Two-loop corrections to the Higgs trilinear coupling in models with extended scalar sectors

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based on
and work in preparation
with Shinya Kanemura

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Outline

1. Introduction: why we study $\lambda_{hhh}$

2. Non-decoupling effects at one loop

3. Our two-loop calculation in the 2HDM

4. Some numerical results
INTRODUCTION
Investigating the Higgs trilinear coupling $\lambda_{hhh}$

Probing the shape of the Higgs potential

- Since the Higgs discovery, the existence of the Higgs potential is confirmed, but at the moment we only know:
  - the location of the EW minimum: $v \simeq 246$ GeV
  - the curvature of the potential around the EW minimum: $m_h \simeq 125$ GeV

However what we still don’t know is the shape of the Higgs potential, which depends on $\lambda_{hhh}$

- $\lambda_{hhh}$ determines the nature of the EWPT!
  - $\mathcal{O}(20\%)$ deviation of $\lambda_{hhh}$ from its SM prediction needed to have a strong first-order EWPT
  - necessary for EWBG

[Grojean, Servant, Wells ’04], [Kanemura, Okada, Senaha ’04]
Investigating the Higgs trilinear coupling $\lambda_{hhh}$

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Alignment with or without decoupling

- Aligned scenarios already seem to be favoured $\rightarrow$ Higgs couplings are SM-like at tree-level
- Non-aligned scenarios (e.g. in 2HDMs) could be almost entirely excluded in the close future using synergy of HL-LHC and ILC!
  - Alignment through decoupling? or alignment without decoupling?
- If alignment without decoupling, Higgs couplings like $\lambda_{hhh}$ can still exhibit large deviations from SM predictions because of BSM loop effects $\rightarrow$ still allowed by experimental results
Investigating the Higgs trilinear coupling $\lambda_{hhh}$

Current limits (LHC) on $\kappa_\lambda \equiv \lambda_{hhh}/\lambda_{hhh}^{SM}$ are (at 95% CL)

- **Double $h$ production**: $-5.0 < \kappa_\lambda < 12.1$ (ATLAS) and $-11 < \kappa_\lambda < 17$ (CMS)
  
  ![Diagram](image1)

  see [ATL-PHYS-PROC-2018-117] (ATLAS), [CMS-HIG-17-008] (CMS)

- **Single $h$ production**: $-3.2 < \kappa_\lambda < 11.9$ (ATLAS)

  ![Diagram](image2)

  see [ATL-PHYS-PUB-2019-009] (ATLAS)
Future measurements prospects for the Higgs trilinear coupling $\lambda_{hhh}$

$$\left( \kappa_3 = \frac{\lambda_{hhh}}{\lambda_{hhh}^{SM}} \right)$$

[Higgs@FC report, 1905.03764]
Radiative corrections to the Higgs trilinear coupling and non-decoupling effects
The Two-Higgs-Doublet Model (2HDM)

- CP-conserving 2HDM, with softly-broken $\mathbb{Z}_2$ symmetry ($\Phi_1 \to \Phi_1, \Phi_2 \to -\Phi_2$) to avoid tree-level FCNCs
- 2 $SU(2)_L$ doublets $\Phi_{1,2}$ of hypercharge $1/2$

$$V^{(0)}_{2\text{HDM}} = m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - m_3^2 (\Phi_2^\dagger \Phi_1 + \Phi_1^\dagger \Phi_2) + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_2^\dagger \Phi_1|^2 + \frac{\lambda_5}{2} (\Phi_2^\dagger \Phi_1)^2 + \text{h.c.}$$

- 7 free parameters in scalar sector:
  $$m_3^2, \lambda_i \ (i = 1 \cdots 5), \tan \beta \equiv \langle \Phi_2^0 \rangle / \langle \Phi_1^0 \rangle$$

($m_1^2, m_2^2$ eliminated with tadpole equations, and $\langle \Phi_1^0 \rangle + \langle \Phi_2^0 \rangle = v^2 = (246 \text{ GeV})^2$)

- Doublets expanded in terms of mass eigenstates:
  $h, H$: CP-even Higgses, $A$: CP-odd Higgs, $H^\pm$: charged Higgs

- $\lambda_i \ (i = 1 \cdots 5)$ traded for mass eigenvalues $m_h, m_H, m_A, m_{H\pm}$ and CP-even mixing angle $\alpha$

- $m_3^2$ replaced by a $\mathbb{Z}_2$-symmetry soft-breaking mass scale $M^2 = 2m_3^2 / s_{2\beta}$
Non-decoupling effects in $\lambda_{hhh}$ at one loop

First studies of the one-loop corrections to $\lambda_{hhh}$ in the 2HDM in [Kanemura, Kiyoura, Okada, Senaha, Yuan '02] and [Kanemura, Okada, Senaha, Yuan '04]

- $\lambda_{hhh}$ up to leading one-loop corrections (for $s_{\beta-\alpha} = 1$)
  \[
  \lambda_{hhh} = \frac{3m_h^2}{v} + \frac{1}{16\pi^2} \left[ -\frac{48m_t^4}{v^3} + \sum_{\Phi=H,A,H\pm} \frac{4n_\Phi m_\Phi^4}{v^3} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^3 \right] + \ldots
  \]

- Masses of additional scalars $\Phi = H, A, H\pm$ in 2HDM can be written as $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$ ($\tilde{\lambda}_\Phi$: some combination of $\lambda_i$)

- Power-like dependence of BSM terms $\propto m_\Phi^4$, and
  \[
  \left( 1 - \frac{M^2}{m_\Phi^2} \right)^3 \rightarrow \begin{cases} 
  0, & \text{for } M^2 \gg \tilde{\lambda}_\Phi v^2 \\
  1, & \text{for } M^2 \ll \tilde{\lambda}_\Phi v^2
  \end{cases}
  \]
Non-decoupling effects in $\lambda_{hhh}$ at one loop

$$
\lambda_{hhh} = \frac{3m_h^2}{v} + \frac{1}{16\pi^2} \left( -\frac{48m_t^4}{v^3} + \sum_{\Phi=H,A,H^\pm} \frac{4n_\Phi m_\Phi^4}{v^3} \left(1 - \frac{M^2}{m_\Phi^2}\right)^3 \right) + \cdots
$$

- Huge deviations possible, without violating unitarity!
- → non-decoupling effects
- [see also K. Sakurai’s talk yesterday]

- Tree level $\propto m_h^2$
- One loop $\propto m_\Phi^4$
- Not a breakdown of perturbative expansion!

Figure from [Kanemura, Okada, Senaha, Yuan '04]
One-loop calculations of $\lambda_{hhh}$

- Complete diagrammatic, OS-scheme, calculations been performed for a number of BSM models with extended sectors (with singlets, doublets, triplets)

- One-loop calculations available for 2HDMs, HSM, IDM in program $H$-COUP [Kanemura, Kikuchi, Sakurai, Yagyu '17], [Kanemura, Kikuchi, Mawatari, Sakurai, Yagyu '19] [see talks by K. Sakurai and K. Mawatari]

Non-decoupling effects found for a range of BSM models at one loop
⇒ What happens at two loops? New huge corrections?
⇒ We derive dominant two-loop corrections to $\lambda_{hhh}$ in a 2HDM [J.B., Kanemura '19]

Note: a few works exist at two loops, in MSSM [Brucherseifer, Gavin, Spira '14], NMSSM [Mühlleitner, Nhung, Ziesche '15], and IDM [Senaha '18], but with different motivations (more details in backup)
Our two-loop calculation of $\lambda_{hhh}$ in the Two-Higgs-Doublet Model
Setup of our effective-potential calculation

**Step 1:** calculate $V_{\text{eff}}^{\overline{\text{MS}}}$ → **Step 2:** $\lambda_{hhh} = \left. \frac{\partial^3 V_{\text{eff}}}{\partial h^3} \right|_{\text{min.}}$ → **Step 3:** convert from $\overline{\text{MS}}$ to OS scheme

- $\overline{\text{MS}}$-renormalised two-loop effective potential is

$$V_{\text{eff}} = V^{(0)} + \kappa V^{(1)} + \kappa^2 V^{(2)}$$

(\(\kappa \equiv \frac{1}{16\pi^2}\))

- $V^{(2)}$: 1PI vacuum bubble diags., and we want to study the leading two-loop BSM corrections from *additional scalars* and *top quark*, so we only need

  - $V^{(2)}_{SSS}$
  - $V^{(2)}_{SS}$
  - $V^{(2)}_{FFS}$

- **Subleading contributions** from $h$, $G$, $G^\pm$, and light fermions neglected

- **Scenarios without mixing**: aligned 2HDM ($s_{\beta-\alpha} = 1$) ⇒ evade exp. constrains!

  (loop-induced deviations from alignment also neglected)
\( \lambda_{hhh} \) at two loops in the 2HDM

In [JB, Kanemura '19], we considered for the first time \( \lambda_{hhh}^{(2)} \) in the 2HDM:

\( \rightarrow \) 15 new BSM diagrams appearing in \( V^{(2)} \) in the 2HDM w.r.t. the SM case

2HDM

SM
$\lambda_{hhh}$ at two loops in the 2HDM

- We assume $H, A, H^\pm$ to have a degenerate mass $m_\Phi$
  $\rightarrow$ 3 mass scales in the calculation: $m_t, m_\Phi, M$ ($\rightarrow$ simpler analytical expressions)
- In the $\overline{\text{MS}}$ scheme

$$
\delta^{(2)} \lambda_{hhh} = \frac{16 m_\Phi^4}{v^5} \left(4 + 9 \cot^2 2\beta \right) \left(1 - \frac{M^2}{m_\Phi^2}\right)^4 \left[-2M^2 - m_\Phi^2 + (M^2 + 2m_\Phi^2) \log m_\Phi^2\right] \\
+ \frac{192 m_\Phi^6 \cot^2 2\beta}{v^5} \left(1 - \frac{M^2}{m_\Phi^2}\right)^4 \left[1 + 2 \log m_\Phi^2\right] \\
+ \frac{96 m_\Phi^4 m_t^2 \cot^2 \beta}{v^5} \left(1 - \frac{M^2}{m_\Phi^2}\right)^3 \left[-1 + 2 \log m_\Phi^2\right] + \mathcal{O} \left(\frac{m_\Phi^2 m_t^4}{v^5}\right)
$$
Decoupling behaviour of the $\overline{\text{MS}}$ expressions

Decoupling theorem [Appelquist, Carazzone ’75] → corrections from additional BSM states should decouple if said states are taken to be very massive

$$m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$$

To have $m_\Phi \to \infty$, then we must take $M \to \infty$, otherwise the quartic couplings grow out of control

$$\delta^{(2)} \lambda_{hhh} = \frac{16 m_\Phi^4}{v^5} \left( 4 + 9 \cot^2 2 \beta \right) \left( 1 - \frac{M^2}{m_\Phi^2} \right)^4 \left[ -2M^2 - m_\Phi^2 + (M^2 + 2m_\Phi^2) \log m_\Phi^2 \right]$$

$$\delta^{(1)} \lambda_{hhh} = \frac{16 m_\Phi^4}{v^3} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^3 + \frac{192 m_\Phi^6 \cot^2 2 \beta}{v^5} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^4 \left[ 1 + 2 \log m_\Phi^2 \right]$$

$$+ \frac{96 m_\Phi^4 m_t^2 \cot^2 2 \beta}{v^5} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^3 \left[ -1 + 2 \log m_\Phi^2 \right] + O \left( \frac{m_\Phi^2 m_t^4}{v^5} \right)$$

Fortunately all of these terms go like

$$(m_\Phi^2)^{n-1} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^n = \frac{(\tilde{\lambda}_\Phi v^2)^n}{M^2 + \tilde{\lambda}_\Phi v^2} \xrightarrow{M \to \infty} 0$$

$$(m_\Phi^2)^{n-1} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^n \xrightarrow{\tilde{\lambda}_\Phi v^2 \text{ fixed}} 0$$
Decoupling behaviour and $\overline{\text{MS}}$ to OS scheme conversion

To express $\delta^{(2)} \lambda_{hhh}$ in terms of physical parameters ($v_{\text{phys}}, M_t, M_A = M_H = M_{H\pm} = M_\Phi$), we replace

$\overline{\text{MS}}$ scheme: $\{m_H, m_A, m_{H\pm}, m_t, v\} \longrightarrow$ OS scheme: $\{M_H, M_A, M_{H\pm}, M_t, v_{\text{phys}} = (\sqrt{2}G_F)^{-1/2}\}$

A priori, $M$ is still renormalised in $\overline{\text{MS}}$ scheme, because it is difficult to relate to physical observable... but then, expressions do not decouple for $M_{\Phi}^2 = M^2 + \tilde{\lambda}_\Phi v^2$ and $M \to \infty$!

This is because we should relate $M_\Phi$, renormalised in OS scheme, and $M$, renormalised in $\overline{\text{MS}}$ scheme, with a one-loop relation $\rightarrow$ then the two-loop corrections decouple properly.

We give a new "OS" prescription for the finite part of the counterterm for $M$ by requiring that the decoupling of $\delta^{(2)} \hat{\lambda}_{hhh}$ (in OS scheme) is apparent using a relation $M_{\Phi}^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$

$$
\delta^{(2)} \hat{\lambda}_{hhh} = \frac{48M_{\Phi}^6}{v_{\text{phys}}^5} \left(1 - \frac{\tilde{M}^2}{M_{\Phi}^2}\right)^4 \left\{ 4 + 3 \cot^2 2\beta \left[ 3 - \frac{\pi}{\sqrt{3}} \left( \frac{\tilde{M}^2}{M_{\Phi}^2} + 2 \right) \right] \right\} + \frac{576M_{\Phi}^6}{v_{\text{phys}}^5} \cot^2 2\beta \left(1 - \frac{\tilde{M}^2}{M_{\Phi}^2}\right)^4
+ \frac{288M_{\Phi}^4 M_t^2}{v_{\text{phys}}^5} \cot^2 \beta \left(1 - \frac{\tilde{M}^2}{M_{\Phi}^2}\right)^3 + \frac{168M_{\Phi}^4 M_t^2}{v_{\text{phys}}^5} \left(1 - \frac{\tilde{M}^2}{M_{\Phi}^2}\right)^3 - \frac{48M_{\Phi}^6}{v_{\text{phys}}^5} \left(1 - \frac{\tilde{M}^2}{M_{\Phi}^2}\right)^5 + O \left( \frac{M_{\Phi}^2 M_t^4}{v_{\text{phys}}^5} \right)
$$

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Numerical results
Numerical results

In the following we show results for the BSM deviation $\delta R$:

$$
\delta R \equiv \frac{\Delta \lambda^{2\text{HDM}}_{hhh}}{\lambda^{SM}_{hhh}} = \frac{\lambda^{2\text{HDM}}_{hhh} - \lambda^{SM}_{hhh}}{\lambda^{SM}_{hhh}}
$$
Decoupling behaviour

\[ \delta R \equiv \frac{\lambda_{2HDM}^{hh} hhh}{\lambda_{SM}^{hh} hhh} - 1 \]

- \( \delta R \) size of BSM contributions to \( \lambda_{hhh} \):

- Radiative corrections from additional scalars + top quark indeed decouple properly for \( \tilde{M} \to \infty \)

- \( \tilde{M} \) controls decoupling of BSM scalars in 2HDM in OS scheme!
Non-decoupling effects

\[ M_H = M_A = M_{H^\pm} = M_\Phi \]
\[ \tilde{M} = 0 \]
\[ s_{\beta - \alpha} = 1 \]
\[ t_\beta = 1.1 \]

\[ \delta R \equiv \frac{\lambda_{h_2 h h}^{2HDM}}{\lambda_{h h h}^{SM}} - 1 \]

- Other limit of interest: \( \tilde{M} = 0 \rightarrow \) maximal non-decoupling effects

- \( \delta^{(1)} \lambda_{h h h} \rightarrow \propto M_\Phi^4 \)
- \( \delta^{(2)} \lambda_{h h h} \rightarrow \propto M_\Phi^6 \)

- For \( \tilde{M} = 0 \), \( \tan \beta = 1.1 \), tree-level unitarity is lost around \( M_\Phi \approx 600 \text{ GeV} \) [Kanemura, Kubota, Takasugi '93]
Maximal BSM allowed deviations

(at two loops)

\[ \delta R \equiv \frac{\lambda_{h h h}^{2HDM}}{\lambda_{h h h}^{SM}} - 1 \]

◨ Here: Maximal deviation \( \delta R \)

\((1\ell+2\ell)\) while fulfilling perturbative unitarity, in \((\tan \beta, M_\Phi)\) plane

\[ M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2 \]

◨ One cannot take \( M_\Phi \to \infty \) with \( \tilde{M} = 0 \) without breaking unitarity

◨ At some point \( \tilde{M} \) must be non-zero

\[ \left( 1 - \frac{\tilde{M}^2}{M_\Phi^2} \right)^n < 1 \]
Maximal BSM allowed deviations

(at two loops)

\[ %]
\[ \delta R \equiv \frac{\lambda_{h h h}^{2HDM}}{\lambda_{h h h}^{SM}} - 1 \]

▷ Here: Maximal deviation \( \delta R \)

\( (1\ell+2\ell) \) while fulfilling perturbative unitarity, in \((\tan \beta, M_\Phi)\) plane

\[ M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2 \]

▷ One cannot take \( M_\Phi \to \infty \) with \( \tilde{M} = 0 \) without breaking unitarity

▷ At some point \( \tilde{M} \) must be non-zero

\[ \left( 1 - \frac{\tilde{M}^2}{M_\Phi^2} \right)^n < 1 \]
Maximal BSM allowed deviations

(at two loops)

\[ \delta R \equiv \frac{\lambda_{h,h,h}^{2HDM}}{\lambda_{h,h,h}^{SM}} - 1 \]

- Here: Maximal deviation \( \delta R \) (1\(\ell\)+2\(\ell\)) while fulfilling perturbative unitarity, in \((\tan\beta, M_\Phi)\) plane

\[ M_\Phi^2 = \tilde{M}^2 + \lambda_\Phi v^2 \]

- One cannot take \( M_\Phi \to \infty \) with \( \tilde{M} = 0 \) without breaking unitarity

- At some point \( \tilde{M} \) must be non-zero \( \to \) reduction factor

\[ \left( 1 - \frac{\tilde{M}^2}{M_\Phi^2} \right)^n < 1 \]
Summary

- **First two-loop calculation of** $\lambda_{hhh}$ **in 2HDM**, in a scenario with alignment

- Two-loop corrections to $\lambda_{hhh}$ remain smaller than one-loop contributions, at least as long as perturbative unitarity is maintained → **typical size** $10 - 20\%$ **of one-loop contributions**

  - Non-decoupling effects found at one loop are **not drastically changed**
  - In the future perspective of a precise measurement of $\lambda_{hhh}$, computing corrections beyond one loop will be **necessary**

- Precise calculation of Higgs couplings ($\lambda_{hhh}$, etc.) can allow **distinguishing aligned scenarios with or without decoupling**
Thank you for your attention!
Backup
An example of experimental limits on $\lambda_{hh}$

Example of current limits on $\kappa_\lambda$ from the ATLAS search of $hh \rightarrow b\bar{b}\gamma\gamma$

(taken from [ATLAS collaboration 1807.04873])
Momentum dependence (at one loop)

\[ \Delta \lambda_{hhh}^{\text{THDM}} / \lambda_{hhh}^{\text{SM}} \quad \% \]

\[ M=0 \, (\text{Max. Non-Decoupling Case}) \]
\[ m_\Phi = 450 \text{GeV} \]
\[ m_h = 120 \text{GeV}, \sin^2(\alpha - \beta) = 1 \]

\[ \Delta \Gamma_{hhh}^{\text{loop}} / \Gamma_{hhh}^{\text{tree}} \quad \% \]
\[ m_h = 100 \text{GeV} \]
\[ m_t = 178 \text{GeV} \]

\[ \Delta \lambda_{hhh} \quad \text{THDM/SM} \quad \% \]

\[ 300 \quad 400 \quad 500 \quad 600 \quad 700 \quad 800 \quad 900 \]
\[ \sqrt{q^2} \text{ (GeV)} \]

\[ 0 \quad 50 \quad 100 \quad 150 \quad 200 \quad 250 \]

\[ 300 \quad 400 \quad 500 \quad 600 \quad 700 \quad 800 \quad 900 \]
\[ \sqrt{q^2} \text{ (GeV)} \]

\[ -10 \quad -5 \quad 0 \quad 5 \quad 10 \]

\[ \Delta \Gamma_{hhh} \quad \text{loop/(q^2)/tree} \quad \% \]

\[ m_h = 100 \text{GeV} \]
\[ m_t = 178 \text{GeV} \]

\[ (\text{scalar part}) \]

\[ (\text{top quark loop}) \]

figures from [Kanemura, Okada, Senaha, Yuan '04]

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Radiative corrections to the Higgs trilinear coupling

- Higgs three-point function, $\Gamma_{hhh}(p_1^2, p_2^2, p_3^2)$, requires a diagrammatic calculation, with non-zero external momentum.

- Instead it is much more convenient to work with an effective Higgs trilinear coupling $\lambda_{hhh}$

\[ \mathcal{L} \ni -\frac{1}{6} \lambda_{hhh} h^3 \rightarrow \lambda_{hhh} = \left. \frac{\partial^3 V_{\text{eff}}}{\partial h^3} \right|_{\text{min}} \]

- $\Gamma_{hhh}$ and $\lambda_{hhh}$ can be related as

\[ -\Gamma_{hhh}(0, 0, 0) = \hat{\lambda}_{hhh} = \left( \frac{Z_h^{OS}}{Z_h^{MS}} \right)^{3/2} \lambda_{hhh} = \left( 1 + \frac{3}{2} \frac{d}{dp^2} \Pi_{hh}(p^2) \bigg|_{p^2 = M_h^2} \right) \lambda_{hhh} \]

\[ Z_h^{OS, MS} : \text{OS/MS WFR constants; } \Pi_{hh}(p^2) : \text{finite part of Higgs self-energy at ext. momentum } p^2 \]

- Taking $\Gamma_{hhh}(p_1^2, p_2^2, p_3^2) \simeq \Gamma_{hhh}(0, 0, 0)$ is a good approximation

\[ \rightarrow \text{ shown for } \lambda_{hhh} \text{ at one loop in } [\text{Kanemura, Okada, Senaha, Yuan '04}] \text{ (difference is only a few %)} \]

\[ \rightarrow \text{ no study including external momentum exists at two loops, but in the case of two-loop Higgs mass calculations, momentum effects are known to be subleading} \]
Setup of our effective-potential calculation – details

▶ OS result is obtained as

\[ \hat{\lambda}_{hhh} = \left( \frac{Z_{OS}^h}{Z_{MS}^h} \right)^{3/2} \times \lambda_{hhh} \]

\text{inclusion of WFR}

\text{translated to OS ones}

▶ Let’s suppose (for simplicity) that \( \lambda_{hhh} \) only depends on one parameter \( x \), as

\[ \lambda_{hhh} = \lambda^{(0)}_{hhh}(x_{\text{MS}}) + \kappa \delta^{(1)} \lambda_{hhh}(x_{\text{MS}}) + \kappa^2 \delta^{(2)} \lambda_{hhh}(x_{\text{MS}}) \quad (\kappa = \frac{1}{16\pi^2}) \]

and

\[ x_{\text{MS}} = X_{\text{OS}} + \kappa \delta^{(1)} x + \kappa^2 \delta^{(2)} x \]

then in terms of OS parameters

\[ \lambda_{hhh} = \lambda^{(0)}_{hhh}(X_{\text{OS}}) + \kappa \left[ \delta^{(1)} \lambda_{hhh}(X_{\text{OS}}) + \frac{\partial \lambda^{(0)}_{hhh}(X_{\text{OS}})}{\partial x} \delta^{(1)} x \right] \]

\[ + \kappa^2 \left[ \delta^{(2)} \lambda_{hhh}(X_{\text{OS}}) + \frac{\partial \delta^{(1)} \lambda_{hhh}(X_{\text{OS}})}{\partial x} \delta^{(1)} x + \frac{\partial \lambda^{(0)}_{hhh}(X_{\text{OS}})}{\partial x} \delta^{(2)} x + \frac{\partial^2 \lambda^{(0)}_{hhh}(X_{\text{OS}})}{\partial x^2} (X_{\text{OS}})(\delta^{(1)} x)^2 \right] \]
Setup of our effective-potential calculation – details

- OS result is obtained as

\[
\hat{\lambda}_{hhh} = \left( \frac{Z_{h}^{OS}}{Z_{h}^{MS}} \right)^{3/2} \times \lambda_{hhh} \quad \text{(inclusion of WFR)}
\]

- MS parameters translated to OS ones

Let’s suppose (for simplicity) that \(\lambda_{hhh}\) only depends on one parameter \(x\), as

\[
\lambda_{hhh} = \lambda_{hhh}^{(0)}(x^{MS}) + \kappa \delta^{(1)} \lambda_{hhh}(x^{MS}) + \kappa^2 \delta^{(2)} \lambda_{hhh}(x^{MS})
\]

\(\kappa = \frac{1}{16\pi^2}\)

and

\[
x^{MS} = X^{OS} + \kappa \delta^{(1)} x + \kappa^2 \delta^{(2)} x
\]

then in terms of OS parameters

\[
\lambda_{hhh} = \lambda_{hhh}^{(0)}(X^{OS}) + \kappa \left[ \delta^{(1)} \lambda_{hhh}(X^{OS}) + \frac{\partial \lambda_{hhh}^{(0)}(X^{OS})}{\partial x} \delta^{(1)} x \right]
\]

\[
+ \kappa^2 \left[ \delta^{(2)} \lambda_{hhh}(X^{OS}) + \frac{\partial \delta^{(1)} \lambda_{hhh}(X^{OS})}{\partial x} \delta^{(1)} x + \frac{\partial \lambda_{hhh}^{(0)}(X^{OS})}{\partial x} \delta^{(2)} x + \frac{\partial^2 \lambda_{hhh}^{(0)}(X^{OS})}{\partial x^2} (\delta^{(1)} x)^2 \right]
\]

because we neglect \(m_h\) in the loop corrections and \(\lambda_{hhh}^{(0)} = 3m_h^2/v\) (in absence of mixing)
## Existing works at two loops

<table>
<thead>
<tr>
<th>Model [ref.]</th>
<th>Included Corrections</th>
<th>Eff. pot. approx.</th>
<th>Typical size</th>
<th>Motivation</th>
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<tr>
<td>MSSM [Brucherseifer, Gavin, Spira '14]</td>
<td>$O(\alpha_s \alpha_t)$</td>
<td>Yes</td>
<td>$O(\sim 10%)$</td>
<td>Reach similar accuracy as $m_h$</td>
</tr>
<tr>
<td>NMSSM [Mühlleitner, Nhung, Ziesche '15]</td>
<td>$O(\alpha_s \alpha_t)$</td>
<td>Yes</td>
<td>$O(\sim 5 - 10%)$</td>
<td>Reach similar accuracy as $m_h$</td>
</tr>
<tr>
<td>IDM [Senaha '18]</td>
<td>$O(\lambda_\Phi^3)$ (partial)</td>
<td>Yes</td>
<td>$O(\sim 2%)$</td>
<td>Effect on strength of EWPT</td>
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