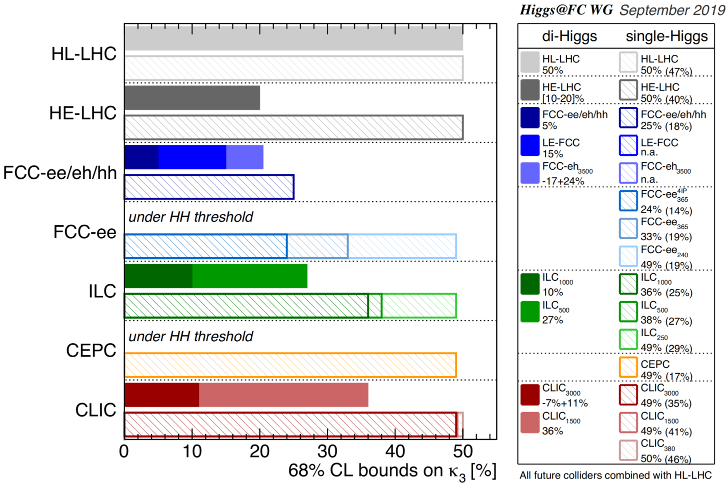

- Single $h$ production: $-3.2 < \kappa_\lambda < 11.9$ (ATLAS)

  see [ATL-PHYS-PUB-2019-009] (ATLAS)



# Future measurements prospects for the Higgs trilinear coupling $\lambda_{hhh}$

$$\left(\kappa_3 = \frac{\lambda_{hhh}}{\lambda_{hhh}^{\text{SM}}}\right)$$




[Higgs@FC report, 1905.03764]

# Radiative corrections to the Higgs trilinear coupling and non-decoupling effects

# The Two-Higgs-Doublet Model (2HDM)

- CP-conserving 2HDM, with softly-broken $\mathbb{Z}_2$ symmetry $(\Phi_1 \to \Phi_1, \Phi_2 \to -\Phi_2)$ to avoid tree-level FCNCs
- 2 $SU(2)_L$ doublets $\Phi_{1,2}$ of hypercharge $1/2$

$$V^{(0)}_{\text{2HDM}} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2) + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\Big((\Phi_2^\dagger\Phi_1)^2 + \text{h.c.}\Big)$$

- 7 free parameters in scalar sector:

$$m_3^2,\ \lambda_i\ (i = 1\cdots5),\ \tan\beta \equiv \langle\Phi_2^0\rangle/\langle\Phi_1^0\rangle$$

($m_1^2$, $m_2^2$ eliminated with tadpole equations, and $\langle\Phi_1^0\rangle + \langle\Phi_2^0\rangle = v^2 = (246\text{ GeV})^2$)

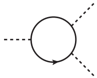
- Doublets expanded in terms of mass eigenstates:
  $h$, $H$: CP-even Higgses, $A$: CP-odd Higgs, $H^\pm$: charged Higgs
- $\lambda_i$ $(i = 1\cdots5)$ traded for mass eigenvalues $m_h$, $m_H$, $m_A$, $m_{H^\pm}$ and CP-even mixing angle $\alpha$
- $m_3^2$ replaced by a $\mathbb{Z}_2$-symmetry soft-breaking mass scale $M^2 = 2m_3^2/s_{2\beta}$

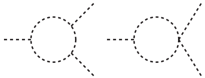
## Non-decoupling effects in $\lambda_{hhh}$ at one loop

First studies of the one-loop corrections to $\lambda_{hhh}$ in the 2HDM in [Kanemura, Kiyoura, Okada, Senaha, Yuan '02] and [Kanemura, Okada, Senaha, Yuan '04]

- $\lambda_{hhh}$ up to leading one-loop corrections (for $s_{\beta-\alpha}=1$)

$$\lambda_{hhh} = \frac{3m_h^2}{v} + \frac{1}{16\pi^2}\left[\underbrace{-\frac{48m_t^4}{v^3}}_{\text{SM-like}} + \sum_{\Phi=H,A,H^\pm}\underbrace{\frac{4n_\Phi m_\Phi^4}{v^3}\left(1-\frac{M^2}{m_\Phi^2}\right)^3}_{\text{BSM}}\right] + \cdots$$





- Masses of additional scalars $\Phi = H, A, H^\pm$ in 2HDM can be written as $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$ ($\tilde{\lambda}_\Phi$: some combination of $\lambda_i$)
- Power-like dependence of BSM terms $\propto m_\Phi^4$, and

$$\left(1-\frac{M^2}{m_\Phi^2}\right)^3 \to \begin{cases} 0, & \text{for } M^2 \gg \tilde{\lambda}_\Phi v^2 \\ 1, & \text{for } M^2 \ll \tilde{\lambda}_\Phi v^2 \end{cases}$$

# Non-decoupling effects in $\lambda_{hhh}$ at one loop

$$\lambda_{hhh} = \frac{3m_h^2}{v} + \frac{1}{16\pi^2}\left[-\frac{48m_t^4}{v^3} + \sum_{\Phi=H,A,H^\pm}\frac{4n_\Phi m_\Phi^4}{v^3}\left(1-\frac{M^2}{m_\Phi^2}\right)^3\right] + \cdots$$

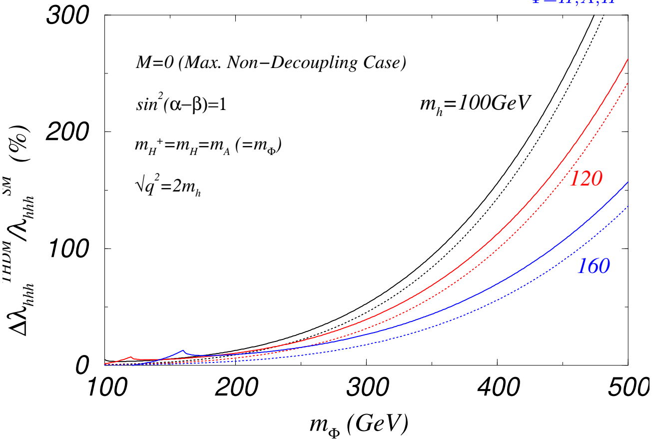



figure from [Kanemura, Okada, Senaha, Yuan '04]

- **Huge deviations possible, without violating unitarity!**
  **$\rightarrow$ non-decoupling effects**

  [see also K. Sakurai's talk yesterday]

- Tree level $\propto m_h^2$
- One loop $\propto m_\Phi^4$
- **Not a breakdown of perturbative expansion!**

# One-loop calculations of $\lambda_{hhh}$

- Complete diagrammatic, OS-scheme, calculations been performed for a number of BSM models with extended sectors (with singlets, doublets, triplets)

- One-loop calculations available for 2HDMs, HSM, IDM in program `H-COUP` [Kanemura, Kikuchi, Sakurai, Yagyu '17], [Kanemura, Kikuchi, Mawatari, Sakurai, Yagyu '19] [see talks by K. Sakurai and K. Mawatari]

**Non-decoupling effects found for a range of BSM models at one loop**
$\Rightarrow$ **What happens at two loops? New huge corrections?**
$\Rightarrow$ **We derive <u>dominant</u> two-loop corrections to $\lambda_{hhh}$ in a 2HDM** [J.B., Kanemura '19]

*Note*: a few works exist at two loops, in MSSM [Brucherseifer, Gavin, Spira '14], NMSSM [Mühlleitner, Nhung, Ziesche '15], and IDM [Senaha '18], but with **different motivations** (*more details in backup*)

# Our two-loop calculation of $\lambda_{hhh}$ in the Two-Higgs-Doublet Model

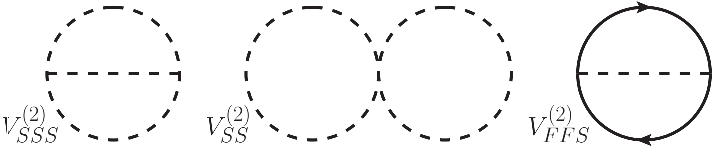
# Setup of our effective-potential calculation

**Step 1:** calculate $\underbrace{V_{\text{eff}}}_{\overline{\text{MS}}}$ $\rightarrow$ **Step 2:** $\underbrace{\lambda_{hhh} = \left.\frac{\partial^3 V_{\text{eff}}}{\partial h^3}\right|_{\text{min.}}}_{\overline{\text{MS}}}$ $\rightarrow$ **Step 3:** convert from $\overline{\text{MS}}$ to OS scheme

- $\overline{\text{MS}}$-renormalised two-loop effective potential is

$$V_{\text{eff}} = V^{(0)} + \kappa V^{(1)} + \kappa^2 V^{(2)} \qquad \left(\kappa \equiv \frac{1}{16\pi^2}\right)$$

- $V^{(2)}$: 1PI vacuum bubble diags., and we want to study the leading two-loop BSM corrections from **additional scalars** and **top quark**, so we only need

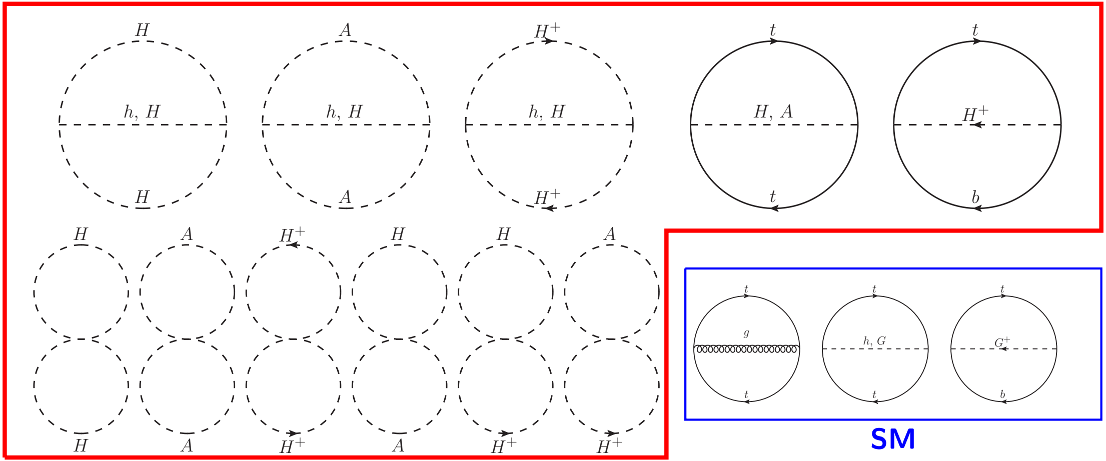



- **Subleading contributions** from $h$, $G, G^{\pm}$, and light fermions **neglected**
- **Scenarios without mixing**: aligned 2HDM ($s_{\beta-\alpha} = 1$) $\Rightarrow$ evade exp. constrains!
(loop-induced deviations from alignment also neglected)

# $\lambda_{hhh}$ at two loops in the 2HDM

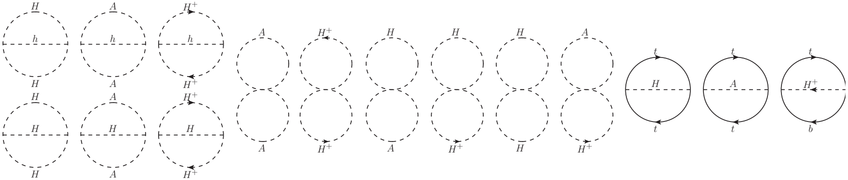
In [JB, Kanemura '19], we considered for the first time $\lambda^{(2)}_{hhh}$ in the 2HDM:
$\rightarrow$ 15 new BSM diagrams appearing in $V^{(2)}$ in the 2HDM w.r.t. the SM case

**2HDM**




**SM**

# $\lambda_{hhh}$ at two loops in the 2HDM




- We assume $H$, $A$, $H^\pm$ to have a degenerate mass $m_\Phi$
  $\to$ 3 mass scales in the calculation: $m_t$, $m_\Phi$, $M$ ($\to$ simpler analytical expressions)
- In the $\overline{\mathrm{MS}}$ scheme

$$\delta^{(2)}\lambda_{hhh} = \frac{16 m_\Phi^4}{v^5}\left(4 + 9\cot^2 2\beta\right)\left(1 - \frac{M^2}{m_\Phi^2}\right)^4 \left[-2M^2 - m_\Phi^2 + (M^2 + 2m_\Phi^2)\,\overline{\log}\, m_\Phi^2\right]$$
$$+ \frac{192 m_\Phi^6 \cot^2 2\beta}{v^5}\left(1 - \frac{M^2}{m_\Phi^2}\right)^4 \left[1 + 2\,\overline{\log}\, m_\Phi^2\right]$$
$$+ \frac{96 m_\Phi^4 m_t^2 \cot^2\beta}{v^5}\left(1 - \frac{M^2}{m_\Phi^2}\right)^3 \left[-1 + 2\,\overline{\log}\, m_\Phi^2\right] + \mathcal{O}\left(\frac{m_\Phi^2 m_t^4}{v^5}\right)$$

## Decoupling behaviour of the $\overline{\text{MS}}$ expressions

- Decoupling theorem [Appelquist, Carazzone '75] $\rightarrow$ corrections from additional BSM states should decouple if said states are taken to be very massive

$$m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$$

- To have $m_\Phi \rightarrow \infty$, then we must take $M \rightarrow \infty$, otherwise the quartic couplings grow out of control

$$\delta^{(1)}\lambda_{hhh} = \frac{16 m_\Phi^4}{v^3}\left(1 - \frac{M^2}{m_\Phi^2}\right)^3$$

$$\begin{aligned}\delta^{(2)}\lambda_{hhh} = {} & \frac{16 m_\Phi^4}{v^5}\left(4 + 9\cot^2 2\beta\right)\left(1 - \frac{M^2}{m_\Phi^2}\right)^4\left[-2M^2 - m_\Phi^2 + (M^2 + 2m_\Phi^2)\,\overline{\log}\, m_\Phi^2\right] \\ & + \frac{192 m_\Phi^6 \cot^2 2\beta}{v^5}\left(1 - \frac{M^2}{m_\Phi^2}\right)^4\left[1 + 2\,\overline{\log}\, m_\Phi^2\right] \\ & + \frac{96 m_\Phi^4 m_t^2 \cot^2\beta}{v^5}\left(1 - \frac{M^2}{m_\Phi^2}\right)^3\left[-1 + 2\,\overline{\log}\, m_\Phi^2\right] + \mathcal{O}\left(\frac{m_\Phi^2 m_t^4}{v^5}\right)\end{aligned}$$

- Fortunately all of these terms go like

$$(m_\Phi^2)^{n-1}\left(1 - \frac{M^2}{m_\Phi^2}\right)^n \underset{m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2}{=} \frac{(\tilde{\lambda}_\Phi v^2)^n}{M^2 + \tilde{\lambda}_\Phi v^2} \xrightarrow[\tilde{\lambda}_\Phi v^2 \text{ fixed}]{M \rightarrow \infty} 0$$

## Decoupling behaviour and $\overline{\text{MS}}$ to OS scheme conversion

- To express $\delta^{(2)}\lambda_{hhh}$ in terms of physical parameters ($v_{\text{phys}}$, $M_t$, $M_A = M_H = M_{H^\pm} = M_\Phi$), we replace

$$\overline{\text{MS}}\text{ scheme:}\Big\{\underbrace{m_H, m_A, m_{H^\pm}}_{m_\Phi}, m_t, v\Big\} \longrightarrow \text{OS scheme:}\Big\{\underbrace{M_H, M_A, M_{H^\pm}}_{M_\Phi}, M_t, v_{\text{phys}} = (\sqrt{2}G_F)^{-1/2}\Big\}$$

- A priori, $M$ is still renormalised in $\overline{\text{MS}}$ scheme, because it is difficult to relate to physical observable
  … but then, expressions do not decouple for $M_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$ and $M \to \infty$!
- This is because we should relate $M_\Phi$, renormalised in OS scheme, and $M$, renormalised in $\overline{\text{MS}}$ scheme, with a **one-loop relation** $\to$ then the two-loop corrections decouple properly
- We give a new "OS" prescription for the finite part of the counterterm for $M$ by requiring that the decoupling of $\delta^{(2)}\hat{\lambda}_{hhh}$ (in OS scheme) is apparent using a relation $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$

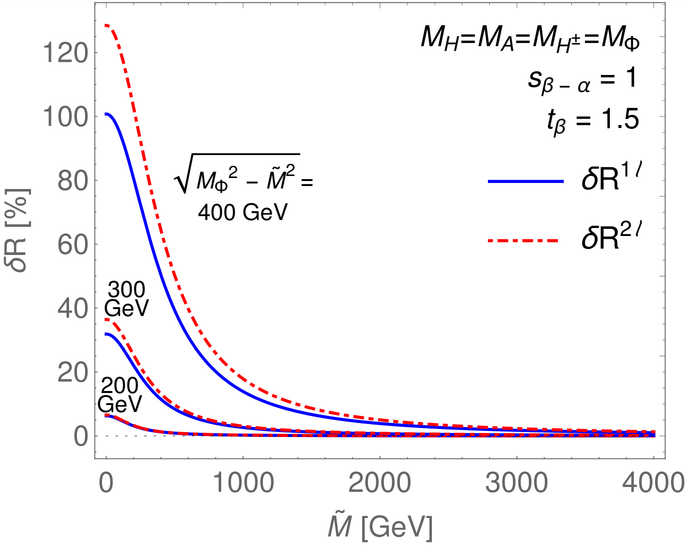
$$\begin{aligned}\delta^{(2)}\hat{\lambda}_{hhh} = {}& \frac{48M_\Phi^6}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^4\left\{4+3\cot^2 2\beta\left[3-\frac{\pi}{\sqrt{3}}\left(\frac{\tilde{M}^2}{M_\Phi^2}+2\right)\right]\right\}+\frac{576M_\Phi^6\cot^2 2\beta}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^4\\ &+\frac{288M_\Phi^4M_t^2\cot^2\beta}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^3+\frac{168M_\Phi^4M_t^2}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^3-\frac{48M_\Phi^6}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^5+\mathcal{O}\left(\frac{M_\Phi^2M_t^4}{v_{\text{phys}}^5}\right)\end{aligned}$$

# Numerical results

In the following we show results for the BSM deviation $\delta R$:

$$\delta R \equiv \frac{\Delta\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}} = \frac{\lambda_{hhh}^{\text{2HDM}} - \lambda_{hhh}^{\text{SM}}}{\lambda_{hhh}^{\text{SM}}}$$

## Decoupling behaviour



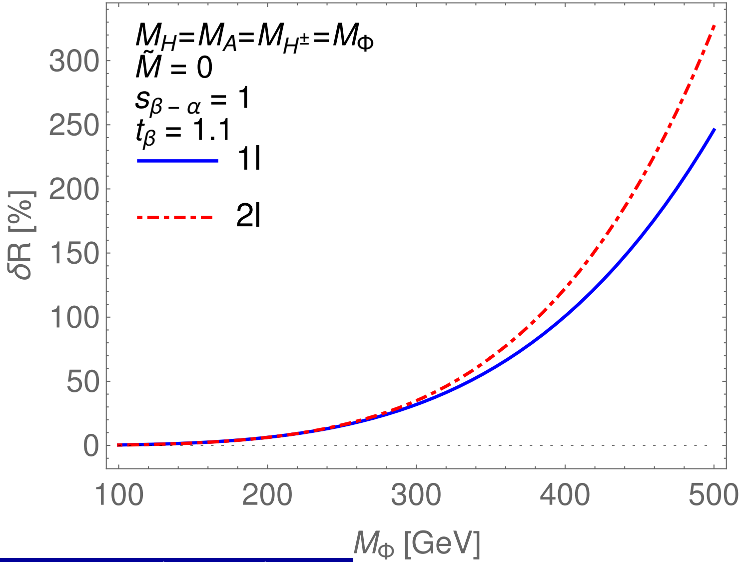

- $\delta R$ size of BSM contributions to $\lambda_{hhh}$:

$$\delta R \equiv \frac{\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}} - 1$$

- Radiative corrections from additional scalars + top quark indeed decouple properly for $\tilde{M} \to \infty$

- $\tilde{M}$ controls decoupling of BSM scalars in 2HDM in OS scheme!

## Non-decoupling effects

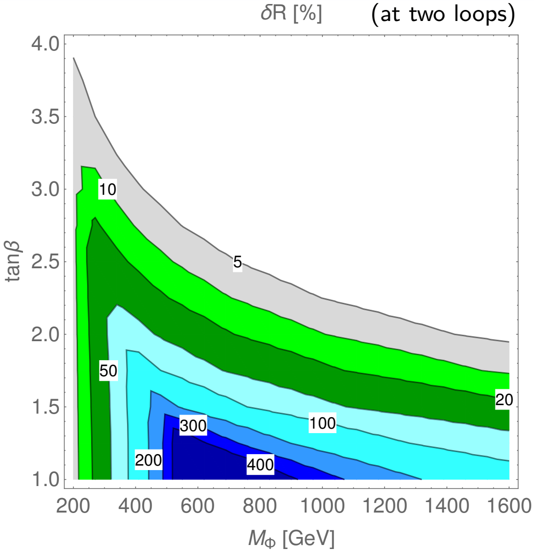



$$\delta R \equiv \frac{\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}} - 1$$

- ▷ Other limit of interest: $\tilde{M} = 0 \rightarrow$ maximal non-decoupling effects
- ▷ $\delta^{(1)}\hat{\lambda}_{hhh} \rightarrow \propto M_\Phi^4$
- ▷ $\delta^{(2)}\hat{\lambda}_{hhh} \rightarrow \propto M_\Phi^6$
- ▷ For $\tilde{M} = 0$, $\tan\beta = 1.1$, tree-level unitarity is lost around $M_\Phi \approx 600$ GeV [Kanemura, Kubota, Takasugi '93]

# Maximal BSM allowed deviations

(at two loops)




$$\delta R \equiv \frac{\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}} - 1$$

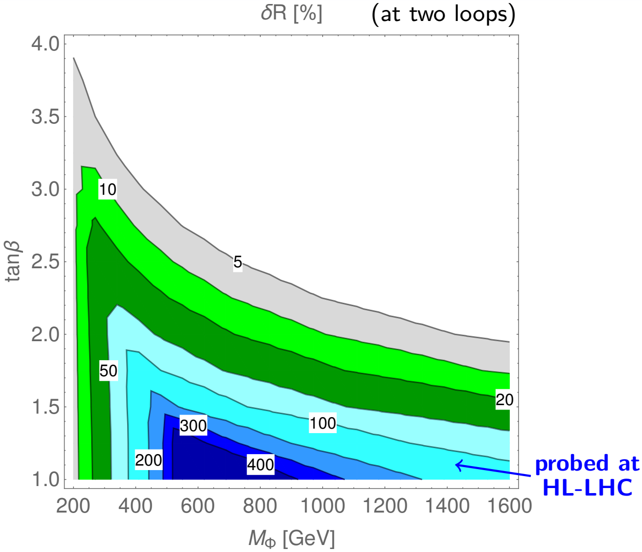
- Here: Maximal deviation $\delta R$ $(1\ell{+}2\ell)$ while fulfilling perturbative unitarity, in $(\tan\beta, M_\Phi)$ plane

$$M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$$

- One cannot take $M_\Phi \to \infty$ with $\tilde{M} = 0$ without breaking unitarity
- At some point $\tilde{M}$ must be non-zero $\to$ reduction factor

$$\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^n < 1$$

# Maximal BSM allowed deviations

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- Here: Maximal deviation $\delta R$ ($1\ell$+$2\ell$) while fulfilling perturbative unitarity, in $(\tan\beta, M_\Phi)$ plane

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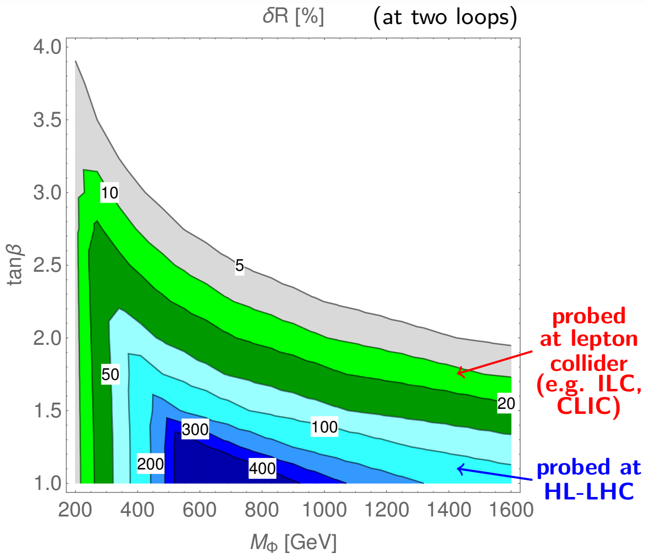


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# Summary

- **First two-loop calculation of $\lambda_{hhh}$ in 2HDM**, in a scenario with alignment

- Two-loop corrections to $\lambda_{hhh}$ remain smaller than one-loop contributions, at least as long as perturbative unitarity is maintained $\rightarrow$ **typical size $10-20\%$ of one-loop contributions**

$\Rightarrow$ non-decoupling effects found at one loop are **not drastically changed**

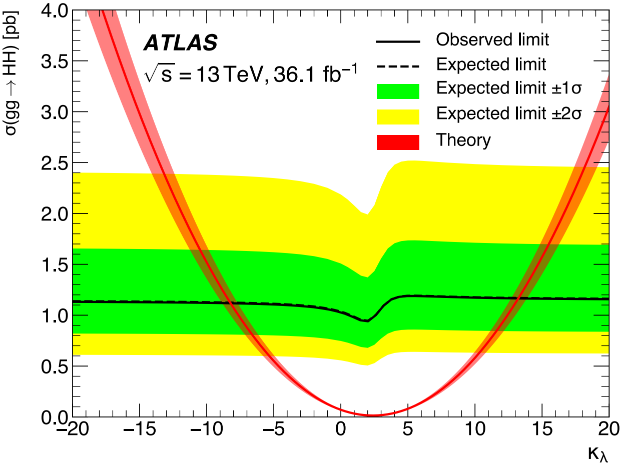
$\Rightarrow$ in the future perspective of a precise measurement of $\lambda_{hhh}$, computing corrections beyond one loop will be **necessary**

- Precise calculation of Higgs couplings ($\lambda_{hhh}$, etc.) can allow distinguishing aligned scenarios **<u>with</u> or <u>without</u> decoupling**

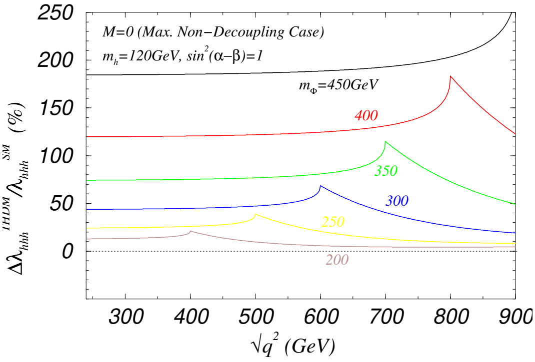
Thank you for your attention!

# Backup

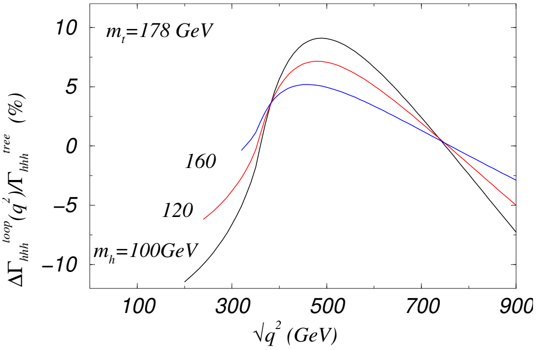
# An example of experimental limits on $\lambda_{hhh}$




Example of current limits on $\kappa_\lambda$ from the ATLAS search of $hh \to b\bar{b}\gamma\gamma$
(*taken from* [ATLAS collaboration 1807.04873])




(*scalar part*)



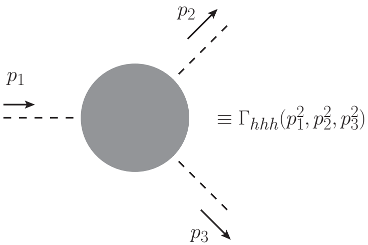

(*top quark loop*)

figures from [Kanemura, Okada, Senaha, Yuan '04]

# Radiative corrections to the Higgs trilinear coupling

- Higgs three-point function, $\Gamma_{hhh}(p_1^2, p_2^2, p_3^2)$, requires a diagrammatic calculation, with non-zero external momentum
- Instead it is much more convenient to work with an effective Higgs trilinear coupling $\lambda_{hhh}$

$$\mathcal{L} \supset -\frac{1}{6}\lambda_{hhh}h^3 \quad \rightarrow \quad \underbrace{\lambda_{hhh} = \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min.}}}_{\overline{\text{MS}}\text{ result}}$$




- $\Gamma_{hhh}$ and $\lambda_{hhh}$ can be related as

$$-\Gamma_{hhh}(0,0,0) = \underbrace{\hat{\lambda}_{hhh}}_{\text{OS result}} = \left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2} \underbrace{\lambda_{hhh}}_{\substack{\overline{\text{MS}}\text{ result} \\ \text{expressed in terms of} \\ \text{OS parameters}}} = \left(1 + \frac{3}{2}\frac{d}{dp^2}\Pi_{hh}(p^2)\Big|_{p^2=M_h^2}\right)\lambda_{hhh}$$

$Z_h^{\text{OS},\overline{\text{MS}}}$: OS/$\overline{\text{MS}}$ WFR constants; $\Pi_{hh}(p^2)$: finite part of Higgs self-energy at ext. momentum $p^2$

- Taking $\Gamma_{hhh}(p_1^2, p_2^2, p_3^2) \simeq \Gamma_{hhh}(0,0,0)$ is a good approximation
  - $\rightarrow$ shown for $\lambda_{hhh}$ at one loop in [Kanemura, Okada, Senaha, Yuan '04] (difference is only a few %)
  - $\rightarrow$ no study including external momentum exists at two loops, but in the case of two-loop Higgs mass calculations, momentum effects are known to be subleading

## Setup of our effective-potential calculation – details

- OS result is obtained as

$$\hat{\lambda}_{hhh} = \underbrace{\left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2}}_{\text{inclusion of WFR}} \times \underbrace{\lambda_{hhh}}_{\substack{\overline{\text{MS}}\text{ parameters}\\ \text{translated to OS ones}}}$$

- Let's suppose (for simplicity) that $\lambda_{hhh}$ only depends on one parameter $x$, as

$$\lambda_{hhh} = \lambda_{hhh}^{(0)}(x^{\overline{\text{MS}}}) + \kappa\delta^{(1)}\lambda_{hhh}(x^{\overline{\text{MS}}}) + \kappa^2\delta^{(2)}\lambda_{hhh}(x^{\overline{\text{MS}}}) \qquad \left(\kappa = \frac{1}{16\pi^2}\right)$$

and

$$x^{\overline{\text{MS}}} = X^{\text{OS}} + \kappa\delta^{(1)}x + \kappa^2\delta^{(2)}x$$

then in terms of OS parameters

$$\lambda_{hhh} = \lambda_{hhh}^{(0)}(X^{\text{OS}}) + \kappa\left[\delta^{(1)}\lambda_{hhh}(X^{\text{OS}}) + \frac{\partial\lambda_{hhh}^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x\right]$$
$$+\kappa^2\left[\delta^{(2)}\lambda_{hhh}(X^{\text{OS}}) + \frac{\partial\delta^{(1)}\lambda_{hhh}}{\partial x}(X^{\text{OS}})\delta^{(1)}x + \frac{\partial\lambda_{hhh}^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(2)}x + \frac{\partial^2\lambda_{hhh}^{(0)}}{\partial x^2}(X^{\text{OS}})(\delta^{(1)}x)^2\right]$$

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because we neglect $m_h$ in the loop corrections and $\lambda^{(0)}_{hhh} = 3m_h^2/v$ (in absence of mixing)

# Existing works at two loops

| Model [ref.] | Included Corrections | Eff. pot. approx. | Typical size | Motivation |
|---|---|---|---|---|
| MSSM [Brucherseifer, Gavin, Spira '14] | $\mathcal{O}(\alpha_s \alpha_t)$ | Yes | $\mathcal{O}(\sim 10\%)$ | Reach similiar accuracy as $m_h$ |
| NMSSM [Mühlleitner, Nhung, Ziesche '15] | $\mathcal{O}(\alpha_s \alpha_t)$ | Yes | $\mathcal{O}(\sim 5 - 10\%)$ | Reach similiar accuracy as $m_h$ |
| IDM [Senaha '18] | $\mathcal{O}(\lambda_\Phi^3)$ (partial) | Yes | $\mathcal{O}(\sim 2\%)$ | Effect on strength of EWPT |